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ESTIMATES OF ERROR INTRODUCED WHEN ONE-DIMENSIONAL INVERSE HEAT TRANSFER TECHNIQUES ARE APPLIED TO MULTI-DIMENSIONAL PROBLEMS

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ABSTRACT

A study of the errors introduced when one-dimensional inverse heat conduction techniques are applied to problems involving two-dimensional heat transfer effects was performed. The geometry used for the study was a cylinder with similar dimensions as a typical container used for the transportation of radioactive materials. The finite element analysis code MSC P/Thermal was used to generate synthetic test data that was then used as input for an inverse heat conduction code. Four different problems were considered including one with uniform flux around the outer surface of the cylinder and three with non-uniform flux applied over 360°, 180°, and 90° sections of the outer surface of the cylinder. The Sandia One-Dimensional Direct and Inverse Thermal (SODDIT) code was used to estimate the surface heat flux of all four cases. The error analysis was performed by comparing the results from SODDIT and the heat flux calculated based on the temperature results obtained from P/Thermal. Results showed an increase in error of the surface heat flux estimates as the applied heat became more localized. For the uniform case, SODDIT provided heat flux estimates with a maximum error of 0.5% whereas for the non-uniform cases, the maximum errors were found to be about 3%, 7%, and 18% for the 360°, 180°, and 90° cases, respectively.

NOMENCLATURE

$F_{s\text{-}f}$	view factor between surface and fire
T_f	absolute fire temperature
T_s	absolute surface temperature
a	arbitrary constant
c_p	specific heat
h_r	radiation heat transfer coefficient
k	thermal conductivity
q''	heat flux
r_i	inner diameter
r_f	distance from the center of the cask to the fire
r_o	outer diameter
Greek	
ε_s	surface emissivity
ε_f	fire emissivity
ρ	density
θ	angle (in degrees)
σ	Stefan-Boltzmann constant

INTRODUCTION

When an object is directly exposed to a fire, direct measurements of the surface temperature and heat flux are difficult. When thermocouples are attached to a surface that is exposed to a fire, they do not provide a true reading of surface temperature because the thermocouple bead responds differently to rapid changes in heat flux than the surface to which it is attached. Another alternative, heat flux gauges, change the surface characteristics and thus alter the measurement. Therefore, an indirect method has often been used to estimate these surface conditions and this is generally known as the inverse heat conduction problem [1, 2, 3, 4]. Inverse methods permit estimation of surface heat flux without changing the surface or geometry of the object. This is accomplished through the use of the temperature history at interior locations. Interior locations are not exposed to rapid changes in radiant energy, thus permitting more accurate temperature measurements to be made.

The Sandia National Laboratories Transportation Technology Department has used the inverse heat conduction technique to estimate the thermal boundary conditions of containers used for the transportation of radioactive material while undergoing a thermal test. This test typically consists of a 30-minute fully engulfing fire similar

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to those found in severe transportation accidents. After the test, the Sandia One-Dimensional Direct and Inverse Thermal (SODDIT) code is used for heat flux estimation. SODDIT [5] is a FORTRAN 77-based program designed to solve a wide variety of one-dimensional, transient, thermal diffusion problems, including the inverse heat conduction problem, in a variety of coordinate systems. The purpose of this study was to investigate what level of accuracy could be expected from this one-dimensional code when a two-dimensional scenario is presented to it.

Koski *et al.* [6] did a study in which the SODDIT code was used to estimate fire heat fluxes to the surface of a pipe calorimeter. It was found that SODDIT started to underestimate the peak heat flux calculated by the two-dimensional finite element code Topaz2D after the first few minutes and that the difference between the two increased with time. This paper presents a closer look at such errors.

DESCRIPTION OF THE STUDY

Generally, casks for the transportation of radioactive materials are cylindrical, therefore a cylindrical shell geometry was selected for this study (see Fig. 1). The inner radius of the cask, r_i, is 0.6604 m (26 in), the outer radius, r_o, is 0.762 m (30 in), and the length, z, is 10 m (32.8 ft.). These casks are usually made from steel, so the properties of carbon steel were used for this study.

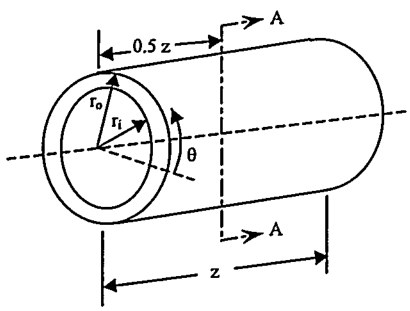


Figure 1. Problem geometry

The purpose of this study was to analyze the error introduced when the one-dimensional inverse heat conduction option of the SODDIT code is used to estimate the heat flux incidence at the outer surface of the cask during a 30-minute fully engulfing fire test. Because of the one-dimensionality of SODDIT, multidimensional heat transfer effects are not taken into account by the code, and this results in heat flux estimates with errors. The magnitude of these errors will be presented and discussed for four different cases of heat flux input to the two-dimensional model of the above geometry consisting of section A-A. The assumption that only two-dimensional thermal effects occur at this location of the cylinder was considered valid because the cask is very long and it is fully engulfed during the fire test. Thus, the problem can be treated as an infinite cylinder, which reduces it from a three- to a two-dimensional problem.

The major heat transfer mode in a fire is known to be thermal radiation. Therefore, in order to simulate typical fire conditions and generate synthetic test data, radiation heat transfer was assumed to occur from a fire at 1273K to the outer surface of the cask. Furthermore, the mode in which the heat is transferred to the cask is not important for this study since SODDIT estimates the heat flux to the surface of the body, regardless of the nature of the heat source. For this study, the interior surface of the cask was assumed to be perfectly insulated.

In addition to the assumptions already mentioned (*i.e.*, two-dimensional thermal effects, constant and uniform flame temperature, radiation heat transfer on the outer surface, and insulated inner surface), constant properties were assumed for the carbon steel cask. That is, thermal conductivity, k, was 71.176 W/m-K; density, ρ, was 7,860 kg/m^3; and specific heat, c_p, was 460.15 J/kg-K. All of these values are for 298K. Constant emissivity values of 0.9 for the fire and of 0.8 for the outer surface of the cask were also assumed. The chosen emissivity value for the surface of the cask is an assumed average value since this surface is often covered by a layer of soot particles during the fire due to the combustion processes.

The above-mentioned assumptions in this paper were intended to reduce the number of variables in the study so that the errors caused by the one-dimensional nature of SODDIT could be well isolated.

Cases Studied

Four cases of heat flux incidence at the outer surface of the cask were studied. One of uniform heat flux around the surface and the other three non-uniform with a localized peak of the form of a sine function applied over 360°, 180°, and 90° (see Table 1). This heat flux profile was achieved by use of a view factor, *F*, of the form:

$$F = \sin(a^*\theta) \tag{1}$$

where "a" is an arbitrary constant. As mentioned before, the heat is assumed to be transferred by radiation from a fire at constant temperature (1273K) to the cask that is originally at 311K.

The four cases were modeled and analyzed with the finite element code MSC PATRAN/Thermal (P/Thermal) to generate the pseudo-experimental data used as input for the inverse calculations in SODDIT. Since SODDIT is a one-dimensional code, every angle of every case (as specified in Table 2) was analyzed independently from the others, then recombined for the error analysis.

PATRAN Model

The P/Thermal model consisted of 580 elements with 5 elements through the wall as shown in Fig. 2.

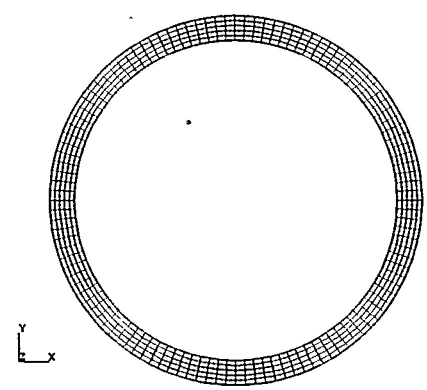
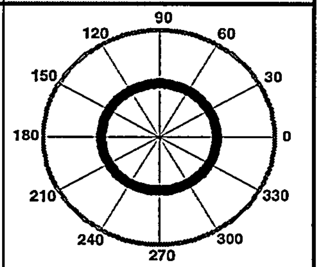
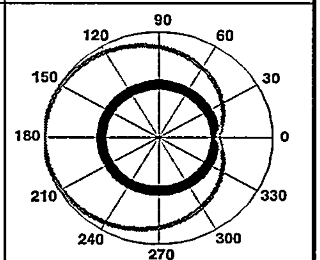
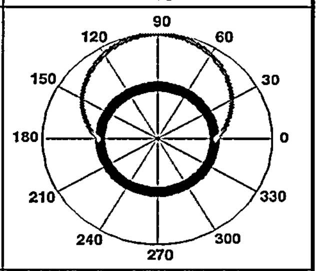
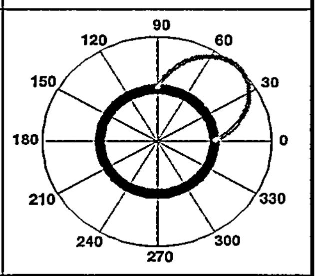


Figure 2. P/Thermal computer model

Table 1. Description of the studied cases

Case No.	Case Description	Applied View Factor ■ Container ▬ View Factor
1	Uniform flux around the surface. View factor = 1	
2	Non-uniform flux over 360°. View factor = sin(θ/2) (a = ½)	
3	Non-uniform flux over 180° and insulated elsewhere. View factor = sin(θ) (a = 1)	
4	Non-uniform flux over 90° and insulated elsewhere. View factor = sin(2*θ) (a = 2)	

The same model was used for the four cases with pertinent modifications to the view factor and application region. The transient run of all four models lasted 30 minutes, the duration of a fully engulfing fire test. The temperature histories from some of the interior and exterior nodes were retrieved from the analyses. The interior node temperature data were used as input for SODDIT. The exterior node data were also used to calculate separately the heat flux and compare it against the output from SODDIT.

The uniform flux data (case 1) was used to determine the accuracy of SODDIT for one-dimensional scenarios because heat was transferred in the radial direction only. Therefore, only data from the interior and exterior nodes at one angle (0°) was retrieved. On the other hand, the angle from which data was retrieved on the non-uniform models varied from model to model. Nevertheless, all non-uniform cases had something in common: the data were retrieved from the same location on the applied heat curve. That is, data was collected from one-half, three-eighths, one-fourth, and one-eighth of the peak of the applied heat function. Table 2 presents the examined angles for the non-uniform cases shown in Table 1.

Table 2. Studied angles (degrees) for the nonuniform cases

Case Number	Approximate Location on the Curve: sin(a*θ) 1/8	1/4	3/8	1/2 (peak)
2	46.55	90	133.45	180
3	21.72	46.55	68.27	90
4	12.41	21.72	34.14	46.55

The angles shown in Table 2 do not represent exactly the mentioned fraction of the curve because there was no node at some of these points of the curve. However, the selected locations were close enough for the purpose of this study since the sine function was evaluated at the angle of the selected nodes. No angle was studied beyond the peak of the sine curve due to problem symmetry.

Summary of the Procedure

The steps that were taken to visualize the errors from SODDIT as the heat flux was concentrated into progressively smaller regions are presented below. All cases were modeled using P/Thermal and the fire simulations were set to last 30 minutes.

Case 1 (uniform flux) was modeled and the temperature history of the interior and exterior node at 0° was retrieved. Then the data of the interior node was read into SODDIT. The heat flux and the estimated outside temperature data from SODDIT and P/Thermal were imported into a spreadsheet. The error analysis for this case was performed by comparing the output from SODDIT against the theoretical ("directly-calculated") heat flux values using the temperature output from P/Thermal. Because P/Thermal does not provide a direct heat flux output, a formula was used for the calculation of the heat flux at each time step [7]:

$$q"=\frac{\sigma}{\left[\frac{1-\varepsilon_s}{\varepsilon_s}+\frac{1}{F_{s-f}}+\frac{1-\varepsilon_f}{\varepsilon_f}\left(\frac{r_o}{r_f}\right)\right]}\left(T_f^4-T_s^4\right) \tag{2}$$

where $q"$ is the heat flux, σ is the Stefan-Boltzmann constant, ε_s and ε_f are the surface and fire emissivities, respectively, F_{s-f} is the view factor between the surface and the fire, r_o and r_f are the outer radius of the cask and the distance from the center of the cask to the fire, respectively, and T_s and T_f are the absolute surface and fire temperatures, respectively. For the purpose of this study r_f was set equal to 0.763 m. The error was calculated as:

$$\%E=\left|\frac{q"_{SODDIT}-q"_{theoretical}}{q"_{theoretical}}\right|\times 100 \tag{3}$$

The simulation for case 2 was then run and the temperature history data of the inner and outer node retrieved at the angles specified in Table 2. Then the inner node temperature data was read into SODDIT and its heat flux output compared against the theoretical results of Equation 2. Similarly, this procedure was repeated for cases 3 and 4.

The results obtained from these steps are presented and discussed in the following section of the paper. Please refer to Tables 1 and 2 in case of doubt about the studied cases while reading the next section. For the remaining portion of the paper the case number and the location on the curve, as fraction and/or actual angle, will be the form of reference.

RESULTS AND DISCUSSION

Case 1

A view factor of unity was used to obtain the desired heat flux uniformity for this case. The heat flux estimated by SODDIT is presented in Fig. 3. This figure shows how well SODDIT can predict the surface heat flux with use of only the interior node temperature data (SODDIT-1 node input) when compared to using the inner and outer node temperature data (SODDIT-2 nodes input). Therefore, for the remainder of the paper only the interior temperature data will be used for the error analysis since it is the most reliable data that can be collected from real experimentation.

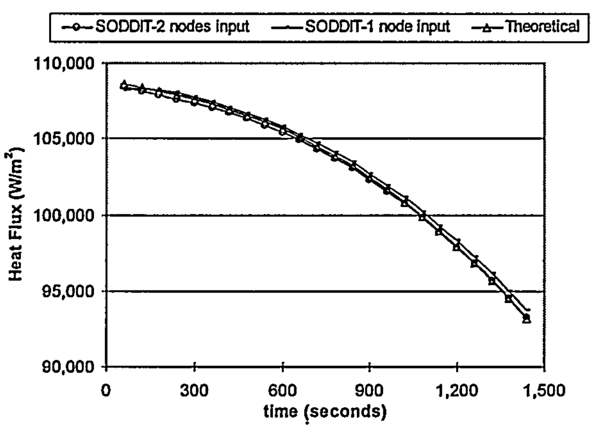


Figure 3. Heat flux estimates for case 1

The major difference between the two estimates from SODDIT occurred during the first few time steps. This was due to the instantaneous high heat flux imposed at the initial time step. Note that in actual experimental data, the initial thermal radiation from the fire increases gradually, minimizing the initial transient observed in these simulations. The solutions from SODDIT did not reach the 1,800 seconds (30 minutes) because of the code dependence on future time steps. The error of SODDIT using only the interior node temperature history is shown in Fig. 4. As expected on the one-dimensional case, SODDIT did a good job of estimating the surface heat flux.

Case 2

Case 2 was the first of the non-uniform cases in which SODDIT was used under circumstances where two-dimensional thermal effects existed. The surface heat flux was estimated at four different locations

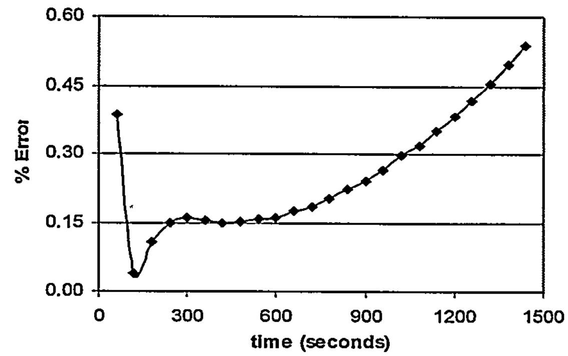


Figure 4. Error of SODDIT for case 1, one node

as specified in Table 2. The view factor, $F_{s\text{-}f}$, was set to be sin(θ/2) so that there was only one peak of applied flux (at 180°). The temperature history of the surface of the cask at each of the studied angles is presented in Fig. 5. The transient behavior of the heat flux around the outer surface is shown in Fig. 6.

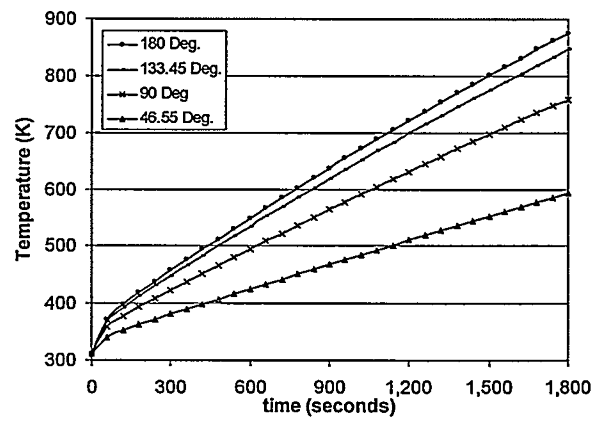


Figure 5. Temperature history of outer surface – case 2

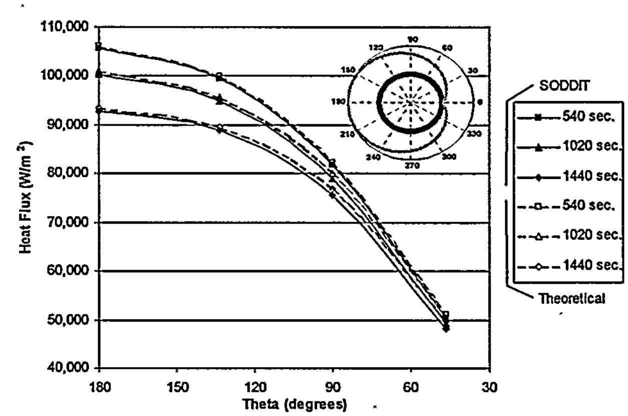


Figure 6. Heat flux as a function of theta and time – case 2

As expected, the magnitude of the heat flux was reduced with time due to the increase of the surface temperature. Although, the outer surface temperature remained such that all heat flux curves kept a sine-like shape. The errors of SODDIT for this case are shown in Fig. 7. Errors were larger for regions away from the applied heat flux peak. Also, all case 2 estimates had higher errors than for case 1 (uniform case). However, errors in case 2 were still small, which indicates that the use of SODDIT may be appropriate for similar real-life problems.

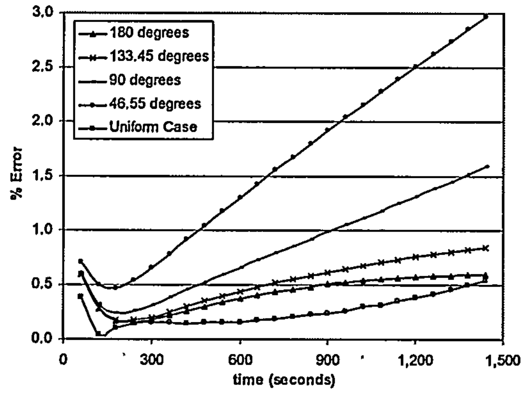


Figure 7. Error on estimates from SODDIT - case 2

<u>Case 3</u>

In this case, the sine curve of heat flux was applied over 180° with peak at 90°. This was done using a view factor, $F_{s\text{-}f}$, of sin(θ). Since the applied heat flux was more localized for this case, higher errors from SODDIT were expected. A temperature history plot of the outer surface is not presented for this case because of its similarity to the one from case 2 (Fig. 5). The transient behavior of the heat flux as a function of theta is presented in Fig. 8. The calculated errors of SODDIT for the different locations of this case are presented in Fig. 9. Errors in this case were found to be higher than in cases 1 and 2. However, errors remained below seven percent demonstrating that, for a similar real-life problem, the use of SODDIT may still be suitable depending on the accuracy required by the problem.

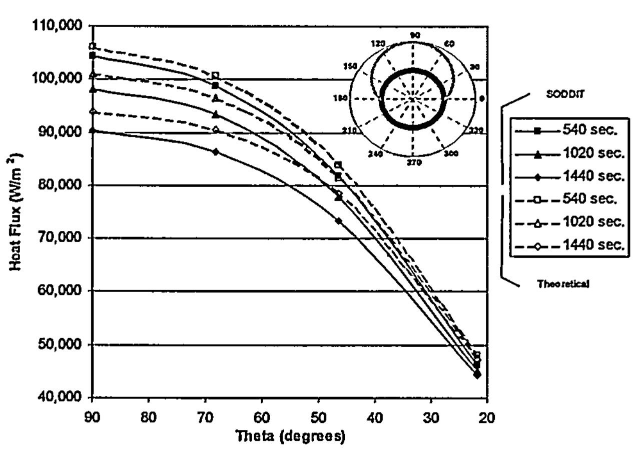


Figure 8. Heat flux as a function of theta and time – case 3

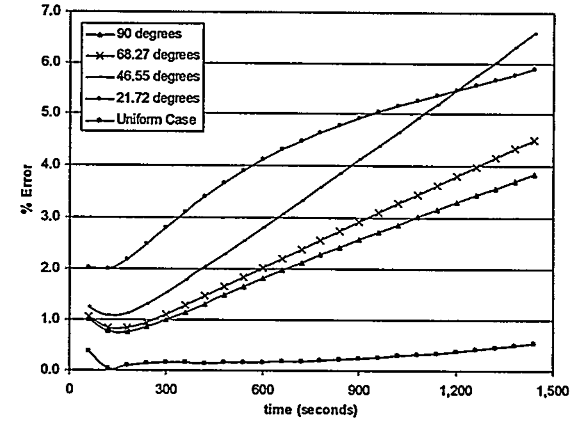


Figure 9. Error on estimates from SODDIT - case 3

<u>Case 4</u>

This case was the last one considered for purposes of this study. A view factor, $F_{s\text{-}f}$, of sin(2*θ) was applied between 0° and 90° so that there was only one peak of applied flux (at 45°). Even though this case is not representative of any of the tests normally performed on this type of transportation container, it provided an insight into the accuracy of SODDIT on difficult multidimensional problems. As with case 3, a temperature history plot of the outer surface is not presented for this case since a similar plot was obtained for case 2. The heat flux incidence on the outer surface for this case is presented in Fig. 10. This figure shows the large difference between the results from SODDIT and the theoretical ones for this case. As expected, using SODDIT to estimate the heat flux for this case resulted in larger errors than those found in any of the previously studied cases. Figure 11 shows the magnitudes of these errors. The error history at 12.41° showed a lower magnitude than the rest of the studied positions after 480 seconds. Similarly, the flux estimates of case 3 at 21.72° had less error than at 46.55° after 1,200 seconds (see Fig. 9). This was not expected since it was thought that error would be higher as the angle of study was decreased. Therefore, in order to better understand this unexpected behavior, an error study was performed for one node above and three below (angle wise) of the node at 12.41°. The results of this study are presented in Fig. 12.

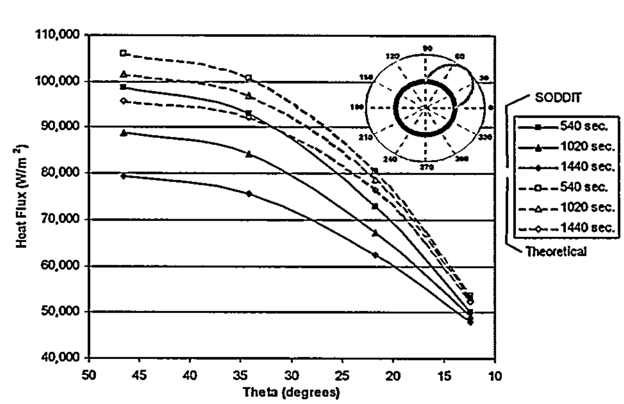


Figure 10. Heat flux as a function of theta and time – case 4

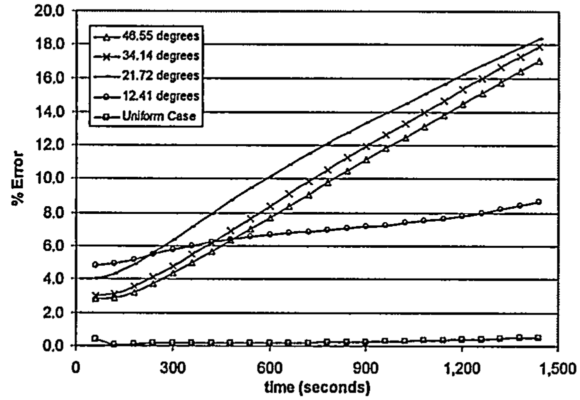


Figure 11. Error on estimates from SODDIT - case 4

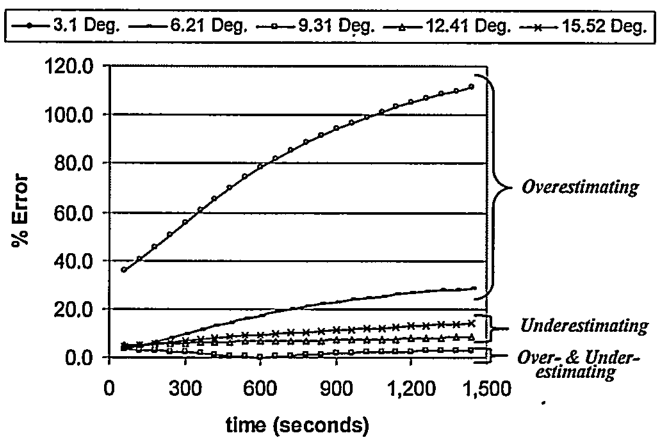


Figure 12. Error at nodes close to the node at 12.41°

As illustrated in Fig 12, it was found that there is a transition from underestimation to overestimation at 9.31°. This was understood to happen because, at locations very close to the non-heated area, the applied heat flux approached zero and the temperature difference through the wall was mainly due to the heat transferred by conduction in the theta direction. Therefore, the temperature rise that SODDIT sensed was not due to the heat flux applied in the radial direction as SODDIT assumed, thus resulting in overestimates of the surface heat flux.

CONCLUSIONS

The results presented in this paper are very important for understanding the level of accuracy that can be expected when using SODDIT for multidimensional thermal problems. With only the interior temperature history data, SODDIT produced acceptable surface heat flux estimates for the studied multidimensional scenarios. This opens the door for the utilization of this code in applications where two-dimensional fire effects occur. However, more studies are needed to assess the results from SODDIT when it is used in multidimensional cases of complex geometries and/or heat flux conditions. Maximum errors of ≈0.5%, ≈3%, ≈7%, and ≈18% were found for the uniform case and for cases with heat flux peaks covering 360°, 180°, 90°, respectively.

SODDIT estimates showed higher errors in case 4 because the peak heat flux was closer to the non-heated regions and the circumferential heat transfer was of higher magnitude. Case 4 also demonstrated the existence of a transition region where SODDIT switches from under- to over-estimating the surface heat flux.

The inverse heat transfer technique is useful for the estimation of the surface heat flux since no surface instrumentation other than a thermocouple is needed and the errors found with the SODDIT code are acceptable for many engineering analyses.

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