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# HIGHWAY SAFETY
# COMPUTER SIMULATION EXERCISES

## The New England University Transportation Center

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6. AUTHOR(S)
Lawrence C. Barr
Paul J. Ossenbruggen

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
University of New Hampshire
Department of Civil Engineering
Kingsbury Hall
Durham, NH 03824

8. PERFORMING ORGANIZATION REPORT NUMBER
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9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
University Transportation Center
Massachusetts Institute of Technology
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Transportation engineering texts cover the topic of geometric design of highways but provide little information on highway safety, vehicle crashworthiness, and occupant safety. The objective of this project is to develop exercises using interactive computer animation to more effectively teach highway safety in transportation engineering. With computer animation, the student can clearly visualize the harm done to the individual in an accident. In addition, model parameters can be quickly and easily changed to allow the student to investigate alternative accident scenarios and perform sensitivity analyses. Educational exercises were developed using the computer programs Working Model and Mathematica. Lecture material, class exercises, homework problems, and computer simulations on the following five areas are included in this report in a workbook format: Geometric design of a banked curve, occupant safety and head impact analysis, safe car following theory, hazardous highway identification, and queuing theory and merging dynamics.

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# HIGHWAY SAFETY COMPUTER SIMULATION EXERCISES

## Workbook Outline

INTRODUCTION

SECTION I. BANKED CURVE

- Lecture Material
- Class Exercises
- Working Model Simulations

SECTION II. OCCUPANT SAFETY - HEAD IMPACT

- Lecture Material
- Homework Problems
- Working Model Simulations

SECTION III. SAFE CAR FOLLOWING MODEL

- Lecture Material
- Homework Problem and Class Exercise
- Working Model Simulations

SECTION IV. HAZARDOUS HIGHWAY IDENTIFICATION

- Lecture Material
- Class Exercise

SECTION V. QUEUING THEORY AND MERGING MODELS

- Lecture Material
- Class Exercises
- Monte Carlo Simulation Project

# INTRODUCTION

# INTRODUCTION

Transportation engineering texts cover the topic of geometric design of highways but provide little information on highway safety, vehicle crashworthiness, and occupant safety. The objective of this project is to develop exercises using interactive computer animation to more effectively teach highway safety in transportation engineering. After developing several of these exercises, they have been classroom tested during the 1996-1997 academic year, and informative feedback was received from students.

With computer animation, the student can clearly visualize the harm done to the individual in an accident. In addition, model parameters can be quickly and easily changed to allow the student to investigate alternative accident scenarios and perform sensitivity analyses. Educational exercises were developed using the computer program Working Model by Knowledge Revolution. This powerful program, available on both Macintosh and Windows platforms, is used to simulate roadway accidents using destructive test simulations. Working Model uses numerical analysis techniques to calculate the motion of interacting bodies, and it permits the construction of complex systems and computes their motion for a wide variety of constraint conditions. On Macintosh computers, Working Model simulations can be played back as Quick Time movies.

In order to insure the validity of the impact and crash simulations, data from physical tests using human cadavers and anthropomorphic test dummies were compared to the results obtained from computer simulation models. A literature survey of government documents and biomedical research articles was conducted to assess the impact response and injury tolerance of vehicular occupants involved in accidents. The literature search provided sufficient test data to verify the validity of the Working Model simulations, and it also provided interesting information on safety analysis and design for the students.

A description of each computer simulation exercise is presented below.

## <u>Geometric Design of a Banked Curve</u>

This exercise analyzes the effects of geometric design characteristics (roadway friction, radius of curvature, and superelevation grade), travel speed, and vehicle design (height, width) on vehicle safety and stability. By varying these parameters either in combination or one at a time, the stability threshold -- that is, the point at which the vehicle begins to slide off the road or tips over -- can be determined. The simulation is intended to provide a thorough understanding of the dynamics of curvilinear motion as well as an appreciation of the conditions which lead to roadway hazards and accidents.

In the Working Model simulation, the student should vary the following roadway and vehicle design parameters and document the conditions required for dynamic equilibrium, sliding off the road, and vehicle tipping:

- vehicle speed
- vehicle height
- vehicle weight
- radius of curvature
- superelevation grade (i.e., banking angle)
- roadway friction

## <u>Occupant Safety -- Head Impact</u>

This problem investigates the effects of mass and velocity on impact forces to the head. Head injury criteria and tolerance limits have been developed by biomedical researchers using various methods. Working Model simulations were developed for two of the methods. The first method employs drop tests of human cadaver heads onto a rigid steel plate from various heights to evaluate the extent of head injuries. The second technique involves striking a crash test dummy in the head with an impactor of varying mass and velocity. In the first simulation, the head is dropped onto a steel plate, and the contact force and effective head acceleration is measured. In the second exercise, a pendulum (a steel ball on a rigid rod) strikes a simulated head resulting in a contact force due to the impact. From impact studies and research conducted on anthropomorphic dummies and human cadavers documented in the biomedical literature, an assessment of injury severity can be made based on the impact force to the head. The effectiveness of helmets in preventing head injury can also be investigated in these simulations.

The following parameters can be varied in the simulations:

- impactor (pendulum) mass
- impactor velocity
- drop height of head above steel plate
- duration of impact

## Safe Car Following Model

This problem is intended to provide an understanding of the basic principles of traffic flow by introducing the student to a general model of a vehicular stream for the simple case of identically spaced vehicles on an exclusive right of way. The determinant of this basic traffic flow model is the car following rule adopted by drivers in an attempt to maximize their speeds while maintaining an acceptable level of safety. As a general rule, the spacing between vehicles should be such that if a sudden deceleration becomes necessary for the leading vehicle, the following vehicle has sufficient time and distance to be able to react to the situation and decelerate safely without colliding with the leading vehicle.

The Working Model simulation consists of the simple case of two vehicles in a vehicle stream. A parametric study can be performed using the following traffic flow variables to determine a relationship between vehicle spacing, speed, and deceleration:

- initial speed of the two vehicles
- deceleration rate of the leading vehicle
- deceleration rate of the following vehicle
- perception-reaction time of the following vehicle
- safety margin after stop

Thus, for each set of variables selected, it is possible to compute the safe car following distance, that is, the spacing necessary for the following car to be able to avoid a collision by anticipating a potential stopping maneuver by the car ahead. The combined choice of particular values for these variables has important implications with respect to the level of safety provided by a traffic system's operation. From a knowledge of vehicle speed and spacing, the other primary elements of a traffic stream -- flow and density -- can be easily determined.

**Occupant Safety -- Accident Reconstruction**

This exercise compares the extent of injuries on vehicle occupants using seat belts, air bags, and no restraints in automobile destructive collision tests. The Working Model computer simulation includes a crash test dummy that has biomechanical features similar to the General Motors Hybrid III dummy, which has been specified as the standardized occupant for Federal Motor Vehicle Safety Standard 208 frontal rigid barrier crash testing. Thus, the simulation exercise accurately depicts real-world destructive crash test conditions. Biomechanical response data -- for example, head acceleration and impact force -- can be measured, and the results can be compared to the injury criteria developed from extensive government testing of vehicle collisions to assess the extent of injuries sustained by the occupant.

The student can vary vehicle speed and run several computer simulations using no restraints, simulated seat belts, and a simulated air bag. In this manner, the student can gain an understanding of the conditions that result in severe or fatal injuries, and he can also evaluate the effectiveness of automobile safety restraints (seat belts, air bags) in preventing injuries.

Two additional exercises which do not use the Working Model computer simulation program are also included in the final project report. The **Hazardous Highway Identification** exercise acquaints the students with the use of actual traffic count and accident data to determine if a highway location can be classified as hazardous. The **Merging Vehicle Model** uses a Monte Carlo computer simulation to analyze an important aspect of traffic flow and highway safety, namely the interaction of vehicles on a minor road wishing to join the traffic stream on a major road at an unsignalized intersection. The purpose of the right-hand turn merge model is to realistically simulate conditions encountered in the field.

A package of highway safety educational materials, including Quick Time movies of the computer simulation exercises described above as well as homework problems and lecture material, is being made available through the Internet. The web site for this material can be found at http://bitbucket.unh.edu/HighwaySafety.html. A major advantage of having the educational material available on the Internet is the minimal time commitment and expense to potential users who wish to test the usefulness of the material. Widespread accessibility is also a primary benefit.

It is hoped that the results of this project will provide engineering students with a better appreciation of the analysis and design process required for highway safety. The unique features of the project are to:

- use computer simulation for the analysis of complex dynamic systems.
- reinforce the importance of the use of Newtonian physics in engineering design.
- offer basic insights into the extent and importance of the highway safety problem (for example, to give the student a wider perspective and better appreciation of the engineering and design process, material from the biomedical research literature is incorporated into the exercises).
- critically analyze the engineering design process for occupant safety.

# Section I

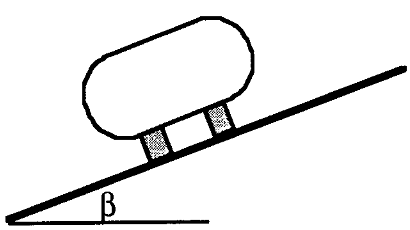
# BANKED CURVE

# BANKED CURVE

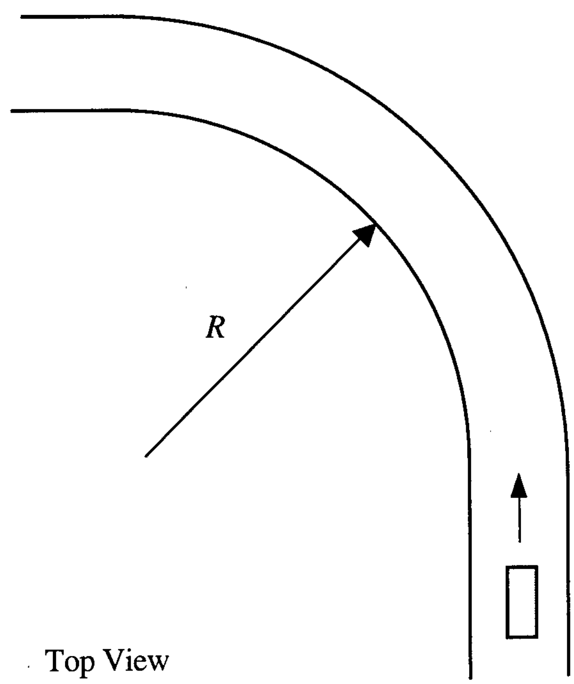
## Theoretical Background and Governing Equations

Automotive engineers, civil engineers who design highways, and engineerws who study traffic accidents and their prevention must analyze and measure the motions of vehicles under different conditions. By using the concepts of force, mass, and acceleration as well as the equations of curvilinear motion, engineers can relate the forces acting on vehicles to their motions. They can then analyze, for example, the effects of banking and curvature on the velocity at which a car can safely be driven on a curved road (such as a freeway off-ramp).

This section presents a computer simulation of a vehicle on a banked curve (diagrammed below) in which students can study the effects of vehicle speed, road banking angle, radius of curvature, coefficient of friction, and vehicle design (height and width) on the vehicle's motion on a curved roadway.




Cross-Sectional View




Top View

For a roadway cross-section, the only force in the normal direction is due to the side friction between the vehicle's tires and the pavement, which resists the tendency of the vehicle to slide. To minimize this tendency, highway design provides for the banking, or superelevation, of the cross-section of the roadway. As shown on the previous page, the cross-section is tilted by an angle ß so that the component of the vehicle's weight along the tilted pavement surface also resists the sliding tendency of the vehicle.

Since a vehicle on a curved road such as an off-ramp moves in a circular path, it has a normal component of acceleration that depends on its velocity. The necessary normal component of force is exerted by friction between the tires and the road, and the friction force cannot be greater than the product of $\mu_s$ (the coefficient of static friction) and the normal force for vehicle stability. By assuming, for example, that the friction force is equal to this value ($\mu_s N$), the maximum speed at which vehicles can enter the off-ramp without losing traction can be determined.

Dynamic Equilibrium

From Newton's Second Law, the governing equations are

$$\Sigma F_y = ma_t = 0$$

$$\Sigma F_x = ma_n = mV^2/R$$

$$\Sigma M_c = 0$$

The free-body and inertia diagrams of a vehicle on a banked curve are shown on the next page. A and B are the normal forces exerted on the tires. The total normal force acting on the car is

$$N = A + B$$

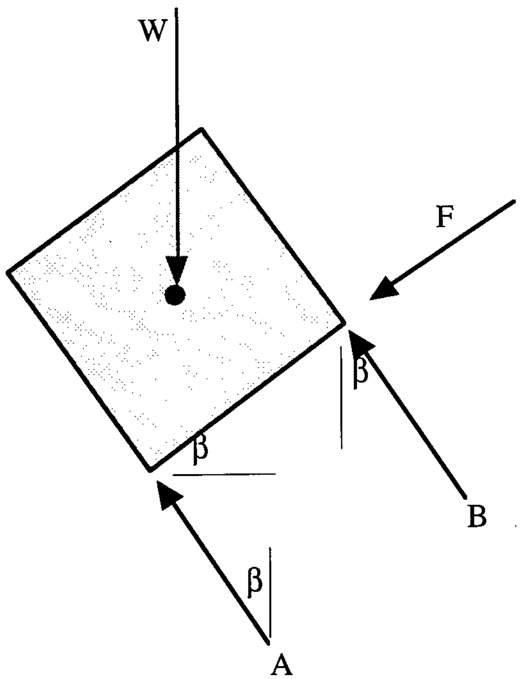
From the free-body and inertia diagrams, the equilibrium equations are:

$$A\cos\beta + B\cos\beta - F\sin\beta = W$$

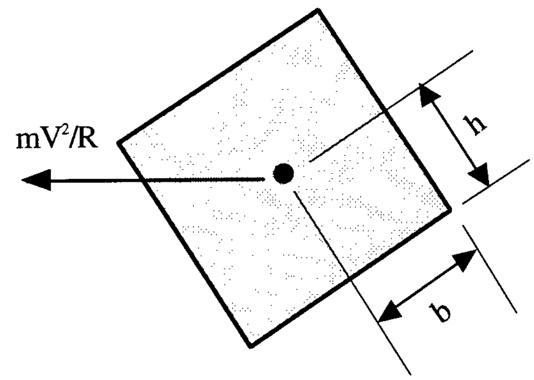
$$A\sin\beta + B\sin\beta + F\cos\beta = \frac{WV^2}{gR}$$

$$-bA + bB + hF = 0$$

Free-Body Diagram




Inertia Diagram




In Matrix Form:

$$\begin{pmatrix} \cos\beta & \cos\beta & -\sin\beta \\ \sin\beta & \sin\beta & \cos\beta \\ -b & b & h \end{pmatrix} \begin{pmatrix} A \\ B \\ F \end{pmatrix} = \begin{pmatrix} W \\ \frac{WV^2}{gR} \\ 0 \end{pmatrix}$$

Solving for A, B, and F, we get:

$$A = \frac{W\left(b\cos\beta + \frac{b\sin\beta\ V^2}{gR} - \frac{V^2\ h\cos\beta}{gR} + h\sin\beta\right)}{2b}$$

$$B = \frac{W\left(b\cos\beta + \frac{b\sin\beta\ V^2}{gR} + \frac{V^2\ h\cos\beta}{gR} - h\sin\beta\right)}{2b}$$

$$F = \frac{\frac{(2\,WV^2)\,b\cos\beta}{gR} - 2\,Wb\sin\beta}{2\,b} = \frac{WV^2\cos\beta}{gR} - W\sin\beta$$

The total normal force is:

$$N = A + B = \frac{W\left(2\,b\cos\beta + \frac{b\sin\beta\,(2\,V^2)}{gR}\right)}{2\,b} = \frac{WV^2\sin\beta}{gR} + W\cos\beta$$

By collecting terms and simplifying, A and B can be rewritten as:

$$A = \frac{1}{2}\sin\beta\left(\frac{WV^2}{gR} + \frac{h\,W}{b}\right) + \frac{1}{2}\cos\beta\left(W - \frac{WV^2\,h}{gR\,b}\right)$$

$$B = \frac{1}{2}\cos\beta\left(\frac{h\,WV^2}{b\,gR} + W\right) + \frac{1}{2}\sin\beta\left(\frac{WV^2}{gR} - \frac{W\,h}{b}\right)$$

The conditions necessary for equilibrium are:

$$\begin{aligned} A &\geq 0 \\ B &\geq 0 \\ F &\geq 0 \\ F &\leq \mu_s N \end{aligned}$$

If one or more of these conditions is violated, the vehicle is not in dynamic equilibrium. The following unstable situations can occur:

1. If $F \geq \mu_s N$, $F < 0$, and $A \geq 0$, the car will slide down the road (i.e., toward the inside of the curve).

2. If $F \geq \mu_s N$, $F > 0$, and $A \geq 0$, the car will slide up and off the road rather than negotiating the the curve safely.

3. If $F < \mu_s N$, $A < 0$, and $F > 0$, the car will tip over in a clockwise rotation.

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1. Bedford, Anthony and Fowler, Wallace, *Engineering Mechanics: Dynamics*, Addison-Wesley Publishing Company, Inc., 1995.

2. Papacostas, C.S., and Prevedouros, P.D., *Transportation Engineering and Planning,* Second Edition, Prentice Hall, Inc., 1993.

In class, we formulated a mathematical model for a vehicle traveling around a banked curve.

The forces necessary to keep the vehicle in dynamic equilibrium are:

1. A = normal force on the left tire.
2. N = normal force on both tires, N = A + B. (B= force of right tire.)
3. F = lateral force.

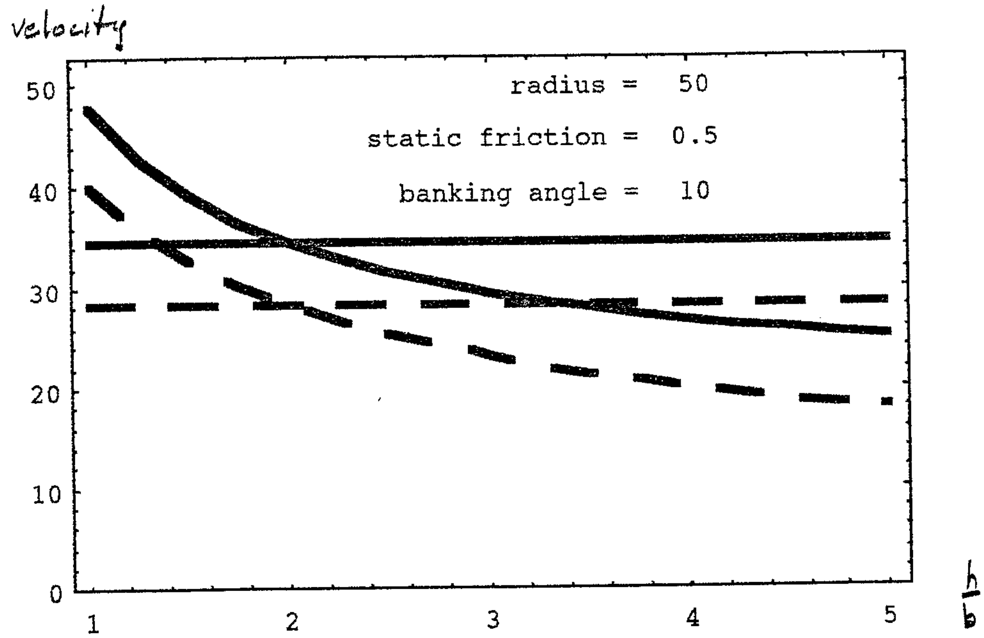
The vehicle will not slide if $|F| < fN$ where f = coefficient of side friction on tires and will not tip if $A > 0$.

The computer simulations show the effects of

- vehicle speed
- road banking angle
- coefficient of friction f
- vehicle design or vehicle height to weight ratio (h/w)
- radius of curvature

**The following shows a progression of runs where a model parameter was changed.**

| **Run** | h (feet) | w (feet) | f | v (mph) | Banking Angle (degrees) | Radius (feet) | Comment |
|---|---|---|---|---|---|---|---|
| Run1 | 15 | 5 | 0.1 | 20 | 0 | 50 | slides |
| Run2 | 15 | 5 | 0.1 | 10 | 0 | 50 | OK |
| Run3 | 15 | 5 | 0.15 | 20 | 0 | 50 | slides |
| Run4 | 15 | 5 | 0.2 | 30 | 0 | 50 | slides and tips |
| Run5 | 15 | 5 | 0.1 | 30 | 15 5 | 50 | slides and tips |
| Run6 | 15 | 5 | 0.15 | 20 | 15 | 50 | OK |
| Run7 | 15 | 5 | 0.15 | 0 | 15 | 50 | slides |
| Run8 | 5 | 5 | 0.15 | 0 | 15 | 50 | slides |
| Run9 | 15 | 5 | 0.15 | 75 | 15 | 300 | slides |
| Run10 | 15 | 5 | 0.15 | 64 | 15 | 300 | OK |

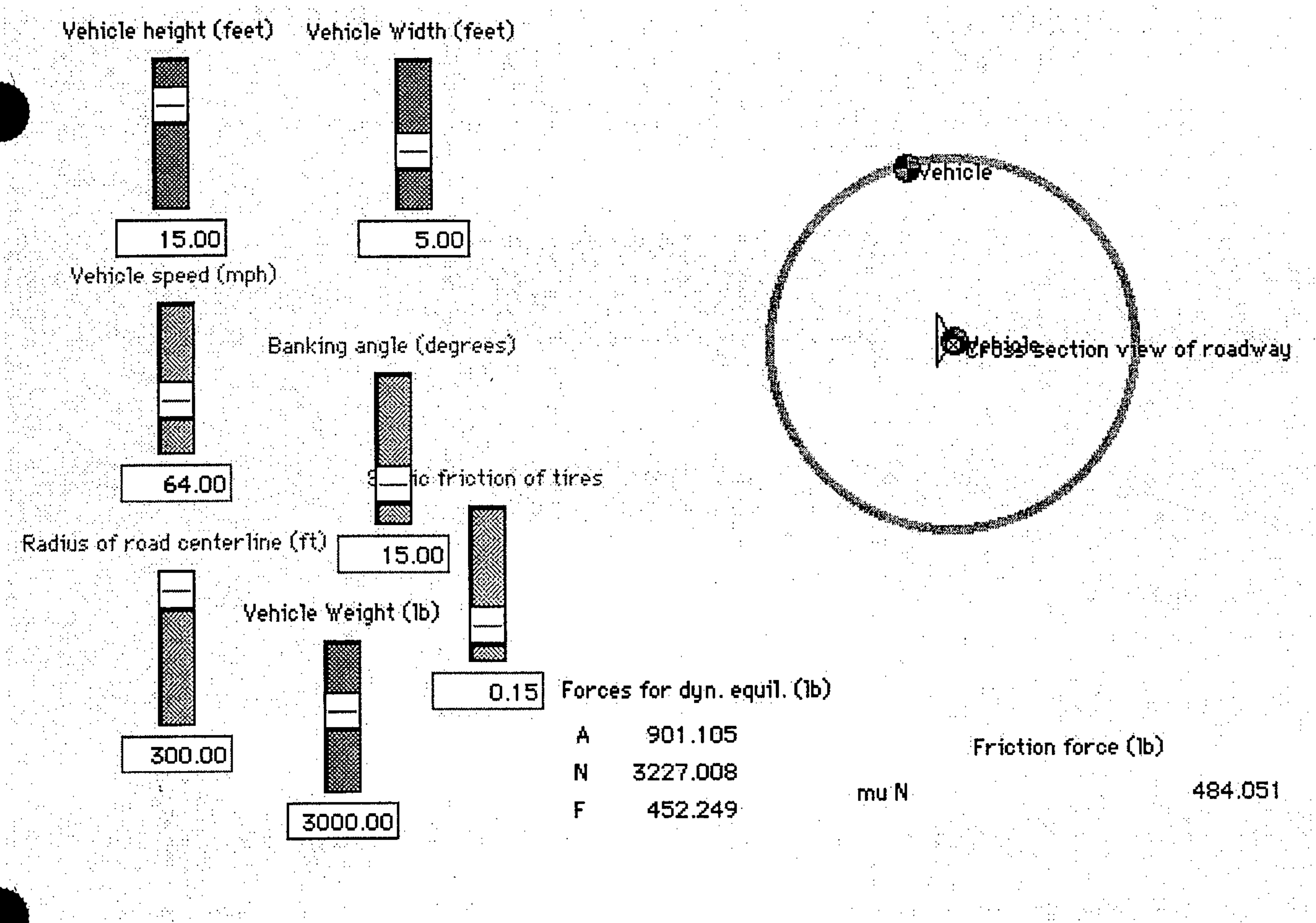



Conditions:

If v < vslip and v < vtip, then dynamic equilibrium.

If v >= vslip and vslip <= vtip, then vehicle will slide.

If v >= vtip and vtip < vslip, then vehicle will tip.



Vehicle height (feet)
Vehicle Width (feet)
15.00
5.00
Vehicle speed (mph)
64.00
Banking angle (degrees)
15.00
ic friction of tires
0.15
Radius of road centerline (ft)
300.00
Vehicle Weight (lb)
3000.00
Forces for dyn. equil. (lb)
A 901.105
N 3227.008
F 452.249
Friction force (lb)
mu N 484.051
Vehicle
Vehicle
ross section view of roadway

Banked Curve Movie 3

Start.

Vehicle height (feet) 15.00

Vehicle Width (feet) 5.00

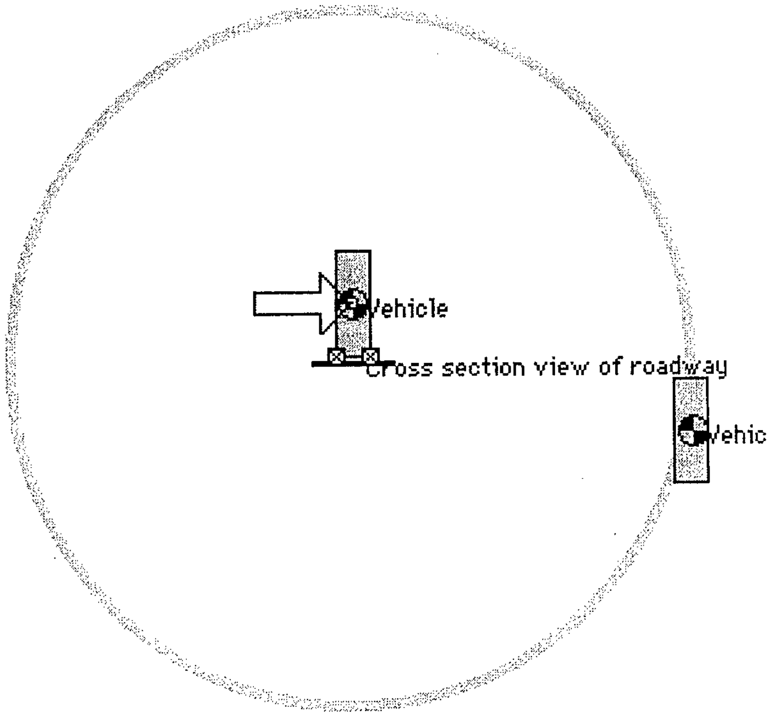
Vehicle speed (mph) 20.00

Banking angle (degrees) 0.00

Static friction of tires 0.15

Radius of road centerline (ft) 50.00

Vehicle Weight (lb) 3000.00




Forces for dyn. equil. (lb)

| | |
|---|---|
| A | 325.909 |
| N | 3000.000 |
| F | 745.342 |

Friction force (lb)

mu N 450.000

# Banked Curve Movie 3

Slides.

Vehicle height (feet) 15.00

Vehicle Width (feet) 5.00

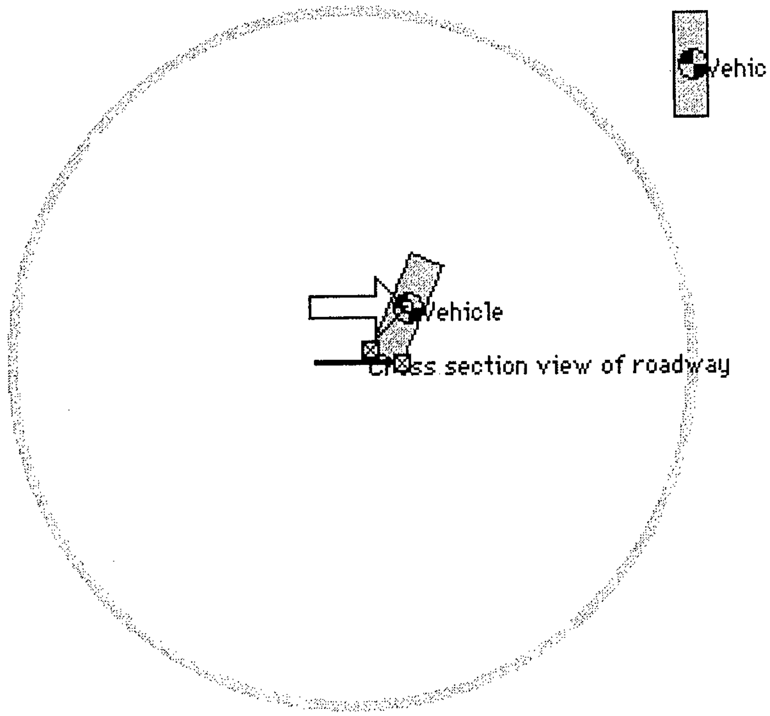
Vehicle speed (mph) 20.00

Banking angle (degrees) 0.00

Static friction of tires 0.15

Radius of road centerline (ft) 50.00

Vehicle Weight (lb) 3000.00




Forces for dyn. equil. (lb)

| | |
|---|---|
| A | 325.909 |
| N | 3000.000 |
| F | 745.342 |

Friction force (lb)

mu N 450.000

Run 6

Vehicle height (feet): 15.00

Vehicle Width (feet): 5.00

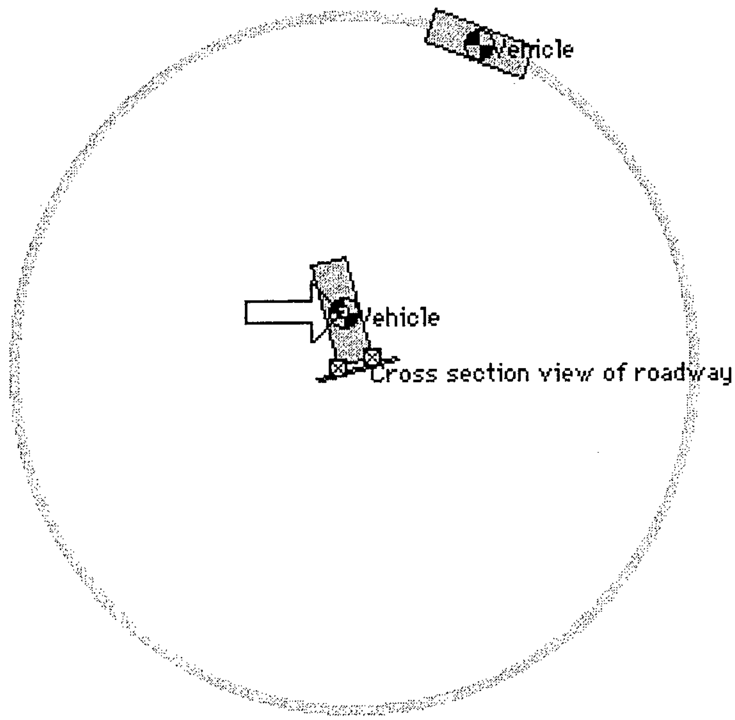
Vehicle speed (mph): 20.00

Banking angle (degrees): 15.00

Static friction of tires: 0.15

Radius of road centerline (ft): 50.00

Vehicle Weight (lb): 3000.00




Forces for dyn. equil. (lb)

| | |
|---|---|
| A | 1634.364 |
| N | 3090.686 |
| F | -56.512 |

Friction force (lb)

mu N 463.603

*Run 8*

Vehicle height (feet)

5.01

Vehicle Width (feet)

5.00

Vehicle speed (mph)

0.00

Banking angle (degrees)

15.00

S[illegible]ic friction of tires

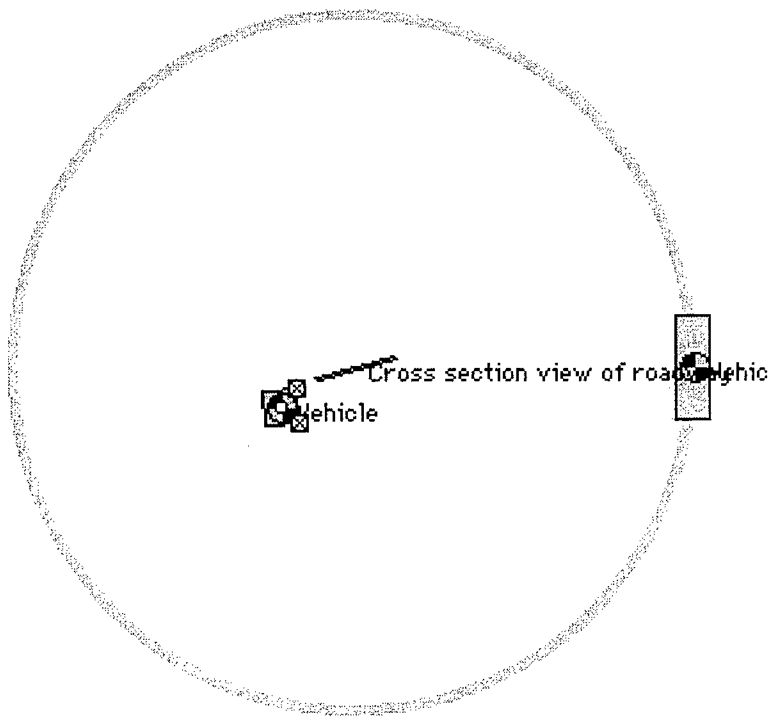
0.15

Radius of road centerline (ft)

50.00

Vehicle Weight (lb)

3000.00




Forces for dyn. equil. (lb)

| | |
|---|---|
| A | 1895.906 |
| N | 2897.777 |
| F | -776.457 |

Friction force (lb)

mu N 434.667

# Section II

# OCCUPANT SAFETY - HEAD IMPACT

# OCCUPANT SAFETY - HEAD IMPACT

## Theoretical Background and Governing Equations

Principle of Work and Energy

$$\text{Work} = \int F\, d s$$

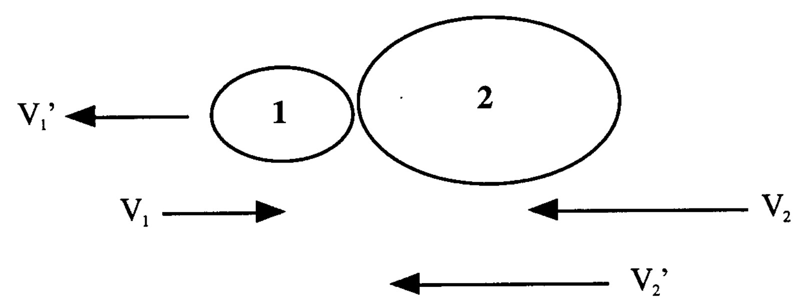
$$\text{Kinetic Energy} = \frac{mV^2}{2}$$

$$\text{Work} = \int_0^{-h} -mg\, d y = \frac{mV^2}{2}$$

$$-mg\,(-h) = \frac{mV^2}{2}$$

$$V = \pm\sqrt{2gh}$$

Impacts and Collision




By the Conservation of Linear Momentum:

$$m_1 V_1 + m_2 V_2 = m_1 V_1' + m_2 V_2'$$

where the "primes" denote the velocities of objects 1 and 2 after they collide with each other.

The Coefficient of Restitution, e, is defined as:

$$e = \frac{V_2' - V_1'}{V_1 - V_2}$$

In a ***Perfectly Plastic Collision***, the velocities of the objects after impact are the same, so the objects remain together after impact. That is,

$$e = 0 \Longrightarrow V_2' = V_1'$$

At the other extreme is a ***Perfectly Elastic Collision*** in which no energy is lost during the impact and the coefficient of restitution, e, is equal to 1. Therefore, the total kinetic energy is the same before and after impact:

$$\frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 = \frac{1}{2} m_1 (V_1')^2 + \frac{1}{2} m_2 (V_2')^2$$

Principle of Impulse and Momentum

$$\int_{t_1}^{t_2} F\, dt = F_{ave}\ (t_2 - t_1) = m_1\ (V_1 - V_1')$$

$$F_{ave} = \frac{m_1\ (V_1 - V_1')}{\Delta t}$$

where $\Delta t$ is the duration of the impact and $F_{ave}$ is the average force sustained by the object as a result of the collision.

The effective acceleration of the object caused by the impact can then easily be calculated using Newton's Law (F = ma):

$$\text{Effective Acceleration (in } g's) = \frac{F_{ave}}{m_1\ g}$$

Biomechanics of Head Injury - Indices of Injury Severity

To quantify the severity of an injury sustained in an automobile crash, researchers have developed several standard indices of injury severity. The basic criterion for most evaluations of head impact trauma is the Wayne State University Tolerance Limit. The data on which this criterion is based were obtained from animal tests involving frontal hammer blows and air blasts to the exposed brain, and from drop tests of human cadaver heads. These tests indicated that as the time exposure to cranial pressure pulses increased, the tolerable intensity decreased. In other words, the Wayne State Tolerance Curve shows that very intense head acceleration is tolerable if it is very brief, but that much less is tolerable if the pulse duration exceeds 10 or 15 milliseconds.

The indices most commonly found in the literature and used in practice are given below. The Gadd Severity Index and the Head Injury Criterion are both based on the Wayne State University Tolerance Limit. Both indices are derived from an acceleration response. The Abbreviated Injury Scale is an empirically-based categorical scale that assigns an injury severity rating on the basis of the observed injuries sustained by the experimental subject following the test.

1. Gadd Severity Index (GSI)

$$GSI = \int_0^t a^n \, dt$$

where n is an empirically-based exponential weighting factor equal to 2.5.
The use of this exponential weighting factor effectively gives more weight to the high portions of the acceleration pulse which contribute more to head injury than the lower portions of the pulse.
A GSI value of 1000 is generally considered to be the threshold level or tolerance limit for serious head injury.

2. Head Injury Criterion (HIC)

The HIC is a mathematical refinement of the Gadd Severity Index and is given by:

$$HIC = \left( \frac{\int_{t_1}^{t_2} a \, dt}{t_2 - t_1} \right)^{2.5} (t_2 - t_1)$$

An HIC value of 1500 or greater is typically associated with extremely severe head injury. Below an HIC value of 1500, the probability of finding a severe brain injury is less than 50 percent.

3. Abbreviated Injury Scale (AIS)

The AIS is a categorical index ranging from 0 to 6, where

| | |
|---|---|
| 0 | = No injury |
| 1 | = Minor injury |
| 2 | = Moderate injury |
| 3/4 | = Serious/Severe injury |
| 5/6 | = Critical/Fatal injury |

Examples of the most common AIS 1-3 injuries are neck or back pain and minor concussion. Examples of common AIS 4-6 injuries include concussion (unconscious 12 hours), fractured vertebrae, and cerebellur lesion.

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March 3, 1997

# CIE 754

## HOMEWORK PROBLEM

The basic criterion for most evaluations of head impact trauma is the Wayne State University Tolerance Limit, shown below. The curve shows that very intense head acceleration is tolerable if it is very brief, but that much less is tolerable if the impact duration exceeds 10 or 15 msec. The impact data on which the tolerance curve is based were derived in part from drop tests of human cadaver heads. Being far more humane, current transportation engineers prefer to use a Working Model simulation of these drop tests to gain an understanding of the conditions that cause serious head injury in an impact.

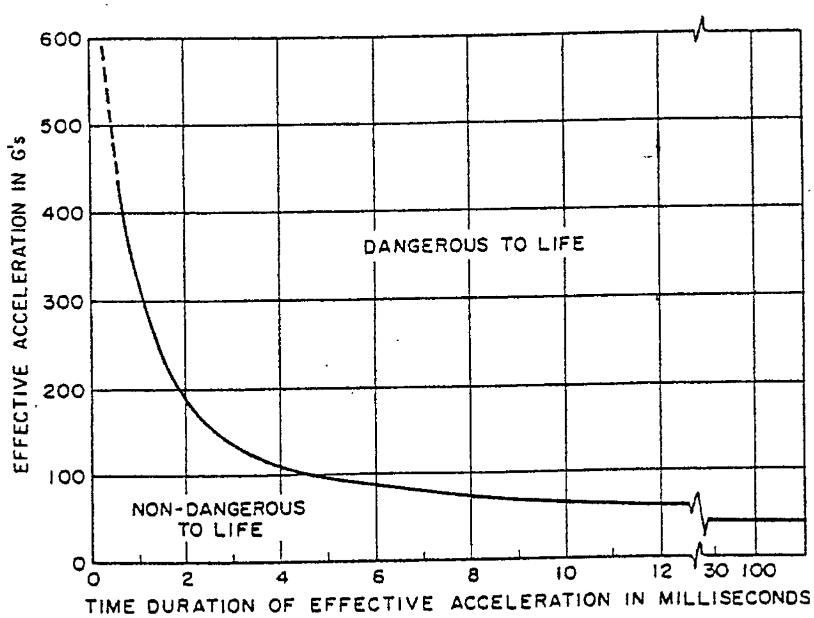
The objectives of this homework problem are to:

1. Determine the effects of impact velocity and duration on effective head acceleration and resulting head injury.

2. Determine the effect of wearing a helmet on reducing head injuries.

Conduct a parametric analysis by choosing various values for drop height, h, and impact duration, $\Delta t$. Calculate the resulting effective head acceleration (in g's). Using the Wayne State Tolerance Curve, determine whether the conditions you selected will result in a serious head injury. Assume a head mass of 5 kg. You might also consider two values for the coefficient of restitution -- say 0.8 and 0.5 -- to see if this makes any difference.

For some of the conditions that result in serious, possibly fatal, head injury, rerun the drop test with a helmet on the head. Assume the mass of the helmet is 2 kg. The padding in the helmet can be simulated with a low coefficient of restitution, so use e = 0.2 for the collision of the head with the helmet.



10/10

Homework Problem #13 (for Larry Barr)
Due: 3/12/97
Drop Test - NO Helmets

g ($m/s^2$) = 9.81
$m_{head}$ (kg) = 5

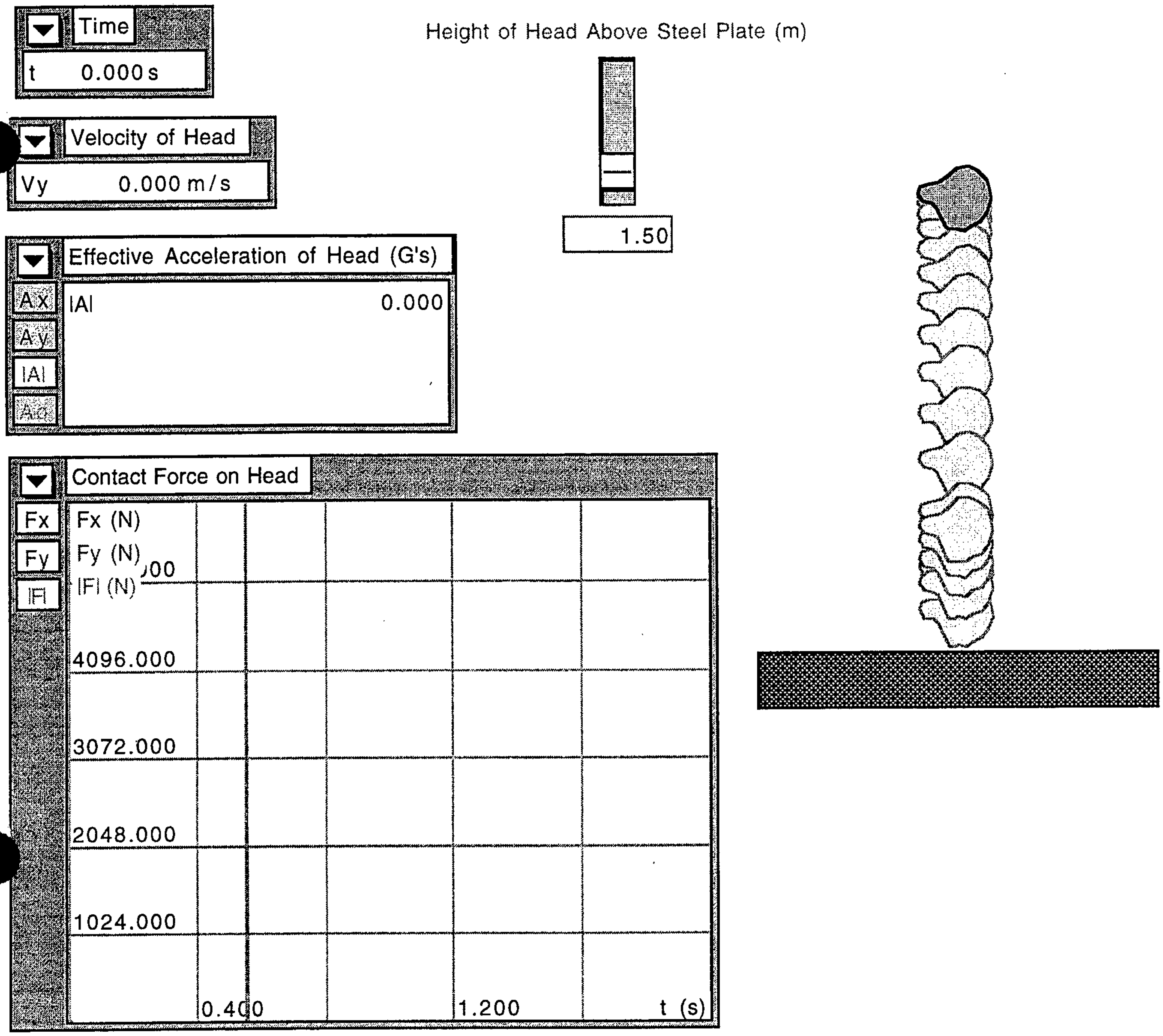
| Coefficient of restitution = 0.8 | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Time Duration = 0.001 | | | | | | | Time Duration = 0.002 | | | | | | |
| sample | height (m) | initial velocity (m/s) | velocity after impact (m/s) | $F_{avg}$ (N) | $F_{avg}$ (in G's) | outcome | sample | height (m) | initial velocity (m/s) | velocity after impact (m/s) | $F_{avg}$ (N) | $F_{avg}$ (in G's) | outcome |
| 1a | 0.05 | 0.990 | -0.792 | 8914 | 181.7 | safe | 1b | 0.05 | 0.990 | -0.792 | 4457 | 90.9 | safe |
| 2a | 0.10 | 1.401 | -1.121 | 12606 | 257.0 | safe | 2b | 0.10 | 1.401 | -1.121 | 6303 | 128.5 | safe |
| 3a | 0.15 | 1.716 | -1.372 | 15440 | 314.8 | *marginal* | 3b | 0.15 | 1.716 | -1.372 | 7720 | 157.4 | safe |
| 4a | 0.20 | 1.981 | -1.585 | 17828 | 363.5 | *fatal* | 4b | 0.20 | 1.981 | -1.585 | 8914 | 181.7 | *marginal* |
| 5a | 0.25 | 2.215 | -1.772 | 19933 | 406.4 | *fatal* | 5b | 0.25 | 2.215 | -1.772 | 9966 | 203.2 | *fatal* |

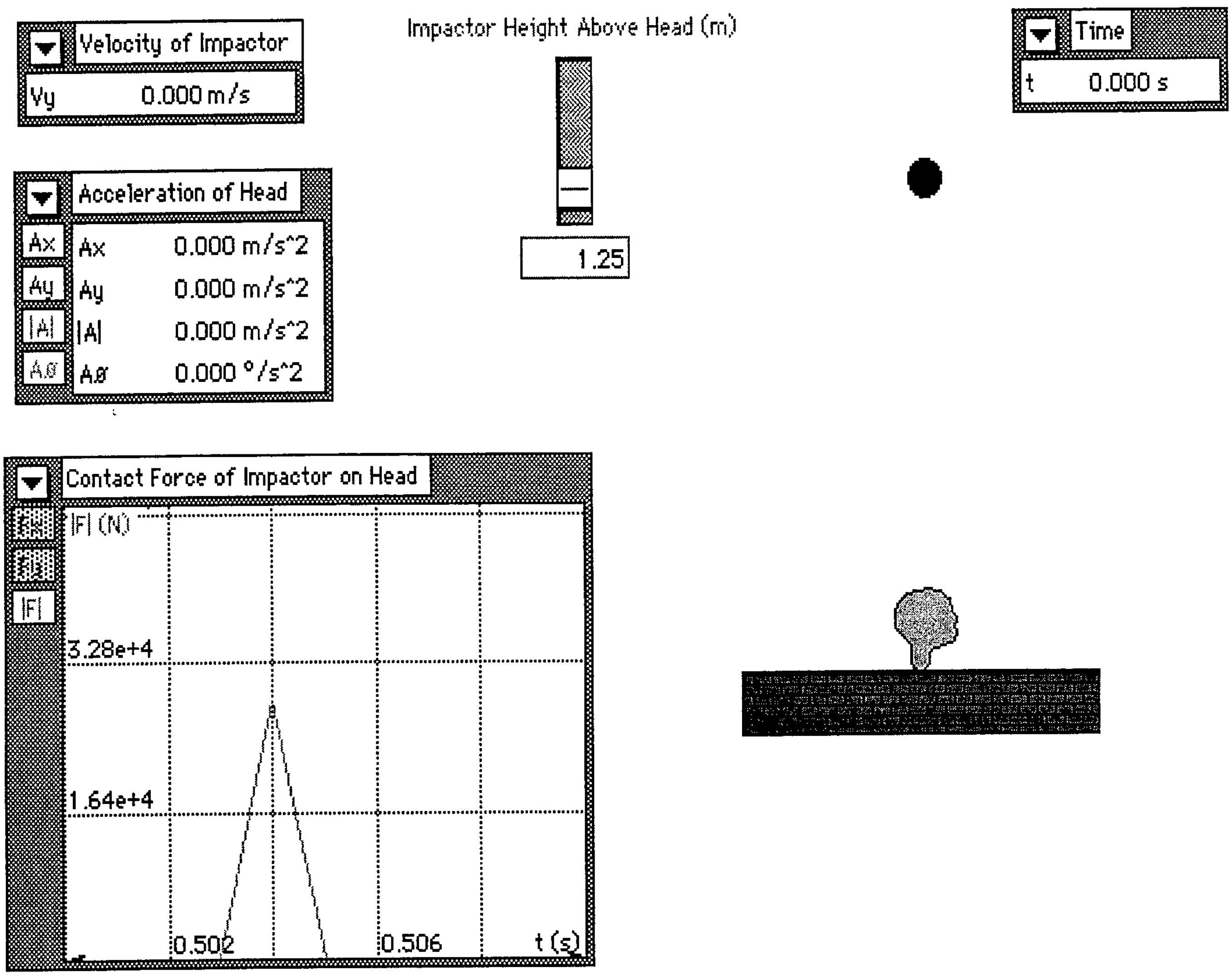
| Coefficient of restitution = 0.5 | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Time Duration = 0.001 | | | | | | | Time Duration = 0.002 | | | | | | |
| sample | height (m) | initial velocity (m/s) | velocity after impact (m/s) | $F_{avg}$ (N) | $F_{avg}$ (in G's) | outcome | sample | height (m) | initial velocity (m/s) | velocity after impact (m/s) | $F_{avg}$ (N) | $F_{avg}$ (in G's) | outcome |
| 1c | 0.05 | 0.990 | -0.495 | 7428.4 | 151.4 | safe | 1d | 0.05 | 0.990 | -0.495 | 3714 | 75.7 | safe |
| 2c | 0.10 | 1.401 | -0.700 | 10505 | 214.2 | safe | 2d | 0.10 | 1.401 | -0.700 | 5253 | 107.1 | safe |
| 3c | 0.15 | 1.716 | -0.858 | 12866 | 262.3 | safe | 3d | 0.15 | 1.716 | -0.858 | 6433 | 131.2 | safe |
| 4c | 0.20 | 1.981 | -0.990 | 14857 | 302.9 | *marginal* | 4d | 0.20 | 1.981 | -0.990 | 7428 | 151.4 | safe |
| 5c | 0.25 | 2.215 | -1.107 | 16610 | 338.6 | *fatal* | 5d | 0.25 | 2.215 | -1.107 | 8305 | 169.3 | safe |

## CIE 754 Transportation Take Home Exam
## Mid-term 1997

5. A man is involved in a legal dispute with his insurance company over a recent car accident in which he sustained a severe concussion. The insurance company is withholding payment on some damages because they claim he was speeding and driving recklessly at the time his car slid off the road and collided with a telephone pole. The man maintains that he was not exceeding the 30 mph posted speed limit and that the accident occurred because of poor weather and reduced visibility. You are asked to provide your expert technical opinion to support or refute the man's claim that he was not speeding.

Using the Wayne State Tolerance Curve as the criteria for head injury, discuss how you would determine whether the man's claim that he was not speeding is justifiable. Clearly state any assumptions you make. For example, since the material properties of the impacting surface are unknown, assuming a coefficient of restitution of 0 and 1 will bound the problem. A rigorous analysis is **not** required, but the governing equations and an example calculation should be included in your discussion.


Time
t 0.000 s
Velocity of Head
Vy 0.000 m/s
Effective Acceleration of Head (G's)
Ax
Ay
|A|
|A| 0.000
Height of Head Above Steel Plate (m)
1.50
Contact Force on Head
Fx
Fy
|F|
Fx (N)
Fy (N)
|F| (N)
4096.000
3072.000
2048.000
1024.000
0.400
1.200
t (s)


Velocity of Impactor
Vy 0.000 m/s
Impactor Height Above Head (m)
1.25
Time
t 0.000 s
Acceleration of Head
Ax 0.000 m/s^2
Ay 0.000 m/s^2
|A| 0.000 m/s^2
Aø 0.000 °/s^2
Contact Force of Impactor on Head
|F| (N)
3.28e+4
1.64e+4
0.502
0.506
t (s)

Time

t s

Velocity of Head

Vy m/s

Effective Acceleration of Head (G's)

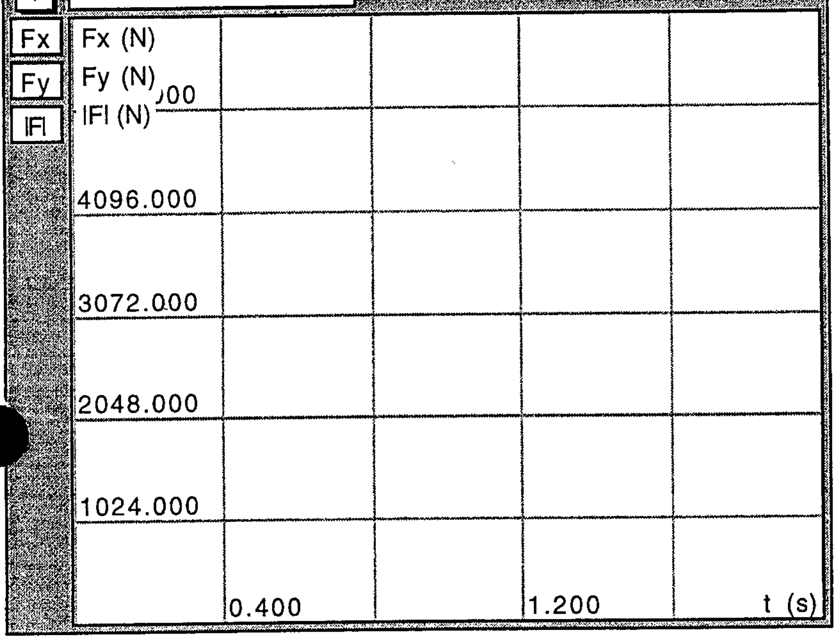
Ax
Ay
|A|
Aø

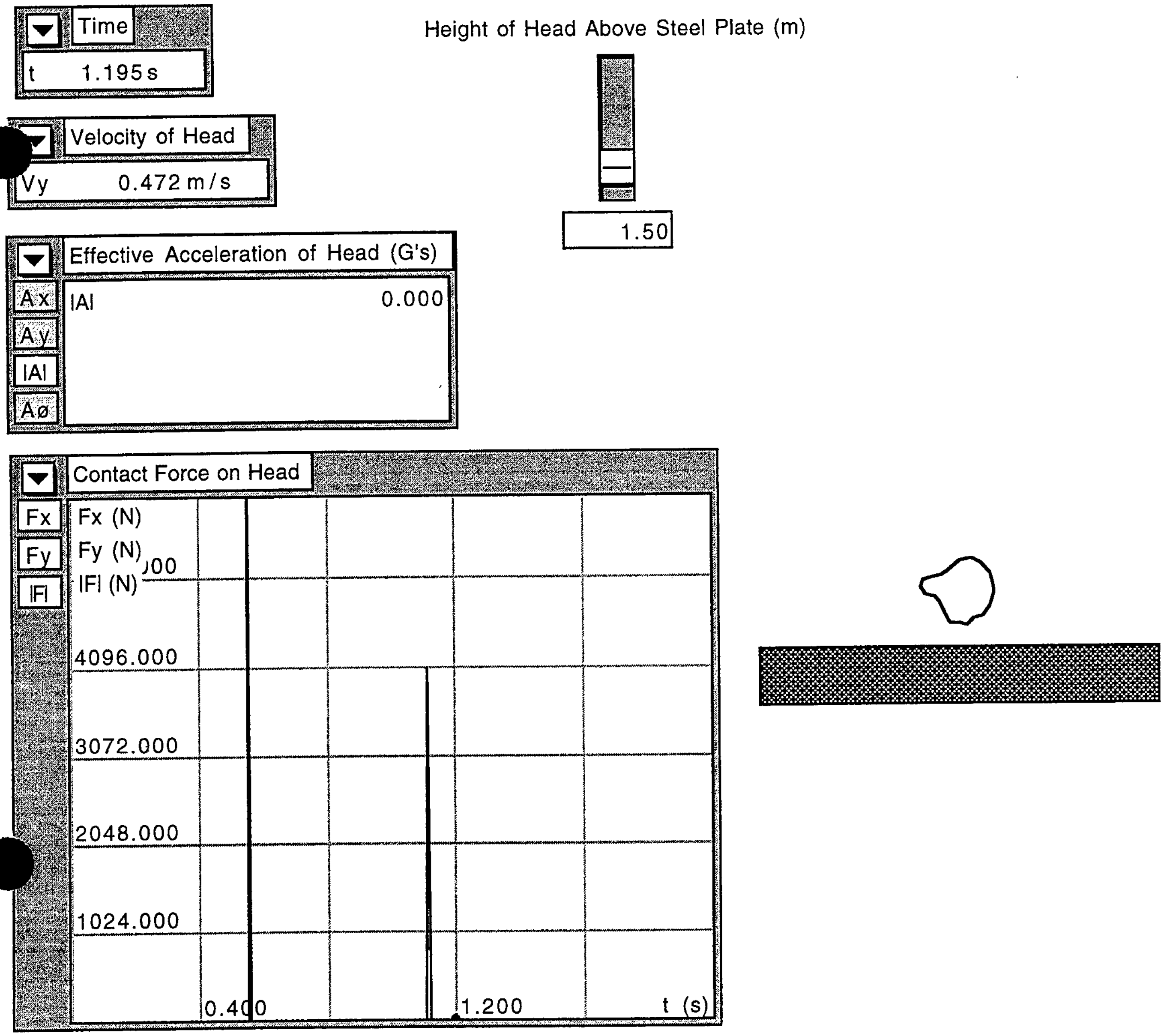
|A|

Height of Head Above Steel Plate (m)

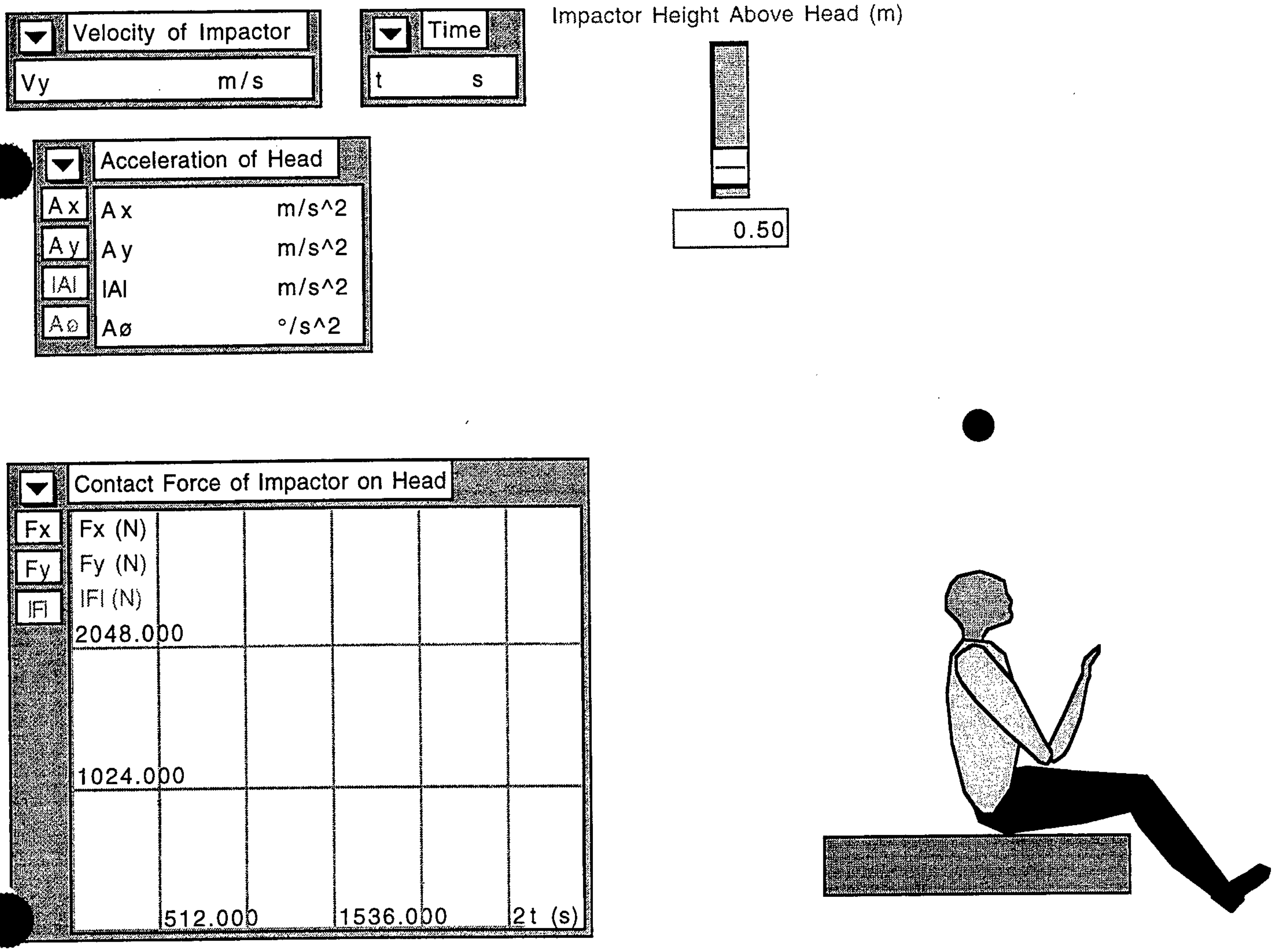
1.50

Contact Force on Head



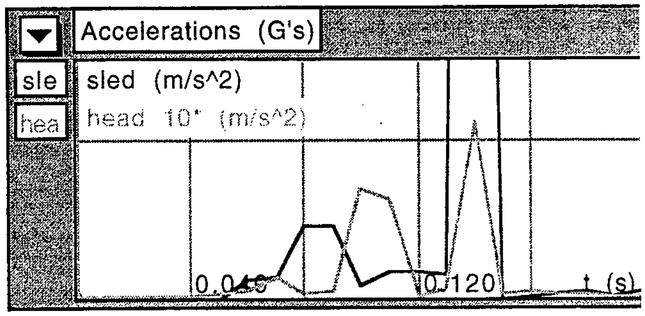


Time
t 1.195 s
Velocity of Head
Vy 0.472 m/s
Effective Acceleration of Head (G's)
Ax
Ay
|A|
Aø
|A| 0.000
Contact Force on Head
Fx
Fy
|F|
Fx (N)
Fy (N)
|F| (N)
4096.000
3072.000
2048.000
1024.000
0.400
1.200
t (s)
Height of Head Above Steel Plate (m)
1.50

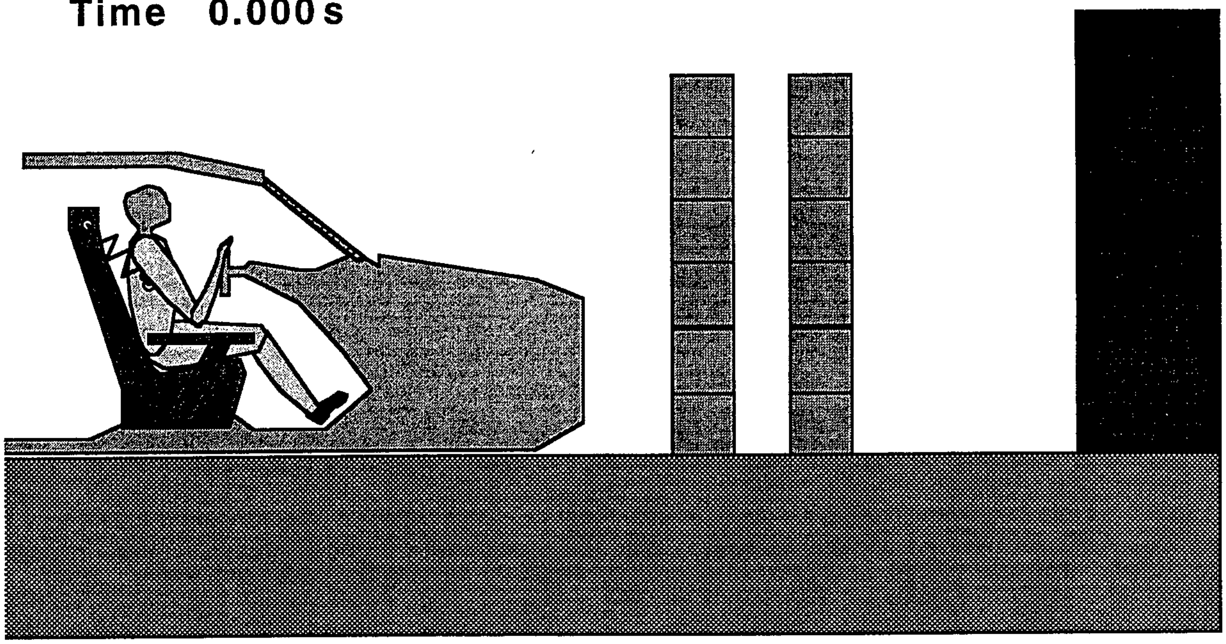

Velocity of Impactor
Vy m/s
Time
t s
Impactor Height Above Head (m)
0.50
Acceleration of Head
Ax m/s^2
Ay m/s^2
|A| m/s^2
Aø °/s^2
Contact Force of Impactor on Head
Fx (N)
Fy (N)
|F| (N)
2048.000
1024.000
512.000
1536.000
2t (s)

**Contact Force of Head**

| | |
|---|---|
| Fx | 0.000 N |
| Fy | 0.000 N |
| \|F\| | 0.000 N |




**Time 0.000 s**



# Section III

# SAFE CAR FOLLOWING MODEL

# SAFE CAR FOLLOWING MODEL

## Theoretical Background and Governing Equations

Using the principles of rectilinear motion, acceleration is the rate of change of velocity with respect to time:

$$a = \frac{dV}{dt}$$

Applying the chain rule yields

$$a = \frac{dV}{dx}\left(\frac{dx}{dt}\right) = \frac{dV}{dx} \times V$$

Rearranging and integrating from initial conditions $x_0$ and $V_0$ to final conditions x and V gives:

$$\int_{x_0}^{x} a\, dx = \int_{V_0}^{V} V\, dV$$

$$a\,(x - x_0) = \frac{1}{2}\,(V^2 - V_0^2)$$

$$a = \frac{V^2 - V_0^2}{2\,(x - x_0)}$$

The change in position, $x$-$x_0$, is the vehicle's braking distance:

$$D_b = x\cos\alpha$$

where $\alpha$ is the incline angle of the road.
For a level road, $\alpha = 0$ and $D_b = x$. Thus,

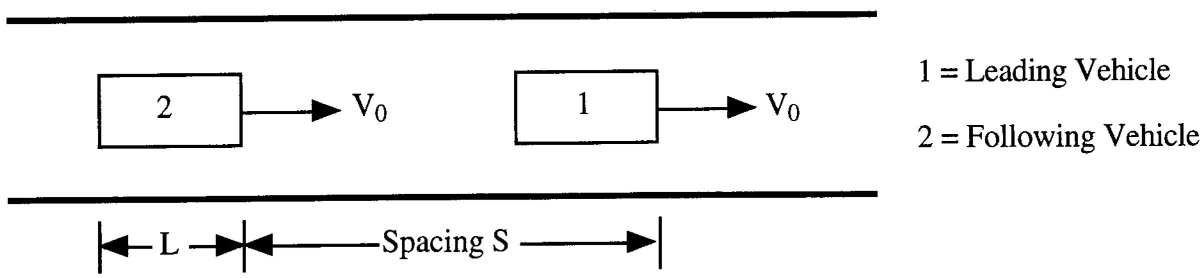
$$a = \frac{V^2 - V_0^2}{2x}$$

For a vehicle decelerating to a stop, the final velocity V is equal to zero, and the deceleration rate d is the negative acceleration. So,

$$a = -d = \frac{0 - V_0^2}{2x}$$

$$d = \frac{V_0^2}{2x} \Rightarrow x = \frac{V_0^2}{2d}$$

Consider the case of two vehicles following each other on a long stretch of roadway. Both vehicles are traveling at the same speed.




For the safe car following model, the spacing between vehicles should be such that if the leading vehicle suddenly decelerates, the following vehicle has sufficient time and distance to perceive the situation, react to it, and decelerate safely without colliding with the leading vehicle.

Using the following notation, a relationship between spacing, speed, and deceleration can be developed:

| | | |
|---|---|---|
| $V_0$ | = | initial speed of the two vehicles |
| $d_1$ | = | deceleration rate of the leading vehicle |
| $d_2$ | = | deceleration rate of the following vehicle |
| $\delta$ | = | perception-reaction time of the following vehicle |
| $x_0$ | = | safety margin between vehicles after stop |
| L | = | length of vehicle |
| N | = | number of vehicles in a train (= 1 for a single, unarticulated vehicle) |

From above, the braking distance of the leading vehicle under constant deceleration is

$$x_1 = \frac{V_0^2}{2 d_1}$$

Including perception-reaction time, the total distance that would be covered by the following vehicle is

$$x_2 = \frac{V_0^2}{2 d_2} + \delta V_0$$

In terms of initial spacing S, the length of the vehicle NL, and the safety margin $x_0$,

$$x_2 = -NL + S - x_0 + x_1$$

Substituting for $x_1$ and $x_2$ and solving for S gives the vehicle spacing necessary to avoid collision:

$$S = \frac{V_0^2}{2\,d_2} + \delta V_0 + NL - \frac{V_0^2}{2\,d_1} + x_0$$


Reference

Papacostas, C.S., and Prevedouros, P.D., *Transportation Engineering and Planning,* Second Edition, Prentice Hall, Inc., 1993.

# CIE 754, Homework Exercise 1

This exercise is intented to provide a basic and simple introduction to the Working Model computer simulation program. Teamwork is encouraged for this homework exercise. Get a partner or partners if you wish, or do it on your own.

Stopping a Vehicle with a Working Model Simulation

Read Section 11.7 (Anchor as a Velocity Constraint) and Appendix B (Formula Language Reference) from the Working Model User's Manual. The manual is available in the Discovery Clusters. Ask for it.

You will use the following tools: Workspace; Numbers and Units from the View menu; the rectangle from the toolbox; and Properties from the Window menu.

1. Follow the instructions in Section 11.7 of the User's Manual to create a vehicle, i.e., a rectangle and anchor.

2. Give the vehicle an initial velocity, $V_x = 10$ ft/sec, using Properties from the Window menu. (*Note:* The units are defined by selecting Numbers and Units from the View menu).

3. To stop the vehicle, use an "If" statement for the rectangle. The following statement will work:

   *if(time > 2,if(time > 10,100 - 10*time,0), 10)*

4. Select the Time and Velocity options from the Measure menu for obvious reasons.

**Print the screen and hand in.**

# CIE 754 Transportation Engineering and Planning
## Homework Assignment #7

1. Draw a scatter plot of Speed vs. Density for the Chicago expressway data shown below.

| Speed (mph) | Density (vpm) |
|---|---|
| 51 | 13 |
| 48 | 22 |
| 50 | 32 |
| 43 | 42 |
| 40 | 43 |
| 28 | 60 |
| 23 | 71 |
| 15 | 81 |
| 12 | 100 |
| 11 | 120 |

2. Generate a data set using the Working Model simulation "Safe Car Following Model" for each of the speeds given above. Calculate the traffic density from the information given from the Model. Assume a median brake reaction time of 0.6 seconds, a lead vehicle deceleration rate of 24 ft/sec$^2$, and a following vehicle deceleration rate of 8 ft/sec$^2$. Plot these data on the scatter plot in #1.

Questions:

A. The empirical Chicago expressway data fit a piece-wise linear model reasonably well. Do the data from the Working Model simulation exhibit a linear relationship? Can you conclude that drivers behave as the model predicts?

B. Perform a sensitivity analysis. Determine if a different brake reaction time, lead vehicle deceleration, or following vehicle deceleration will give a better agreement between the empirical and Model data.

## CHICAGO EXPRESSWAY DATA

| Speed (mph) | Density (vpm) |
|---|---|
| 11 | 120 |
| 12 | 100 |
| 15 | 81 |
| 23 | 71 |
| 28 | 60 |
| 40 | 43 |
| 43 | 42 |
| 50 | 32 |
| 48 | 22 |
| 51 | 13 |

## SAFE CAR FOLLOWING MODEL

**$\delta$ = 0.6 sec** **N = 1**
**dl = 24 ft/sec^2** **L = 20 ft**
**df = 8 ft/sec^2** **xo = 3 ft**

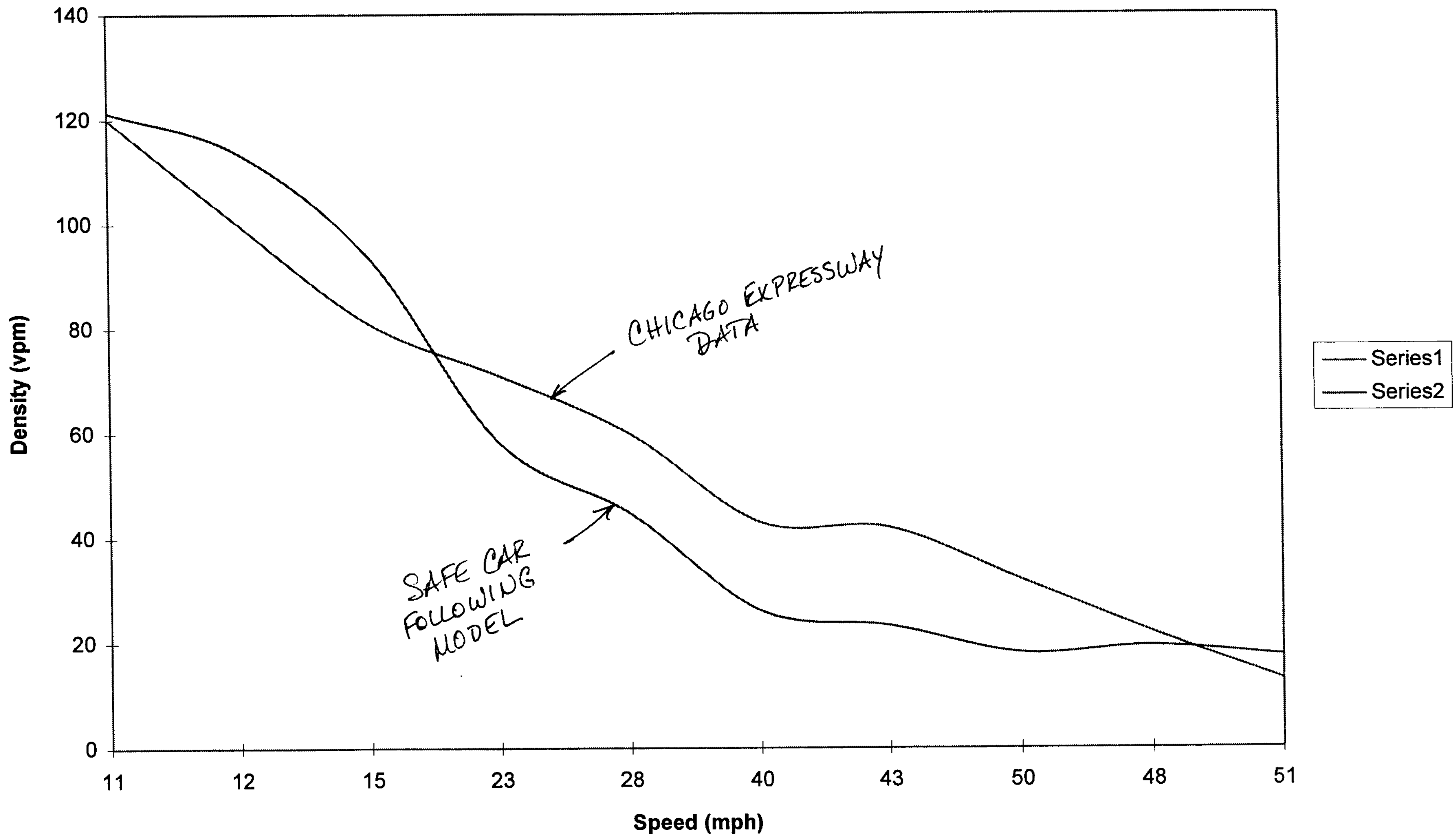
| Speed (mph) | Speed (ft/sec) | Spacing (ft) | Density (vpm) |
|---|---|---|---|
| 11 | 16.13333333 | 43.52518519 | 121.3090761 |
| 12 | 17.6 | 46.46666667 | 113.6298422 |
| 15 | 22 | 56.36666667 | 93.67238321 |
| 23 | 33.73333333 | 90.65407407 | 58.24338348 |
| 28 | 41.06666667 | 117.9096296 | 44.78005755 |
| 40 | 58.66666667 | 201.6074074 | 26.18951391 |
| 43 | 63.06666667 | 226.5651852 | 23.30455138 |
| 50 | 73.33333333 | 291.0740741 | 18.13971243 |
| 48 | 70.4 | 271.7466667 | 19.42986115 |
| 51 | 74.8 | 301.0066667 | 17.54113973 |

**$\delta$ = 1.2 sec** **N = 1**
**dl = 12 ft/sec^2** **L = 20 ft**
**df = 10 ft/sec^2** **xo = 3 ft**

| Speed (mph) | Speed (ft/sec) | Spacing (ft) | Density (vpm) |
|---|---|---|---|
| 11 | 16.13333333 | 44.52903704 | 118.5743136 |
| 12 | 17.6 | 46.70133333 | 113.0588706 |
| 15 | 22 | 53.43333333 | 98.8147224 |
| 23 | 33.73333333 | 72.96281481 | 72.36562917 |
| 28 | 41.06666667 | 86.33392593 | 61.15788137 |
| 40 | 58.66666667 | 122.0814815 | 43.2498028 |
| 43 | 63.06666667 | 131.825037 | 40.05308945 |
| 50 | 73.33333333 | 155.8148148 | 33.88637984 |
| 48 | 70.4 | 148.7813333 | 35.48832291 |
| 51 | 74.8 | 159.3853333 | 33.12726391 |

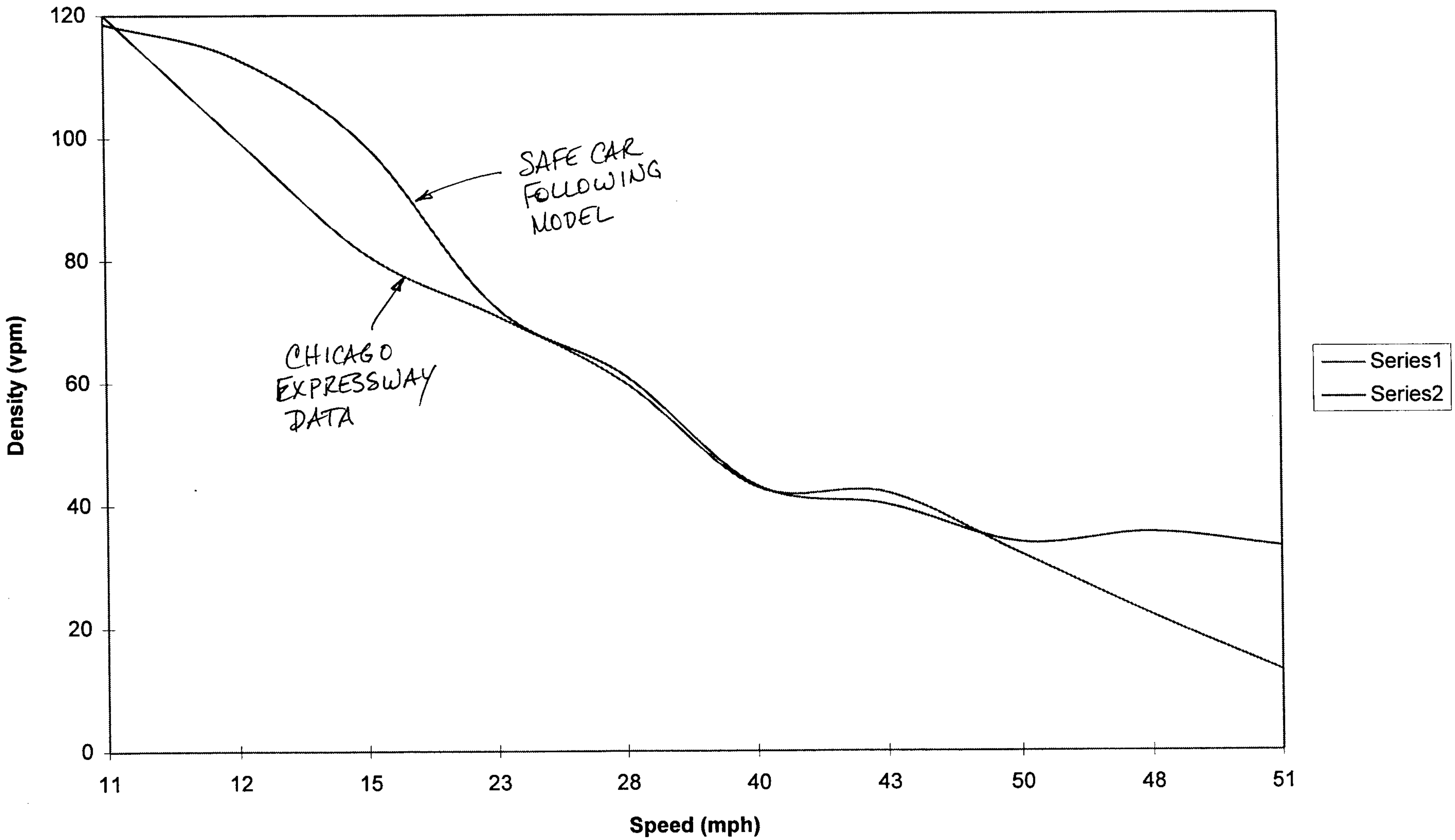
$\delta = 0.6$ sec
$d_l = 24$ ft/sec$^2$
$d_f = 8$ ft/sec$^2$

**SPEED-DENSITY RELATIONSHIP**



$\delta = 1.2$ sec.
$d_l = 12$ ft/sec$^2$
$d_f = 10$ ft/sec$^2$

**SPEED-DENSITY RELATIONSHIP**



# ■ Safe Spacing Model

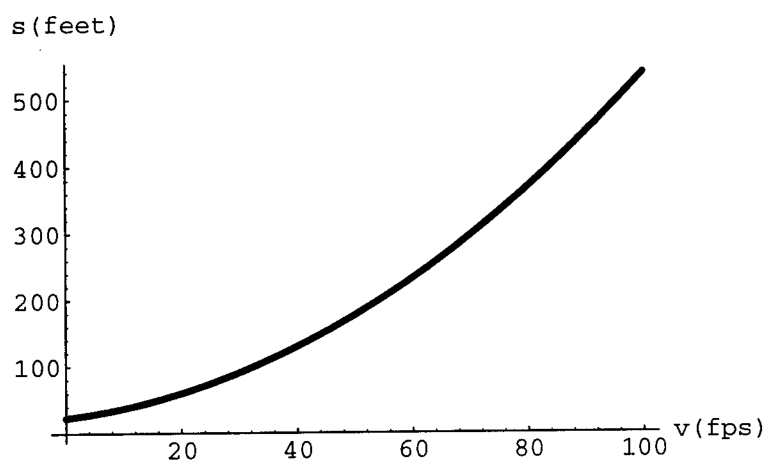
■ s = spacing,
v = velocity
df = deceleration of following vehicle
dl = deceleration of lead vehicle
n = number of vehicles in train
l = length of vehicle
x0 = safety distance margin after stop
delta = perception time

```
Clear[s, xl, xf]
dl = 24;
df = 8;
delta = 1;
n = 1;
l = 20;
x0 = 3;
```

$$xl[v\_, dl\_] := \frac{v^2}{2\, dl}$$

$$xf[v\_, df\_] := v\, delta + \frac{v^2}{2\, df}$$

```
s[v_, df_, dl_] := xf[v, df] - xl[v, dl] + n l + x0
s[v_] := s[v, df, dl]
s[v_] := s[v, df, dl]
fig1 = Plot[s[v], {v, 0, 100}, AxesLabel → {"v(fps)", "s(feet)"},
  PlotStyle → {Thickness[0.01]}];
```




```
Clear[dl, df, n, l, x0, delta, v]
s[v]
```

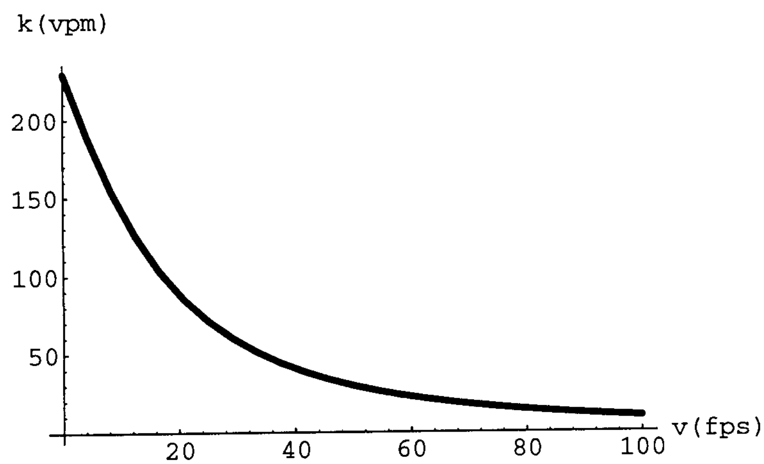
$$l\, n + delta\, v + \frac{v^2}{2\, df} - \frac{v^2}{2\, dl} + x0$$

## ■ Density relationship

**d = perception time = 1 second**
**s = x0 + L = 22**

```
k[v_] := 5280/s[v]
k[v]
dl = 24;
df = 8;
delta = 1;
n = 1;
l = 20;
x0 = 3;
fig2 =
 Plot[k[v], {v, 0, 100}, AxesLabel → {"v(fps)", "k(vpm)"}, PlotStyle → {Thickness[0.01]}];
```

$$\frac{5280}{23 + v + \frac{v^2}{24}}$$



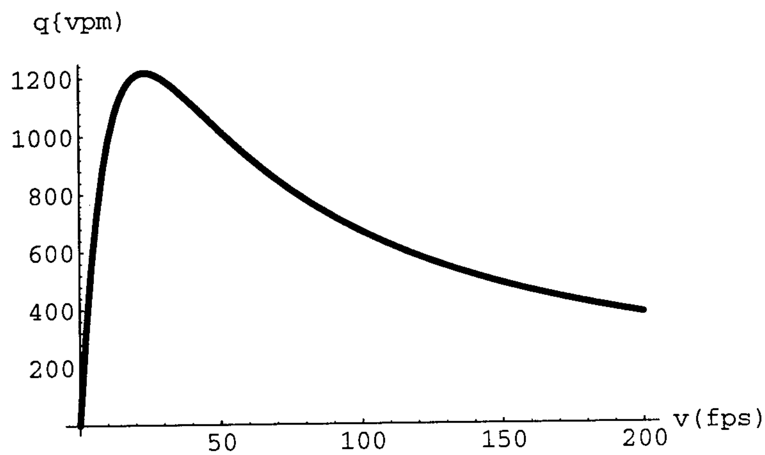
## ■ Flow, q = k v

```
k[v]
```

$$\frac{5280}{23 + v + \frac{v^2}{24}}$$

```
q[v_] := v k[v] 3600 / 5280
q[v]
fig3 =
 Plot[q[v], {v, 0, 200}, AxesLabel → {"v(fps)", "q{vpm)"}, PlotStyle → {Thickness[0.01]}];
```

$$\frac{3600\, v}{23 + v + \frac{v^2}{24}}$$



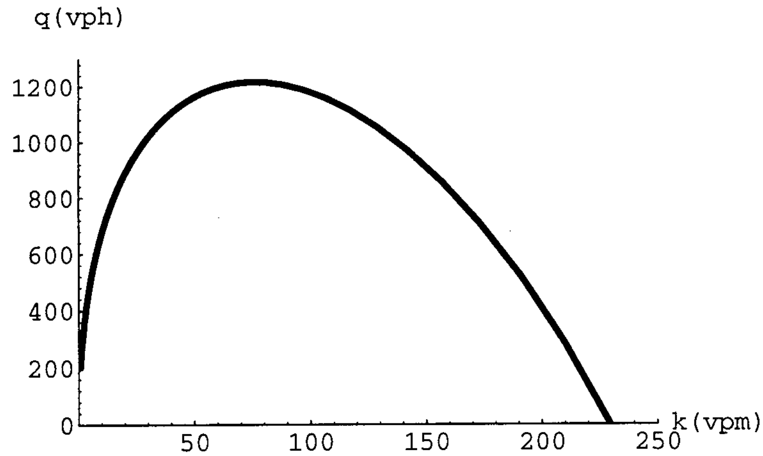

## ■ Flow Density Curve

```
k[v]
q[v]
```

$$\frac{5280}{23 + v + \frac{v^2}{24}}$$

$$\frac{3600\, v}{23 + v + \frac{v^2}{24}}$$

```
data = N[Table[{k[v], q[v]}, {v, 400, 0, -2}]];
fig4 = ListPlot[data, PlotJoined → True, PlotRange → {{0, 250}, {0, 1300}},
  AxesLabel → {"k(vpm)", "q(vph)"}, PlotStyle → {Thickness[0.01]}];
```



Time

t 2.000 s

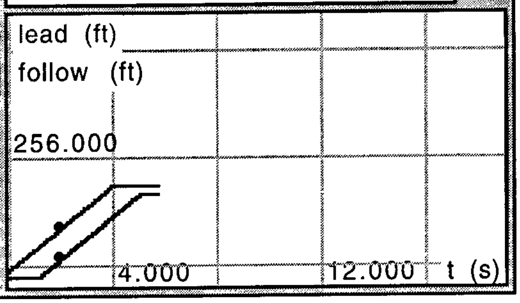
Vehicle spacing, ft

| | |
|---|---|
| spa | 26.042 |
| add | 1.121 |
| spa | 82.083 |

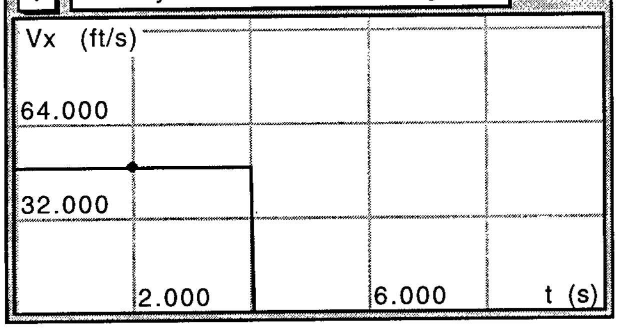
Positions of Lead and Following Cars

lea

fol




Velocity of Lead and Following Cars




Lead & following car velocities

50.00

Stop time

4.00

Perception reaction time

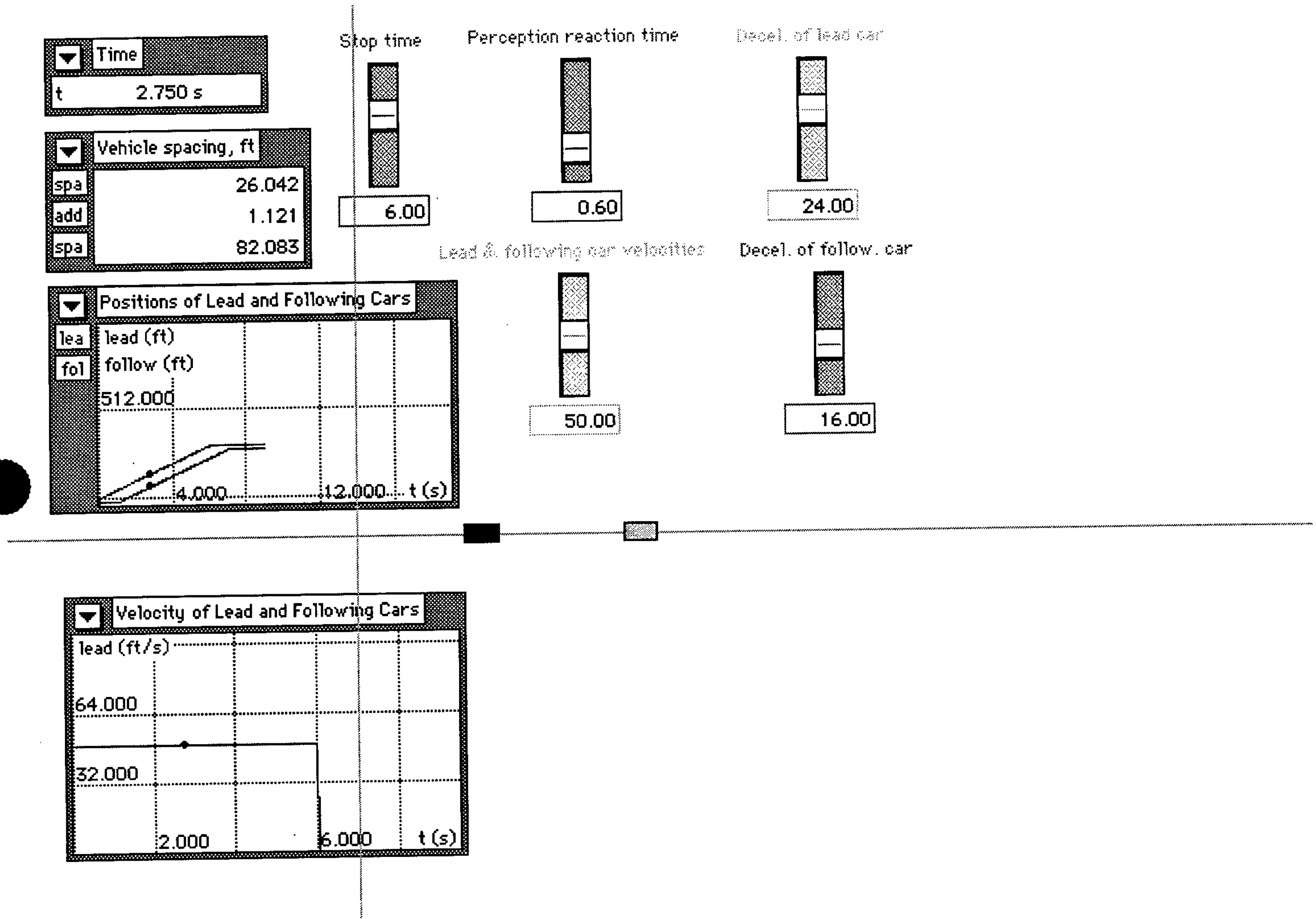
0.60

Decel. of lead car

24.00

Decel. of follow. car

16.00


Time
t 2.750 s
Vehicle spacing, ft
spa 26.042
add 1.121
spa 82.083
Stop time
6.00
Perception reaction time
0.60
Decel. of lead car
24.00
Lead & following car velocities
50.00
Decel. of follow. car
16.00
Positions of Lead and Following Cars
lea lead (ft)
fol follow (ft)
512.000
4.000
12.000
t (s)
Velocity of Lead and Following Cars
lead (ft/s)
64.000
32.000
2.000
6.000
t (s)

Time

t 0.000 s

Vehicle spacing, ft

| | |
|---|---|
| spa | 26.042 |
| add | 1.121 |
| spa | 82.083 |

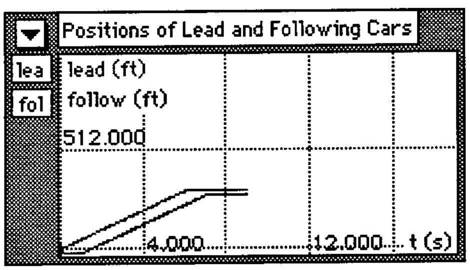
Stop time 6.00

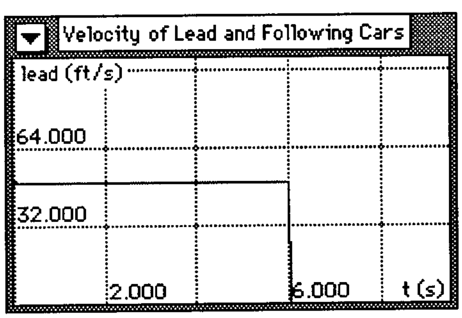
Perception reaction time 0.60

Decel. of lead car 24.00

Lead & following car velocities 50.00

Decel. of follow. car 16.00



# Section IV

# HAZARDOUS HIGHWAY IDENTIFICATION

# HAZARDOUS HIGHWAY IDENTIFICATION

## Theoretical Background and Governing Equations

To facilitate the comparison of results obtained from the analysis of accidents at a particular location with those of other locations, one or more accident rates are used. These accident rates are determined on the basis of exposure data, such as traffic volume, and the length of road section being considered. Commonly used rates are: (1) Rate per million of entering vehicles and (2) Rate per 100 million vehicle-miles.

The ***Rate per Million of Entering Vehicles (RMEV)*** is the number of accidents per million vehicles entering the particular location under study during the study period. This rate is very often used as a measure of accident rates at intersections. Mathematically, it is expressed as:

$$RMEV = \frac{A \times 1{,}000{,}000}{V}$$

where

$A$ = Total number of accidents or number of accidents by type (e.g., fatal, injury, property damage, etc.) occurring in one year at the study location

$V$ = Average daily traffic (ADT) x 365

The ***Rate per 100 Million Vehicle Miles (RMVM)*** is the number of accidents per 100 million vehicle miles of travel. This rate is commonly used as a measure of accident rates on a stretch of highway with similar traffic and geometric characteristics. It is obtained from the expression:

$$RMVM = \frac{A \times 100{,}000{,}000}{VMT}$$

where

$A$ = Number of total accidents or number of accidents by type at the location under study, during a given period

$VMT$ = Total vehicle miles of travel during the given period

= ADT x (number of days in study period) x (length of road)

A commonly used technique to determine highway accident patterns is ***Expected Value Analysis***. This is a mathematical method used to identify locations with abnormal accident characteristics. It is used to compare sites with similar features (e.g., geometric design characteristics, traffic volume, traffic control devices), since the analysis does not take exposure levels into account. Expected value analysis is carried out by determining the average number of a specific type of accident occurring at several locations with similar geometric and traffic characteristics. This average value, adjusted for a given level of confidence, indicates the "expected" value for the specific type of accident. Locations with accident counts higher than the expected value are considered to be sites which are more representative of, or susceptible to, the occurrence of that specific type of accident. The expected value can be obtained from the expression:

$$EV = \overline{x} \pm ZS$$

where

$EV$ = expected range of accident frequency

$\overline{x}$ = average number of accidents per location

$S$ = estimated standard deviation of accident frequencies

$Z$ = the number of standard deviations corresponding to the required confidence level

Reference

Garber, Nicholas J. and Hoel, Lester A., *Traffic and Highway Engineering,* Second Edition, PWS Publishing Company, 1997.

## Hazardous Highway Identification Exercise

In Table 1, C and $C_I$ = average annual number of fatal and personal injury crashes in Durham, NH, respectively. In Table 3, C and $C_I$ are the total number of crashes observed in year 1992. L = length of highway in miles.

a) Complete Table 2 by calculating the RMVM and $RMVM_I$ for highway locations given in Table 1.

b) The critical accident rate method is used to identify highway locations with an abnormal accident rate experience, in other words, to classify a highway location as hazardous. In order to illustrate the method, RMVM is used as a measure of effectiveness. The critical accident rate is calculated as an upper-level confidence level using statewide accident and traffic statistics. It is calculated as $RMVM_{cr} = \overline{RMVM} + Z\, S_{RMVM}$ where $\overline{RMVM}$, $S_{RMVM}$ and Z are the sample average, sample standard deviation and standard normal random variable, respectively. The values of Z, for example, are 1.645 for a 95% and 2.576 for the 99.5% upper confidence levels. A highway segment average is denoted as RMVM; therefore, if RMVM > $RMVM_{cr}$, then the segment is classified as hazardous; otherwise, it is classified as safe. Assuming Z = 0, classify the highways given in Table 1.

Table 1. Hazardous Highway Location Classifications for Fatal and Personal Injury Crashes in Durham, NH

| Location | ADT | L | C | $C_I$ |
|---|---|---|---|---|
| Route 4 East | 15,470 | 2.25 | 0.86 | 6.4 |
| Route 4 West | 15,470 | 3 | 0.29 | 3.4 |
| Route 108 North | 10,000 | 1 | 0.0 | 3.4 |
| Route 108 South | 9,290 | 3 | 0.0 | 4.3 |

Table 2. RMVM Measures for Durham and U.S. Highways

| Location | C | RMVM | $C_I$ | $RMVM_I$ |
|---|---|---|---|---|
| Route 4 East | 0.86 | | 6.43 | |
| Route 4 West | 0.29 | | 3.43 | |
| Route 108 North | 0.0 | | 3.43 | |
| Route 108 South | 0.0 | | 4.29 | |

Table 3. RMVM Measures for NH and U.S. Highways[1]

| Location | C | RMVM | $C_I$ | $RMVM_I$ |
|---|---|---|---|---|
| | System Category: Urban Principal Arterial for 1992 | | | |
| New Hampshire | 16 | 1.59 | 1,195 | 118.91 |
| U.S. | 5,246 | 1.52 | 488,228 | 141.85 |
| | System Category: Urban Total Systems for 1992 | | | |
| New Hampshire | 29 | 0.78 | 1,722 | 171.34 |
| U.S. | 15,202 | 1.12 | 781,631 | 227.09 |
| | System Category: Total Systems for 1992 | | | |
| New Hampshire | 110 | 1.09 | 6,850 | 68.04 |
| U.S. | 34,928 | 1.56 | 2,216,245 | 98.95 |

[1] Federal Highway Administration, Highway Safety Performance - 1992, Publication Number FHWA-SA-95-030.

# Section V

# QUEUING THEORY AND MERGING MODELS

# QUEUING THEORY AND MERGING MODELS

## Theoretical Background and Governing Equations

### Queuing Models

The congestion that exists on urban highways results in the formation of queues on expressway on-ramps and off-ramps, at signalized intersections, and on arterial roadways, where moving queues may occur. The theory of queuing concerns the use of mathematical models to describe the processes that cause queues to form; these models can be used to determine the probability that an arrival will be delayed, the waiting time for all arrivals, the expected waiting time of an arrival that waits, and so forth.

Several models have been developed that can be applied to traffic situations such as the merging of ramp traffic to freeway traffic, interactions at pedestrian crossings, and sudden reduction of capacity on highways. The solution to a queuing problem entails the assessment of a system's performance, which is described by a set of measures of performance. These may include the number of customers served per unit time, the average delay per customer, the average and maximum length of the waiting lines, the percent of time each service counter is idle, the cost of operating the system, etc. An overview of the fundamental queuing theory relationships for a specific type of queue, the single-channel or single-server system, is presented here.

A queue is formed when arrivals wait for a service or an opportunity, such as the arrival of an accepted gap in a main traffic stream or the collection of tolls at a tollbooth. A proper and thorough analysis of the effects of such a queue requires that the following characteristics of the queue be defined:

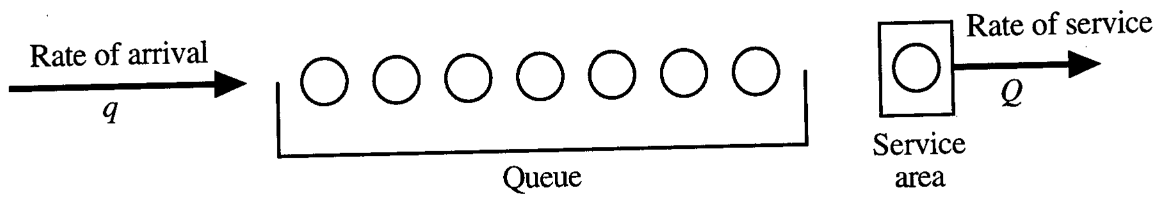
- **Arrival Distribution.** Arrivals can be described as either a deterministic or a random distribution. Light-to-medium traffic is usually described by a Poisson distribution, and this is generally used in queuing theories related to traffic flow.
- **Service Method.** The methods used in servicing arrivals include ***first-in, first-out,*** where units are served in order of their arrivals, and ***last-in, first-out,*** where the service is reversed to the order of arrival. The service method can also be based on priority, where arrivals are directed to specific queues of appropriate priority levels (e.g., carpool lanes on freeways). Queues are then serviced in order of their priority level.
- **Characteristics of Queue Length.** The maximum number of units in the queue is specified as either finite or infinite. Finite queues are sometimes necessary when the waiting area is limited. ***Undersaturated queues*** are those in which the arrival rate is less than the service rate, and ***oversaturated queues*** are those in which the arrival rate is greater than the service rate. The length of an undersaturated queue may vary but will

reach a steady state with the arrival of units. The length of an oversaturated queue, however, will continue to increase with the arrival of units and never reach a steady state.

- **Service Distribution.** This distribution is usually considered as random and is typically described by a Poisson or a negative exponential distribution.

Single-Channel, Undersaturated, Infinite Queues

One of the most basic queuing problems is the single-channel, first-in, first-out system with Poisson arrivals and exponentially distributed customer service times. When in the system, customers are assumed to be patient, that is, they do not leave prematurely. The system is assumed to have an unlimited holding capacity; there is no upper limit on the number of customers that can be in the queue. A schematic of a single-channel queue in which the rate of arrival is $q$ vehicles per hour (vph) and the service rate is $Q$ vph is shown below.




For an undersaturated queue, $Q > q$. Assuming that both the rate of arrivals and the rate of service are random, the following relationships can be developed.

1. Probability of $n$ units in the system, $P(n)$

$$P(n) = \left(\frac{q}{Q}\right)^n \left(1 - \frac{q}{Q}\right)$$

where $n$ is the number of units in the system, including the unit being serviced.

2. The expected number of units in the system, $E(n)$

$$E(n) = \frac{q}{Q - q}$$

3. The expected number of units waiting to be served (that is, the mean queue length) in the system, $E(m)$

$$E(m) = \frac{q^2}{Q(Q-q)}$$

4. Average waiting time in the queue, *E(w)*

$$E(w) = \frac{q}{Q(Q-q)}$$

5. Average waiting time of an arrival, including queue and service time, *E(v)*

$$E(v) = \frac{1}{Q-q}$$

6. Probability of spending time *t* or less in the system

$$P(v \leq t) = 1 - e^{-\left(1-\frac{q}{Q}\right)qt}$$

7. Probability of waiting for time t or less in the queue

$$P(w \leq t) = 1 - \frac{q\, e^{-\left(1-\frac{q}{Q}\right)qt}}{Q}$$

8. Probability of more than *N* vehicles being in the queue, that is, *P(n>N)*

$$P(n > N) = \left(\frac{q}{Q}\right)^{N+1}$$

## Merging Vehicle Model

An important aspect of traffic flow and highway safety is the interaction of vehicles as they join, leave, or cross a traffic stream. Examples of these include ramp vehicles merging onto an expressway stream, freeway vehicles leaving the freeway onto frontage roads, vehicles on minor roads wishing to join the traffic stream on a major road at an unsignalized intersection, and the changing of lanes by vehicles on a multilane highway. The most important factor a driver considers in making any one of these maneuvers is the availability of a gap that, in the driver's judgment, is adequate to complete the maneuver. A gap is defined as the time or space headway in a major traffic stream which is evaluated by a driver in a minor traffic stream who wants to merge into the major stream; merging is the process by which a vehicle in one traffic stream joins another traffic stream moving in the same direction. A driver who intends to merge must evaluate the gaps that become available to determine which gap, if any, is large enough to accept the vehicle. In accepting that gap, the driver believes that he or she can complete the merging maneuver and safely join the major stream within the length of the gap. This concept is referred to as ***gap acceptance***.

Consider a stretch of a major roadway that is lightly travelled and free of traffic controls. The use of gap acceptance to determine the delay of vehicles on a minor road wishing to merge onto the major traffic stream depends on the distribution of arrivals of main stream vehicles at the area of merge. An observer may notice that vehicle arrivals do not occur at regular intervals; thus, it is generally accepted that for light to medium traffic flow on a highway, the arrival of vehicles is randomly distributed. Furthermore, vehicle arrivals in light to medium traffic can usually be described by the Poisson distribution. Therefore, the probability of $n$ arrivals occurring in any interval of time $t$ can be obtained from the expression:

$$P\,(N = n) = \frac{\mu^n\ e^{-\mu}}{n!}$$

where

$N$ = random variable = $[0, 1, 2, \ldots\ldots, \infty]$

= event that $n$ vehicles arrive in time period $t$

$\mu = qt$

$q$ = average rate of arrival

$t$ = time period

The vehicle on the minor road will merge into the major roadway traffic stream only if there is a gap of $t$ seconds equal to or greater than its *critical gap* (i.e., the acceptable average minimum gap). This will occur when no vehicles arrive in time $t$. So,

$$P(N = 0) = \frac{\mu^0 \, e^{-\mu}}{0!} = e^{-\mu} = e^{-qt}$$

Now, if we define a new random variable $T$ as the time between arrivals, then

$P(T > t) =$ Probability that no vehicle arrives in time $t$

$= P(N = 0)$

$= e^{-qt}$

The cumulative distribution is then given by

$$F_T(t) = P(T < t) = 1 - P(T > t) = 1 - e^{-qt}$$

The probability density function, $f_T(t)$ is defined as:

$$f_T(t) = \frac{d\,F_T(t)}{dt} = \frac{d(1 - e^{-qt})}{dt}$$

$$f_T(t) = q e^{-qt}$$

This is called an ***exponential distribution.***

Example

Suppose the traffic flow in each lane of a two-lane major roadway is 300 vph. A vehicle on a minor road at an unsignalized intersection wants to merge into the traffic stream by making a right turn onto the major road.

(a) Given a critical merge time of 3.5 seconds, what is the probability of a merge?

Arrival rate = $q$ = 300 vehicles per hour = 0.0833 vehicle per second

$$f_T(t) = q e^{-qt}$$

$P(T > t) =$ Probability the time gap is greater than 3.5 sec

$$P(T > 3.5) = \int_{3.5}^{\infty} 0.0833\, e^{-0.0833\, t}\, dt = e^{-0.0833(3.5)} - e^{-\infty}$$

$$P\,(T > 3.5) = \frac{1}{e^{0.292}} = 0.747 \approx 75\ \%$$

Thus, when $q$ = 300 vph on the major road, a driver will merge 3 out of 4 times with a gap time of 3.5 seconds.

(b) What is the probability of **no** merge?

$$P\,(T < 3.5) = 1 - 0.747 = 0.253 \approx 25\ \%$$

(c) What is the probability a vehicle will merge on the second gap?

$$P\,(\text{no merge})\ P(\text{merge}) = P\,(T < 3.5)\,P\,(T \geq 3.5) = (0.25)\,(0.75) = 0.1875 = 18.75\ \%$$

In general, we can say,

$$P\,(N = n) = P\,(\text{merge on gap n}) = p^{n-1}\ (1 - p)$$

where $p$ is the probability of no merge.

(d) Let N = the number of gaps before merging. What is the expected number of gaps occurring on the major road before a driver on the minor road successfully merges into the major traffic stream?

$$E(N) = \sum_{n=1}^{\infty} np^{n-1}\ (1 - p) = (1 - p) \sum_{n=1}^{\infty} np^{n-1}$$

$$\frac{1}{1 - p} = \sum_{n=0}^{\infty} p^{n}$$

Differentiating both sides with respect to p, we get

$$\frac{1}{(1 - p)^2} = \sum_{n=1}^{\infty} np^{n-1}$$

Substitution into the equation for $E(N)$ gives:

$$E(N) = \frac{1 - p}{(1 - p)^2} = \frac{1}{1 - p}$$

$$E(N) = \frac{1}{1 - 0.25} = \frac{1}{0.75} = 1.33$$

Thus, 1.33 gaps are expected before a successful merge occurs.

(e) What is the probability of merging into the major stream by making a left turn from the minor road?

In this case, the time between arrivals must exceed the critical merge time in each lane of the major roadway. Thus, the probability of a successful merge is:

$$P(T \geq 3.5)P(T \geq 3.5) = 0.75^2 = 0.56 = 56\%$$

References

1. Garber, Nicholas J. and Hoel, Lester A., *Traffic and Highway Engineering,* Second Edition, PWS Publishing Company, 1997.
2. Papacostas, C.S., and Prevedouros, P.D., *Transportation Engineering and Planning,* Second Edition, Prentice Hall, Inc., 1993.

## ■ Merging Dynamics

Discussion questions:

1. Does speed of the major flow vehicle affect merging decisions?

Derive a model to answer the question.

2. Do you feel it is valid to use a critical headway of hcr = 3 seconds as decision parameter in determining a merge? Why?

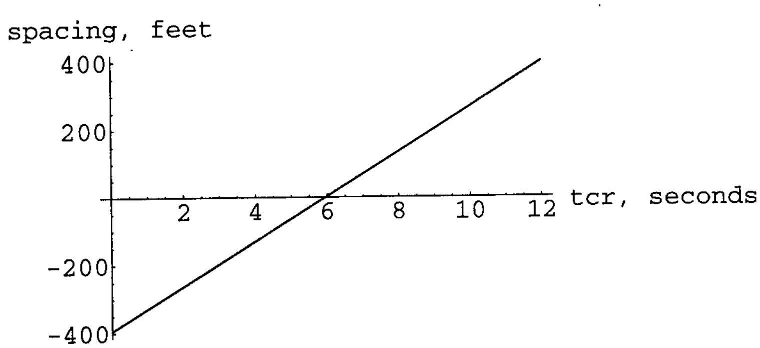
3. What about a perception-reaction time? Should it be added to hcr?

l = vehicle length = 20 feet
u = vehicle speed on major road
a = acceleration of merging vehicle 7 feet per second
tm = time of merging vehicle to reach u.
tt = time for merging car to reach major road centerline
xm = location of merging vehicle on major road at tm.
x2 = location of following vehicle 2 on major road at tm.
s = car spacing at speed u and flow q.

Assumptions:
1. Merging vehicle will begin to accelearte when x1 = 0
where x1 = location of first vehicle on major road.
2. Merging vehile is 2 car lenths from major centerline or R = 24
where R = radius of circular path that the merging car uses in its merge.

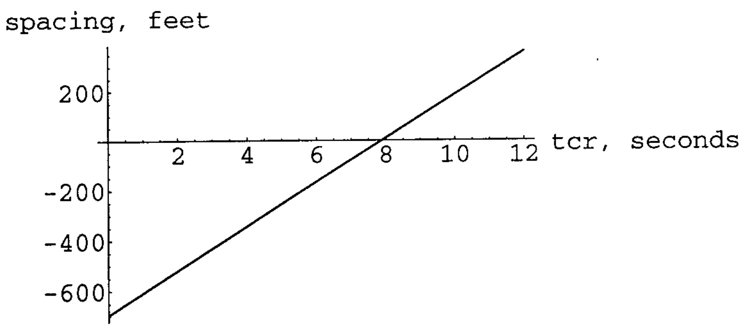
## ■ Speed u = 45 mph

```
Spacing at 45 mph = 792

Time headway = 12 at flow = 300 vph
```



## ■ Speed u = 60 mph

```
Spacing at 60 mph = 1056

Time headway = 12 at flow = 300 vph
```




Discussion questions:

1. Does speed of the major flow vehicle affect merging decisions?

2. Do you feel it is valid to use a critical headway of hcr = 3 seconds as decision parameter in determining a merge? Why?

3. What about a perception-reaction time? Should it be added to hcr?

## ■ Waiting Time of Lead Vehicle on Minor Road

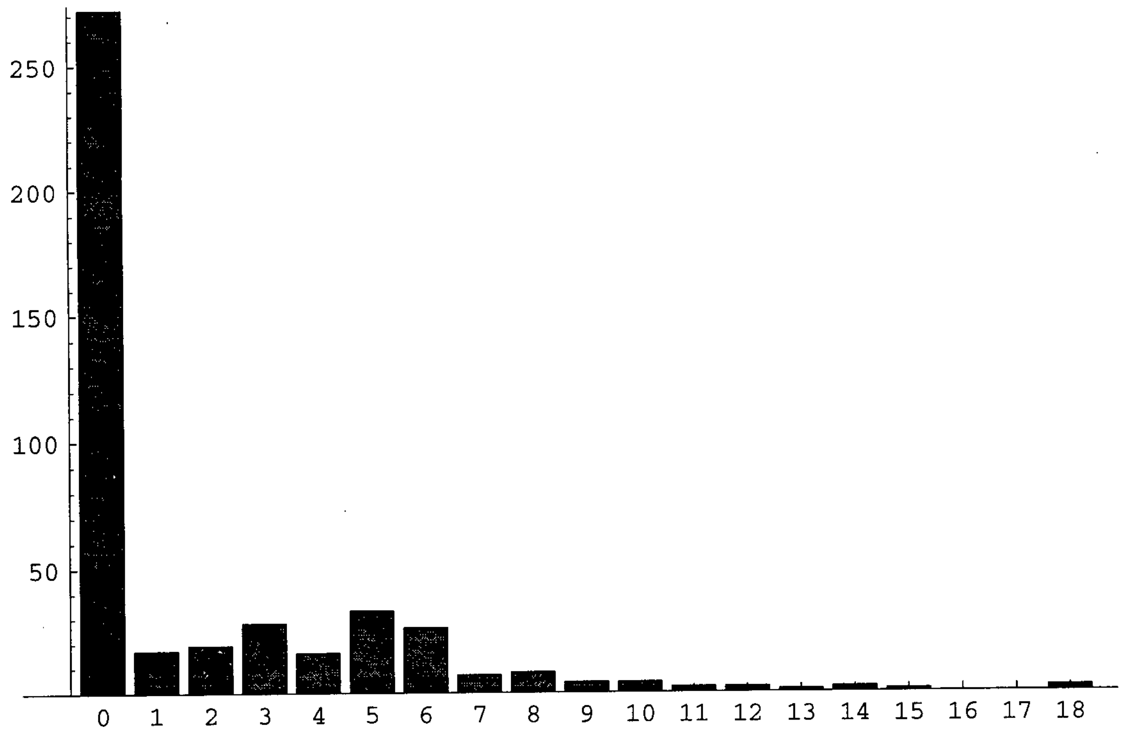
Assumptions:

The queuee is assumed to have an infinite number of vehicles in line.
If one vehicle merges, then the next vehicle takes its place and is ready to merge.

If H > hct, then one vehicle merges.
If H > 2 hcr, then two vehicles merge,
and so on .

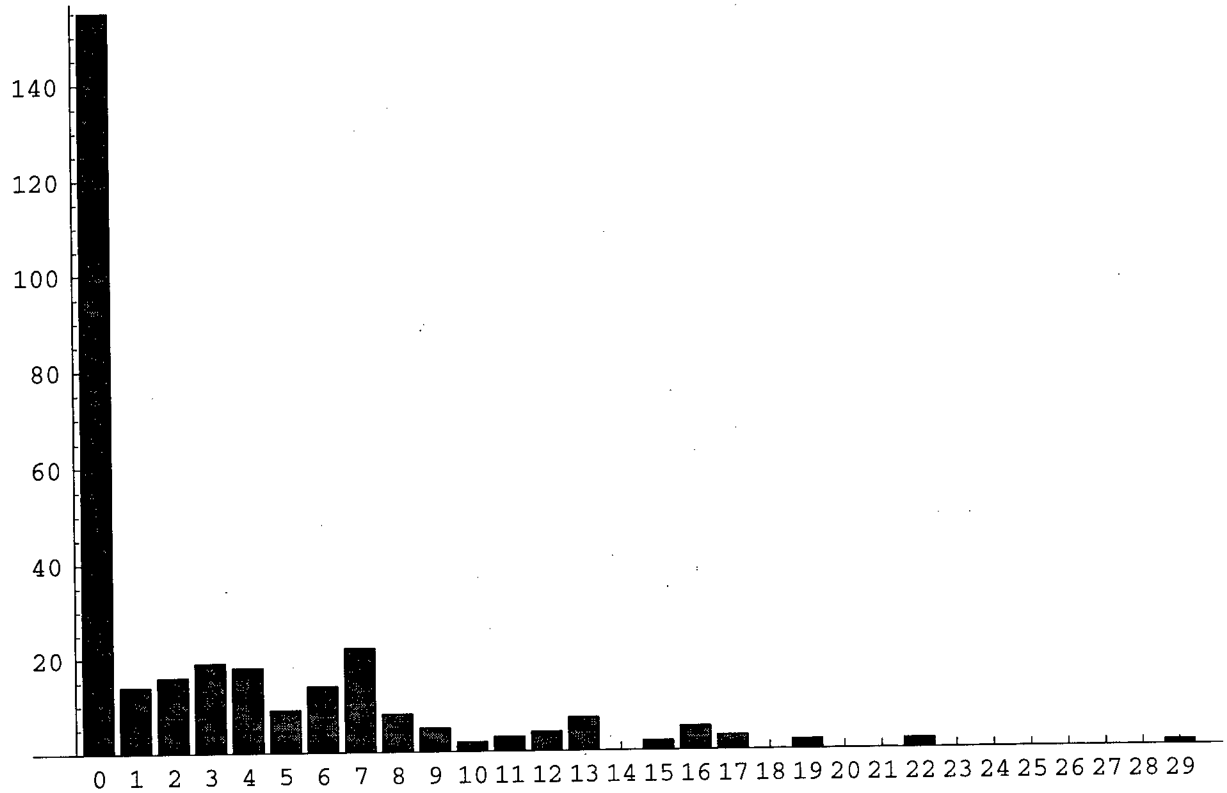
If H > hcr, then H the second vehicle for the second vehicle is H - hcr.

```
hcr = 6;
HistogramWait[Queue[H, hcr], 1]
```




- Graphics -

```
hcr = 8;
HistogramWait[Queue[H, hcr], 1];
```



```
<< "Statistics`DataManipulation`"
<< "Graphics`Graphics`"
<< Graphics`MultipleListPlot`
```

## ■ Functions

## ■ Merging Dynamics

## ■ Data Generated with Monte Carlo Simulation

## ■ Waiting Time of Lead Vehicle on Minor Road

## ■ The Merge as a Random Event

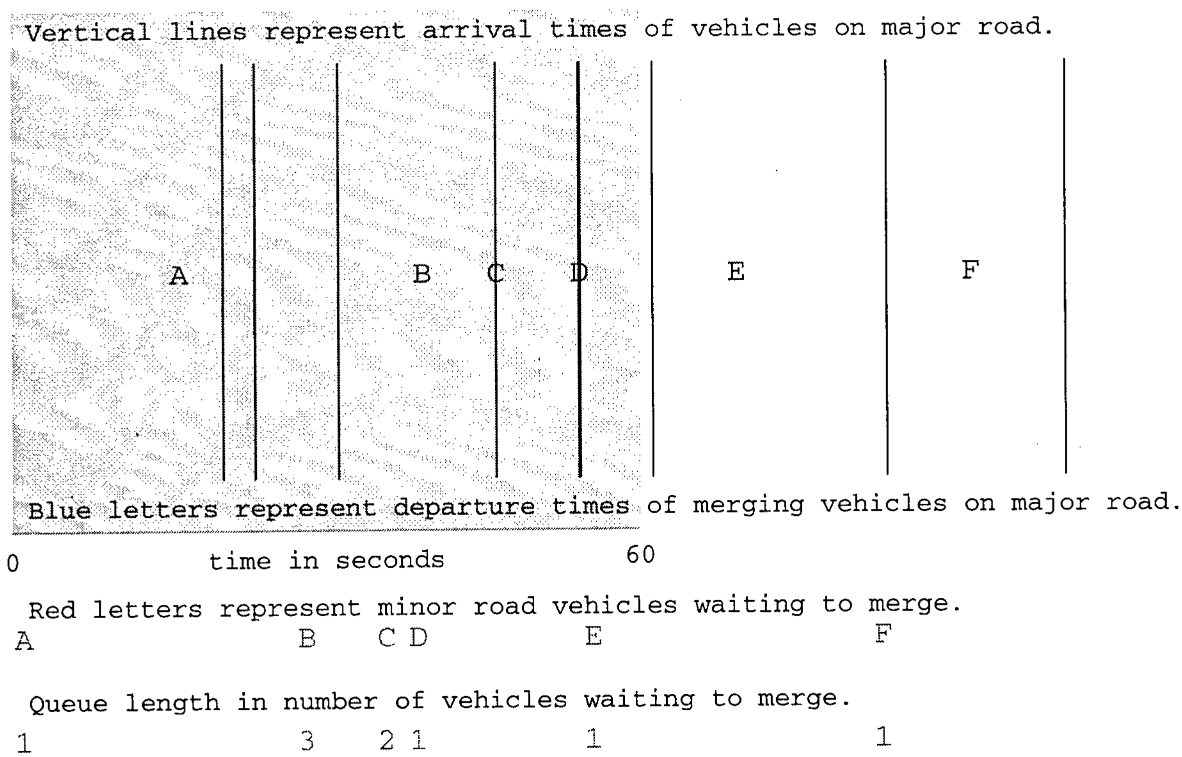
```
duration = 60;
{h, sh} = Table[Exponential[q, duration]];
{H, sH} = Table[Exponential[Q, 1.7 duration]];
hcr = 8;
{arrive, major, leave, depart, measure} = QueueLength[sh, sH, hcr, duration];
hcr = 6;
{arrive, major, leave, depart, measure} = QueueLength[sh, sH, hcr, duration];
```

Maximum number of vehicles in queue for this simulation run = 3

Critical headway, $h_{cr}$ = 8 seconds

Flow on major road, Q = 420 vph

Flow on minor road, q = 228 vph

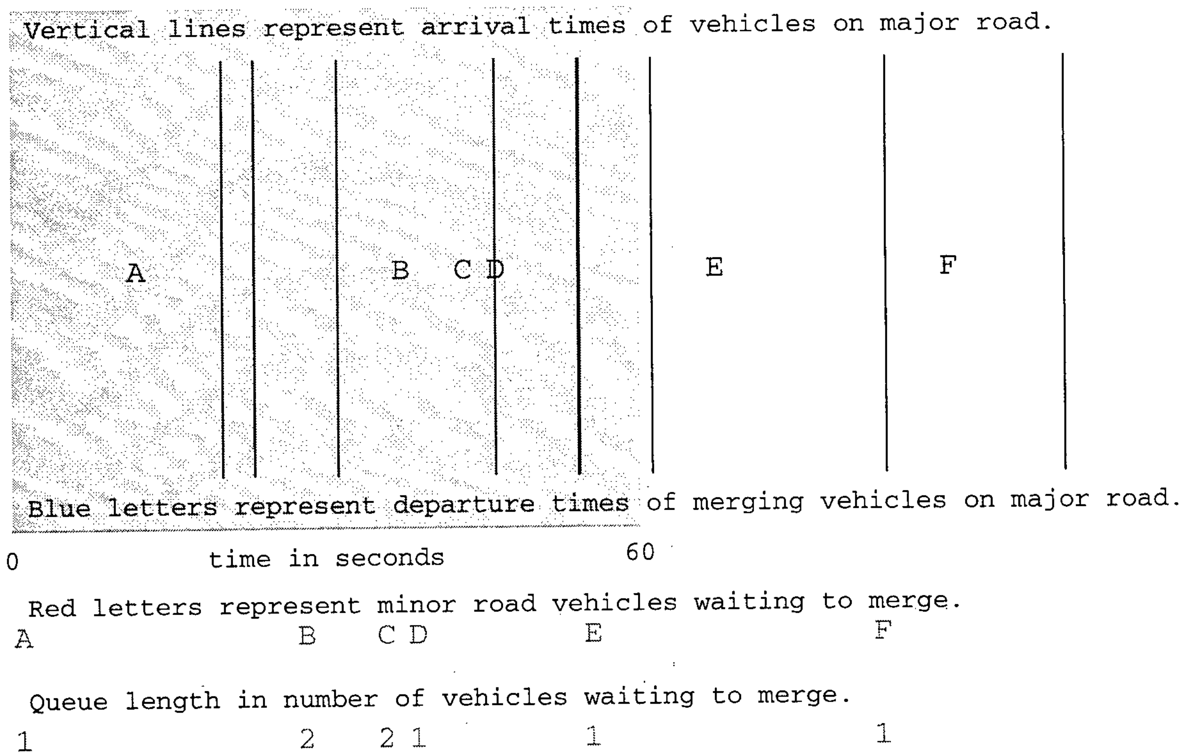



Maximum number of vehicles in queue for this simulation run = 2

Critical headway, $h_{cr}$ = 6 seconds

Flow on major road, Q = 420 vph

Flow on minor road, q = 228 vph




## ■ Maximum Queue Length

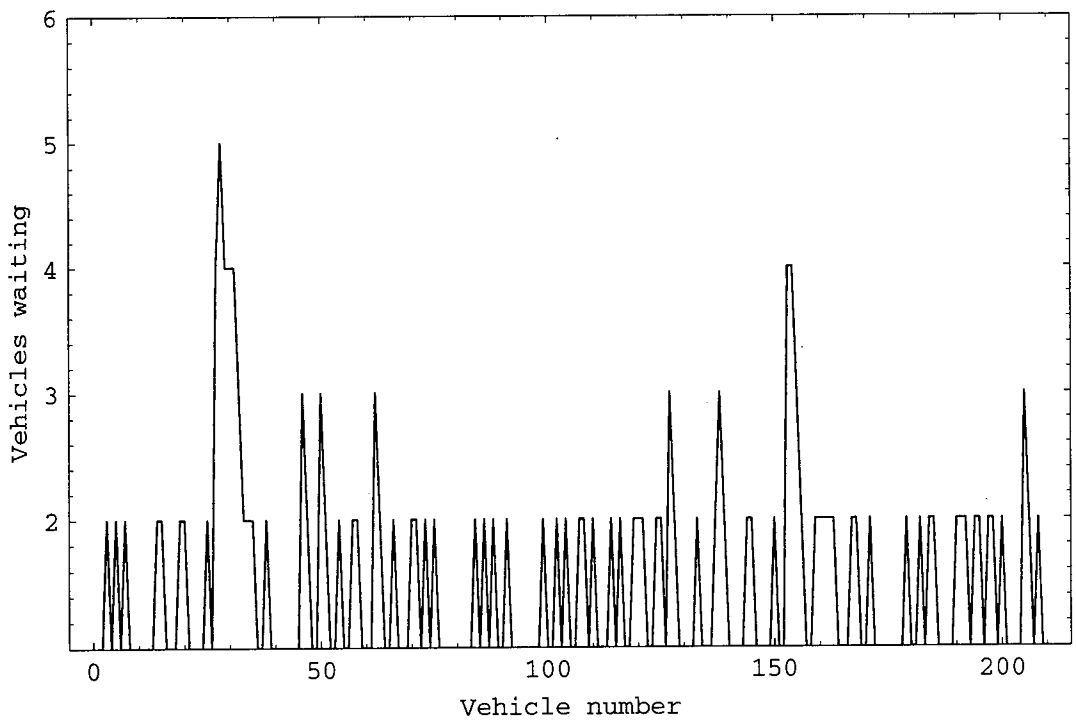
```
duration = 3600;
{h, sh} = Table[Exponential[q, duration]];
{H, sH} = Table[Exponential[Q, 1.7 duration]];
hcr = 6;
{max , list} = MaxQueueLengthPlot[sh, sH, hcr, duration, Q, q];
hcr = 8;
{max , list} = MaxQueueLengthPlot[sh, sH, hcr, duration, Q, q];
```

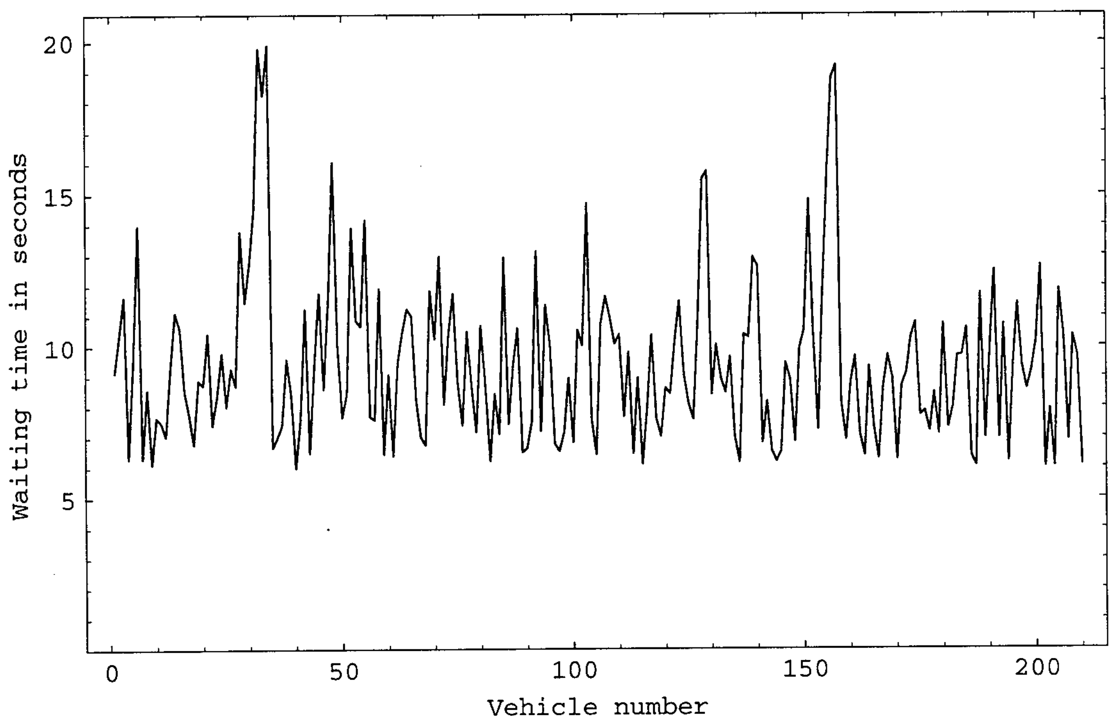
Critical headway, $h_{cr}$ = 6 seconds

Flow on major road, Q = 420 vph

Flow on minor road, q = 228 vph

Maximum number of vehicles in queue for this simulation run = 5




Minimum waiting time of vehicles in queue for this simulation run = 6.009 seconds

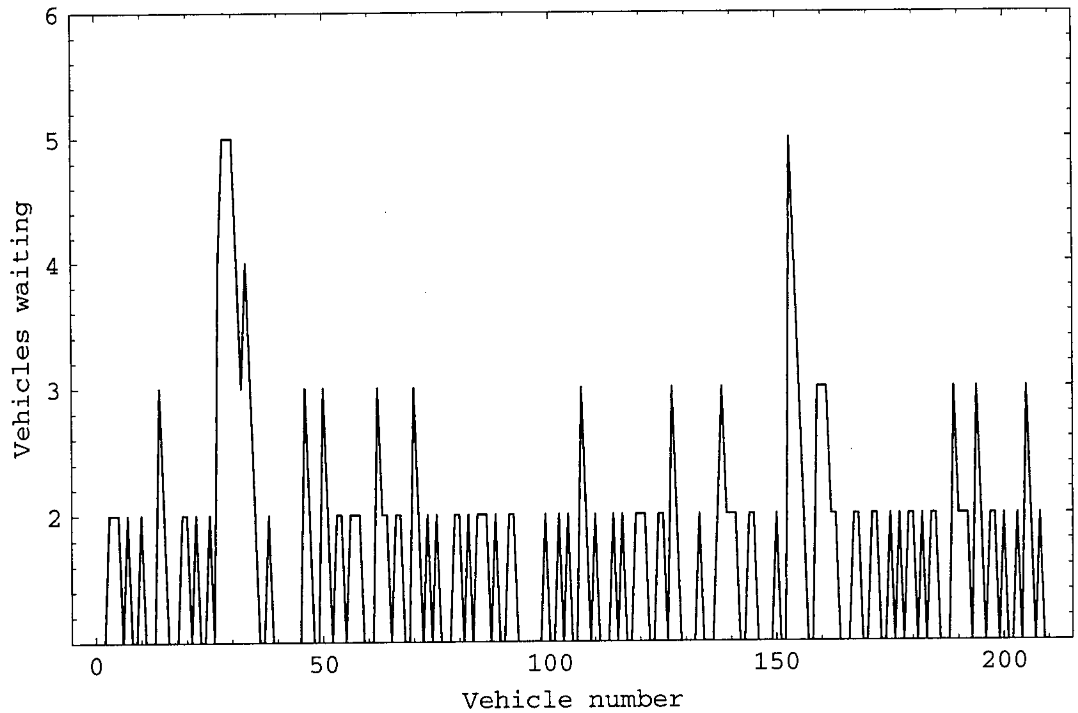
Maximum waiting time of vehicles in queue for this simulation run = 19.9 seconds

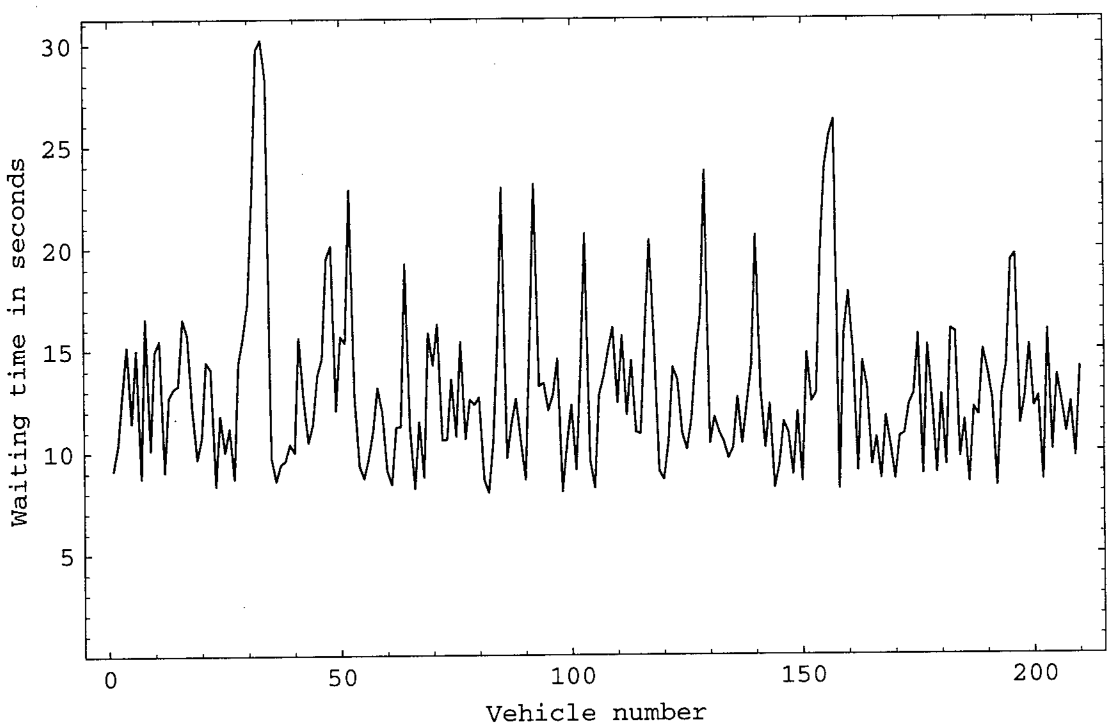
Critical headway, $h_{cr}$ = 8 seconds

Flow on major road, Q = 420 vph

Flow on minor road, q = 228 vph

Maximum number of vehicles in queue for this simulation run = 5




Minimum waiting time of vehicles in queue for this simulation run = 8.002 seconds

Maximum waiting time of vehicles in queue for this simulation run = 30.24 seconds

## ■ Sensitivity Analysis

# Mini-Project

## ■ Purpose

The purpose of this document is to assist you with the mini-project. The mini-project has the following education goals. You will learn:

1. How to write an engineering report.
2. How to design and conduct a field experiment.
3. The benefits of mathematical modeling in transportation planning and analysis.
4. How to write a computer program to perform a Monte Carlo simulation.
5. How to use statistical analysis for model calibration and testing.
6. Learn team work.

## ■ The Format for the Engineering Report

Introduction

Write a problem statement. The statement explains in brief terms what you plan to accomplish.

Nomenclature

A description of model variables.

Model Description

Describe the analysis method including governing equations

Network diagramand other sketches to show what you are

Monte Carlo Simulation

Description of the method including the governing equations.

Sample output.

Description of Field Test

Purpose of the test including description of the site.

Describe the variables that you measured and why they were chosen.

Describe sampling methods used and practical limitations.

*The method of choosing the critical headway time* $h_{cr}$*used by Garber and Hoel is determined as the intersection of the number of rejected and accepted headway gaps. It seems reasonable to use this approach for determining if their approach is reasonable.*

Model Testing

Describe the purpose of the model testing.

Describe the statistical test used.

Conclusion

Is the Monte Carlo simulation model useful?

If so, a description of model limitations and practical limitations for its use.

Appendix

Place field measurement data and any information that seems pertinent, Excel spreadsheet for example, are placed in the Appendix.

# Right-hand Turn Merge Model and Results

by

Paul J. Ossenbruggen

CiE 754 Transportation Planning and Analysis

April 1997

# Introduction

The purpose of this document is to give you guidance on preparing your report. I used the same format that I require you to use. It is my first draft; therefore, it may have typographical errors and my writing may not be clear. Any problems that you find, please inform me. Revision is a key element of good writing.

As important, the results that I generated with my model is my interpretation of the problem. It may be flawed. I welcome you to challenge my basic assumptions and results. As a professional, you will be faced with problems that do not have textbook solutions. I encourage you to bring up any problem or idea for classroom discussion. If you feel uncomfortable, talk to me privately.

The purpose of my right-hand turn merge model is to realistically simulate the conditions in the field. If this is possible, then the model can be used with confidence. It has practical advantages. For one, it is less expensive to run computer simualtions than to conduct field tests. The basic assumptions used in developing the model and assigning model parameters are critical.

# Nomenclature

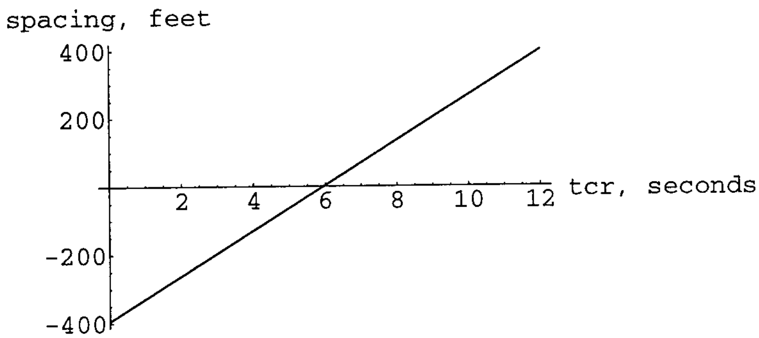
a = acceleration of the vehicle that merges from a minor road to a major road.
h = time headway of minor road.
H = time headway of major road.
hcr, $h_{cr}$= critical merge time for the driver.
n = number of observations.
q = flow in vph on minor road.
Q = flow in vph on major road.
u = average speed on major road.

# Merging Dynamics

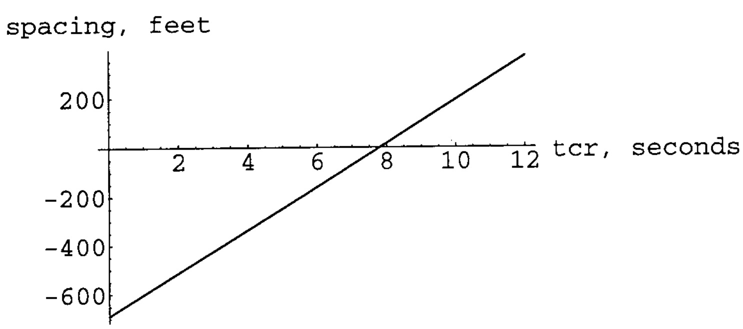
The textbook method of determining $h_{cr}$does not mention speed as an important factor. The purpose of the section is to show that speed on the major road is important. I assume that the first vehicle on the minor road will accelerate from rest to the average vehicle speed on the major road.

Assumptions:
1. Merging vehicle will begin to accelearte when x1 = 0 where x1 = location of first vehicle on major road.
2. Merging vehicle is R = 24 feet where R = radius of circular path that the merging car uses in its merge.

```
Average vehicle spacing at 45 mph = 792 feet

Time headway = 12 seconds at flow = 300 vph
```




```
Average vehicle spacing at 60 mph = 1056 feet

Time headway = 12 seconds at flow = 300 vph
```




The results show that average speed on the major road affects a driver's ability to merge. It seems that the speed of the major road is directly proportional to the critical merge time $h_{cr}$.

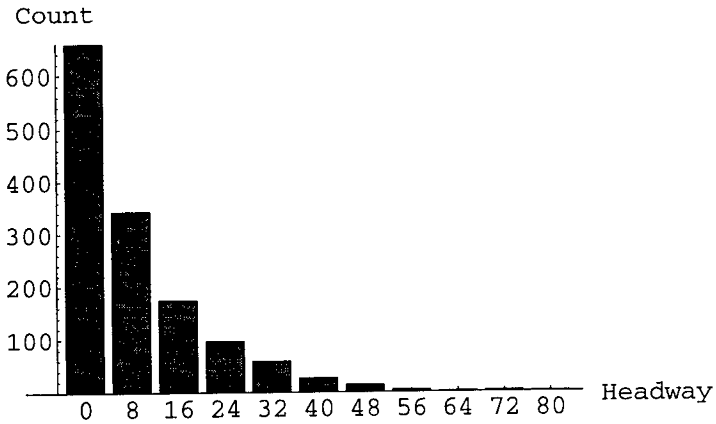
# Data Generated with Monte Carlo Simulation

The flow on the major and minor roads are assumed to have an exponential distributions.
Major road

```
Flow = 300 vph

Number of observations = 1374
```

$\bar{h}$ = 11.8 seconds

$s_h$ = 11.64



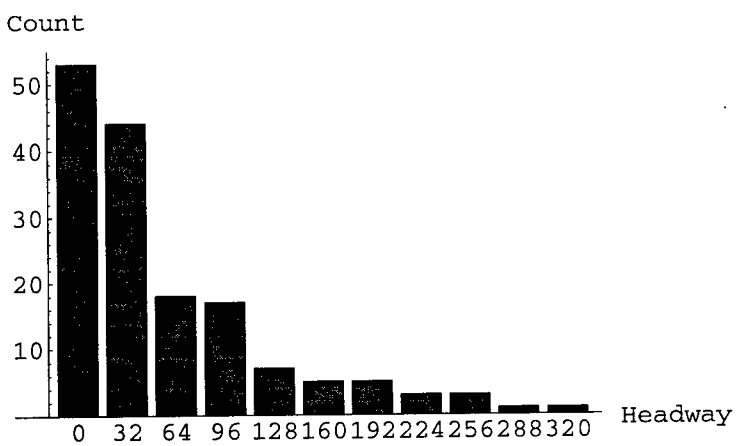

Minor Road

Flow = 50 vph

Number of observations = 157

$\bar{h}$ = 68.99 seconds

$s_h$ = 69.16




These data are used for all the following analyses. Exceptions are noted.

# Waiting Time of Lead Vehicle on a Minor Road

The purpose of this analysis is to show the effect of the assignment of $h_{cr}$ on merging.

Assumptions:

The queue is assumed to have an infinite number of vehicles in line.
If one vehicle merges, then the next vehicle takes its place and is ready to merge.

If H > $h_{cr}$, then one vehicle merges.
If H > 2 $h_{cr}$, then two vehicles merge,
and so on .

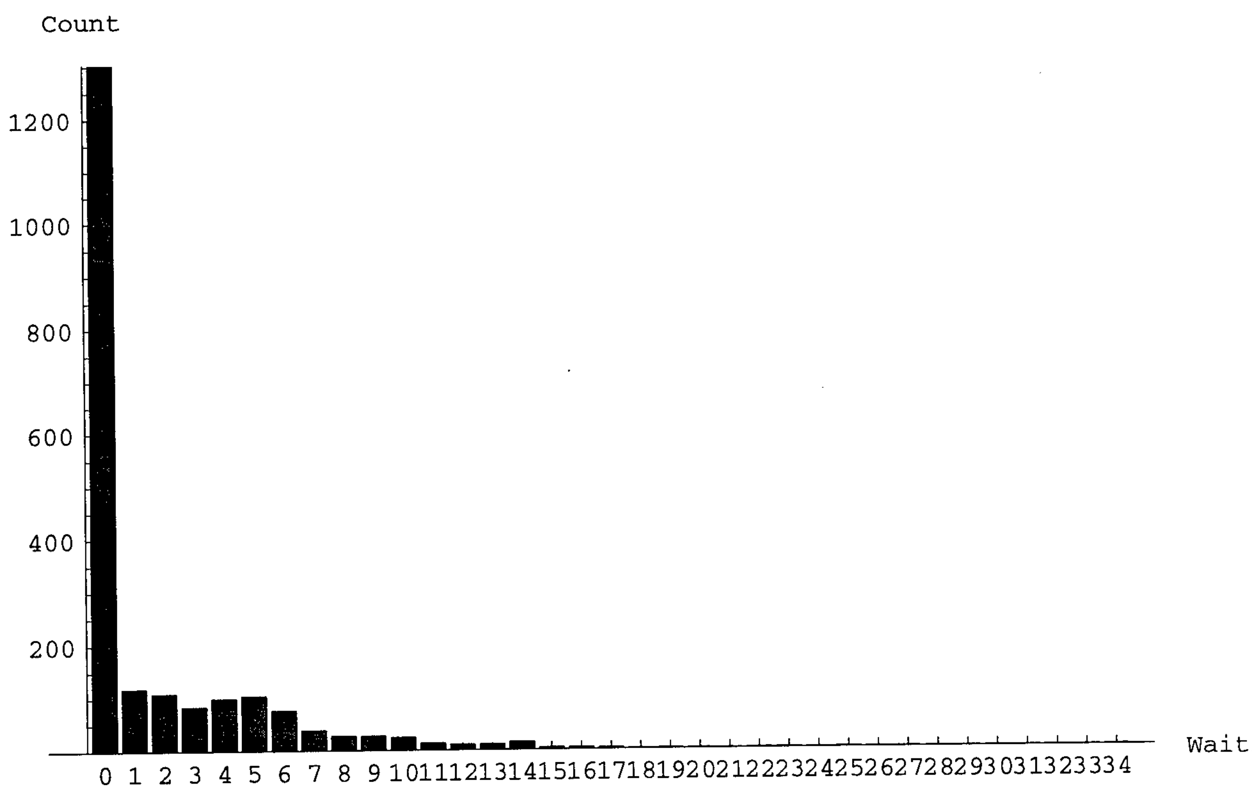
If H > $h_{cr}$, then H the second vehicle for the second vehicle is H - $h_{cr}$.

$h_{cr}$ = 6 seconds

Potential number of vehicles that can merge in an ∞ length queue = 2069

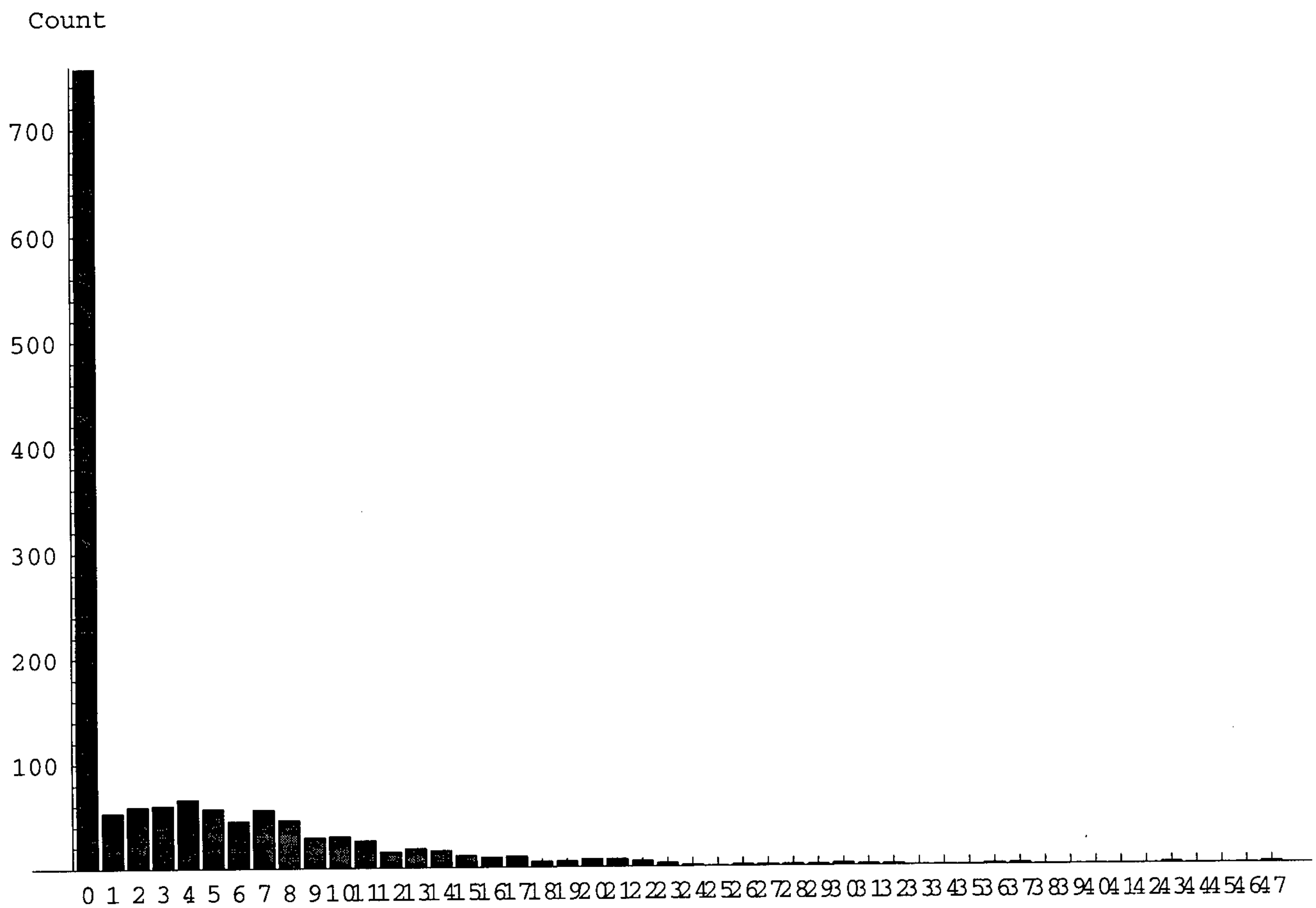
$\bar{w}$ = average wait time in queue = 1.839 seconds

$s_w$ = 3.311 seconds



$h_{cr}$ = 8 seconds

```
Potential number of vehicles that can merge in an ∞ length queue = 1407

w̄ = average wait time in queue = 3.529 seconds

sw = 5.656 seconds
```




The histograms show that a great percentage of the lead vehicles merge into the major road stream without waiting. The calculation of the average wait times includes the drivers who merge immediately and have zero wait times.

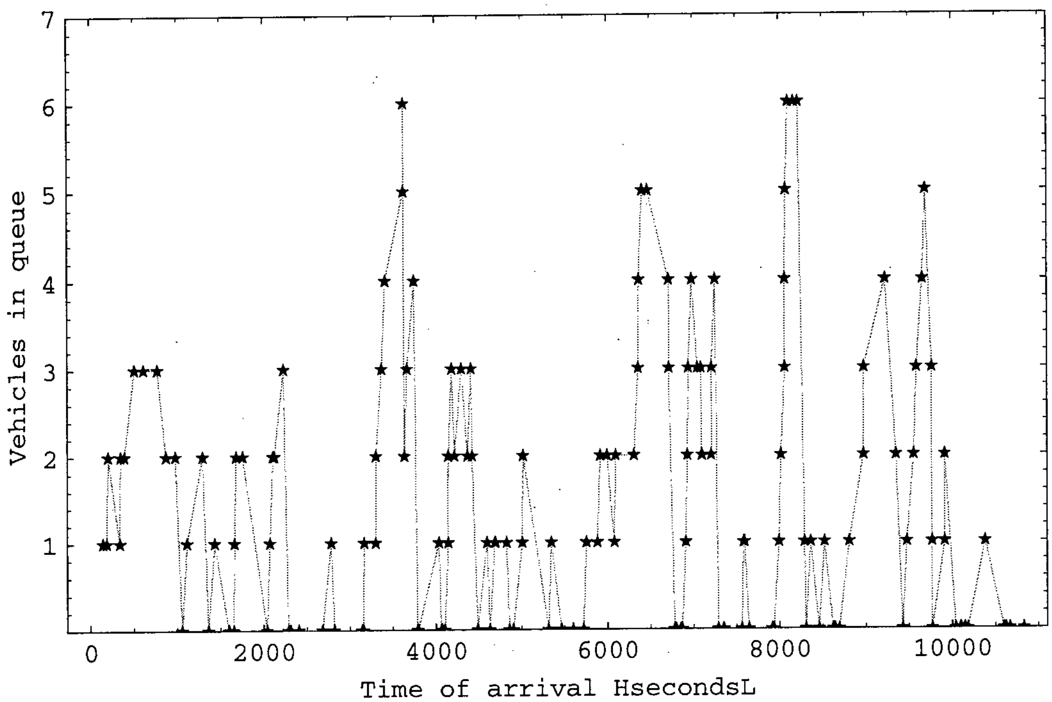
# Queue Length

The purpose of this analysis is determine the effect of $h_{cr}$ on merging. The queue length is analyzed.

Critical headway, $h_{cr}$ = 6

Flow on major road, Q = 300 vph

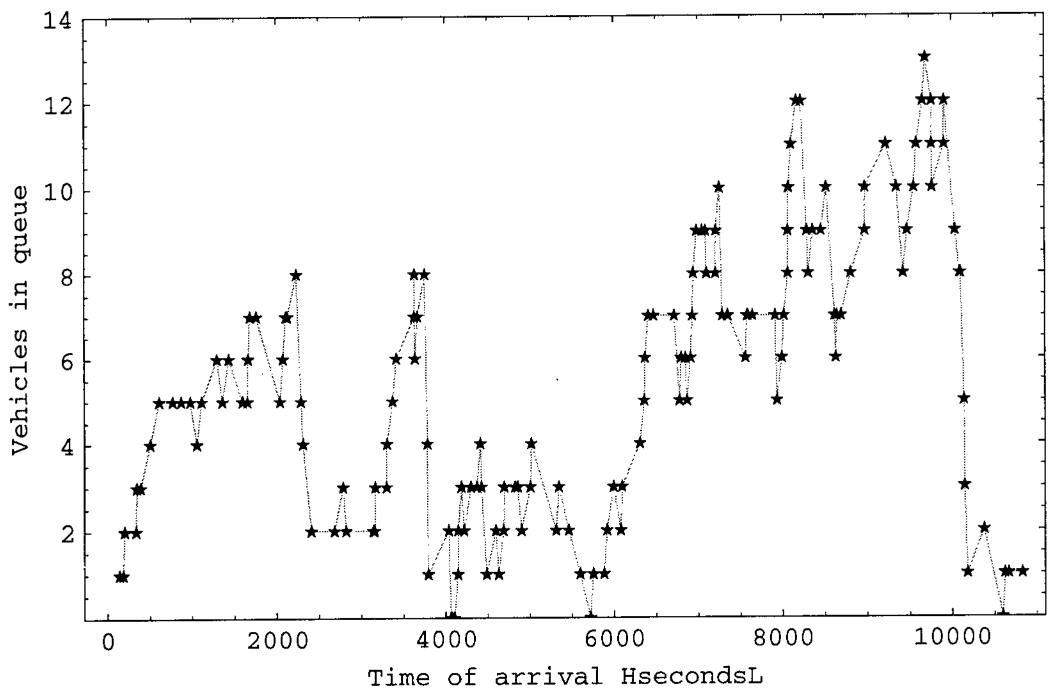
Flow on minor road, q = 50 vph




Critical headway, $h_{cr}$ = 8

Flow on major road, Q = 300 vph

Flow on minor road, q = 50 vph



Vehicles in queue
14
12
10
8
6
4
2
0
2000
4000
6000
8000
10000
Time of arrival HsecondsL

# Sensitivity Analysis

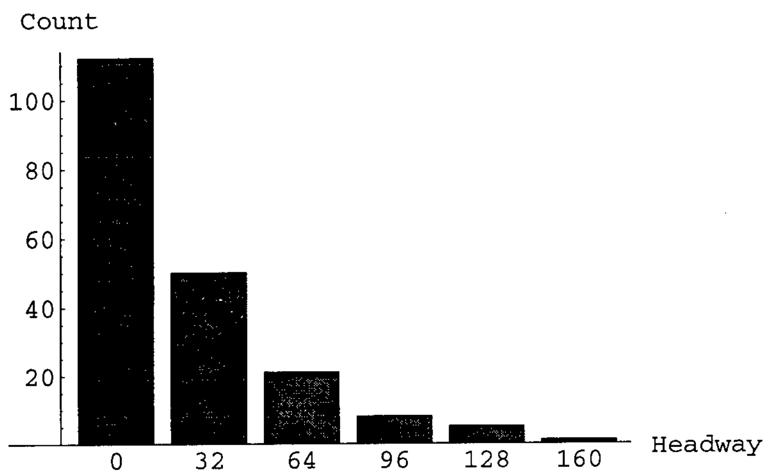
The effect of traffic flow on the minor road is investigated by doubling its flow from 50 vph to 100 vph. The same data generated with the Monte Carlo simulation for the major road with flow of 300 vph is used in the following analysis.

Flow = 100 vph

Number of observations = 197

$\bar{h}$ = 37.08 seconds

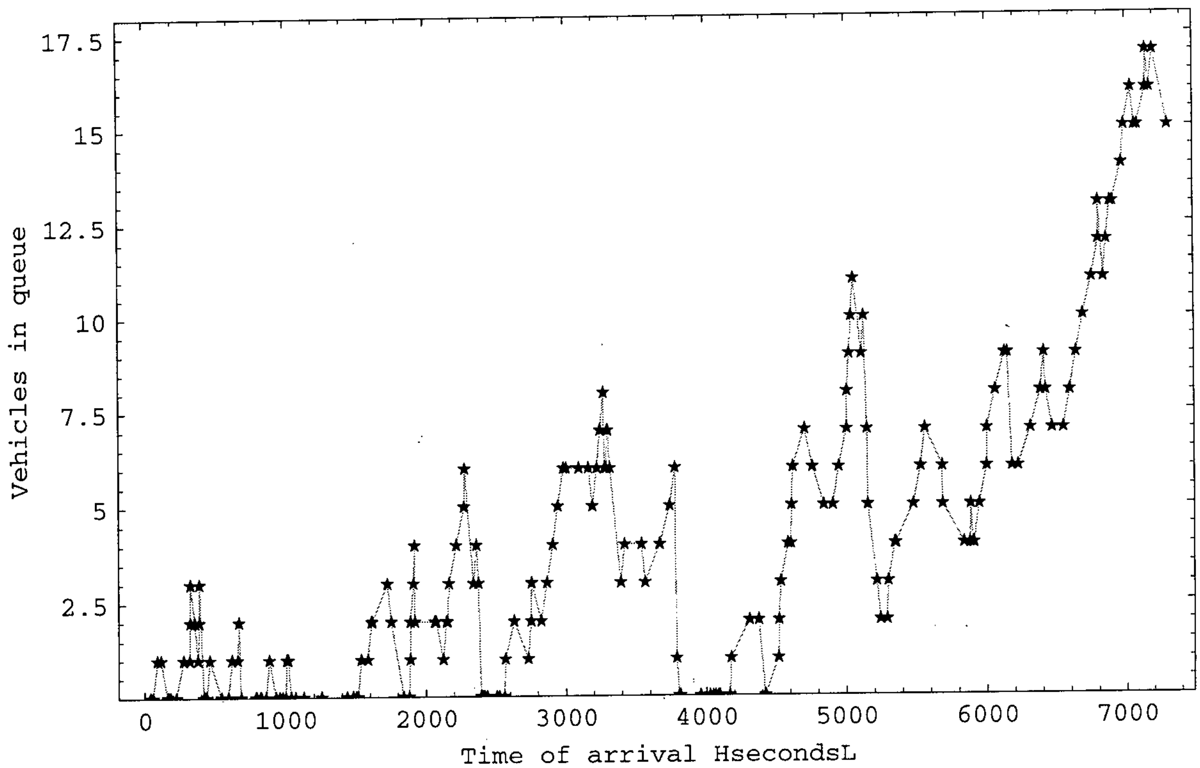
$s_h$ = 33.75




Critical headway, $h_{cr}$ = 6

Flow on major road, Q = 300 vph

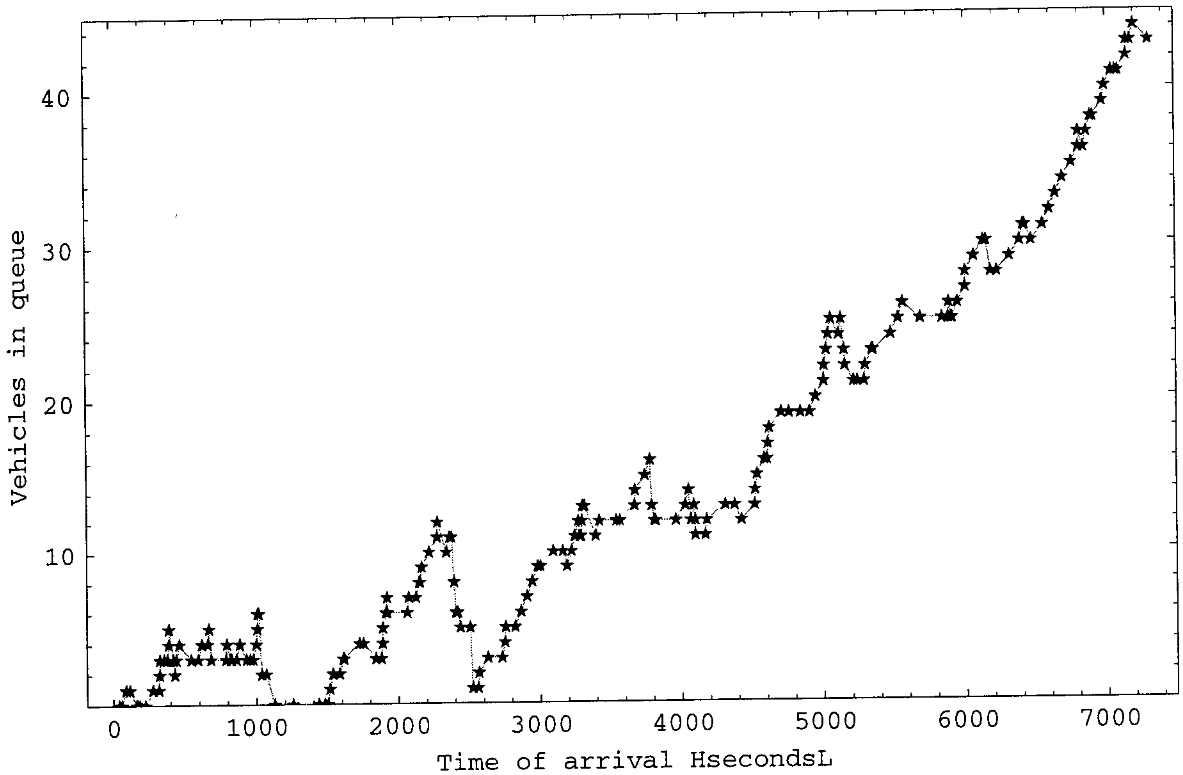
Flow on minor road, q = 100 vph


17.5
15
12.5
10
7.5
5
2.5
Vehicles in queue
0
1000
2000
3000
4000
5000
6000
7000
Time of arrival HsecondsL

Critical headway, $h_{cr}$ = 8

Flow on major road, Q = 300 vph

Flow on minor road, q = 100 vph




The assignments of $h_{cr}$ and q, the flow of the minor road, are important. At q = 100 vph, the queue lengths on the minor road, approach infinity.

# Conclusions

The important model parameters include q, Q and $h_{cr}$.

# Appendix

The following are Mathematica functions that I used to generate and plot data.

```
Clear[t, average, stdev, Histogram, HistogramWait]

t[q_, duration_] :=
 Block[{j, s, t, sum, tlist},
  s = 0;
  tlist = {};
  sum = {};
  For[j = 1, s < duration, j++,
```

$$t = \frac{3600 \text{ Log}\left[\frac{1}{1-\text{Random}[\,]}\right]}{q};$$

```
   AppendTo[tlist, t];
   AppendTo[sum, s = s + t]];
  Return[{tlist, sum}]]
```

$$\text{average}[x\_List] := \frac{\sum_{j=1}^{\text{Length}[x]} x[[j]]}{\text{Length}[x]}$$

$$\text{stdev}[x\_List, \text{xbar}\_] := \sqrt{\sum_{j=1}^{\text{Length}[x]} \frac{(x[[j]] - \text{xbar})^2}{\text{Length}[x] - 1}}$$

```
Clear[Merge]
Merge[Q_, u_, a_, tcr_, R_] := Block[{l = 20, tm, tt, xm, x1, x2, s, s1, s2},
```

$$\text{tm} = \frac{1.47\,u}{a};$$

$$\text{tt} = \sqrt{\frac{N[\frac{\pi}{2}]\,R}{a}};$$

$$\text{xm} = a\,\text{tt} + 0.5\,a\,(\text{tm} - \text{tt})^2 + N\left[\frac{\pi}{2}\right] R;$$

```
  x1 = N[1.47 u tm];
```

$$s = \frac{5280\,u}{Q};$$

```
  x2 = -1.47 u tcr + 1.47 u tm;
  s1 = x1 - xm;
  s2 = xm - x2;

  Return[{tcr, s2}]]
Clear[Histogram]

Histogram[q_, x_List, dx_] :=
 Block[{freq, midpts, m, xbar},

  freq = BinCounts[x, {-0.5, Max[x], dx}];
  m = Length[freq];
  midpts = Table[dx (j - 1), {j, 1, m}];
  xbar = average[x];
```

```
  Print[StringForm["Flow = `` vph", q]];
  Print[StringForm["Number of observations = ``", Length[x]]];
  Print[StringForm["h̄ = `` seconds", NumberForm[xbar, 4]]];
  Print[StringForm["sh = ``", NumberForm[stdev[x, xbar], 4]]];
  BarChart[Transpose[{freq, midpts}],
    AxesLabel -> {"Headway", "Count"}, PlotRange → {0, Max[freq] + 2}]
 ]

Clear[PrepareQueue, Queue]
PrepareQueue[H_List, hcr_] :=
       Block[{j, wlist, num},
         wlist = {};
         wait = 0.0;
         Do[If[H[[j]] < hcr,
            AppendTo[wlist, H[[j]]],
            num = Floor[H[[j]]/hcr];
         AppendTo[wlist, Flatten[{Table[0.0, {i, 1, num}], H[[j]] - num hcr}]]],
   {j, 1, Length[H]}]; Return[Flatten[wlist]]]

Queue[H_, hcr_] :=
        Block[{j, wait, wlist, newlist, drop},

         newlist = PrepareQueue[H, hcr];
         newlist = AppendTo[newlist, 0.0];
         wlist = {};
         wait = 0.0;
         Do[If[newlist[[j]] ≤ hcr && newlist[[j]] ≠ 0.0,
           wait = wait + newlist[[j]],
           AppendTo[wlist, wait];
         wait = 0.0], {j, 1, Length[newlist]}];
         drop = Position[wlist, 0];
         wlist = Delete[wlist, drop];
         Return[wlist]]


Clear[HistogramWait]

HistogramWait[w_List, dw_] :=
 Block[{freq, midpts, diag, m, xbar},

  freq = BinCounts[w, {-0.5, Max[w], dw}];
  m = Length[freq];
  midpts = Table[dw (j - 1), {j, 1, m}];
  xbar = average[w];
  Print[StringForm["Potential number of
      vehicles that can merge in an ∞ length queue = ``", Length[w]]];
  Print[
   StringForm["w̄ = average wait time in queue = `` seconds", NumberForm[xbar, 4]]];
  Print[StringForm["sw = `` seconds", NumberForm[stdev[w, xbar], 4]]];
  BarChart[Transpose[{freq, midpts}],
    AxesLabel -> {"Waiting Time", "Count"}, PlotRange → {0, Max[freq] + 2}]
 ]

Clear[QueueLength]
```

```
QueueLength[sh_List, sH_List, hcr_] :=
 Block[{leave, num = 0},
  numlist = {};
  Do[
  For[k = 1, sh[[i]] >= sH[[k]], k++, Null];
  leave = Floor[(sH[[k]] - sh[[i]])/hcr];
   If[(num + 1 - leave) > 0, num = num + 1 - leave, num = 0];
  AppendTo[numlist, num],
   {i, 1, Length[sh]}
  ];
  Return[numlist]
  ]

Clear[QueuePlot]

QueuePlot[time_List, cnt_List, hcr_, Q_, q_] :=
 Block[{list1, list2},
  Print[StringForm["Critical headway, hcr = ``", hcr]];
  Print[StringForm["Flow on major road, Q = `` vph", Q]];
  Print[StringForm["Flow on minor road, q = `` vph", q]];
  list1 = list2 = Transpose[{time, cnt}];
  MultipleListPlot[list1, list2,
   PlotRange -> {0, Max[cnt] + 1},
   PlotJoined -> {False, True},
   SymbolShape -> {PlotSymbol[Star]},
   PlotStyle -> {GrayLevel[0.5]},
   Frame -> True,
   FrameLabel -> {"Time of arrival (seconds)", "Vehicles in queue"}]
  ]
```