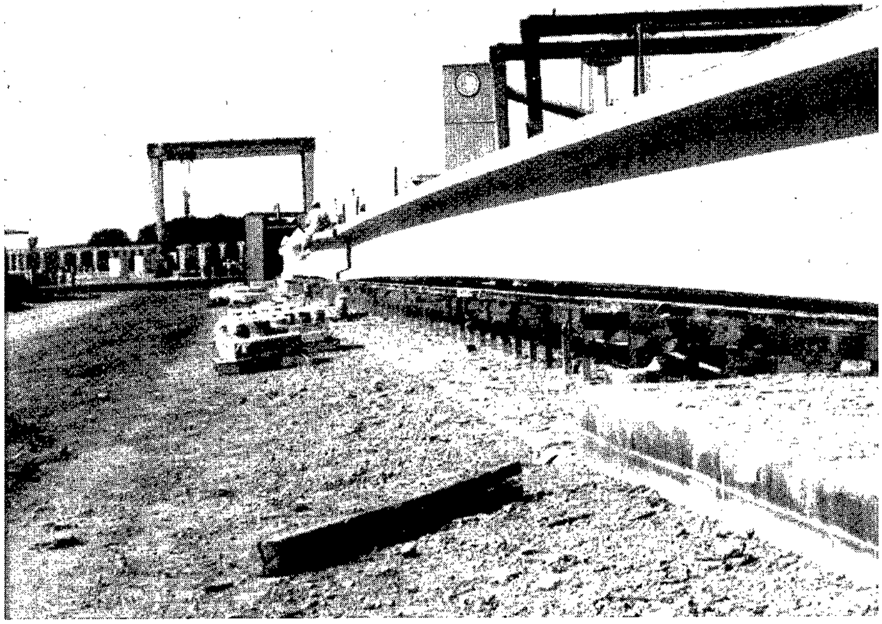
Final Report 1998-08

PB98-154594



A Camber Study of Mn/DOT Prestressed Concrete I-Girders

UNIVERSITY OF MINNESOTA
CENTER FOR
TRANSPORTATION
STUDIES

mpa minnesota prestress association

This research was made possible with the support and contributions from the Minnesota Prestress Association. Many of its members took a role in the fabrication and testing of the girder specimens.

Technical Report Documentation Page

1. Report No.
1998-08

2.

3. Re
PB98-154594

4. Title and Subtitle
A CAMBER STUDY OF Mn/DOT PRESTRESSED CONCRETE I-GIRDERS

5. Report Date
January 1998

6.

7. Author(s)
Douglass Woolf
Catherine French

8. Performing Organization Report No.

9. Performing Organization Name and Address
Department of Civil Engineering
University of Minnesota
500 Pillsbury Dr. S.E.
Minneapolis, MN 55455-0220

10. Project/Task/Work Unit No.

11. Contract (C) or Grant (G) No.
(C) 69098 TOC # 83

12. Sponsoring Organization Name and Address
Minnesota Department of Transportation
395 John Ireland Boulevard Mail Stop 330
St. Paul, Minnesota 55155

13. Type of Report and Period Covered
Final Report 1993-1997

14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract (Limit: 200 words)

This project investigated the relationship between predicted and measured girder cambers. For more than three years, researchers collected camber data on girders of various depths and lengths from the time of strand release until shipment to the job site.

Researchers used three camber prediction methods to compare with the measured values: PCI method, Branson's time-step approach, and "CRACK" analysis program by Ghali et. al. The Branson time-step approach resulted in the closest predictions to the measured cambers. The PCI method, although simple, gave reasonable long-term camber results compared with the more detailed methods.

17. Document Analysis/Descriptors

prestressed bridge girders
field measurements
camber
long-term effects

18. Availability Statement
No restrictions. Document available from:
National Technical Information Services,
Springfield, Virginia 22161

19. Security Class (this report)
Unclassified

20. Security Class (this page)
Unclassified

21. No. of Pages
130

22. Price

A CAMBER STUDY OF Mn/DOT PRESTRESSED CONCRETE I-GIRDERS

Final Report

Prepared by

Douglass Woolf
Catherine E. French

Department of Civil Engineering
University of Minnesota

January 1998

Published by

Minnesota Department of Transportation
Office of Research Administration
200 Ford Building Mail Stop 330
117 University Avenue
St. Paul, Minnesota 55117

ACKNOWLEDGMENTS

This project was sponsored by the Minnesota Department of Transportation. The authors also express appreciation to Elk River Concrete for their cooperation and assistance throughout the project. The views expressed herein are those of the authors and do not necessarily reflect the views of the sponsors.

Table of Contents

LIST OF TABLES

LIST OF FIGURES

EXECUTIVE SUMMARY

The objective of this project was to investigate the relationship between predicted and measured girder cambers. For over three years, camber data was collected on girders of various depths (45 to 81 in.) and lengths (span-to-depth ratios from 17.3 to 24.8). The data was collected from the time of release until shipment to the job site.

Measured cambers were observed to differ even for similar girders cast together on the same precasting bed. Differences ranged up to 0.5 in. (2 to 10%). In addition, solar radiation had an effect on the results. Over the course of a day, camber measured for a single girder had a variance of 0.5 in.

An analytical parametric study was conducted to evaluate the factors affecting short-term camber. The parameters investigated included: modulus of elasticity; moment of inertia (gross or transformed); and variations in concrete density, initial strand stress, member length, and harp point locations. Two of the factors, which had the largest influence on camber, were variations in the prestress force and section stiffness.

Three camber prediction methods were used to compare with the measured values. The prediction methods were: PCI method, Branson's time-step approach, and "CRACK" analysis program by Ghali et al. The Branson time-step approach resulted in the closest predictions to the measured cambers. The PCI Method, although simple, gave reasonable long-term camber results compared with the more detailed methods.

Chapter One - Introduction

Prestressing is the introduction of a force into a structural member to counteract tension stresses in the concrete. When concrete is cast around a tensioned strand, it is termed "Pretensioning." When these strands are placed eccentric to the center of gravity of the member, they produce moments in the section used to counteract the effects of gravity loads and increase the capacity of the section. As is well-known, concrete is about one tenth as strong in tension as it is in compression. By using prestressing forces in the section, more of the concrete compressive capacity can be utilized.

Not only does more of the concrete strength become utilized, but the deflection of the member is decreased. Deflections in the member are related to the moments. If a moment counteracts the gravity moments, then the deflection it produces will also counterbalance the deflection produced by the gravity deflections (Figure 1.1 a-d, [1]). This prestress deflection is often larger than the deflection produced by the self weight of the member alone. As a result of this, an upward deflection is produced, otherwise known as camber.

Camber for the section can be calculated fairly well at early ages after release, as will be shown later in the paper; however, there are several time-dependent factors that are not as easily determined for long-term camber estimations. Variations in the concrete material properties, such as strength and stiffness, from the actual to the design properties add to a designer's uncertainty in calculating long term deflections. The material differences will be discussed in the chapter on short-term analysis. The time-dependent factor variations will be discussed in the long-term analysis chapter.

As will be discussed in the following chapter, data was collected on actual Minnesota Department of Transportation (Mn/DOT) girders to be placed in various bridges. The data was collected over a three year period on precast "I" shaped girder sections. The girders were all constructed at Elk River Concrete Products in Elk River, MN. All of the data was collected at the plant while they were stored until shipment.

All of the members that were observed in the field were pretensioned. Pretensioned members are usually prefabricated in a plant and then brought to the job site. The process involves placing strands at their proper locations in a steel frame/form and then stressing them to the desired force using a hydraulic jack. Typically, more than one member was cast on a bed. Thus, strands for these members should have about the same stresses when released. At Elk River Concrete, the steel stirrups and regular reinforcement are tied into place around the stressed strands. From this step, the remainder of the formwork is put into place and the concrete is poured. After 18 hours of curing, the forms are removed and the strands are cut in a systematic fashion. The same strands are cut simultaneously from the ends of each girder on the bed. One exception to the curing period is that if the member is poured over a weekend or holiday, it may cure over a longer period of time (approximately 3 days).

The curing process involves covering the girders with a tarp and using steam to modify the temperature, if necessary. In the summer, the thermal heat of hydration is usually adequate to produce the desired curing temperature under the tarp without the addition of steam. The use of Type III cement and high range water reducer admixtures enable desired release strengths to be achieved at early ages, allowing for rapid reuse of the formwork. A high concrete slump is important for placement of the mix in the concrete formwork. By using a water reducer admixture, a higher initial slump is achieved for the same strength concrete. Thus, a higher strength concrete could be used by the plant to achieve an earlier release strength with the same slump as a concrete mix with less strength and no admixture. The 28-day strength of this concrete mix will generally be a few thousand psi above that called for in the design in order to ensure meeting the Mn/DOT requirements for testing cylinders at release.

After reaching the release strength, the strands are cut and the loads that were held by the frame are transferred to the concrete member. The net result is to induce a net compressive force and a moment into the member (the moment may produce tension in certain areas of the girder).

As mentioned earlier, these moments can be related to deflections and the net result usually produces an upward camber at release from the pretensioning bed.

Several terms should be defined in order to help in understanding the determination of deflections for prestressed girder members. Short-term deflections are deflections that occur immediately after the strands have been cut. Determination of the stress in the strand immediately after release of the member needs to be considered in order to determine this deflection. In order to determine this strand stress, elastic shortening losses need to be considered. Elastic shortening prestress loss is the loss of prestress due to immediate shortening of the member length due to concrete strains induced by the prestress force to the concrete.

There are several factors that need to be considered in determining the long-term deflection for the member. Long-term deflections are deflections that occur anytime after the short-term deflection through the life of the structure. Creep effects are one of the largest contributors to time dependent changes in the section. Creep is the time dependent strain of the concrete under sustained compressive loading. Creep has a two part effect on the section. First, it decreases the prestress force in the strands. This tends to reduce the camber or increase the downward deflection in the member. The second effect of creep is to increase the member curvature due to non-uniform strains in the concrete. (Greater creep strains occur in regions of greater compressive stresses). If the net result of all loads is to produce camber, then this portion of the creep effect will increase the camber in the section. How much the creep factor dominates over the other factors depends on how much these two separate effects of creep cancel or add to one another [9]. In general, most of the methods used for determining concrete creep are based on a time function multiplied by an ultimate creep strain.

Shrinkage of the concrete is also an important time dependent factor that changes the camber of the section. Shrinkage is the change in concrete volume due to loss of water in the section. When this shortening of the member occurs, prestressing force is lost due to a slackening of the strand. There are several methods available for determining the time dependent shrinkage of the section. Most of these require knowledge of the volume to surface ratio of the cross section as well as the relative humidity to which the member is subjected.

Strand relaxation is the last important time dependent factor to be considered. Relaxation is the reduction in stress at constant strain. This relaxation usually becomes significant at strand stresses greater than 50 % of the ultimate stress capacity (ACI 209R-92). Most strands are stressed to around 75 % of the ultimate stress capacity at the plant where the field data was collected. Several strands are produced with different grades of steel and relaxation properties. The plant where the field members were produced uses mostly half-inch diameter 270 ksi low-relaxation (stabilized) strands. Relaxation for these strands is not as large as for the case of stress- relieved strands.

It should be realized that all of the factors, to some degree, are dependent upon one another. When the concrete shrinks, the member shortens. When the member shortens, the stresses in the strands reduce. The reduced stress causes less concrete stress and results in less creep in the member. With reduced strain, relaxation of the steel is reduced. Many of the time dependent methods try to account for factor interdependency in some simplified fashion. The PCI Design Handbook adjusts only the relaxation factor to account for interdependent losses due to creep, shrinkage, and relaxation. The CEB-FIP model code adjusts the creep factor to account for these interdependent losses.

Purpose

The main objective of this project was to investigate the relationship between calculated and observed girder camber. If the camber is not as predicted, then there is a possibility that service deflection limits, such as those of the AASHTO specification, may not be met. [1, page 425] In addition, variations in member camber necessitate the use of a thickened haunch over certain girders in order to maintain a level driving surface on the bridge.

Because of these reasons, this project was conducted at the University of Minnesota to study camber in girder members. For over three years, data was collected on various length and depth girders. The members ranged in depth from 45 - 81 inches deep, with a maximum span length of about 139 feet. The span to depth ratio ranged from 17.3 - 24.8 for all of the members that were studied. Data was collected from the time of release of these members until shipment to the job site. The amount of storage time varied considerably for these members depending on the construction schedule.

The listing of bridge numbers that were studied and general information are included in Table 2.1 of Chapter 2. Mn/DOT classifies each of its bridge projects with a job number. It then assigns a number for each type of girder with the same design, then a number for each member. The number organization is printed onto the side of each girder produced. The last set of girders investigated has the most complete information available and is mentioned frequently throughout the report as Bridge 19045 girder members.

In order to begin the analysis of the methods used in the prediction of short and long-term camber, a parametric study for short-term effects was conducted. This study served the purpose of finding those factors that affected the short term camber the most. The spreadsheet given in Appendix A, related to short-term deflection calculations, has additional information on the material properties of particular members used in the parametric study. The sensitivity analysis laid a foundation for evaluating the short term camber calculations. It was also reasonable to use this study for evaluating the material properties used in the long-term analysis. The parametric study considered factors affecting the camber such as methods for determining the modulus of elasticity and the moment of inertia of the section. The study also investigated variations in concrete density, strand initial stress, length of member, and draping locations in order to evaluate differences between observed and assumed member properties.

After performing the parameteric study for the short term analysis, the actual data was compared to various methods for determining camber. For the determination of short-term effects, the PCI method was employed (see Chapter 3 of report, [4]). For the determination of long-term effects, the PCI multipliers (Chapter 4, Table 4.6.2, [4]), Prof. Branson's approximate time step analysis (Chapter 7, [1]), and the time step analysis program "CRACK" developed by Professors Ghali and Elbadry [12] were used. The observations and methods are discussed in the following chapters.

Limitations with Respect to the Data

Support Conditions

Data related to the support conditions of the girders could have been used, but was missing for several of the girders. When stored in the field until transport, the members are placed on pre-established supports. As such, the bearing pad does not necessarily correlate with the storage support conditions. In most cases for the Bridge 19045 beams, the supports were located at a range of 3 to 5 feet from each of the ends. This meant that not only was the span distance decreased by 6 - 10 feet, but end rotations were also being produced due to the overhanging dead weight. All of these changes in the location of forces increased the short-term camber in the girders by 0.25 - 0.5 inches. For long term camber in Bridge 19045, the ratio between end and actual support conditions deflection had an average value of 0.86 for various conditions that will be discussed later (see Table 4.2). None of the other girders of varying depth, length, and strength had information available regarding the temporary support locations. Therefore, support conditions for the other bridges could not be accounted for in calculations.

Concrete Compressive Strength

Another problem encountered with the field data related to the material testing data. Compressive strengths for the concrete used in these members were measured periodically. The first set of strength data was collected right before the girders were released from the forms and the strands cut. The second set of strength data was taken at some point in time between 14 and 30 days after the release of the member. Thus it became difficult to determine the 28-day strength of the member for use in the determination of the elastic modulus of the concrete. To supplement this data, the use of conventional formulas for estimating concrete strength as outlined in the ACI Committee 209 report - "Prediction of Creep, Shrinkage, and Temperature Effects in Concrete Structures " [2] were considered. The formulas were modified based on information from members that were cast at about that same time with more complete data - including the 28-day strength. It will also be shown in the sensitivity analysis (Appendix A) that as long as the concrete strength was within 500-1000 psi of the actual strength, the member camber was not significantly affected.

Prestress Force

The prestress force that was actually applied to the member was not normally recorded by the plant. The design calls for a stress after seating of 75 % of the ultimate capacity of the strand. For the half inch diameter, low-lax, 270 ksi strength strands that are normally used by this plant, a value of 31 kips/strand (corresponding to $0.75*f_{pu}$) is used in the design determination. When measured in the field, the hydraulic jack has a pressure gauge on it that measures the jacking stress put into the strands. However, this gage can give unreliable readings with respect to the stress actually put into the strands, as compared to the hydraulic pressure developed in the jack. In addition to the gage, the length of elongation of the strands is measured to determine the force induced into the member based on the stress-strain relationship for the strand. Draped strands are elongated to a lesser degree and then pulled into position in order to try to alleviate friction losses due to the draping anchors. Because of difficulties in getting the strain data, the calculations for the analysis were based on the design values of strand stress. This may be in error according to the Mn/DOT inspector for the plant by 2-3% over the design stress even after seating has occurred. This results because the plant tends to induce more stress rather then risk not having enough stress. Although most of the data compares well to that of the calculated results, some of the error in the actual versus calculated camber may be a result of this. Trends in underestimation of initial camber in Bridge 19045 may be a result of this inaccuracy, as can be seen in Appendix A.

High Range Water Reducer Admixture

As mentioned earlier, because of the high range water reducer admixture that was applied to the concrete mix, the determination of the calculated time dependent data was complicated. For example, the creep and shrinkage time distribution formulas in the ACI 209R-92 report mentioned earlier, are based on averaged strength and strength gain properties. Although it is most likely a minor effect, the high range water reducer may play an unknown effect on the measured camber for the section. The members were modeled as best as possible to the code specifications of the method used for the analysis. Admixtures in the concrete mix to help early strength gain, solvency of the mix, etc. are not necessarily known at the time of the design.

All of these limitations to the data are of concern for the project. Some of these issues are more important than others as will be shown in the sensitivity analysis. In the short-term parametric study, the variations with respect to the assumed design values were considered. Variations for the parametric study were chosen in order to best reflect possible explanations for camber differences in the members. In general, a great deal of what is done in the concrete design process with concern to these limitations is based on approximations beyond the control of the design engineer.

Previous Research

Several past research projects regarding camber effects were reviewed. Much of the past research has focused on the development of methods and programs to analyze camber in girder members for long term effects. The following is a brief summary of that research.

In the last several years much research has been done at the University of Minnesota by Mokhtarzadeh et al. with regards to the mechanical properties of high strength concrete. Although the members considered in this study were not designed as high strength members, the actual strengths were considered to be rather high (6800-10,000+ psi) . The designs were at least at the upper end of the normal strength concrete range. As such, much of the information regarding characterizing the strengths of materials was based on this research.

In 1987, Kelly et al. at the University of Texas [6] conducted tests on eight instrumented bridge girders. The beams were 127 feet long, high strength with both low-relaxation and normal stress-relieved strand members, and fully instrumented. Temperature, strain, and deformation were measured over time for these girders. Data was collected on these girders over a 1000 day period with the girders being placed in service for a portion of this time. Tests were conducted to determine both short and long-term material parameters for the Texas region. The collected data was then compared to various methods for determining camber in girder members. The PCI Design Handbook, AASHTO Design method, ACI Committee 209 Recommendations, "PBeam" developed by Suttikan et al., and "Camber" developed by Kelly et al. were used in comparing calculated to measured data. The methods were used for determining losses, short-term deflection, and long-term deflection for the girder members.

The work done in this report was highly detailed and measured not only the deflection, but also all of the material properties including some of the time-dependent properties. Method comparisons were made not only for deflection, but also for prediction of individual prestress loss parameters in the section.

Based on the results, the computer program "Camber" was made taking into account the conclusions of the research. Suggested modifications to different material formulas (modulus of elasticity, creep factors, shrinkage, concrete strength properties, etc.) were implemented in the program. Additionally, they formulated equations, based on their research, for the determination of camber. This procedure uses the initial deflections to determine long-term deflections by using multipliers.

This program was considered for use in analysis for this research, but it was decided that the program and the research were too specific to the Texas area. The research was based on using limestone aggregate in the members, and a climate which is much different than

Minnesota. As such, the research that they performed resulted in changes to the formulas that were not applicable to the Minnesota design spectrum. For example, the Texas research team came to the conclusion that the ACI 363 high strength equation used for calculating the modulus of elasticity of concrete with limestone aggregate should be modified to:

$$E_c = 40000 * (fc')^{1/2} + 1500000 \qquad \text{(From Ref. 6)}$$

The ACI 363 high strength concrete equation as written is:

$$E_c = 40000 * (fc')^{1/2} + 1000000 \qquad \text{(Eq. 1.1, [8])}$$

Kelly et al. had determined this to be a more realistic equation because they found that the ACI underestimated the modulus of elasticity for crushed limestone aggregate. However, Mokhtarzadeh et al. of the University of Minnesota found that equation 1.1 gave higher results for the limestone cylinders they tested. The girders monitored in the present investigation were fabricated with glacial river rock. Because of these differences, it was decided not to use the program "Camber" in the analysis for this project, but many of the generalized conclusions were used.

Branson et al. [7] at the University of Iowa have also done extensive work in the area of prestress loss and deflection for prestressed concrete members. Their study focused on differences between the use of normal weight and light weight materials with respect to time dependent factors. The study also looked at the determination of deflection based on the revised design equations for determining losses and time dependent factors that Branson proposed. This study was used for background information and to some extent was included in the report.

A few years after Branson published this report he proposed a simplified procedure for determining camber that was based on the initial deflection. This method was based on empirical data as well as some assumptions made about the curvature relationship. Prestress losses and concrete creep were calculated at various increments of time and were then used in determining the camber at that time. This method was used in this report and is displayed as the approximate time step procedure.

Ghali et al. have also done a great deal of research in the area of prestressed concrete members. Their research was used in this report along with the computer program “CRACK” that they developed. The program was generalized such that it could be used in the United States even though it was developed in Calgary, Canada with the European model code in mind. The program was recently updated in 1990 and the text [3] based on Ghali’s original research was published in 1986. The program and his research were also included in this report. Further details as to the nature of the work will be referenced through the remainder of this report, especially in Chapters 4 and 5.

Chapter Two – Field Data

Since 1992, data for this project has been collected for the correlation of calculated vs. measured camber. Most of the data has been collected throughout the fall, winter, and into the spring months of a Minnesota season. Information on the field data is provided in Table 2.1. Bridge 49535 and 27624 beams had data collected from the summer and fall months only. As such, the environmental conditions will have some influence on the final camber values due to the effect of such things as temperature and relative humidity on some time dependent factors. Standard temperature and relative humidity are assumed to be 70° F and 40 %, respectively, in the basic ACI loss calculation formulas. (As can be seen from Table 2.1, this assumption does not necessarily apply to this project.)

Issues related to the temperature effects on the girders were analyzed for the girders of Bridge 19045. As shown in Table 2.1, a great deal of variation in temperature and relative humidity was observed. The effects of this will be discussed in the sensitivity analysis section of Chapter 3.

Methods and Procedures

The field data was collected over a three year period with several different people collecting data. Because of this situation, the data was not necessarily consistently collected in the same fashion by all of the various teams. This resulted in several gaps in the tables presented in the next two chapters. Keeping this in mind, the author will discuss only the procedures that were used on the most recent set of girders from Bridge 19045.

Over a six month period of time, data was collected for girders from project 19045. These girders were chosen because of their large span to depth ratio. Measurements were taken much more frequently at early ages because of the rapid camber changes the girder experienced in the first 28 days. On the day of release, camber readings were taken at the time of release, after pick up, and at the preparation area. The pick up readings were taken in order to determine the effect of friction on the bed with respect to the girder camber. In most cases, when the initial camber is measured for design Mn/DOT limits, it is measured when the member is released from the bed. This does not give an accurate assessment of the initial camber and can underestimate the initial camber by as much as 25 %. The average difference between release and pick-up was found to be 10 - 12 %, as observed from the field data.

When the members were placed in the preparation area and then a few days later moved into the storage area, readings were taken using a level and rod. The readings were taken every few days for the first two weeks and then weekly for the next month or two. This decrease in data readings continued until taking about one set of readings per month. The rod was designed with a 90° extension to fit under the outside of the bottom flange of the girder allowing for consistent readings. The girders were marked along the bottom flange at eighth points to help ensure consistent data collection with respect to location along the member. To help insure the accuracy of the data, plots were made of the camber of each member along the entire length for every time period. By taking these precautions, very few errors were made in collecting the data.

In the storage area, the girders were placed perpendicular to a row of concrete slabs at each end. These slabs were located as a function of the span of the member being stored. As mentioned earlier, the supports were located at about three to five feet from the girder ends which would reduce the overall span length by around eight feet. The distance for each end of the girders was measured from the center of the support to the end of the member.

The date and time of the data recording were noted; the temperature and relative humidity were collected from the Becker weather station which is located in that area. This information was then considered in the long term analysis of the member for the purpose of accounting for long term temperature trends in the data.

Chapter Three – Short-Term Analysis

The short term analysis was conducted in two parts. The first part consisted of performing a parametric study of factors that might affect the camber. The second part comprised trying to produce values in the calculations that were close to the actual measured camber. In order to model the measured camber, the actual material properties were used as well as the support conditions if they were known.

It was also decided that shear deformations could be ignored for the bridges analyzed in this study. According to the ACI 209R-92 report mentioned earlier, "shear deformations are normally ignored when computing the deflections of reinforced concrete members; however, with deep beams, shear walls and T-beams under high load, the shear deformation" may be substantial (4.8 of [2]). As can be seen from the types of bridges analyzed in Table 2.1, the span to depth ratio for these members ranged from 17.3 to 24.8. According to the ACI 209R-92 report [2] which deals with material properties of reinforced and prestressed concrete, if the ratio is around 8.7 or less, then the effects of the shear deformations over time become significant. They can cause as much as 23 % of the total deflection at a span to depth ratio equal to 8.7. As the span to depth ratio increases this value rapidly decreases. Given that the range of field data span to depth ratios was about twice as large as this ratio, it can be shown that the shear deformation would not significantly affect the overall camber when considering the prestress camber and the self-weight deformation. This was demonstrated by looking at the immediate effects of shear and bending deformation for the lowest span to depth ratio members (Appendix B). In this case, the shear deformations affected the overall camber by around 1 %. Even though the shear deformation increases more rapidly than the bending deformation with respect to time, it should still be insignificant in light of the other assumptions that have been made [2].

Sensitivity Analysis

The following pages contain explanations and summary tables for the parametric analysis that was conducted. The complete sensitivity analysis spreadsheet is given in Appendix A; a summary is given in Tables 3.1 and 3.2. The sensitivity analysis was based on the "PCI Design Handbook" camber equations [4]. The respective tables and equations for determining camber according to PCI are given in Figures 3.1 and 3.2. Both Figure 4.10.14 and design aid 11.1.4 [4] were used in determining the initial camber for the girders analyzed (see Figures 3.1 and 3.2). All of the members analyzed had a certain number of straight strands as well as a few two-point depressed strands. Thus, the strand configuration was typical with respect to the PCI Design Handbook. These formulae are based on moment-area techniques and produced reasonable results for the sensitivity analysis and the short-term calculations. With the use of these equations for determining initial camber, member characteristics were varied in order to determine the relevant information for use in the short-term camber. In the following paragraphs, the author will explain how the sensitivity analysis was conducted and what conclusions can be drawn from it.

The parametric study was conducted for three separate girder types of varying sizes and lengths. In Table 3.1, a summary of the calculated and measured cambers for representative girders from each bridge listed are shown. The first row in Table 3.1 relates to data collected on 10 similar girders based on the same design. These girders were standard Mn/DOT 72 inch deep girders with a total length of about 139 feet. The second row relates to a set of 45M (45 inch deep, modified (from standard Mn/DOT section)) girders with a total length of 90 feet. The total number of girders represented for this girder type was 16. The third row shows data collected on four 45M girders with a length of 65 feet.

The following paragraphs explain Tables 3.1 and 3.2 in detail. The base case as well as each variation and the measured results will be expanded upon. Conclusions related to each particular variation will be drawn at the end of their explanation. Finally, overall explanations with respect to the short term analysis will be drawn.

Overall trends for Tables 3.1 and 3.2 will be noted before looking at each individual variation. Table 3.2 is similar to Table 3.1, but it displays the percentage change from the base case data. This table leads to interesting trends in the data that relate to the size, length, and strand density in the member. Changing any of these parameters has an effect on other parameters. As the steel density decreases, the error in using the gross moment of inertia decreases. In a similar manner, as the length decreases, the difference due to changing the drape point by one foot or changing the overall length of the member increases. If the length decreases, then the difference in camber due to changing the concrete density also decreases.

The base data was used to model the structure as best as possible given design data rather than the actual properties of the section. The data used for the base case varied in a number of respects from procedures typically used by Mn/DOT. Mn/DOT engineers typically use the gross section modulus, I_g, rather than the transformed section modulus (Variation #5). For Mn/DOT calculations, the ACI 318 8.5.1 is used for modulus of elasticity, setting $w_c = 155$ lb/ft^3 (this value for the weight is also used in the self weight calculations). For the base case given in Tables 3.1 and 3.2, the modulus of elasticity and girder weight were investigated with Variations #3 and 6. At release, Mn/DOT engineers use the nominal release strength in the calculation of the modulus of elasticity (Variation #2). The prestress assumed after seating, is taken as 0.75 f_{pu} (this assumption was also used in the base case). The strand modulus was taken as 28,500 ksi for the base case, whereas Mn/DOT uses a value of 28,000 ksi (Variation #1, for which case a value of 29,000 ksi used).

According to the Mn/DOT structural drawings, the initial camber predictions for the three bridges investigated were: Bridge 19045 2.5 in. (base case = 2.43 in.); Bridge 27112 1.81 in. (base case = 2.01 in.); Bridge 49535 1.12 in. (base case = 0.97 in.). Comparing the base case data with the predictions according to Mn/DOT for the three bridges listed gives differences of 3, 11 and 15%, respectively. In addition to the reasons listed above for the differences, Mn/DOT includes the effect of the diaphragms (not present at release). Consequently, it was not possible to compare the Mn/DOT estimated cambers with the measured cambers at release.

The control or base case was used from which to compare variations in material, section, and thermal properties for the members. The variations were applied to represent the range of assumptions that may occur in the actual section.

The base data comprised the following assumptions:

- Transformed moment of inertia
- ACI 363 equation for modulus of elasticity
  ($E_c = 40{,}000(f_c')^{1/2} + 1{,}000{,}000$)
- Ec based on the 28-day strength of the concrete.
  (based on the research of Mokhtarzadeh et al.)
- Support conditions located at the ends of the member.
- Initial jacking stress of $0.75(f_{pu})$
- Density of the concrete of 150 lb/ft^3
- Strand elastic modulus of $28.5 * 10^3$ ksi

The transformed moment of inertia accounts for the enhanced stiffness of the girder due to the increased modulus of elasticity of the steel portion of the girder. The transformed moment of inertia is determined using a ratio of each material modulus of elasticity to a reference modulus of elasticity (usually the concrete modulus of elasticity). This ratio is multiplied times the area of the material it references. Lastly, the moment of inertia of the section is determined using the modified area.

Thus, a unified modulus can be used throughout the remainder of the problem in determining the deflections for the material. In addition to increasing the moment of inertia of the section, the location of the center of gravity for the section is changed by the transformation. The transformed steel causes the center of gravity to be "shifted" toward the direction of the largest amount of steel. As can be seen from the results, the importance of calculating the transformed moment of inertia is proportional to the increase in steel density of the section. It should be noted that many texts suggest ignoring the transformed moment of inertia and using the gross moment of inertia of the concrete as an adequate approximation for use in camber determination ([1], page 393).

Appendix C illustrates the transformed moment of inertia calculations used in the sensitivity analysis. The calculation includes all longitudinal steel in the cross section of the member. Even though an averaged transformed moment of inertia is calculated for the member, only the value at centerline was used for the sensitivity analysis. Consideration of a variable transformed moment of inertia along the section could not be justified by the PCI Design Handbook. As can be seen from the results in Appendix C, the variation in the moment of inertia along the beam was moderate. However, when this variation was considered in a finite element analysis of the beam, the deflections were affected only slightly. Thus, using the centerline moment of inertia was adequate.

The steel strand forces were calculated separately for the draped and straight strands. This was done in an attempt to have a better understanding of how much camber was induced by each particular strand group. The camber for each strand group was based on the PCI Design Aid Table 11.1.4 (Fig. 3.2) with slight variations to separate the draped from the straight strands. The gravity deflection of the girder was calculated assuming a simply supported system.

The base case did not include any deflection component resulting from differential temperature effects. It was assumed that the girder had a uniform temperature for short term calculations, since it was heat-cured under a tarp. As can be seen from the sensitivity analysis, if a constant temperature is maintained there is little if any effect on the camber of the member because the increased length of the girder is negligible.

Comparing the variations to the base case leads to the following observations. Variation #1 consisted of changing the steel strength of the strands to that commonly accepted for reinforcing bars (29,000 ksi vs. 28,500 ksi). This variation had an effect on the transformed moment of inertia because the modulus was different. From Table 3.2, it can be seen that for any of the three different girder types analyzed, the effect on camber was minimal (less than 0.07%).

Variation #2 consisted of using the initial compressive strength of the girder in calculating the modulus of elasticity of the member. This also affected the transformed moment of inertia because of the changed modulus of elasticity. Using the initial strength is the method commonly employed in most design offices; however, Mokhtarzadeh et al. found that there was little change between the 1-day and 28-day elastic modulus. Consequently for the base case, the modulus of elasticity was calculated using the 28-day strength for the concrete which was considered to be most accurate. Comparing variation #2 with the base case led to a maximum difference of 5.35%.

Variation #3 consisted of using the ACI 318 equation 8.5.1 for determining the modulus of elasticity. This formula is reasonable for calculating the modulus of elasticity for normal strength concrete, but for high-strength concrete, the ACI 363 equation is more accurate. As a general rule, prestressed concrete will usually be considered high strength concrete. Therefore, the ACI 363 equation in correlation with the 28-day strength was used for the base case. Comparing this variation to the base case yielded a significant change in the camber of up to

14% due to this variation. As can be seen from the results, the use of one formula over the other may very well be determined by the design strength of the concrete. If one compares variation #3 to that of the measured and the base cases in Tables 3.1 and 3.2, it can be seen that the ACI 318 equation appears better suited for the girders of Bridge 49535. It can also be observed that Bridge 19045 results are more closely aligned with the ACI 363 equation. These results are consistent with the actual concrete strengths (Bridge 19045 concrete was about 2,000 to 3,000 psi higher than that of Bridge 49535).

Variation #4 consisted of increasing the assumed initial prestress (after seating) by 5%. According to the Mn/DOT inspector on site at Elk River Concrete Products this was a reasonable variation. This variation had a significant effect on camber for all three of the different types of girders (9 - 13%). As was mentioned earlier, a trend related to the density of the steel in the section was observed. As the density of the steel increases, so does the difference caused by the change in jacking force. Increasing the steel density by 53%, increased the difference with respect to the base case for this variation by 45%.

Variation #5 used the gross moment of inertia rather than the transformed moment of inertia. This caused a significant change in the camber of the girders, even as high as 17% difference between the base and variation. Using the transformed moment of inertia was considered to characterize the material properties the most accurately. As can be seen, a similar trend related to the density of steel was observed when changing from the transformed to the gross moment of inertia.

Variation #6 took into account possible errors in calculating the self-weight of the member by increasing the concrete weight per foot cubed by five pounds. As can be seen from the sensitivity analysis in Table 3.2, the effect on camber was minimal (around 3%). Variation #7 in which the harp points were moved by one foot as well as variation #8 in which the beam length was changed by one foot also had minimal effects with respect to the camber. Variations #7 and 8 had a maximum effect with respect to the base case of 1.2% and 2.5% respectively.

Variation #9 considered changing the support conditions at the initial time step in order to account for the location of the actual supports during curing of the girder. Support locations have a significant effect when comparing the calculated camber to that actually observed in the storage areas. On average, the supports were located three to five feet from each end of the girder. This condition serves to increase the camber in the girder by decreasing the gravity deflection and by applying end moments to the shorter supported section. An estimate of this effect was around 11% for the data with information available.

Variation #10 used the actual concrete strength measured from test cylinders formed in the field. Cylinders were tested at the initial release of the girder and usually at some time around the 28-day strength of the girder. In the analysis, the 28-day strengths were estimated and used in determining the modulus of elasticity for this variation. Strengths were recorded by the Mn/DOT inspector up to a value of 10,000 psi at which point the test was considered complete. This does not aid in determining the actual strengths of the section, but for most cases failure occurred before reaching 10,000 psi. According to the sensitivity analysis, this variation was moderately important (around 4 to 8% camber difference from using the design data). The fact that designers do not know the actual strength ahead of time for the concrete makes it unlikely that this variable can be fine tuned in design.

Detailed information about the method and data collected on measuring temperature changes over a day long period will be discussed in Chapter 5, which involves thermal effects over a one day period. The method used in Chapter 5 is more precise than that used in the sensitivity analysis. However, Variations #11, 12, and 13 did have a purpose in that they roughly investigated the effect of temperature on the section. A coefficient of 6×10^{-6} in/in/ °F was used to determine the strain in the section at a particular location. In variation #11, a constant increase in temperature of twenty degrees was applied throughout the section. From this application a change in the length of the member was determined and calculations for the camber due to this change in length followed. For this constant temperature case, the effect on camber was minimal for all three types of girders observed.

Variation #12 applied a thermal gradient that varied by 20 ° Fahrenheit throughout the depth of the section in a linear fashion. Typically, a linear function for the temperature variance is inaccurate, but it was used to show the possibilities for affecting camber. This thermal gradient was then turned into a curvature calculation for the beam. The change in strain between the top and the bottom of the member due to thermal effects was divided by the depth to get a curvature value. From this point, it was assumed that the curvature would remain constant and thus a conjugate beam analysis could be conducted to determine the camber. As can be seen from the sensitivity analysis, this variation was significant (around 28%).

Variation # 13 consisted of a crude assumption that only the top flange of the girder was heated by a temperature variance of 20 degrees. This would be typical of a hot day when the sun radiated down on the girder with the top flange shading the bottom flange and web. This would result in a shearing action at the top flange - web interface. Finding these forces, a moment for the force was calculated after determining a moment arm located from the center of the stressed top flange to the top of the web. This moment was assumed to be constant along the beam. Knowing this, a camber for this component of temperature was determined and it was discovered that this variation was insignificant with respect to camber.

Ghali and Favre in Reference 3 have suggested a more realistic model for considering the temperature effects of concrete. They suggest that temperature varies in a parabolic fashion when the sun heats the top of the girder. If the concrete molecules were allowed to strain freely then a parabolic strain diagram would be observed. Ghali and Favre wrote that:

> "Stresses are produced because each fiber being attached to adjacent fibers is not free to acquire the full expansion due to temperature. The stresses produced in this way in an individual cross section must be self-equilibrating; in other words the temperature stress in a

statically determinate structure corresponds to no change in the stress resultants (internal forces)." (Ghali [3], page 23).

They go on to state that these self-equilibrating stresses are sometimes referred to as the eigen-stresses for the section. Thus, plane sections will still remain plane with this equilibrating stress (see Figure 3.3). This was verified by experimental research that they conducted. If the plane sections remain plane, then the parabolic strain of pure thermal expansion will produce residual stresses in the section.

Following variation #13 in Tables 3.1 and 3.2, are listed measured cambers taken from the actual field data collected after the girders had been picked up and set down. The values were actually measured to the nearest millimeter at release which correspond with an accuracy of +/- 0.04 in. The measured cambers after the members were moved to the storage locations are also shown in these tables. The members were moved immediately after release. As mentioned earlier, picking up and setting down the girder helped to alleviate friction between the supports and the bed. However, because the beams were lifted by support hangers located at five to six feet from the ends, friction on the bed had the opposite effect. When the beams were set back down, friction prevented the beams from settling down to the estimated camber, that is camber without the effects of friction. Thus, camber without the effects of friction between the bed and the member is somewhere in between the original measurement and the pick up and the set down measurement.

As can be seen in the tables, there are slight differences between the base case and the actual case. In the first girder type that came from Bridge 19045, the actual measured camber was higher than that calculated in the base case. This can be accounted for all alone by changing the initial prestress (after seating) as was suggested by the Mn/DOT inspector. In the detailed parametric study available for Bridge 19045 in Appendix A, the measured camber range was compared to changes in initial prestress and concrete strength required to correlate with the measured range of results. All other parameters for each of the columns were the same as the base case analysis. When one looks at this table, it can be seen that modifying the initial prestress force to account for the variation is the most reasonable method to account for differences in the calculated and measured camber. Girders cast on the same bed with the same prestress had similar camber results as long as the concrete strengths were close. The difference in cambers for girders meeting the previous qualifications was small; it varied from 0 to 7%.

For the representative girder of Bridge 49535, which is the last girder in Table 3.2, the actual camber was lower than that calculated for the base case. Since the Mn/DOT inspector stated that the prestressing force will always be at least what is called for in the design plans -- if not more, the conclusion was drawn that some other factor must account for this variation. The difference in results may be accounted for by the assumption used for the modulus of elasticity. The results would tend to agree with this conclusion. Several of the other factors may play a part in making up the difference between the base and the actual case, but they are less obvious.

The effect of storing the girders on the supports which shortens the girder span was also investigated. The difference in variation between the actual supported and the end supported case was consistent with the difference measured in the field for the representative girder for Bridge 19045. Even though the measured data varied from the base case, the difference between the end supported case and the actual support case was consistent. For Bridge 19045, the actual difference was 0.25 inches, while calculated was 0.26 inches. Bridge 49535 had an actual difference of 0.37 inches.

Comparison to Field Data

The first step in trying to model the short term camber was to use values for the 28-day strength that were as close as possible to that strength. The reasons for using the 28-day strength have already been discussed in the sensitivity analysis explanation. The 28-day strength data was not known for all of the girders that were modeled. Thus, estimates of the 28-day strength had to be made. This could not be done by using the basic strength estimation equations that are presented in many texts because of the curing process used by Elk River, and because of water reducer admixtures added to the concrete. Instead, modified factors were used to get a better approximation of the strength of the member. Engineering sense was also employed where significant deviation occurred with these modified factors. The modified factors were created by back substitution for members with complete strength data. In order to be considered a complete set of strength data, the member had to have an initial, intermediate, and a 28-day strength. This was done for every member that had a complete set of records and then the average for these coefficients was taken. The function [1] with the accepted values is shown below:

$$fc(t) = \left[\frac{t}{\alpha + \beta * t} \right] * fc' \qquad \text{(Eq. 3.1)}$$

Where: t = time in days

α and β = Factors that correspond to the cement type and curing conditions.

α =0.7 for Type III , steam-cured cement

β = 0.98 for Type III, steam-cured cement

fc' = concrete strength at 28 days

fc(t)= concrete strength at any time t

The terms α and β were modified to reflect the actual concrete properties. These two terms are meant to model the cement type and curing conditions of the concrete. They were modified to 0.34 and 1.08 respectively based on the averaged available data. The range of the data calculated values of α was 0.23 to 0.42 and for β the range was 1.03 to 1.20. Typical ranges for these variables, according to the ACI 209R-92 report referred to earlier, are α = 0.05 to 9.25 and β = 0.67 to 0.98. Even though β falls outside of this range, it should be considered that the typical range listed does not account for admixtures to the concrete mix. Thus, it was decided that these factors were reasonable, but should be used in conjunction with engineering judgment.

Even if the strengths have been estimated incorrectly by 500 psi, the effects on the camber for the member will not be greater than about 1.8 - 2.6 percent for the members considered. This error is well within the accuracy of what was measured for camber in the field. Thus, it was determined that the concrete strengths were estimated fairly well. Appendix D contains the detailed spreadsheet showing values used in the analysis of the short term calculations for all of the girders that had data collected on them. In this spreadsheet there are input concrete strengths shown in bold to indicate that they were not measured test data, but approximations made using engineering judgment and the formula given in Equation 3.1.

With information that was obtained from the parametric study, most of the assumptions made in the base case were used for the calculation of short-term effects in all of the members. However, the actual or approximate values of 28-day strength were used rather than the design strength assumed in the base case. It was also decided that the design prestress force assumed in the base case would be used because this was the only information available at the time of analysis. With all of this in mind, the results of the short-term camber study are presented in Tables 3.3 to 3.8. The notation for each beam begins with the beam number (e.g. B11) followed by the series number (e.g. S2). The same series number indicates the same beam design.

Results from the analysis seem to be reasonable according to other studies done in the area of camber and deflections for short term analysis. According to ACI committee 209 [2] with all of the experimental material parameters known, it is reasonable to expect an error of plus or minus 15%. If design material parameters (concrete strength, modulus of elasticity, and time dependent parameters) are used, it is reasonable to expect an error of plus or minus 30 % [2]. These percent errors are with respect to camber at any time period, not only at initial camber.

In Tables 3.3 to 3.8 the difference between the measured and calculated camber has a range of up to 37%. There are at least two explanations for the wide range of variance especially visible in Tables 3.4 to 3.6. One of the main reasons is that friction resistance in the bed was not considered in Tables 3.4 to 3.6. In most cases measurements taken in the field for initial camber are done immediately after release of the member without consideration of picking the member up to relieve frictional restraining forces. Consideration for the friction losses was not deemed important in the early stages of research; thus, a lack of data collection for Tables 3.4 to 3.6.

Tables 3.9 to 3.11 list the measured friction differences between the bed and the girder. In the first row is shown the measured values before the member was picked up by the crane. The second row shows the measured camber after the girder had been picked up and set back down onto the formwork bed. As mentioned previously, the actual case of non-friction affected camber is somewhere in between the two cases presented in Tables 3.9 to 3.11.

From these tables it is easily apparent that the friction at release due to the bed varies by a great deal. The differences in camber due to friction were in the range of 5 to 20 percent with an average of around 12 percent.

If we consider only Tables 3.3, 3.7, and 3.8 for which the camber were measured after pick-up and set down thereby relieving friction with the bed, the resulting range of error between calculated and measured results drops to 17%. These three tables take into account some of the experimental material parameters, but do not take into account all of them (i.e. prestress strand force.) This leads to the second reason for this high percentage of error which was a lack of knowledge of all the experimental material properties. If a more accurate knowledge of the prestress forces, and the actual modulus of elasticity were known, less error in the calculated versus measured results would be possible.

It should be realized that there is a limit to the comparability of the field data to the calculated data; specifically, those members influenced by the friction losses. However, because all of the members were moved off of the bed, partial alleviation of the friction resistance could be assumed in the long term analysis for Bridges 27112 and 04516. Unfortunately since the storage conditions were not known for these members, other differences between the actual and calculated data were introduced. In the next chapter, the long term results along with these differences will be discussed.

Chapter Four – Long-Term Analysis

The accuracy in determining the short-term camber relied on the determination of the correct physical properties of the section without regard to time. In order to determine the long-term camber for a member, several methods have been developed in order to estimate the effect. As mentioned in the past research section of the first chapter, some of these methods depend on the geographical region where they were developed as well as the types of aggregate available in that area. Thus, the evaluation of a method developed for a general case was considered to be important.

The methods investigated included: the Pretressed Concrete Institute Design Guide Multiplier Method, the approximate time step method proposed by Branson et al., and the computer program "CRACK" which was developed by Ghali et al. In order to perform the analysis using the program "CRACK," many values had to be determined. Of the time dependent values, the program required the creep multiplier, shrinkage strain, reduced relaxation stress, and the aging factor for the concrete. There are several methods of determining these values but only a few that determine time step values. These values were determined using the ACI 325 Committee recommendations in "Control of Deflection in Concrete Structures." These values were also determined according to the European model code for which the "CRACK" program was originally developed in Calgary, Canada. Both of these design model methods for determining camber had variations, but they produced results that were comparable for most of the members.

Because of the missing support information, it was decided to only consider Bridge 19045 girders in the analysis of long term effects using all of the methods. In the first few pages of this chapter the camber calculation methods and the models used for determination of the time-dependent factors will be discussed. Results and conclusions regarding the methods examined will follow the explanations.

Methods and Procedures

PCI Design Handbook Multiplier Method:

This was by far the least complicated of the methods employed in determining the long-term effects of camber for the section. This method, or a variation of this method, has been employed in design practice since the development of the multipliers by Martin in 1977 [13]. As will be shown later in this chapter, the method also yielded moderate overall errors with respect to the other methods employed (on average, up to 8% higher error than the other methods). However, it is extremely quick and a good approximation unless a more accurate calculation of the camber is desired. To use the method, the short-term camber is determined and then appropriate multipliers are applied to the short-term deflection. The multipliers are found in Table 4.1 which was taken from the PCI Design Handbook. Leslie Martin made several assumptions and then determined multipliers to be used based on the ACI multiplier equation for nonprestressed concrete members. These are the assumptions that he made:

1. The ACI 318 short term deflection amplification factor for estimating long-term deflections for prestressed concrete was taken equal to 2 as for normal weight concrete. (This factor assumes that there is very little compressive steel in the member.)
2. Initial prestress loss = 8.0%
3. Time dependent loss of prestress is 15.0%
4. Percent of total camber and deflection change at erection = 50%
5. The ratio of noncomposite to composite moment of inertia = 0.65.
6. Assumes erection of the structure occurs at the concrete age of 45 days.

Martin considered that in prestressed members there are losses over time and that the concrete will gain strength over time. With all of the above information, he modified the ACI 318 multiplier as found in the 1971 code for use in prestressed members. He noted that this method is an approximation, but justifies it by stating:

" ...the data on which ...(more complex methods) are based usually has a scatter of at least 15 to 20 % using laboratory controlled specimens....it seems rather futile to use these time-consuming methods for estimating long-time cambers and deflections." [13]

Branson et al. Approximate Time-step Method:

There are several variations of the approximate time step method that have been developed over the years with Branson's being one of the more recognized (cited in References 1, 2, 4, 5, 9 and 11). The method was based on assumptions related to the complex incremental time-step procedure as well as empirical data from tests that have been performed. The most common reason for using this method is to produce estimates of deflection at any given time. Additionally, the method exhibits a less cumbersome process than the incremental time step method. The incremental time-step method uses a general expression that is a good basis for formulating the approximate time step method:

$$\phi_t = \frac{-P_i \cdot e_x}{E_c \cdot I_c} + \sum_{n=0}^{t} \left(P_{n-1} - P_n\right) \cdot \frac{e_x}{E_c \cdot I_c} - \sum_{n=0}^{t} \left(C_n - C_{n-1}\right) \cdot \frac{P_{n-1} \cdot e_x}{E_c \cdot I_c} \qquad \text{(Eq. 4.1)}$$

where: P_i = initial prestress force before losses

e_x = eccentricity of steel from the center of gravity at any location along the span

$n - 1$ = beginning of a particular time step

n = end of the aforementioned time step

C_{n-1} and C_n = creep coefficient at the beginning and end of the time step

$P_n - P_{n-1}$ = prestress loss at a particular time interval

If the curvature is determined for enough sections along the member, a curvature diagram can be constructed. The conjugate beam method or curvature-area technique can be employed to determine the deflection in the section. Obviously this technique is very cumbersome and time consuming, but well suited to a computer program such as "CRACK" or "PBeam."

The approximate methods use a form of this equation in order to simplify the computations. The first step in the method is to relate the curvature to the deflection.

$$\phi = \frac{M}{E_c \cdot I_c} \quad \text{(Eq. 4.2)} \qquad \text{and} \qquad \delta = \phi \cdot k \cdot L^2 \quad \text{(Eq. 4.3)}$$

where:
δ = deflection in the member
k = some function that depends on the member support conditions and the strand pattern
L = length of the member

Thus, the incremental time step function can be rewritten from the general form to find the final deflections at the end of the member life.

$$\phi_e = \frac{-P_i \cdot e_x}{E_c \cdot I_c} + \sum_{n=0}^{t} \left(P_i - P_e\right) \cdot \frac{e_x}{E_c \cdot I_c} - \sum_{n=0}^{t} \left(C_u - 0\right) \cdot \frac{e_x}{E_c \cdot I_c} \cdot \left(\frac{P_i + P_e}{2}\right) \quad \text{(Eq. 4.4)}$$

$$\delta_{et} = -\delta_i + \left(\delta_i - \delta_e\right) - \left(\frac{\delta_i + \delta_e}{2}\right) \cdot C_u \quad \text{(Eq. 4.5a)}$$

$$\delta_{et} = -\delta_e - \left(\frac{\delta_i + \delta_e}{2}\right) \cdot C_u \quad \text{(Eq. 4.5b)}$$

Where:
δ_{et} = final deflection caused by the prestressed strands
δ_i = deflection due to the initial prestress
δ_e = deflection due to the final prestress
C_u = ultimate creep coefficient

Now if the external loads are included in the formulation, then the overall deflection can be determined and used in incremental time steps.

$$\delta_t = -\delta_e - \left(\frac{\delta_i + \delta_e}{2}\right)\cdot C_t + \left(\delta_D + \delta_{SD}\right)\cdot\left(1 + C_t\right) + \delta_L \qquad \text{(Eq. 4.6)}$$

C_t = time incremental creep coefficient

$$C_t = \left(\frac{t^{0.60}}{10 + t^{0.60}}\right)\cdot C_u \qquad \text{(From ACI 209R-92)}$$

δ_e = deflection due to the effective prestress @ t

δ_D = deflection due to the dead load

δ_{SD} = deflection due to the sustained dead load

δ_L = deflection due to the live load

Branson et al. have suggested a similar equation that is based on the above formulation as well as empirical information. This formula takes into consideration compression steel and the age of the concrete.

$$\Delta\delta = \left[\eta + \left(\frac{1+\eta}{2}\right)\cdot k_r\cdot C_t\right]\cdot\delta_{i\left(p_i\right)} + k_r\cdot C_t\cdot\delta_{i(D)} + K_a\cdot k_r\cdot C_t\cdot\delta_{i(SD)} \qquad \text{(Eq. 4.7)}$$

Where: η = Pe/Pi

k_r = 1/(1+As/Aps), As/Aps <= 2
= factor to consider the non-tensioned steel in the section

Ka = factor corresponding to the age of the concrete at loading
= 1.13*t^-0.095 (for steam cured concrete)

For noncomposite beams, the total deflection becomes:

$$\delta = -\delta_{pi}\cdot\left[1 - \frac{\Delta P}{P_o} + \lambda\cdot\left(k_r\cdot C_t\right)\right] + \delta_D\left(1 + k_r\cdot C_t\right) + \delta_{SD}\cdot\left(1 + K_a\cdot k_r\cdot C_t\right) + \delta_L \qquad \text{(Eq. 4.8)}$$

Where: ΔP = total loss of prestress excluding the initial elastic loss

P_o = initial force at transfer after elastic losses

$$\lambda = 1 - \frac{\Delta P}{2\cdot P_o}$$

The previous equations and derivations, Eqs. 4.1 to 4.8, came from References 1 and 10.

Using Equation 4.8, results were determined for each girder of Bridge 19045. This method gave fairly reasonable results when compared to the other methods used in this report, but generally overestimated the cambers at any given time by about 0.1 to 0.4 inches (average of 5 to 11% difference per girder (see Table 4.3)). However, for the early ages of the concrete life, the measured data varied by as much as 1.5 inches (see Figures 4.2 to 4.9). This could be due to temperature effects or it could be due to the averaged data used by the ACI model code for calculating the properties of the girder.

Many of the components of Equation 4.8 were not needed because superimposed dead and live loads were not applied to the section. As such, the factor "K_a" (defined earlier) which relates to the age of the concrete at application of superimposed loads was never used for determining the camber because only self weight was considered in the project.

Since there were mostly tensioned strands in the section, the "k_r" term which accounts for nontensioned steel in the section was also not used. Naaman [5] discusses the fact that this "k_r" term in Branson's formula may be in error with respect to accounting for the nonstressed steel. The "k_r" term was originally intended to account for the reduction in deflections due to nonprestressed steel in the tension zone. Naaman cites Tadros' et al. research in which Tadros came to the conclusion that in certain situations when the equation is used, calculated results with the term "k_r" contradicted the measured variance. The term does not account for the ratio of nontensioned steel to cross-sectional area of the concrete. Thus, Tadros questioned the effectiveness of the equation in accounting for nonstressed steel in reducing the long-term deflections due to creep in the section. As such, engineering judgment should be used when considering the use of this method. Use of this formula should be decided based on its appropriateness to a given design for a member (i.e., consider using a different method if a considerable percentage of the steel in the section is nonstressed in the tension zone).

Program "CRACK" by Amin Ghali et al.

Much of the derivation of this program can be found in Reference 3 and will be summarized in this section of the report. Ghali and Favre use the relationship that "plane sections remain plane" as the basis for much of the formulation to be presented. They state that this assumption is reasonable based on test results determined in their lab. From here they go on to derive a formula in matrix form to relate strain to forces and moments in the section (Equation 4.9). This formulation also allows for any reference point to be used without regard to the center of gravity of the section.

$$\begin{pmatrix} \varepsilon_o \\ \psi \end{pmatrix} = \frac{1}{E(A \times I - B^2)} \begin{pmatrix} I & -B \\ -B & A \end{pmatrix} \begin{pmatrix} N \\ M \end{pmatrix} \qquad \text{(Eq. 4.9)}$$

A = Transformed area for the section
B = Transformed first area moment for the section
I = Transformed moment of inertia for the section
E = Reference modulus of elasticity (usually the concrete modulus of elasticity)
N = Axial force for the section
M = Moment for the section
ε_o = Strain at a reference point
ψ = Curvature for the section

Ghali transforms all of the prestressed member stresses and external forces into one externally applied force and moment for each time step. These forces are transformed, using Equation 4.9, into strain and curvature changes which are used to compute changes in stresses in the section over the time step. This information is then applied to the previous time-step stress, strain, and curvature values. From this combined information, deflections are determined based on the curvature information. In order to determine the changes in force and moment for the time dependent effects, Ghali uses the following expression:

$$\begin{pmatrix} \Delta N \\ \Delta M \end{pmatrix} = \begin{pmatrix} \Delta N \\ \Delta M \end{pmatrix}_{creep} + \begin{pmatrix} \Delta N \\ \Delta M \end{pmatrix}_{shrinkage} + \begin{pmatrix} \Delta N \\ \Delta M \end{pmatrix}_{relaxation} \qquad \text{(Eq. 4.10)}$$

$$\begin{pmatrix} \Delta N \\ \Delta M \end{pmatrix}_{creep} = -\sum_{i=1}^{m} \left[\left(E_{c\_bar}\right) \cdot C_{(t,to)} \cdot \begin{pmatrix} A_c & B_c \\ B_c & I_c \end{pmatrix} \cdot \begin{bmatrix} \varepsilon_o \cdot (t_o) \\ \psi(t_o) \end{bmatrix} \right]_i \qquad \text{(Eq. 4.11)}$$

Where: A_c = Area of the concrete

B_c = First area moment of the concrete

I_c = Second area moment of the concrete

E_{c_bar} = Age adjusted reference modulus of elasticity of the concrete

$$E_{c\_bar} = \frac{E_c(t_0)}{1 + \chi \cdot C_{(t,t0)}}$$

χ = aging coefficient (ACI 209R-92 Report)

$\varepsilon_o \cdot (t_0)$ = Initial strain for the section

$\psi(t_0)$ = Initial curvature for the section

i = the concrete section to be considered

Since the creep force is determined with a multiplier to the strain and curvature, the matrix that Ghali developed could be used to determine the force and moment necessary to resist free creep in the section (Equation 4.11). He also uses expressions to determine the shrinkage and reduced relaxation moment and axial force required to prevent free shrinkage and reduced relaxation for the section (Equations 4.12 and 4.13). Free creep, shrinkage, and reduced relaxation are the creep, shrinkage, and reduced relaxation that would occur if each fiber were uninhibited by other fibers.

$$\begin{pmatrix}\Delta N\\ \Delta M\end{pmatrix}_{shrinkage} = -\sum_{i=1}^{m}\left[\left(E_{c\_bar}\right)\cdot\varepsilon_{cs}\cdot\begin{bmatrix}\varepsilon_o\cdot\left(t_o\right)\\ \psi\left(t_o\right)\end{bmatrix}\right]_i \qquad \text{(Eq. 4.12)}$$

$\varepsilon_{cs}\cdot\left(t,t_0\right)$ = free shrinkage for the period t_0 to t
m = total number of concrete parts

$$\begin{pmatrix}\Delta N\\ \Delta M\end{pmatrix}_{relaxation} = -\sum_{i=1}^{n}\begin{pmatrix}A_{ps}\cdot\Delta\sigma_{pr}\\ A_{ps}\cdot y_{ps}\cdot\Delta\sigma_{pr}\end{pmatrix}_i \qquad \text{(Eq. 4.13)}$$

A_{ps} = Area of the prestressed strands
y_{ps} = distance from the centroid of strands to the center of gravity for the section
$\Delta\sigma_{pr}$= reduced relaxation over the period t_0 to t
n = total number of strand layers

Ghali et al. determined the incremental strain and curvature for the section using Equation 4.9. The resultant restraining forces (axial and moment) for creep, shrinkage and relaxation are determined from Equation 4.10. Additionally the strain and curvature that would result if these properties could occur freely are used. Then these incremental strains and curvatures are added to the previous time steps total strain and curvature. Using these strains and curvatures the deflections can be calculated if the curvatures from the other sections along the girder are known. A more detailed look at this approach to the determination of long-term camber/deflection in prestressed concrete members can be found in Reference 3 (pages 22 to 32).

The program "CRACK" was labor intensive in the production of input files for the program. The external moments and forces caused by all loads must be input for each loading stage. All of the time dependent factors must be input for each stage as well. This program allows for an extremely generalized case to be used as input for the program, unfortunately it is also extremely time consuming and impractical to create the input file necessary for this program. Even if each member was designed similarly, data related to the time step and concrete would not be similar if constructed on different days. In Appendix F of this report, an example of the input as well as the output files for Beam B18-S2 of Bridge 19045 can be seen.

Model Codes Used for Determining Estimates of Material Properties

CEB-FIP 1978 (MC78) Model Code

Two separate model codes were used in the program "CRACK" to determine the time dependent coefficients appropriate as input for the program. The first model code used was the CEB-FIP 1978 (MC78). The program had been originally written in Calgary, Canada and was developed with this code in mind. However, Ghali states that he kept the program generalized enough to be used with any code determined set of material parameters. The European model code relies on empirical equations based on research done on members. The code attempts to account for the following in tables, charts, and equations:

- Ambient environmental conditions
- Relative humidity
- Volume to surface ratio of the member
- Creep which develops in the early ages of the section
- The hardening rate of the concrete

The method gave reasonable results for 6 of the 8 girders compared to the field data the two exceptions were girders B13-S2 & B14-S2 of Bridge 19045. The code provides time-dependent relations for creep, aging, and shrinkage in the concrete, as well as intrinsic, and reduced relaxation multipliers for the strands. The errors that resulted for the two members were a result of the creep function that the European model code uses. This creep equation relies on the reduction of creep due to a ratio between the initial and 28-day modulus of elasticity. As such, the larger the ratio, the larger the creep. If the members are not released until the concrete is several days old, then the method predicts a higher ratio for the initial to 28-day modulus of elasticity.

Mokhtarzadeh, et al. has determined that the modulus of elasticity at 28 days is similar to that of the initial modulus. The results are dominated by the coarse aggregate and sand which comprises approximately 80 % of the concrete. With this information in mind, the equation should be reevaluated for determining the creep of the section. An attempt at compensating for the problem was done by using Equation 4.15 which is recommended by CEB-FIP as an estimate of the modulus ratio portion of Equation 4.14. Using this equation was an attempt to determine a reasonable creep value that could be used in conjunction with the results of Mokhtarzadeh et al.'s research. As will be shown later, the results still did not agree well with that of the field data. The creep equation is presented below:

$$C_{(t,to)} = \frac{E_c(t_o)}{E_c(28)} \cdot \left[\beta_a(t_o) + 0.4 \cdot \beta_d(t, t_o) + C_f \cdot (\beta_f(t) - \beta_f(t_o))\right] \qquad \text{(Eq. 4.14)}$$

$$\beta_a(t_o) = 0.8 \cdot \left(1 - \frac{f_c(t_o)}{f_c(\infty)}\right)$$

(represents creep developed in the early ages of the member)

$$\beta_d(t - t_o) = 1 - \exp \cdot \left[-0.02 \cdot (t - t_o)\right]$$

(A function of the time length of the period under consideration)

$$\beta_f(t) = \left(\frac{t}{t + t_f}\right)^{\frac{1}{3}}$$

(This is a function of the notional thickness. The notional thickness is related to the volume to surface ratio of the member and the ambient environmental conditions. It is found in the tables of the CEB-FIP model code.)

Note: The elastic modulus and the strength ratios for Equation 4.14 may be determined from empirical equations found in the CEB-FIP model code. Using the measured numbers for E_c (t_o) and E_c (28), gave unreasonable results due to the extended curing period of a couple of the girders (B13 and B14) which led to a high ratio of E_c (t_o) to E_c (28) for Bridge 19045. It was decided to use the CEB-FIP formula for the ratio:

$$\frac{E_c(t_o)}{E_c(28)} = \left(\frac{t_o}{4.2 + 0.85 \cdot t_o}\right)^{\frac{1}{2}} \qquad \text{(Eq. 4.15)}$$

Where: t_o = the age of the concrete at loading

Modeling Assumptions used with "CRACK" and *CEB-FIP*

In order to ensure that the CEB-FIP model and "CRACK" were being performed correctly, plots and tables were made to determine how different assumptions affected the plots of camber for Bridge 19045, member B18-S2. The first factor investigated was a time multiplier the CEB-FIP model code used to characterize the member more accurately (Figure 4.1). The equation was intended to modify the temperature and cement characteristics so that the rate of hardening could be accounted for in the material properties. The multiplier is applied to the actual time and then this modified time is used in all of the time dependent equations which relate to the concrete. This time multiplier is an attempt to properly determine the deflection at a given actual time with accelerated strength gain concrete in mind. The formula for the time multiplier appears as follows:

$$t = \frac{\alpha_c}{30} * \sum ((T_i + 10) * \Delta t_i) \qquad \text{(Equation 4.16)}$$

Where: Ti = average daily temperature (in Celsius)
Δti = the number of days during the period of time
α_c = coefficient related to the cement type
= 1 for normal hardening cement
= 2 for rapid hardening cement
= 3 for rapid hardening high-strength cement

The temperature portion of the formula was to be used when the ambient temperature exceeded 20° Celsius (68°). Since the temperature during which girders for Bridge 19045 were tested was mostly below this value, this part of the equation was not considered. This meant that the remainder of the equation would consist of time multiplied by the cement coefficient (α_c). Figure 4.1 shows the use of the cement coefficient multiplier and a comparison to the actual measured data. From these results, it can be seen that using the cement coefficient multiplier in any form for this particular set of girders would affect the results for the CEB-FIP model code method in a negative fashion. The best approximation for the model was to use a cement coefficient multiplier = 1, which corresponds to a normal to slow rate of hardening. Thus a value equal to one was used in the CEB-FIP model code and the program "CRACK."

ACI 209R-92 and 435R-95 Model Used With "CRACK"

The second model code for the material parameters used for "CRACK" and the approximate time step method was the ACI 209R-92 and 435R-95 respective reports that are referenced. The equations of these ACI model codes produced reasonable camber results with respect to those observed and can be seen in the approximate time step analysis calculations found in Appendix E. Equations from these reports were used in the approximate time step analysis in order to determine the losses for a given time step. Since both reports took slightly different approaches to the determination of prestress losses at different time steps, it was decided to use the ACI 435R Report [8] to determine losses. The 435R-95 Report was intended to be used for the determination of deflections in concrete members and is a more current report. Both of these ACI reports use empirical information and averaged values to determine creep and shrinkage for the sections. However, the models enable adjustment of these averaged values by applying factors to characterize the actual section. Factors that applied to the members analyzed were used in the approximate time step method and the program crack when modeled with the ACI codes (see Appendix E).

Additional Remarks Regarding Methods

Included in addition to Branson's approximate time step analysis in Appendix E is a comparison between the ACI, PCI, and AASHTO total loss calculation. The PCI and AASHTO methods produced final prestress losses for Bridge 19045 that were within 1-2 ksi of each other. These methods differ in their equations for determining creep, shrinkage, and relaxation; however, they seem to give approximately the same results. The losses calculated by the ACI method were larger than both of these values by about 4-5 ksi for the same bridge. Part of the reason for this may be that the ACI method takes into account more characteristic properties of the section than the other two methods. The method additionally accounts for the concrete age at loading, the fine aggregate percentage, the slump of the concrete mix, and the thickness of the member. All of these factors were not specifically addressed in the PCI or the AASHTO calculations of the final total losses. Comparing the PCI, AASHTO, and ACI method for determining losses is difficult because no strain gauges were used for this project to determine losses. However, the ACI 209R and 435R method did predict total final losses to be seven percent higher than PCI and AASHTO methods (see Appendix E). The greater losses associated with the ACI methods led to lower long term cambers than would be associated with the other loss calculations (PCI and AASHTO). The ACI prestress loss method was used in determining long term cambers shown in Figures 4.2 through 4.9 according to the CEB-FIP and Approximate Time Steps methods. Please note that in the figures, the initial camber value calculated at release was used in the long-term calculations.

From Figures 4.2 through 4.9, it can be seen that where the ACI 209R and 435R recommended values were used, the camber results were generally higher than measured. The results could be higher due to the fact that the estimated values of ultimate creep and many of the constants used in the other equations are based on averaged data. If the PCI and AASHTO methods had been used to calculate losses, the long-term predictions would have led to greater long-term cambers. Figures 4.2 to 4.9 will be discussed later in this chapter.

Comparison to Field Data

In this section, the results of the field data are compared to the calculated methods discussed in the previous section. It should be noted that the values presented for the measured field data have been adjusted by a percentage to account for the support conditions. A reasonable estimate of this percentage was determined using averaged camber ratios for different support conditions of members B18-S2 and B11-S2 of Bridge 19045 (see Table 4.2). For these two members, the sections were analyzed as simply supported beams, spanning between the actual support locations, with applied end moments to simulate the cantilevered ends using the program "CRACK." By comparing this case of deflection to the case of a fictitious end supported case, a multiplication factor was determined. As can be seen in Table 4.2, the factor was fairly consistent even when parameters were varied such as the method used, the girder member analyzed, and the cement coefficient multiplier considered which accounts for the concrete hardening rate and the average daily temperature (see Equation 4.15).

The percentage difference in camber between the end supported case and the actual supported case had an average value of 85.53% with a standard deviation of ±0.47% (see Table 4.2). By incorporating this adjustment to the measured cambers, all of the members could be treated as if they were simply supported at the ends when calculating long-term camber. Using this approximation to account for the supports simplified the analysis used in the program "CRACK," as well as the other methods used. "CRACK" can handle performing analyses of cantilevered beam members and simply supported members only. All other cases have to be adjusted to these types of members, such as using separate analyses for the overhangs.

Graphs and tables of the four different approaches to measuring camber are displayed in Figures 4.2 through 4.9 and Table 4.3. In general, the methods resulted in reasonable estimations of the camber for the sections considered. As mentioned previously, the initial camber value calculated at release was used in the long-term calculations. This initial calculated value varied considerably from the measured initial camber for many of the girders.

It should be noted that the scale in Figures 4.2 through 4.9 has been exaggerated in the y-direction in order to fully display differences between each of the methods. The graphs show the centerline plots over time for members from Bridge 19045 series of 139 foot long, 72 inch deep girders. Since the centerline plot contained the largest camber values, these were the only values plotted.

Several comments can be made about the graphs and the results that were determined from this analysis. The measured data looks reasonable with time when compared to other research such as that done at the University of Iowa [7] and the University of Texas [6]. Fluctuations in data, or times when the deflections appear to change in their direction of growth over time, were apparent in their data as well. The deflection fluctuations in the girder were partly due to temperature gradients caused by solar heating of the top flange. It can be assumed that this is the largest contributing factor to the consistent "dip" at about 35 days in Figures 4.2 to 4.9. The peak that occurs consistently happened on a partly cloudy afternoon, whereas the dip in these graphs occurs at mid-morning on a cloudy day. It will be shown in Chapter 5 that these circumstances in temperature could have resulted in the "dip" that was as large as one-half inch in camber.

Another possible source for the fluctuation in the data could be due to the level of precision in determining the camber. Because of storage limitations in the field, data had to be collected while setting up the level at one end of the girder for several of the members. Thus, readings were difficult to measure to the millimeter while set up at approximately 140 feet away from the far end. Data was carefully examined after being taken in order to eliminate as much of this problem as possible by looking at plots of the camber along the length for each set of readings. In the next paragraphs, conclusions will be made regarding each of the methods considered.

Results of the PCI Method

In evaluating the PCI method, it was found that the method tended to slightly overestimate the long-term camber -- especially at the 45 day erection stage. As was mentioned earlier, the plots for the PCI method could be oriented with a smoother transition shape passing through the points shown in Figures 4.2 to 4.9 in order to better approximate the actual centerline plot with time. Since no method for determining this shape was suggested by the PCI Design Handbook, only the three points were plotted. This resulted in underestimation of camber in the early ages of the concrete (5 to 45 days). Table 4.3 shows the overall averages for the difference between the modified measured results and the calculated values at discrete time intervals. Since the PCI Multiplier Method only predicts three points in time (the initial, erection (45 days), and the final deflection for the member) a fair assessment of the overall deviation with respect to the measured data was difficult. The overall deviation that is shown in this table represents an absolute value deviation with respect to the measured field data after 45 days. Even with this assessment of the PCI Multiplier Method, it still had the largest overall deviation with respect to the other methods, and in each case overpredicted the camber at erection and final conditions.

Results of Branson's approximate time step method

Branson's approximate time step method gave the lowest long-term camber predictions on average of the four methods considered. It had a range of deviation of ±11% with an average of 8%. This percentage may seem significant, but when a camber of around four inches is considered this is less than a half an inch at maximum deviation from the actual measured data. There was little consistency as to estimating the camber above or below the actual data.

A few qualifications should be made, however, in order to put this interpretation into perspective. Not all of the factors used in Branson's equation were used in the analysis because no superimposed loads were applied to the section. It was mentioned earlier that the coefficient kr, which appears in all factors except the live load deflection factor, has been questioned by Tadros et al. and others (see "Branson et al. Approximate Time-step Method").

Results using the program "CRACK"

The program "CRACK" gave reasonable results when considered in an overall fashion. Figures 4.2 through 4.9 compare the results with the measured data and the other methods. The CEB-FIP and the ACI committee models used with "CRACK" are represented respectively at data points with squares and slashes. The ACI 435R-95 and ACI 209R-92 committee recommendations produced the best overall results of the two model codes used with "CRACK." The largest source of error for the CEB-FIP model code came in the evaluation of girder members B13 and B14 of Bridge 19045 (see Figures 4.4 and 4.5) .

The source of difference was with respect to the creep coefficient for these two members. The CEB-FIP code uses a varying modulus of elasticity with time for determination of creep with respect to the 28-day modulus. These members were not released until three days after the casting date because they were cast on a Friday afternoon. As such, the calculated ratio for the modulus of elasticity with respect to release and 28 days was considerably higher than that of the other members. (Mokhtarzadeh et al. found that these values should be close to one another, consequently the 28-day modulus of elasticity was used at all ages for all methods except CEB-FIP.) Because this value is relied upon heavily in the creep function of the CEB-FIP model code, the model appears to overestimate the creep multiplier. Thus, overestimating the camber for the section. The reason for the overestimation can be understood by looking at the form of Equation 4.14. This problem with determining creep did not occur in the other methods observed as can be seen from the figures listed above. As such, the conclusion can be drawn that this is a unique problem to the model code CEB-FIP.

Results considering members cast on the same bed

As was mentioned in the introduction, more than one member was usually cast on the same bed; thus, both should have about the same strand stress. Even if the assumed initial stress was different than the design, both of these members should have the same stress in the section. If the concrete strengths were close to each other and the geometry of both members were the same, both girders should have similar amounts of deflection. This rule was generally true for all of the girders. This trend can be seen not only for the girders of Bridge 19045, but also in the girders from the other bridges. Plots of the measured camber for members cast on the same bed can be seen in Figures 4.10 -4.13. As can be seen, some discrepancy was noticed in the results due to varying concrete strengths, but can be accounted for. In Figures 4.10 and 4.11, the concrete strengths for each member shown were comparable. The data shown in Figure 4.10 was for two girders with five day concrete compressive strengths of 7900 psi and 8200 psi. Both members shown in Figure 4.11 had concrete strengths of 9550+ . In Figures 4.12 and 4.13, the concrete strengths for B16-S2 and B17-S2 are also shown with a plus sign behind the number. This mark is shown because the concrete cylinder was not tested beyond this strength for compression due to testing equipment limitations. Thus it should be realized that the strength may be much higher than that indicated.

Sensitivity Analysis

After having performed the long-term analysis, a sensitivity analysis was performed for the long-term factors. The program "CRACK" was used for the analysis because it used the most thorough analysis. The ACI Model Code was used with "CRACK" because it gave the best results in the long-term analysis. The sensitivity analysis led to some interesting conclusions regarding the initial project and the current model codes that are used. These results and possible explanations of the results will be discussed in the last chapter. In this section, the results of the long-term parametric study are presented. Table 4.4 lists variations to the base case over a 155 day period from the release of member B18-S2 of Bridge 19045. Girder B18-S2 was chosen for the analysis simply because it had some of the most complete data of the members analyzed. An explanation of the base case, as well as the variations, follows.

The base case for the long-term sensitivity analysis used the material properties assumed for short-term analysis of the base case. One exception to this statement would be that the actual 28-day strength of the member was used in determining the modulus of elasticity for the girder throughout the entire time history as opposed to the design 28-day strength. It was decided, based on the research of Mokhtarzadeh et al., that using the modulus of elasticity calculated from a concrete strength of 28 days would reflect the overall actual modulus of elasticity the most accurately. Ali Mokhtarzadeh et al.'s research found that 95% of the modulus of elasticity is developed in the first day of curing. Thus, leading to the conclusion that varying the modulus of elasticity over the time history is not necessary. Long-term factors for the base case were as follows:

- Relative Humidity was assumed to be 72%
  (from AASHTO charts on average relative humidity in the U. S.)

- Measured slump of the concrete was used (around 5.5 inches)

- Measured % fines were used (about 50% fines)

- ACI averaged ultimate creep factor was used before applying
  material factors (Cu = 2.35)

- ACI averaged shrinkage strain was used before applying material factors ($\varepsilon_s = 780 * 10^{-6} * in / in$)

- Low relaxation stress-relieved strands were assumed (as in design)

These base case assumptions were not only used in the parametric study, but were also the assumptions that were used in the analysis for "CRACK" using the ACI model codes [2,8].

The variations for the long-term analysis were chosen to understand the influence of possible ranges of input to the base case. Variation #1 consisted of increasing the relative humidity from 72% to 82%. The actual relative humidity for the Bridge 19045 girders from release until the end of the time period considered was around 84%. As such, a change of 10% in relative humidity to 82% was used for this variation. By increasing the relative humidity to 82%, a minor decrease in the calculated camber of 2% was noticed. This was consistent with the logic that a larger relative humidity decreases the shrinkage in the member, but the effect was considered to be minor.

Variation #2 consisted of changing the design slump of the concrete mix in order to account for possible mix variations other than the averaged values. The slump test, as summarized from ASTM C-143, is a test for the determination of the water content of a mix. A steel cone shaped mold with a base is filled with concrete and "tamped" with a rod. Tamping is the process of packing the concrete mix into the container to remove possible air pockets in the cement mix. After the concrete completely fills the cone and is leveled, the cone is removed. The difference in height between the top of the cone and the concrete after it has fallen is the slump of the mix. The lower the slump, the stiffer the mix and vice versa. Thus, by increasing the slump by 1 inch in this variation, the mix becomes less stiff. The ACI 209R model accounts for the water content with a factor that is multiplied to the shrinkage and creep coefficients based on the concrete slump. With a higher water content, it was expected that there would be a larger amount of shrinkage in the member. The camber in the parametric study increased an average value of 1.5%, indicating that the effect, according to the ACI model, is minimal.

The next variation was the use of the initial strength based modulus of elasticity over the time period. This variation was unrealistic because the actual modulus of elasticity is actually greater than the initial modulus of elasticity throughout the majority of the time history. The method actually recommended by ACI 209 recommends varying the modulus of elasticity with time. Variation #3 did show how important it was to use a close approximation to the actual modulus of elasticity for the member. The error in using this variation was very high and unrealistic, but did approximate the initial and early stages of camber most accurately (see Figure 4.14). However, it should be noted that this increased accuracy was only seen in the Bridge 19045 girder set. Other measured data (from Bridges 49535 and 27624) indicated that the calculated camber was already greater than measured when the 28-day strength was used. In this case, a lower modulus would induce more error in the initial calculated camber. Because of the lack of actual data with regard to support conditions, these bridge members were not compared for long-term effects to calculation methods.

Variation #4 consisted of changing the percent of fine materials by weight in the mix. Fine materials are normally associated with sand materials. By increasing the fines in the section from 50% to 60%, very little difference in the camber was anticipated by the ACI model. As such, this factor was not considered to be critical as can be seen in Table 4.4.

Variation #5 consisted of changing from the use of the averaged ultimate shrinkage strain ($780 * 10^{-6}$ in/in) to the low end of the recommended shrinkage strain values produced from past testing. This range was outlined in ACI 209R-92 [2] and had a value of $730 * 10^{-6}$ in/in. This low end value had all of the ACI 209R correction factors for shrinkage applied to it that were used in the original analysis for reflecting the member properties. From the long-term sensitivity analysis, it was apparent that this does not cause a significant change. A difference of 0.3% was determined, as can be seen in Table 4.4.

Variation #6 consisted of changing from using low relaxation strands to stress relieved strands. This was done to quantify the differences between the strand types. The results shown in Table 4.4 with regards to the long-term camber sensitivity analysis are clear. This change decreased the camber by 10% and gives an idea of the anticipated effect of using one strand type over the other.

Variation #7 consisted of changing from the use of the averaged ultimate creep multiplier of 2.35 to a value of 2. By making this change in creep factor, the camber decreased by 4% from the base case. This variation had a relatively small effect (<5%) and in all likelihood was not the only contributing source of fluctuation in the results; however, it did come nearest to modeling the actual long-term camber beyond the 28-day period for girder B18-S2. This was not necessarily the case with all the girder members from Bridge 19045, but was a general trend for many of the actual measurements from girders of that bridge.

In conclusion, the variations had little effect with a few exceptions. These exceptions included: using the initial modulus of elasticity, changing the strand type, and changing the ultimate creep factor to 2. This indicates the importance of calculating a reasonable value for the modulus of elasticity of the section. It is this variation that proved to be the most plausible source of error in the long term study. In retrospect, using laboratory measured values for the modulus of elasticity would have been beneficial for comparisons of the methods instead of using an equation estimated value based on the concrete compressive strength.

Chapter Five – Thermal Effects

During long-term camber measurements of Bridge 19045 members, a peak reading followed by a dip was observed (Figures 4.2 and 4.9). Other member series (e.g. Bridges 29535 and 27624) did not experience the same dip; however, they did exhibit some fluctuation of camber with time. The peculiarity in Bridge 19045 members was attributed partly to thermal effects.

In the sensitivity analysis of Chapter 3, gradient or parabolic thermal distributions were observed to have a significant affect on camber in comparison with changes due to ambient air temperatures. For the members of Bridge 19045, data before the peak was taken at 10:00 am on a cloudy day with an ambient temperature of 32^0 F. The peak in the data occurred at 1:00 pm on a partly-sunny day with an ambient temperature of 32^0 F, and the dip in data occurred at 11:00 am on a cloudy day with an ambient temperature of 34^0 F. As evident from these observations, the variations in camber correlated with the degree of solar radiation.

To investigate this phenomenon further, a one-day thermal analysis was conducted on a member from Bridge 86512. The girder had a depth of 72 inches and a length of 128.5 feet (approximately 10 feet shorter than Bridge 19045 members). Over an 8 hour period, the camber was observed to increase by over 1.5 inches (Figures 5.1 and 5.2). The corresponding girder temperatures, measured with a thermometer on the top of the girder, increased from $80\text{-}110^0$ F due to solar radiation (see Figure 5.3). This led to the conclusion that a 34^0 F temperature change due to solar heating of the top flange of the member could cause an additional camber of 0.5 inches for the members studied.

Related Thermal Research

Much work has been done by Kelly et al. [6] in relation to thermal gradients in concrete members. When measuring temperatures internally and externally using thermocouples on the section, Kelly et al. found that in the morning the temperature of the concrete was warmer than the ambient air temperature and that by late afternoon the reverse was true. They also noticed that the temperature of the girder, after already being put into place in the structural system, had a maximum temperature variation along the cross section of about 15° F during a hot summer day. From their plots, it can also be seen that the temperature varied along the span in a nonlinear fashion. However, it should be noted that the temperature variations were determined in the State of Texas which generally is warmer than Minnesota.

Ghali and Favre have suggested a method for accounting for nonlinear thermal gradient changes in concrete members [3, pages 23 -27]. Since the members under consideration were simply supported, no indeterminate thermal effects need be considered. As mentioned previously in Chapter 3, the Ghali et al. method uses both a free thermal expansion factor and a self equilibrating factor to determine the strain in the section. Ghali's thermal analysis was based on using the strain and curvature caused by the nonlinear gradient to calculate the camber in the section (Figure 3.3) . The strains and curvature were determined in a similar fashion to that outlined in Chapter 4 for the program "CRACK."

This method for determining the thermal deflection in the section was checked by using Ghali et al's method. Since Ghali and Favre's text [3] prescribed no function to model the distribution curve of the thermal strains they recommended (Figure 3.1), a function was created that appears similar to that of the strain distribution shown by Ghali and Favre. The shape that they suggest for the thermal distribution was arbitrary, but appears reasonable with respect to data collected by Kelly et al [6]. The results of this analysis led to an estimated maximum thermal increase in camber of 0.25 inches as opposed to the measured thermal increase in camber of around 0.5 inches over one day.

In conclusion, the thermal influences are more than adequate to account for deviations in the camber change over time for the data collected. This was recognized in the one day field data study in addition to calculations that were made to approximate this effect. Thus, data and the results thereof must be assessed with the consideration that thermal effects will result in as much as a 10% change in camber on a given day.

Chapter Six - Conclusions

In this chapter, the overall conclusions to the project will be presented. Because most of the results for the project have been discussed in their respective chapters, this chapter will focus on considering the findings and interpretations of the data collected as well as past interpretations from other research projects.

In Kelly et al.'s work [6], they found that the PCI method generally overestimated camber for the section. This conclusion was also determined in this research (see Figures 4.2 to 4.9). Naaman commented in his text [5] that Branson's approximate time-step method appeared to underestimate the camber. In some cases Branson's approximate approach closely reflected that of the measured camber that was calibrated to a simply supported case. But as previously mentioned, the method has been questioned by Tadros et al. and others for the way it considers partially prestressed members (see Chapter 4). Since none of the members analyzed in this project contained a significant amount of nonprestressed reinforcement, no conclusions were drawn from this research with regards to accuracy in determining camber for partially prestressed members.

The short-term analysis results proved to be quite reasonable when an overall perspective is used. Members from Bridge 27112 had a large difference in camber between the calculated and measured, but that was mainly due to a lack of field data for the pick-up and set-down of the members to relieve possible friction between the bed and the member.

From the study, the following conclusions were found:

- The assumed prestress force will have one of the largest effects on camber in the section. A 5 % variation in prestress force can change the camber by 10-16%.
- An accurate assessment of the actual modulus of elasticity would contribute to a better assessment of the actual camber.
- If consideration for thermal properties is given, it is difficult to predict the slope and size of the gradient. Based on the measured data, estimates of thermal effect range from 8 - 12% for a 20° F variation in temperature over the cross section.

* The effect of solar radiation measured in the field over a one day period was 11% from the beginning of the day to the peak of the camber change. This corresponded to a 30° F temperature change on the top of the girder throughout the day.

* Using the transformed moment of inertia versus the gross moment of inertia of the concrete decreased the camber by 10 to 17% . This range depends on the size of the member and the density of steel.

* Branson's approximate time-step approach resulted in the closest approximations to the measured cambers. This conclusion must be considered with the realization that only a few methods were used and only a few types of prestressed members were incorporated; specifically, the members contained a minimum amount of nonprestressed steel.

* On average, a deviation from the actual camber by using the approximate method or the program "CRACK" with the ACI model code long term factors is expected to be around 10 to 15% variation.

* Friction losses between the bed and the member immediately after release ranged from 5 to 20%, with an average value of 12%. Note that because the lift hooks were located a distance from the end of the member, the friction of the bed caused greater cambers than would have been observed in a frictionless bed.

* Girders cast on the same bed with the same strands and patterns had an initial camber difference of 2 to 10%. One possible explanation for part of this difference is the variance in concrete strengths.

* Measured camber for the members of Bridge 19045 had a scatter in the data of 1 inch in the long term analysis. This scatter was partly a result of material variances (examples: concrete strength, modulus of elasticity, and strand stress). environmental issues also contributed to the scatter of the data (examples: weather on day of casting, and storage location for the member). A plot of this scatter in the data can be seen in Figure 6.1.

* From the Sensitivity Analyses the following had little effect on the calculated camber:

- Changing the steel modulus of elasticity by 500 ksi
- Changing the concrete weight by 5 lb/ft^3
- Changing the length of the beam or location of the draping point by 1 ft
- Constant thermal expansion
- Increasing the relative humidity by 10%
- Increasing the percentage of fines in the mix
- Using a 1 inch larger slump for the concrete mix

* The parameters which had the greatest effect on the calculated cambers were:
  - Changing the initial jacking ratio by 5%
  - Using the transformed moment of inertia vs. gross moment of inertia
  - Assuming 28-day modulus of elasticity at release
  - Changing the assumed ultimate creep (2 vs. 2.35) had a significant effect on long-term cambers

Using best estimations for section and material properties, the ratio of calculated to measured initial cambers differed by up to 20% (Appendix D). This difference increased up to 40% in cases where the camber was measured before pick/up set/down (attributed to friction between the girder and the bed). Although these percentages seem high, they represent camber differences of less than 0.5 in.

Measured cambers were observed to differ even for similar girders cast together on the same bed. These differences ranged up to 0.5 in. (over time these differences increased up to 0.75 in. in some cases). Typically this difference was less than 0.25 in. In addition, solar radiation had a significant effect on the results. Camber measured for a single girder had a variance of 0.5 in. monitored over a single day.

It is recommended that when using the PCI Method, Branson's Approximate Time Step Method, and the program "CRACK" by Ghali et al., careful consideration of the conclusions drawn above should be employed. Further, the modeling equations for the material properties that are used with these methods should be reevaluated in light of new research as to their accuracy. In general, the PCI Method, although simple, gave reasonable long-term camber results compared with the more detailed methods.

References

1. Nawy, Edward G., Prestressed Concrete - A Fundamental Approach. Prentice Hall. Englewood Cliffs, New Jersey. 1989.

2. American Concrete Institute Committee 209. ACI 209R-92 - Prediction of Creep Shrinkage, and Temperature Effects in Concrete Structures. American Concrete Institute. Detroit, Michigan. 1992.

3. Ghali, A. and Favre, R. Concrete Structures: Stress and Deformations. Chapman and Hall. New York, New York. 1986.

4. PCI Industry Handbook Committee. PCI Design Handbook - Precast and Prestressed Concrete (4th ed.). Precast/Prestressed Concrete Institute, Chicago, Illinois. 1992.

5. Naaman, Antoine E. Prestressed Concrete Analysis and Design - Fundamentals. McGraw-Hill, Inc. New York, New York. 1982.

6. Kelly, D.J., Bradberry, T.E. Breen, J.E. Time Dependent Deflections of Pretensioned Beams. Center for Transportation Research. Austin, Texas. 1987.

7. Branson, D.E. Meyers, B.L. Kripanaryann, K.M. Loss of Prestress, Camber, and Deflection of Non-Composite Structures and Composite Structures Using Different Weight Concretes. University of Iowa. Iowa City, Iowa. 1970.

8. American Concrete Institute Committee 435. ACI 435R-95 - Control of Deflections in Concrete Structures. American Concrete Institute. Detroit, Michigan. 1995.

9. Nilson, Arthur H. Design of Prestressed Concrete - Second Edition. John Wiley & Sons Inc. New York, New York. 1987.

10. Libby, James R. Modern Prestressed Concrete: Design Principles and Construction Methods Van Nostrand Reinhold. New York, New York. 1990.

11. Collins and Mitchell. Prestressed Concrete Structures Englewood Cliffs. New Jersey. 1991.

12. Ghali. Users Manual and Computer Program CRACK Department of Civil Engineering. Alberta, Canada. 1991.

13. Martin, Leslie D. PCI Journal "A Rational Method for Estimating Camber and Deflection of Precast Prestressed Members." PCI Journal Publications. January/February 1977.

Table 2.1 - Field Data Information

Bridge Number	Number of Girders Measured	Member Length (feet)	Member Depth (inches)	Range of Release Dates	Last Camber Measurement	Temperature Range* (Fahrenheit)	Relative Humidity Range* (%)
27112	16	95	45	10/23 - 11/17/92	6/19/93	25-64	55-100
4516	5	95	45	11/17 - 11/25/92	12/21/92	25-40	55-100
49535	4	67	45	7/21/93	9/7/93	56-81	37-90
27624	4	139	81	7/27/94	9/22/93	56-74	37-78
19045	10	139	72	11/9 - 11/17/94	4/19/95	30-62	39-99.5

* Note: These values are for days when camber measurements were taken.

Table 3.1 - Camber Sensitivity Results
(Camber in inches)

Steel Density	Girder Type	Base Case	Variation #1 Change steel to 29000 ksi	Variation #2 Eci based on fci	Variation #3 Eci using ACI 318 (eq. 8.5.1)	Variation #4 Changing initial jacking ratio 5%	Variation #5 Changing from Tran. I @ CL to I gross	Variation #6 Changing Concrete weight by 5 pcf increase	Variation #7 Changing draping points by 1 foot	Variation #8 Changing length of beam - 1 foot shorter	Variation #9 Changing the support condition to Storage condition	Variation #10 Using the actual concrete strengths	Variation #11 Consider constant thermal expan, (20 deg.)	Variation #12 Consider gradient thermal expan. (20-0 deg)	Variation #13 Constant thermal expan. in the top flange (20 deg.)	Measured #14 Actual data collected after pick-up & set-down	Measured #15 Actual Data after moving to temp storage area
0.01051	72-139 BR19045	2.43	2.41	2.52	2.20	2.74	2.84	2.35	2.41	2.42	2.68	2.26	2.43	2.99	2.46	2.72	2.97
0.00981	45M-95 BR27112	2.01	2.00	2.11	1.81	2.24	2.30	1.97	1.99	1.99	Data missing	1.85	2.01	2.43	2.07	Data missing	Data missing
0.00685	45M-67 BR49535	0.97	0.97	1.02	0.84	1.06	1.07	0.96	0.96	0.95	Data missing	0.94	0.97	1.18	0.97	0.83	1.20

Table 3.2 - Sensitivity Results (Percentage)
(Percentages based on the assumptions made in the base analysis)

Steel Density	Girder Type	Variation #1 Change steel to 29000 ksi	Variation #2 Eci based on fci	Variation #3 Eci using ACI 318 (eq. 8.5.1)	Variation #4 Changing initial jacking ratio 5%	Variation #5 Changing from Tran. I @ CL to I gross	Variation #6 Changing Concrete weight by 5 pcf increase	Variation #7 Changing draping points by 1 foot	Variation #8 Changing length of beam - 1 foot shorter	Variation #9 Changing the support condition to Storage conditions	Variation #10 Using the actual concrete strengths	Variation #11 Consider constant thermal expansion (20 degr.)	Variation #12 Consider gradient thermal expansion (20-0 deg)	Variation #13 Constant thermal expan. in the top flange (20 degr.)	Measured #14 Actual data collected after pick-up & set-down	Measured #15 Actual Data after moving to temp storage area
0.01051	72-139 BR19045	-0.68	3.74	-9.31	13.07	16.92	-3.20	-0.76	-0.42	10.56	-6.93	0.01	23.36	1.50	12.04	22.34
0.00981	45M-95 BR27112	-0.62	4.65	-10.12	10.99	14.11	-2.16	-1.02	-1.21	Data missing	-7.96	0.01	20.75	2.76	Data missing	Data missing
0.00685	45M-67 BR49535	-0.38	5.35	-13.65	9.00	10.48	-1.13	-1.24	-2.50	Data missing	-3.63	0.02	20.85	2.79	-14.66	23.39

Table 3.3 - Short Term Analysis Results (Bridge #19045; B11-S2 to B20-S2)

BR#19045 (72"-139')	B11-S2	B12-S2	B13-S2	B14-S2	B15-S2	B16-S2	B17-S2	B18-S2	B19-S2	B20-S2
Calculated Values for the members when supported at the ends (inches)	2.26	2.26	2.24	2.23	2.29	2.23	2.18	2.26	2.29	2.28
Actual Values * (inches)	Data Missing	Data Missing	Data Missing	Data Missing	2.44	Data Missing	2.48	2.72	2.48	Data Missing
% Difference					-5.95		-11.94	-16.93	-7.47	

* Data collected after pick-up and set down. The members at this point are actually supported at their ends.

Table 3.4 - Short Term Analysis Results (Bridge #27112; B2-S2-1 to B2-S2-9)

BR#27112 (45M-95')	B2-S2-1	B2-S2-2	B2-S2-3	B2-S2-4	B2-S2-5	B2-S2-6	B2-S2-7	B2-S2-8	B2-S2-9
Calculated Values (inches)	1.88	1.85	1.85	1.86	1.84	1.87	1.84	1.88	1.88
Actual Values ** (inches)	1.37	1.37	1.37	1.37	1.50	1.37	1.63	1.63	1.37
% Difference	36.92	35.39	35.39	35.91	22.85	36.23	13.05	15.08	37.15

** The Actual Data in this case does not account for friction losses.
No data was available for pick-up & set-down of the member

Table 3.5 - Short Term Analysis Results (Bridge #27112; B2-S2-10...B2-S2-26)

BR#27112 (45M-95')	B2-S2-10	B2-S2-11	B2-S2-12	B2-S2-13	B2-S2-14	B2-S2-15	B2-S2-26
Calculated Values (inches)	1.88	1.88	1.89	1.85	1.85	1.88	1.82
Actual Values ** (inches)	1.50	1.50	1.50	1.63	1.50	1.50	1.75
% Difference	25.05	25.26	25.73	13.49	23.32	25.05	3.75

** The Actual Data in this case does not account for friction losses.
No data was available for pick-up & set-down of the member

Table 3.6 - Short Term Analysis Results (Bridge #04516; B2-S2-1 to B2-S2-5)

BR#04516 (45M-95')	B2-S2-1	B2-S2-2	B2-S2-3	B2-S2-4	B2-S2-5
Calculated Values (inches)	2.13	2.13	2.13	2.08	2.17
Actual Values ** (inches)	1.75	1.75	1.87	1.75	1.63
% Difference	21.95	21.95	13.76	18.88	33.13

** The Actual Data in this case does not account for friction losses. No data was available for pick-up & set-down of the member

Table 3.7 - Short Term Analysis Results (Bridge#49535; 1-B1...4-B3)

BR#49535 (45M-67')	1-B1	2-B2	3-B2	4-B3
Calculated Values (inches)	0.88	0.92	0.91	0.94
Actual Values (inches)	0.75	0.83	0.91	0.83
% Difference	17.57	10.96	-0.50	12.92

Table 3.8 - Short Term Analysis Results (Bridge #27624; B8S5N-366...B9S5N-137)

BR#27624 (81M-139')	B8S5N-366	B7S5N-365	B10S5N-368	B9S5N-137
Calculated Values (inches)	2.38	2.30	2.28	2.27
Actual Values (inches)	2.36	2.24	2.24	2.56
% Difference	0.97	2.75	1.89	-11.22

Table 3.9 -Measured Friction Losses in the Bed (Bridge #19045; B15-S2...B19-S2)

BR#19045 (72M-139')	B15-S2	B17-S2	B18-S2	B19-S2
Camber Before Pick-up (inches)	2.28	2.09	1.97	2.40
Camber After Pick-up (inches)	2.44	2.48	2.72	2.48
% Difference	7.02	15.7	27.6	3.2

Table 3.10 - Measured Friction Losses in the Bed (Bridge #49535; 1-B1…4-B3)

BR#49535 (45M-67')	1-B1	2-B2	3-B2	4-B3
Camber Before Pick-up (inches)	0.67	0.73	0.51	0.79
Camber After Pick-up (inches)	0.75	0.83	0.63	0.83
% Difference	10.7	12.0	19.0	4.8

Table 3.11 - Measured Friction Losses in the Bed (Bridge #27624; B8S5N-366…B9S5N-137)

BR#27624 (81M-139')	B8S5N-366	B7S5N-365	B10S5N-368	B9S5N-137
Camber Before Pick-up (inches)	2.17	2.09	1.89	2.09
Camber After Pick-up (inches)	2.36	2.24	2.24	2.56
% Difference	19.6	6.7	15.6	18.3

Table 4.1 PCI Multipliers to Estimate Long-Term Cambers and Deflections
[Reference 4: Table 4.6.2]

	Without Composite Topping	**With Composite Topping**
At erection:		
(1) Deflection (downward) component—apply to the elastic deflection due to the member weight at release of prestress	1.85	1.85
(2) Camber (upward) component—apply to the elastic camber due to prestress at the time of release of prestress	1.80	1.80
Final:		
(3) Deflection (downward) component—apply to the elastic deflection due to the member weight at release of prestress	2.70	2.40
(4) Camber (upward) component—apply to the elastic camber due to prestress at the time of release of prestress	2.45	2.20
(5) Deflection (downward)—apply to elastic deflection due to superimposed dead load only	3.00	3.00
(6) Deflection (downward)—apply to elastic deflection caused by the composite topping	—	2.30

Table 4.2 Camber Reduction Considering Support Conditions (using various modeling assumptions)

Bridge 19045, Girder B18-S2
Cement Coefficient Multiplier = 2.0, using CEB-FIP Model Code

Concrete Age (days)	Calc. Cambers Supported at Ends (inches)	Calc. Cambers Supported at Location Measured Relative to the Ends (inches)	Percent Reduction of Camber for Correlation of Support Conditions
0.75	2.31	2.68	
2.75	3.20	3.73	85.81
15.75	3.68	4.30	85.49
28.75	3.90	4.58	85.34
36.75	4.00	4.69	85.28
61.75	4.19	4.92	85.14
		Average Difference =	85.41

Bridge 19045, Girder B18-S2
Cement Coefficient Multiplier = 1.0, using CEB-FIP Model Code

Concrete Age (days)	Calc. Cambers Supported at Ends (inches)	Calc. Cambers Supported at Location Measured Relative to the Ends (inches)	Percent Reduction of Camber for Correlation of Support Conditions
0.75	2.31	2.68	
2.75	2.98	3.46	86.00
15.75	3.25	3.79	85.76
28.75	3.40	3.97	85.65
36.75	3.46	4.05	85.60
61.75	3.61	4.23	85.49
		Average Difference =	85.70

Bridge 19045, Girder B18-S2
Using ACI 209R & 435R Recommendations

Concrete Age (days)	Calc. Cambers Supported at Ends (inches)	Calc. Cambers Supported at Location Measured Relative to the Ends (inches)	Percent Reduction of Camber for Correlation of Support Conditions
0.75	2.27	2.63	
2.75	2.68	3.11	85.94
15.75	3.23	3.78	85.60
28.75	3.45	4.03	85.44
36.75	3.53	4.14	85.37
61.75	3.70	4.34	85.20
		Average Difference =	85.51

Bridge 19045, Girder B11-S2
Cement Coefficient Multiplier = 1.0, using CEB-FIP Model Code

Concrete Age (days)	Calc. Cambers Supported at Ends (inches)	Calc. Cambers Supported at Location Measured Relative to the Ends (inches)	Percent Reduction of Camber for Correlation of Support Conditions
0.75	2.31	2.69	
5.75	3.07	3.58	85.76
21.75	3.33	3.89	85.55
34.75	3.45	4.04	85.47
42.75	3.51	4.11	85.42
67.75	3.65	4.27	85.33
		Average Difference =	85.51

Overall Average Difference= 85.53

Table 4.3 Deviation between Measured Data and Predictions (Percentage)

Bridge 19045	**Overall Average Deviation for Measured Data from Calculated (percentage)**								
Method of Calculation	B11-S2	B12-S2	B13-S2	B14-S2	B15-S2	B16-S2	B17-S2	B18-S2	Overall Averages
Program "CRACK"									
CEB-FIP Model Code (MC-78	4.13	7.37	22.12	22.30	3.90	15.20	6.38	6.07	10.93
ACI 209R-92 & 435R-95	4.68	7.31	11.05	12.62	5.11	15.43	9.10	8.67	9.25
Approximate Time Step Method (Branson's Method)	9.52	10.81	7.19	8.57	6.97	10.04	5.43	4.98	7.94
PCI Multipliers Method ** (Leslie D. Martin)	5.66	4.99	19.61	21.93	9.26	24.45	16.88	15.88	14.83

**Note: In order to assess the Overall Deviation for the PCI method in a fair manner, the Overall Deviation was determined from the 45 day erection point to the end of data readings. Because the known shape was parabolic initially and approximately linear after this stage, a more accurate assesment could be made.

Table 4.4 Long-Term Camber Sensitivity Analysis

(Using ACI 435R, ACI 209R, and Program "Crack") -- Bridge 19045, Girder B18-S2

Time From Release	Base Case	Variation #1 Increasing RH by 10%	Variation #2 Increasing C. Slump by 1 inch	Variation #3 Using the Initial Mod. of Elasticity (=6113 psi)	Variation #4 Increasing % of Fines From 49.8% to 60%	Variation #5 Using ** Low-Range Ultimate Shr. Strain (730*10^-6in/in)	Variation #6 Using Normal Stress Relieved Strands	Variation #7 Using ** An Ultimate Creep Factor = 2	Modified Measured Camber
0	2.27	2.27	2.27	2.58	2.27	2.27	2.12	2.27	2.72
2	2.68	2.64	2.70	3.03	2.69	2.68	2.46	2.61	3.06
15	3.23	3.16	3.28	3.63	3.25	3.23	2.90	3.09	3.45
28	3.44	3.35	3.50	3.85	3.47	3.44	3.06	3.27	3.20
36	3.52	3.43	3.59	3.94	3.55	3.53	3.13	3.34	3.27
61	3.69	3.59	3.76	4.12	3.72	3.69	3.26	3.48	3.43
97	3.82	3.72	3.90	4.26	3.86	3.83	3.37	3.60	3.50
155	3.99	3.88	4.05	4.44	4.03	4.05	3.51	3.74	3.64
	Deviations From the Base Case								
0		0.00	0.00	13.60	0.00	0.00	-6.53	0.00	19.76
2		-1.27	0.88	13.08	0.40	0.01	-8.02	-2.31	14.46
15		-2.26	1.62	12.36	0.73	0.10	-10.28	-4.43	6.82
28		-2.57	1.80	12.08	0.80	0.15	-10.89	-5.01	-6.97
36		-2.62	1.87	11.96	0.83	0.18	-11.11	-5.22	-7.24
61		-2.68	1.96	11.72	0.86	0.23	-11.56	-5.59	-6.86
97		-2.69	2.03	11.53	0.89	0.29	-11.92	-5.85	-8.38
155		-2.70	1.51	11.42	0.96	1.47	-12.05	-6.15	-8.81
	Average	-2.10	1.46	12.22	0.68	0.30	-10.30	-4.32	0.35

** Note: These cases were additionally modified by the correction factors for non-standard concrete conditions outlined in the ACI Committee Reports.

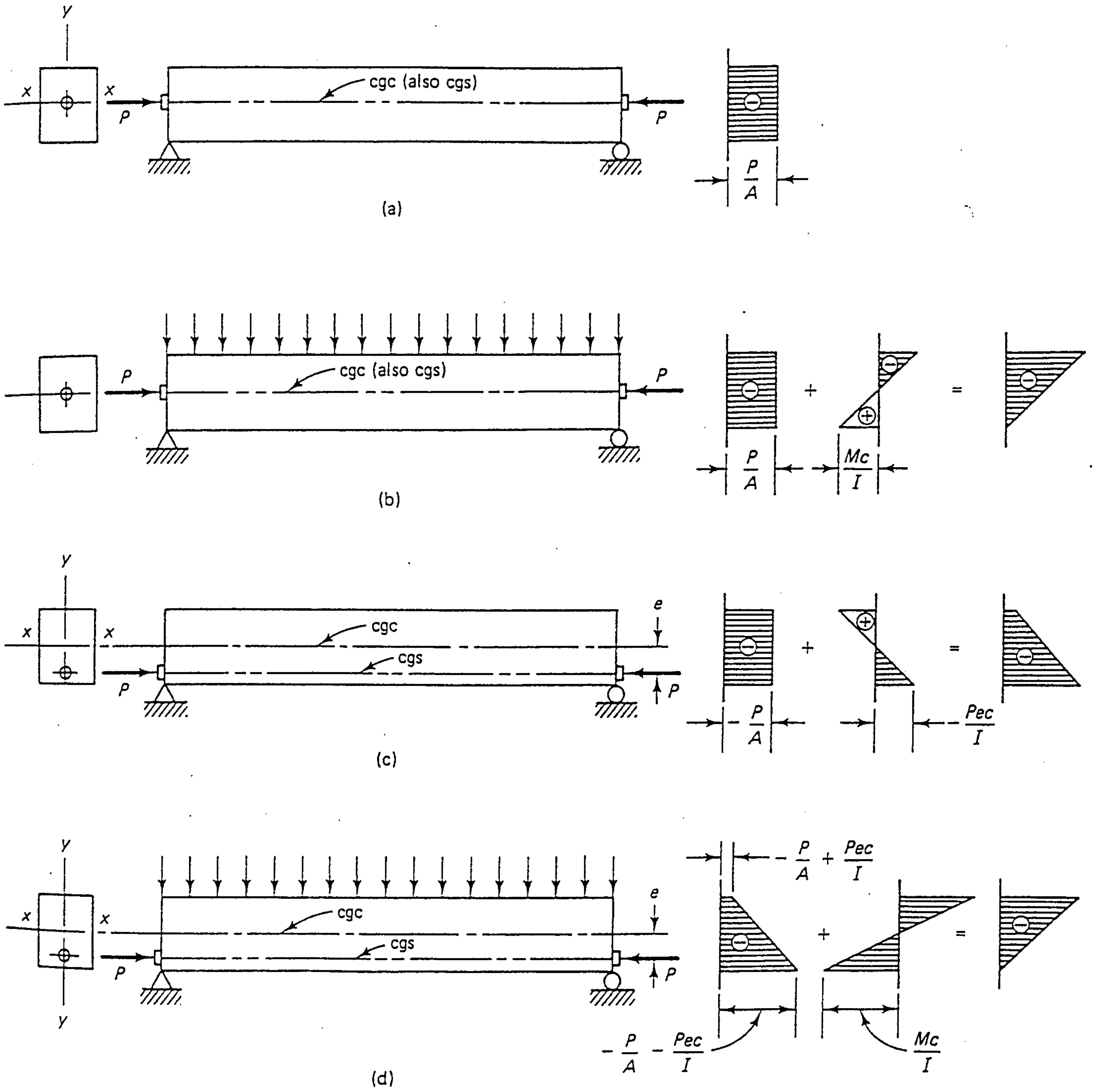
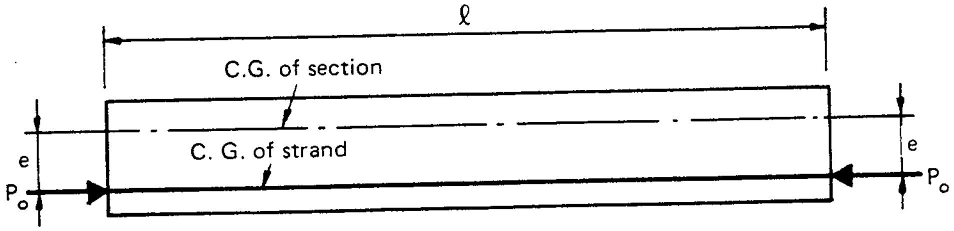
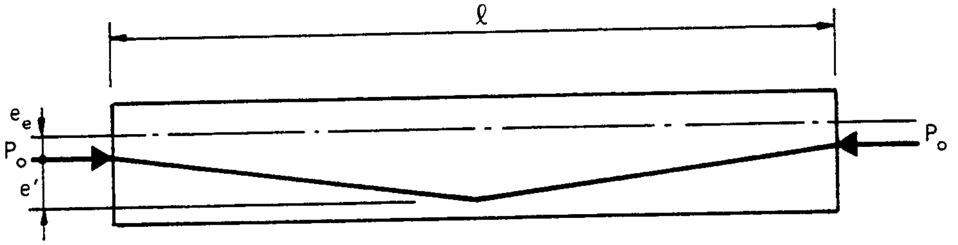


Figure 1.1 Concrete Stress Distribution in Rectangular Beam with Straight Tendon [Reference 1: Figure 1.2]



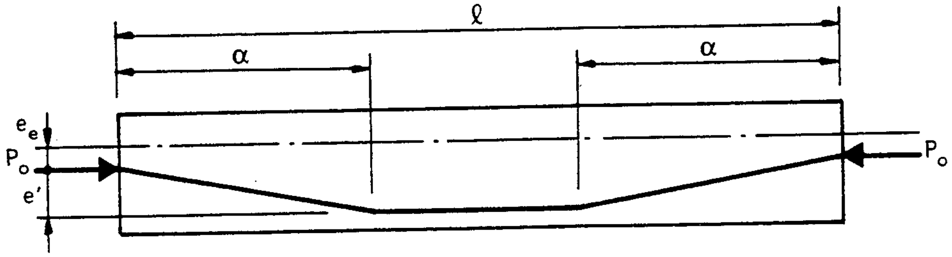
Straight strands

$$\Delta\uparrow \;=\; \frac{P_o e \ell^2}{8EI}$$



Single point depressed

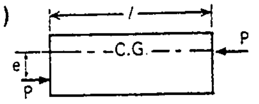
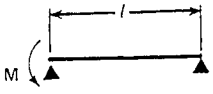
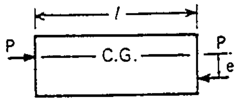
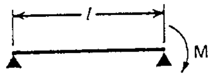
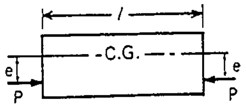
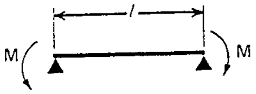
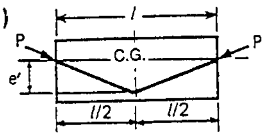
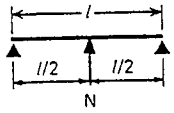
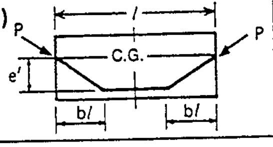
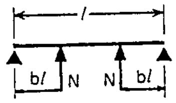
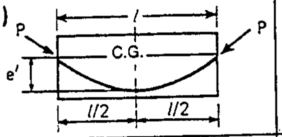
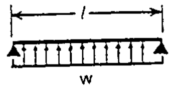
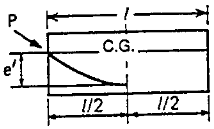
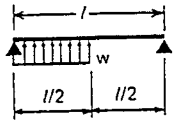
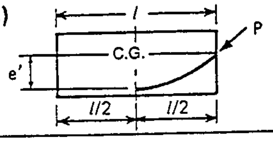
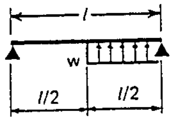
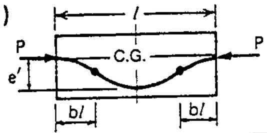
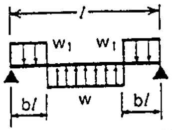
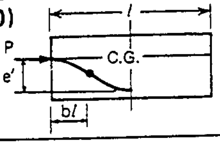
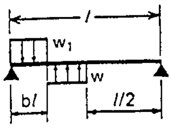
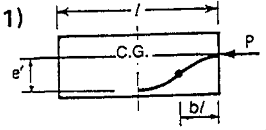
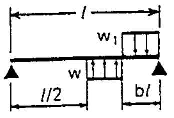
$$\Delta\uparrow \;=\; \frac{P_o e_e \ell^2}{8EI} + \frac{P_o e' \ell^2}{12EI}$$



Two point depressed

$$\Delta\uparrow \;=\; \frac{P_o e_e \ell^2}{8EI} + \frac{P_o e'}{EI}\left(\frac{\ell^2}{8} - \frac{\alpha^2}{6}\right)$$

Figure 3.1 Camber Equations for Typical Strand Profiles
[Reference 4: Figure 4.10.14]

Prestress Pattern	Equivalent Moment or Load	Equivalent Loading	Camber	End Rotation	
			+ ↑	+	+
(1) 	M = Pe		$+\frac{Ml^2}{16\,EI}$	$+\frac{Ml}{3\,EI}$	$-\frac{Ml}{6\,EI}$
(2) 	M = Pe		$+\frac{Ml^2}{16\,EI}$	$+\frac{Ml}{6\,EI}$	$-\frac{Ml}{3\,EI}$
(3) 	M = Pe		$+\frac{Ml^2}{8\,EI}$	$+\frac{Ml}{2\,EI}$	$-\frac{Ml}{2\,EI}$
(4) 	$N = \frac{4\,Pe'}{l}$		$+\frac{Nl^3}{48\,EI}$	$+\frac{Nl^2}{16\,EI}$	$-\frac{Nl^2}{16\,EI}$
(5) 	$= \frac{Pe'}{bl}$		$+\frac{b\,(3-4b^2)\,Nl^3}{24\,EI}$	$+\frac{b\,(1-b)\,Nl^2}{2\,EI}$	$-\frac{b\,(1-b)\,Nl^2}{2\,EI}$
(6) 	$w = \frac{8\,Pe'}{l^2}$		$+\frac{5wl^4}{384\,EI}$	$+\frac{wl^3}{24\,EI}$	$-\frac{wl^3}{24\,EI}$
(7) 	$w = \frac{8\,Pe'}{l^2}$		$+\frac{5\,wl^4}{768\,EI}$	$+\frac{9\,wl^3}{384\,EI}$	$-\frac{7\,wl^3}{384\,EI}$
(8) 	$w = \frac{8\,Pe'}{l^2}$		$+\frac{5\,wl^4}{768\,EI}$	$+\frac{7\,wl^3}{384\,EI}$	$-\frac{9\,wl^3}{384\,EI}$
(9) 	$w = \frac{4\,Pe'}{(0.5-b)\,l^2}$ $w_1 = \frac{w}{b}\,(0.5-b)$		$\left[\frac{5}{8} - \frac{b}{2}\,(3-2b^2)\right]\frac{wl^4}{48\,EI}$	$+\frac{(1-b)\,(1-2b)\,wl^3}{24\,EI}$	$-\frac{(1-b)\,(1-2b)\,wl^3}{24\,EI}$
(10) 	$w = \frac{4\,Pe'}{(0.5-b)\,l^2}$ $w_1 = \frac{w}{b}\,(0.5-b)$		$\left[\frac{5}{16} - \frac{b}{4}\,(3-2b^2)\right]\frac{wl^4}{48\,EI}$	$\left[\frac{9}{8} - b\,(2-b)^2\right]\frac{wl^3}{48\,EI}$	$\left[-\frac{7}{8} + b\,(2-b^2)\right]\frac{wl^3}{48\,EI}$
(11) 	$w = \frac{4\,Pe'}{(0.5-b)\,l^2}$ $w_1 = \frac{w}{b}\,(0.5-b)$		$\left[\frac{5}{16} - \frac{b}{4}(3-2b^2)\right]\frac{wl^4}{48\,EI}$	$\left[\frac{7}{8} - b\,(2-b^2)\right]\frac{wl^3}{48\,EI}$	$\left[-\frac{9}{8} + b\,(2-b)^2\right]\frac{wl^3}{48\,EI}$

* The tabulated values apply to the effects of prestressing. By adjusting the directional rotation, they may also be used for the effects of loads. For patterns 4-11, superimpose on 1, 2, or 3 for other C.G. locations.

Figure 3.2 Camber and Rotation Coefficients for Prestress Force and Loads
[Reference 4: Design Aid 11.1.4]

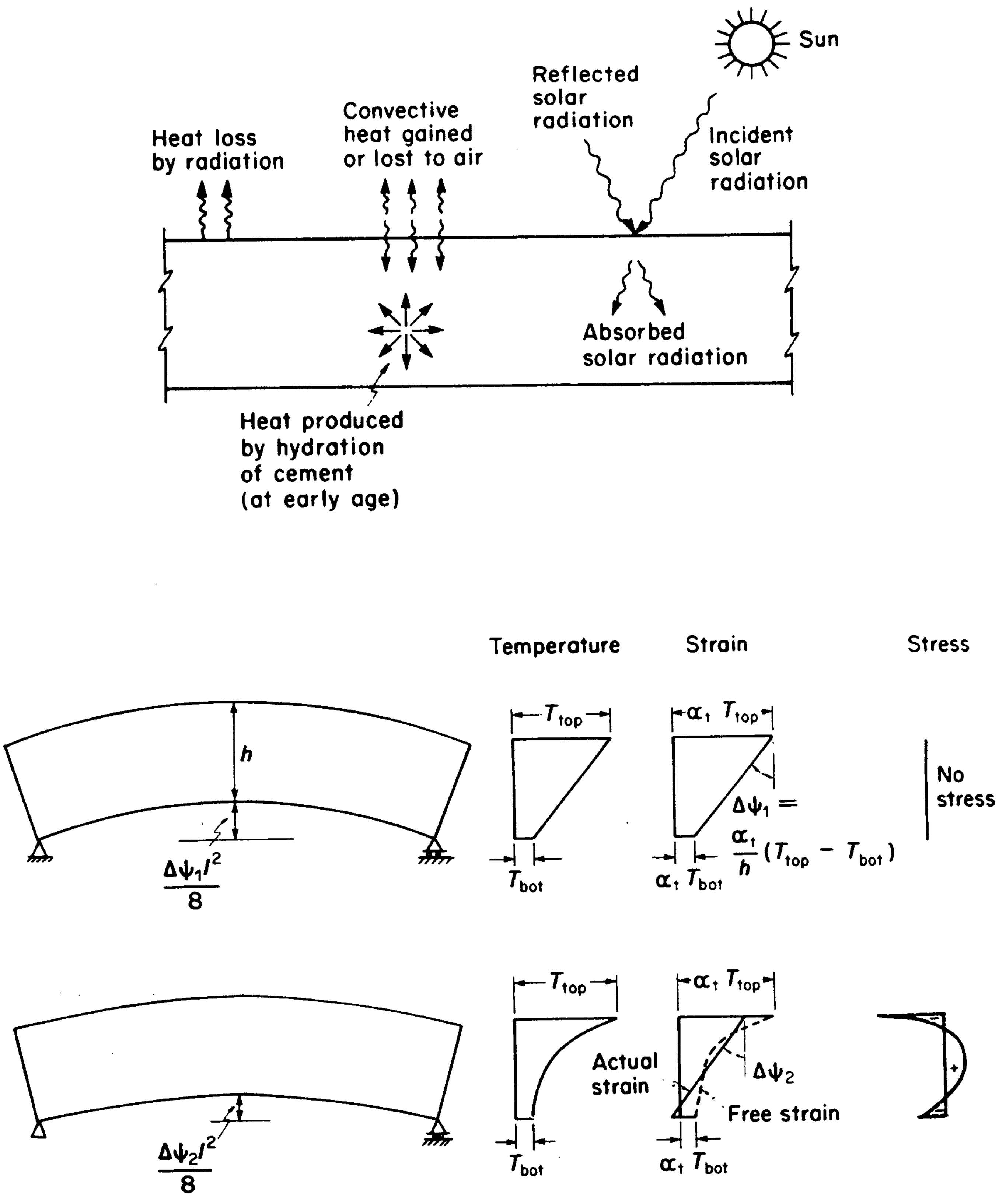


Figure 3.3 Beam Strain and Stress Distribution due to Linear or Nonlinear Temperature Variation through Depth [Reference 3: Figures 9.1 and 9.2]

Fig. 4.1 - Time Mult. Comparison

(Bridge 19045, Girder B18-S2)

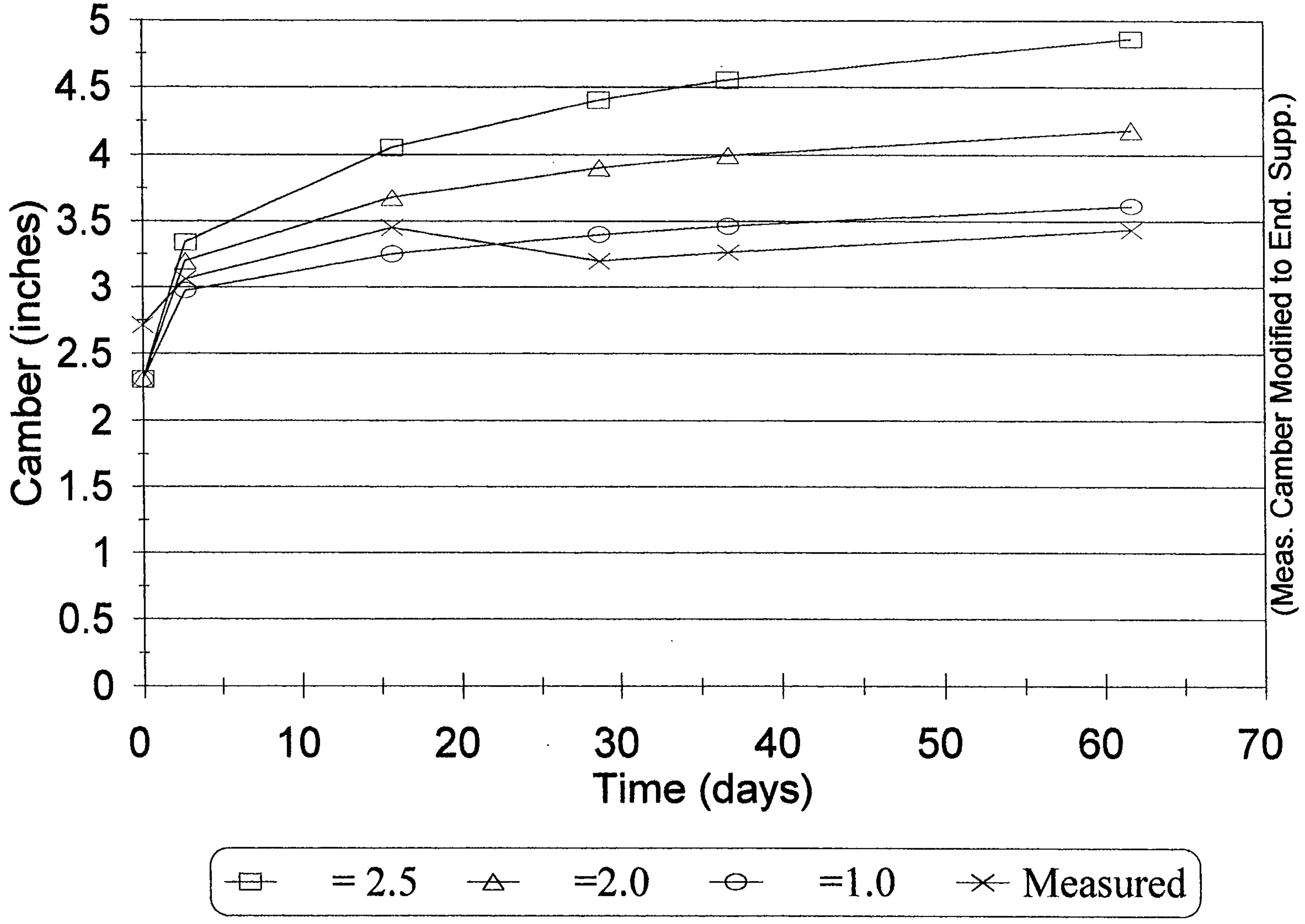


Fig. 4.2 - C.L. Camber - Girder B11-S2

(Methods and Measured Data)

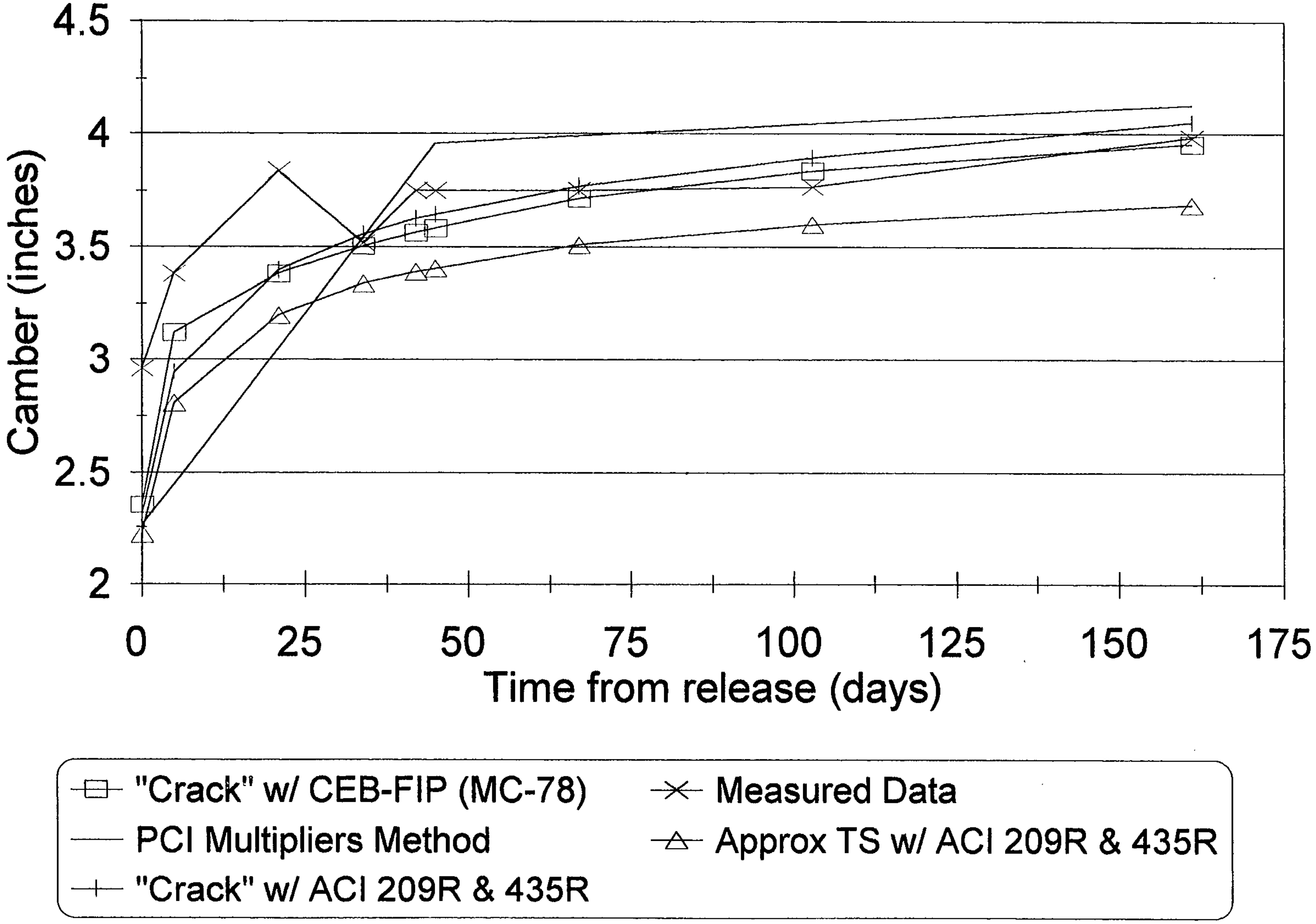


Fig. 4.3 - C.L. Camber - Girder B12-S2

(Methods and Measured Data)

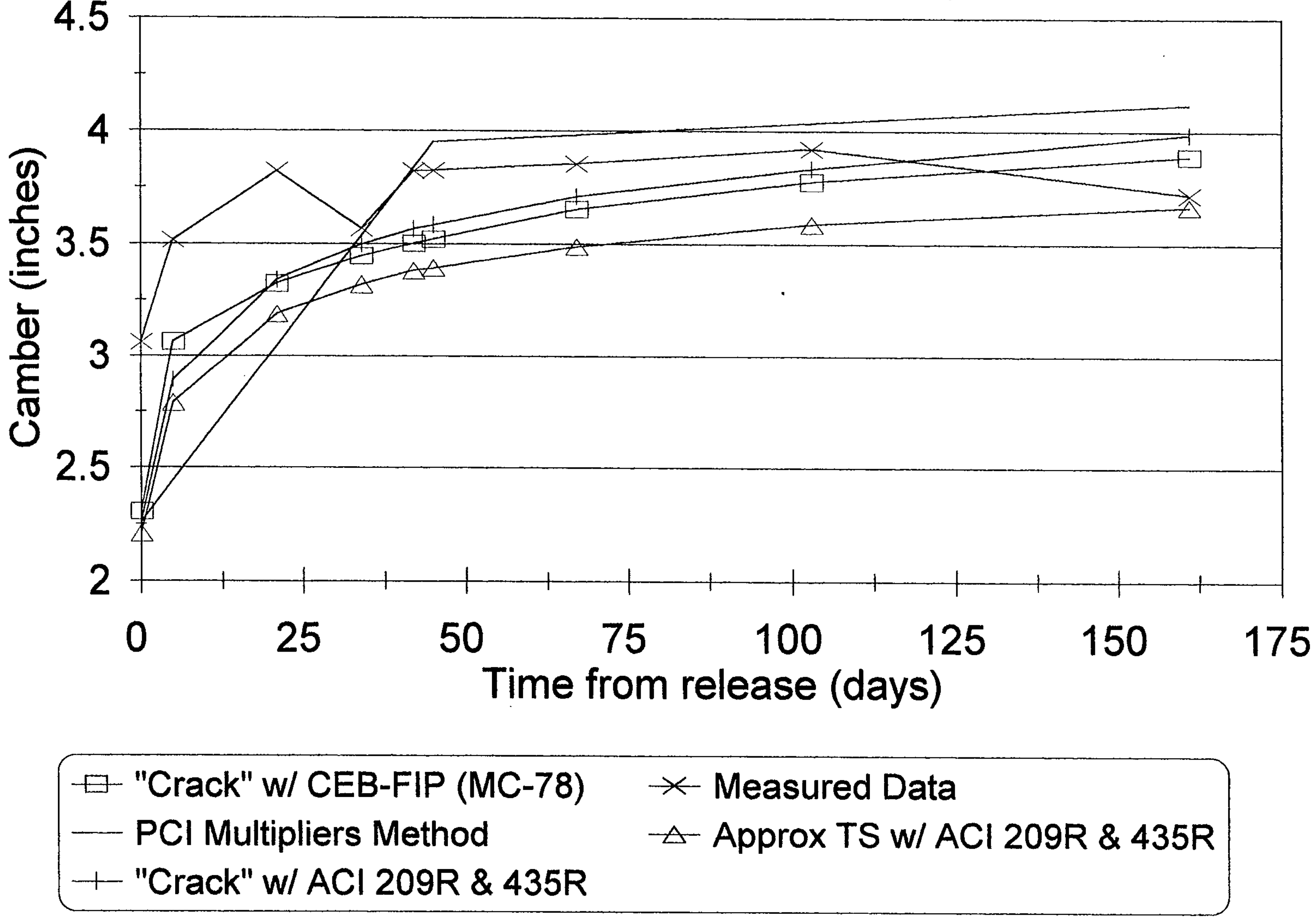


Fig. 4.4 - C.L. Camber - Girder B13-S2

(Methods and Measured Data)

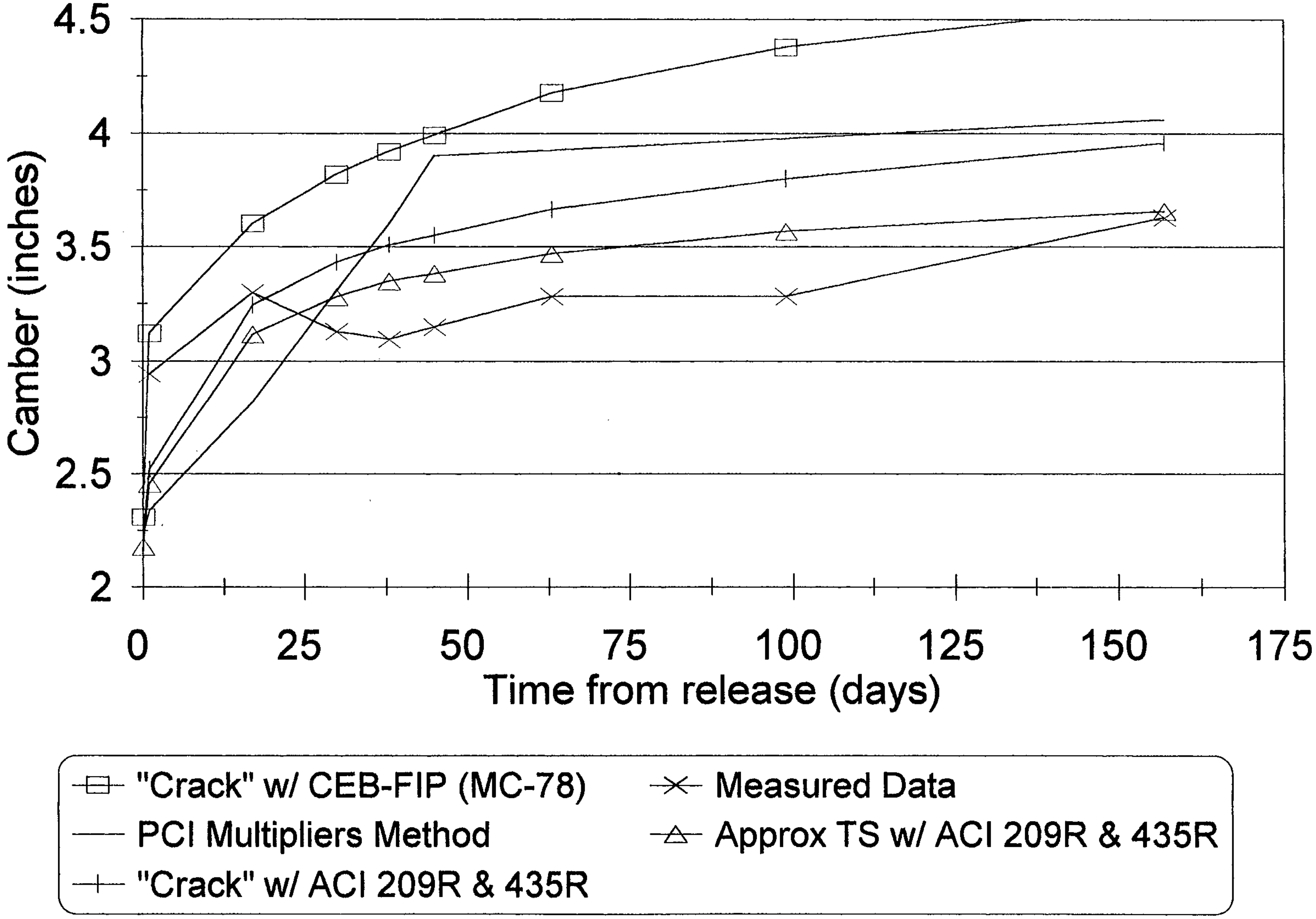


Fig. 4.5 - C.L. Camber - Girder B14-S2
(Methods and Measured Data)

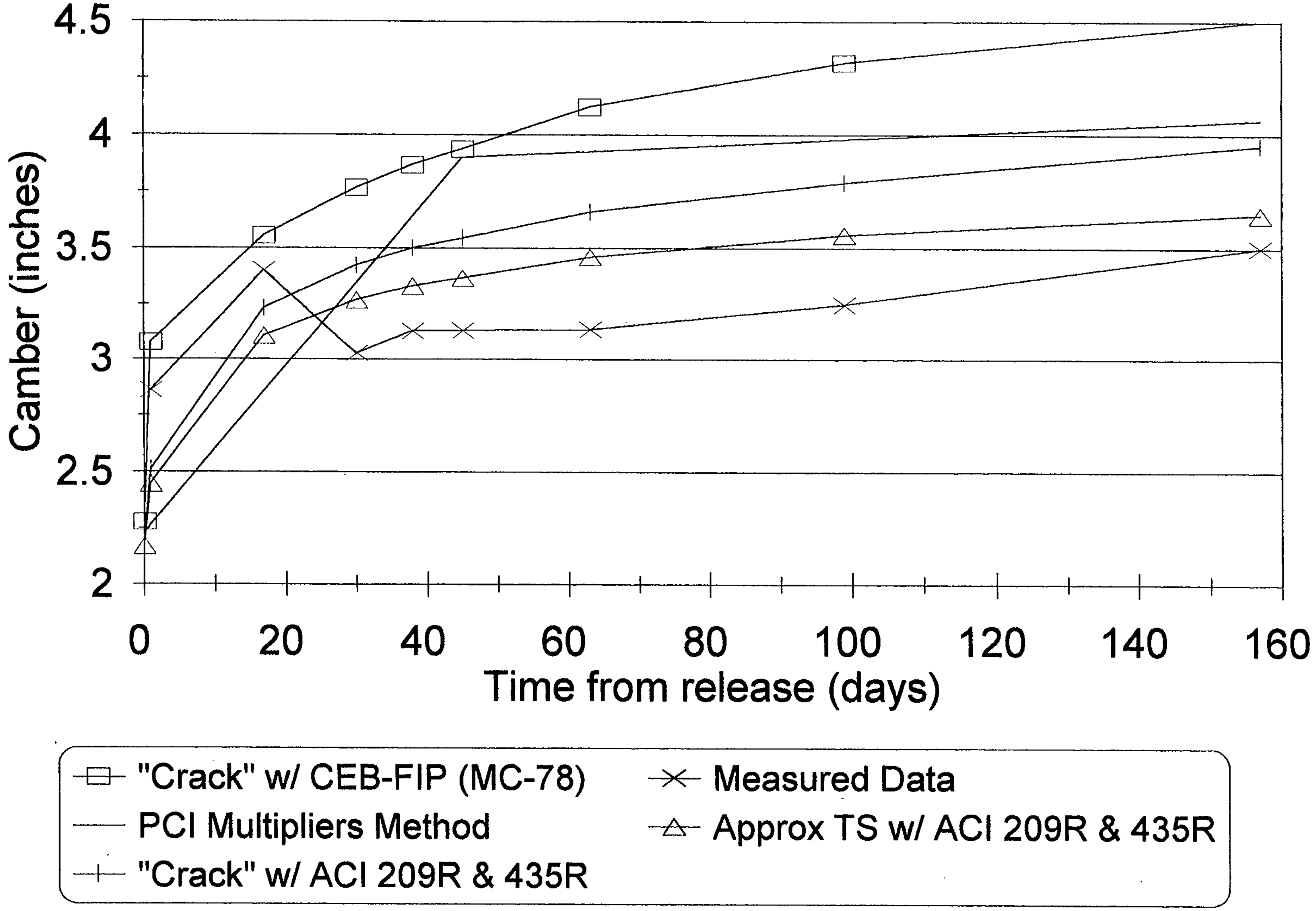


Fig. 4.6 - C.L. Camber - Girder B15-S2
(Methods and Measured Data)

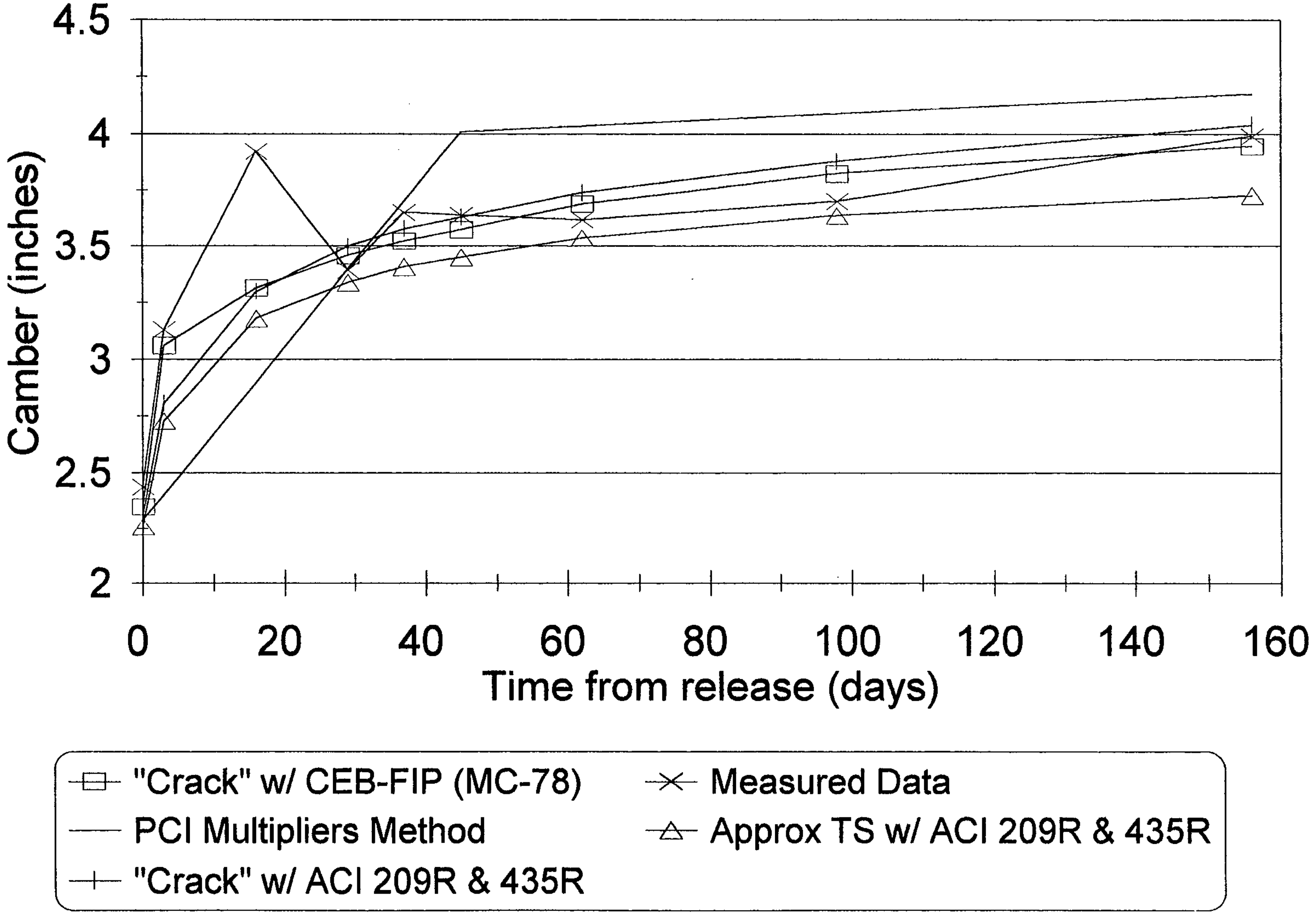


Fig. 4.7 - C.L. Camber - Girder B16-S2

(Methods and Measured Data)

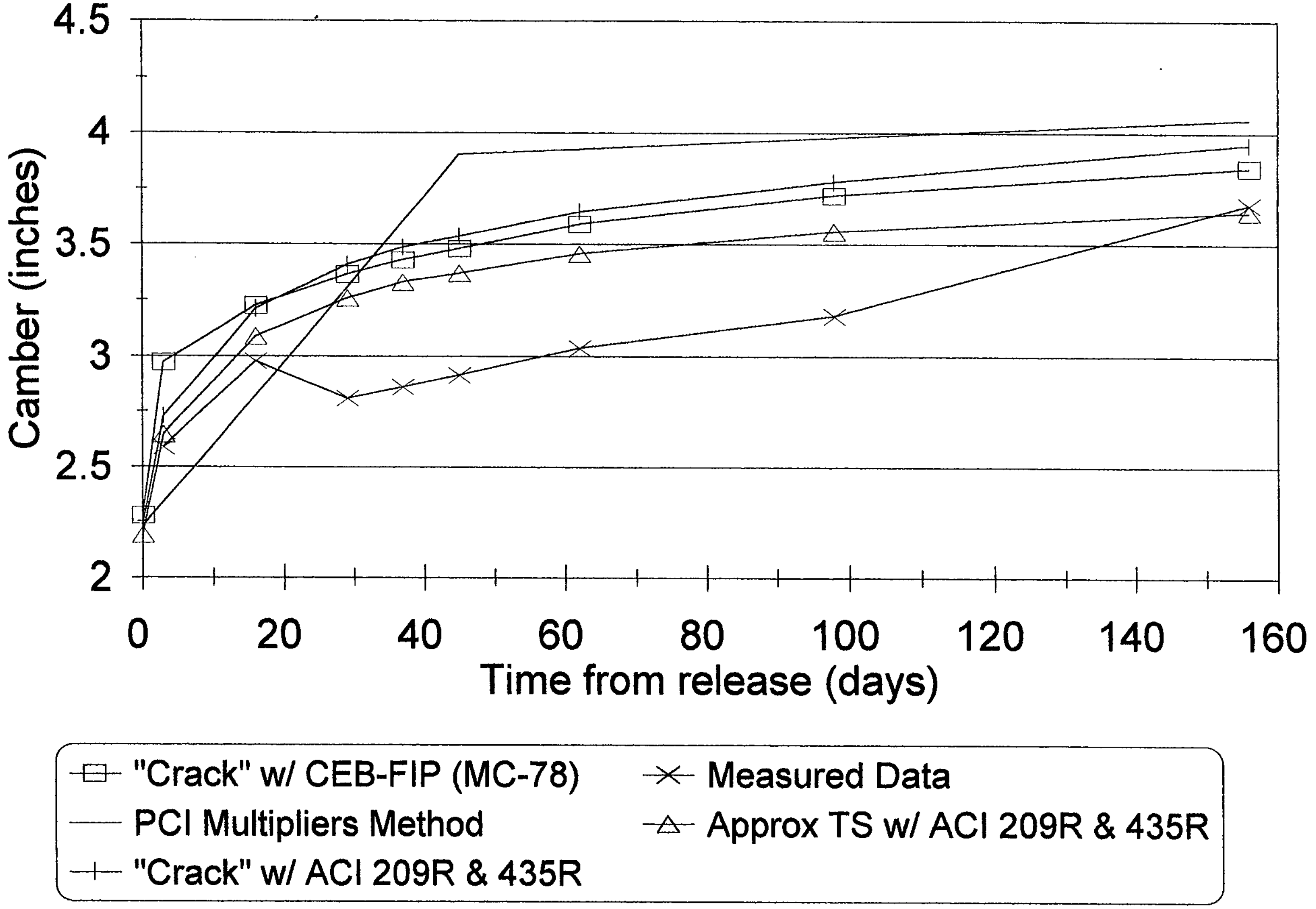


Fig. 4.8 - C.L. Camber - Girder B17-S2

(Methods and Measured Data)

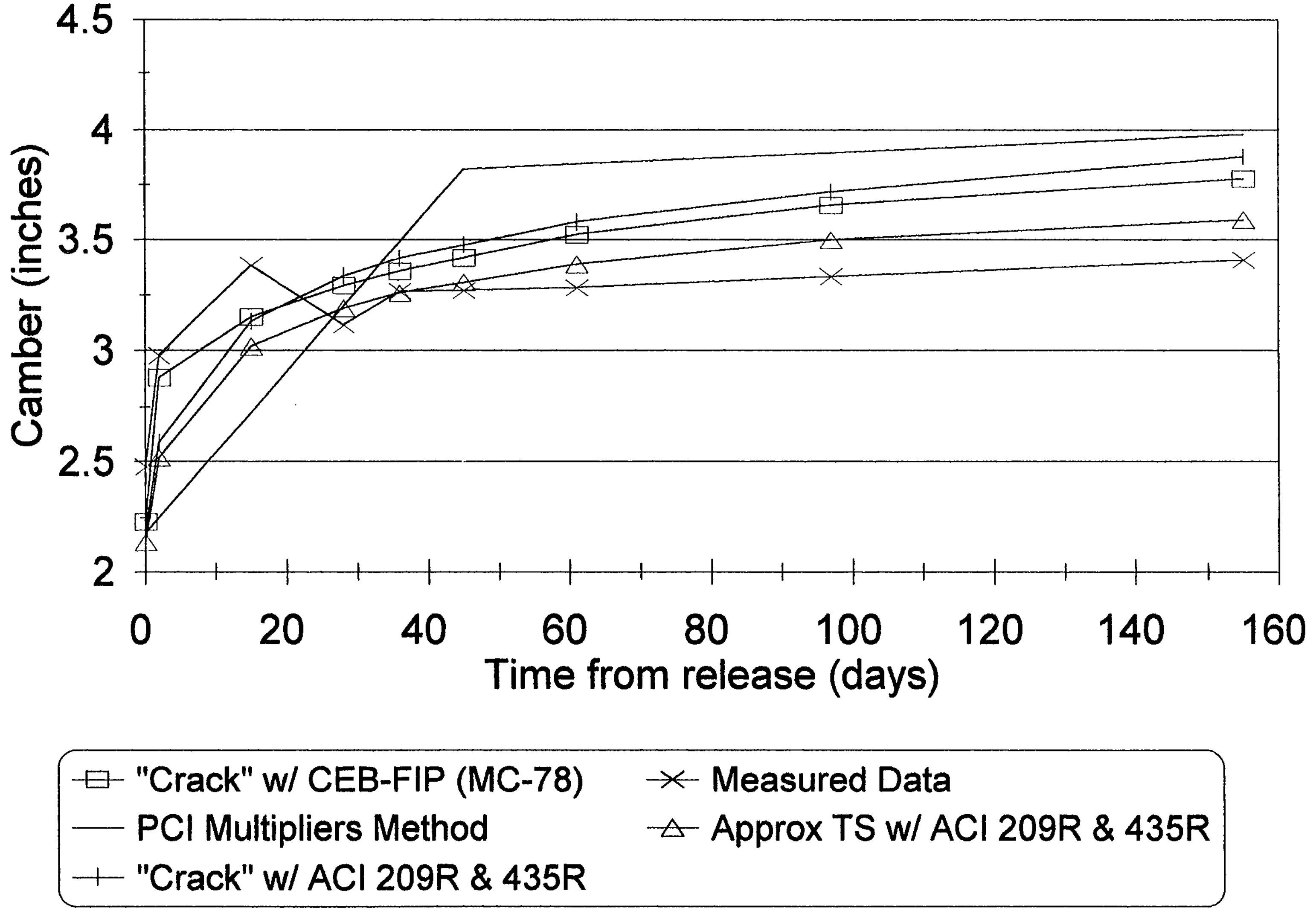


Fig. 4.9 - C.L. Camber - Girder B18-S2

(Methods and Measured Data)

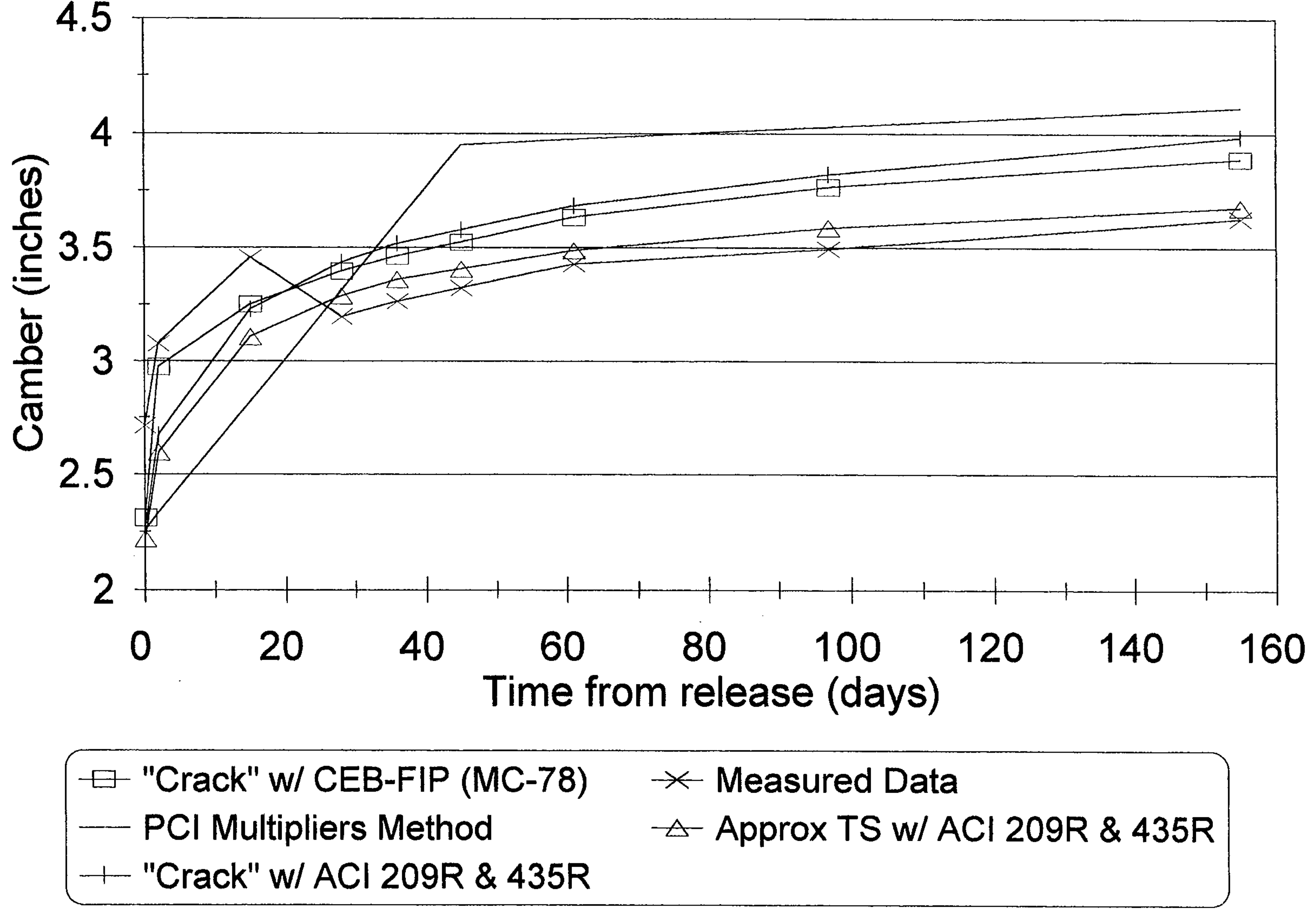


Fig. 4.10 -Measured Camber for Members

(Cast on the Same Bed - 11/9/94)

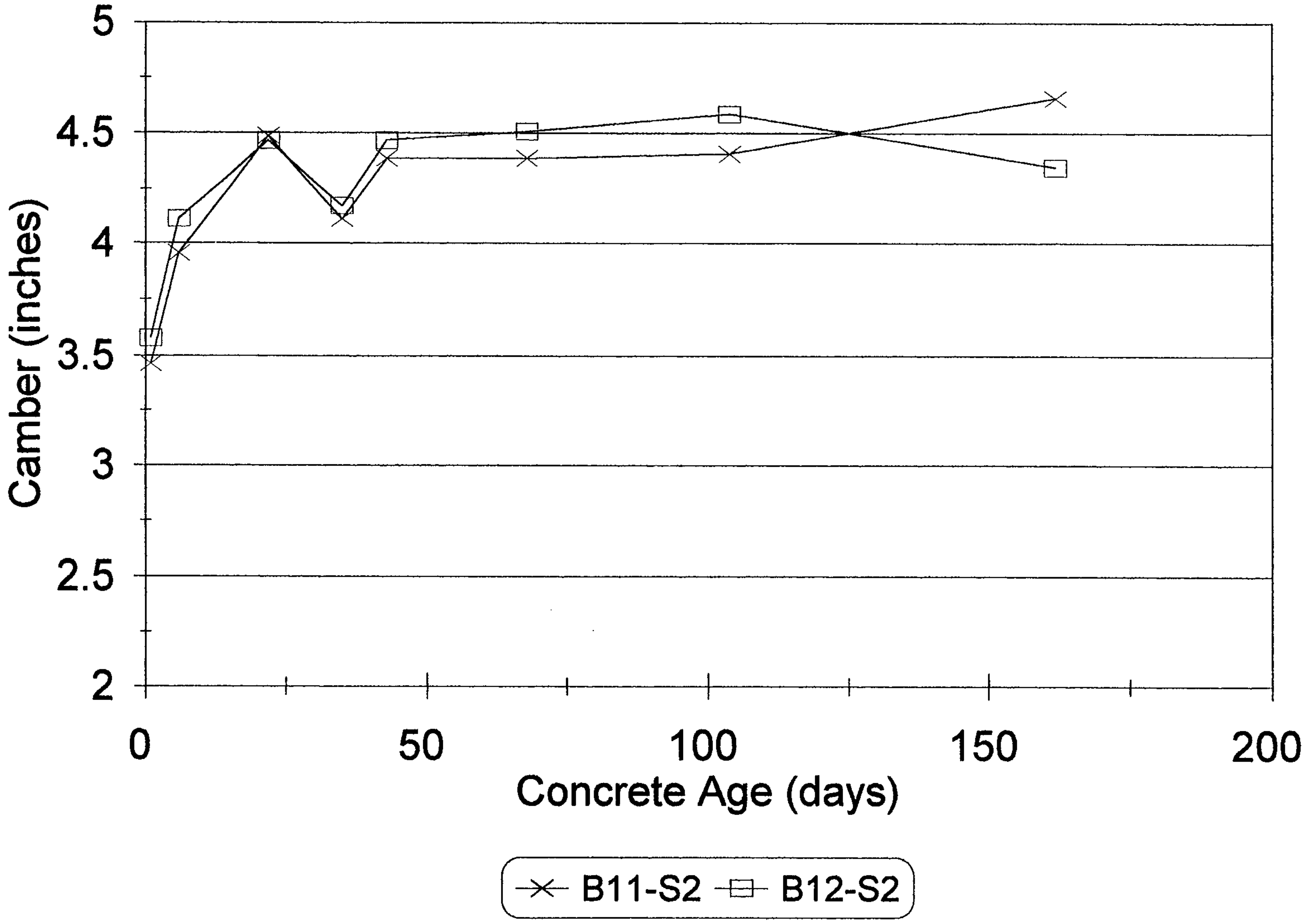


Fig. 4.11 -Measured Camber for Members
(Cast on the Same Bed - 11/11/94)

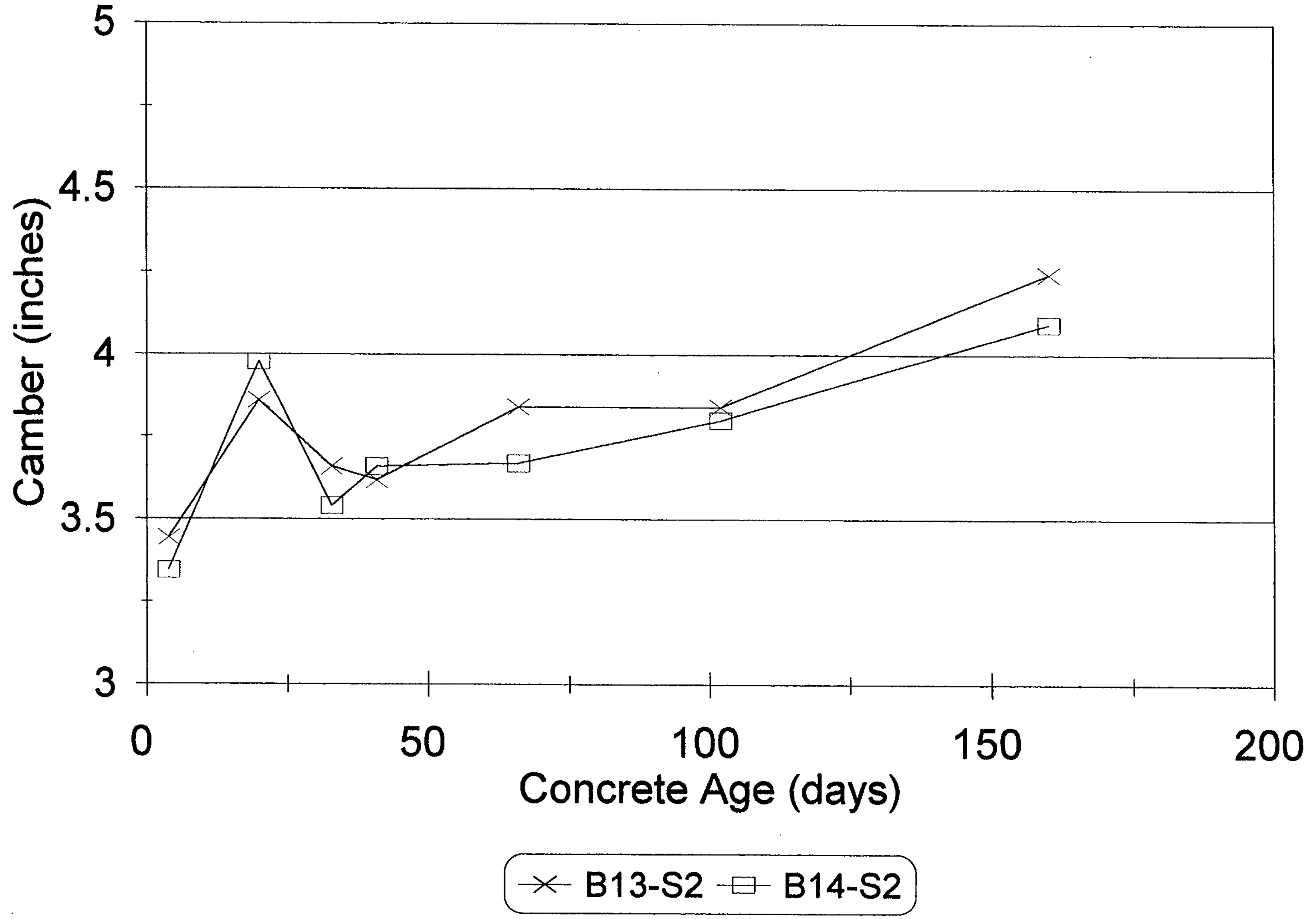


Fig. 4.12 -Measured Camber for Members
(Cast on the Same Bed - 11/14/94)

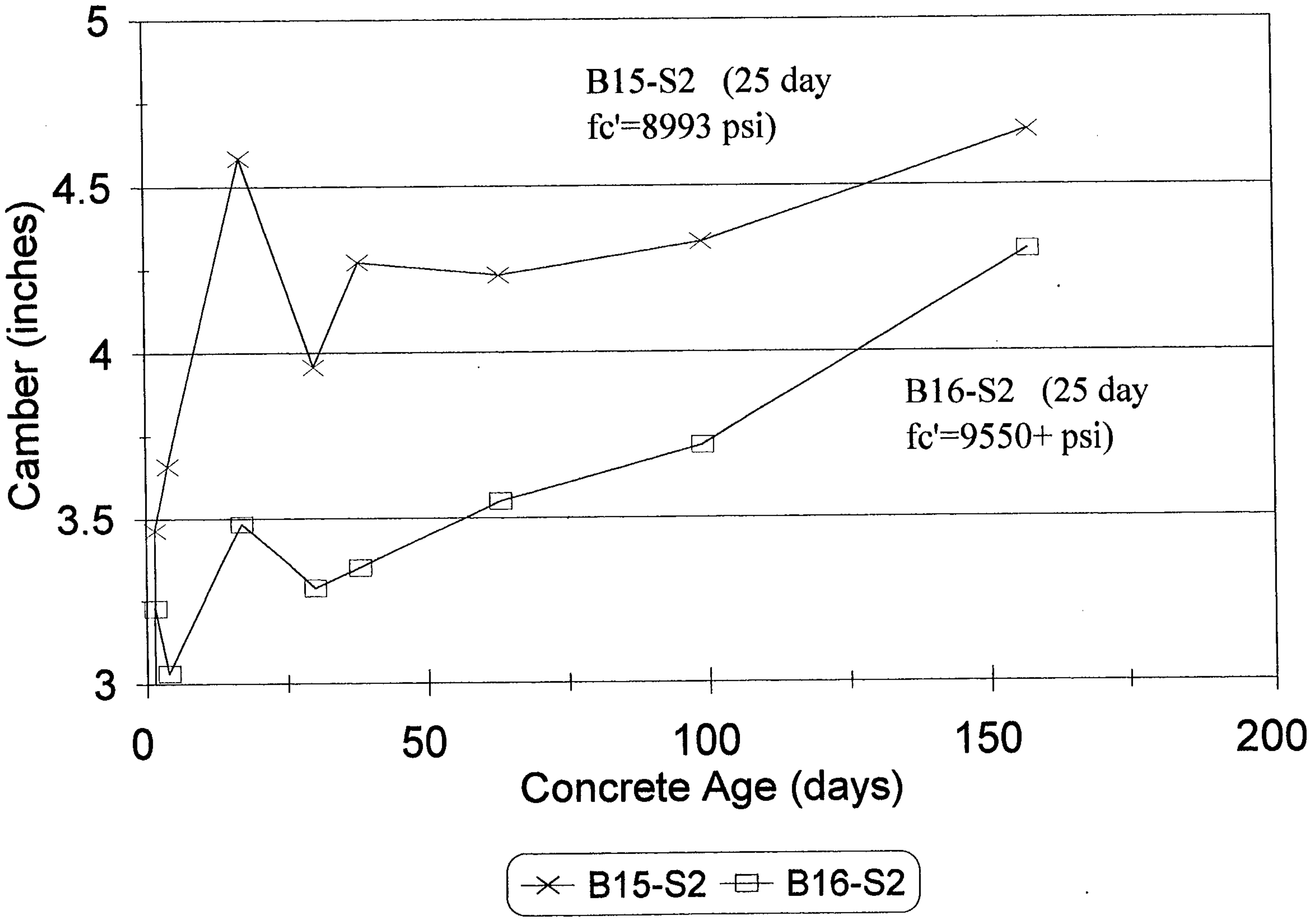


Fig. 4.13 -Measured Camber for Members

(Cast on the Same Bed - 11/15/94)

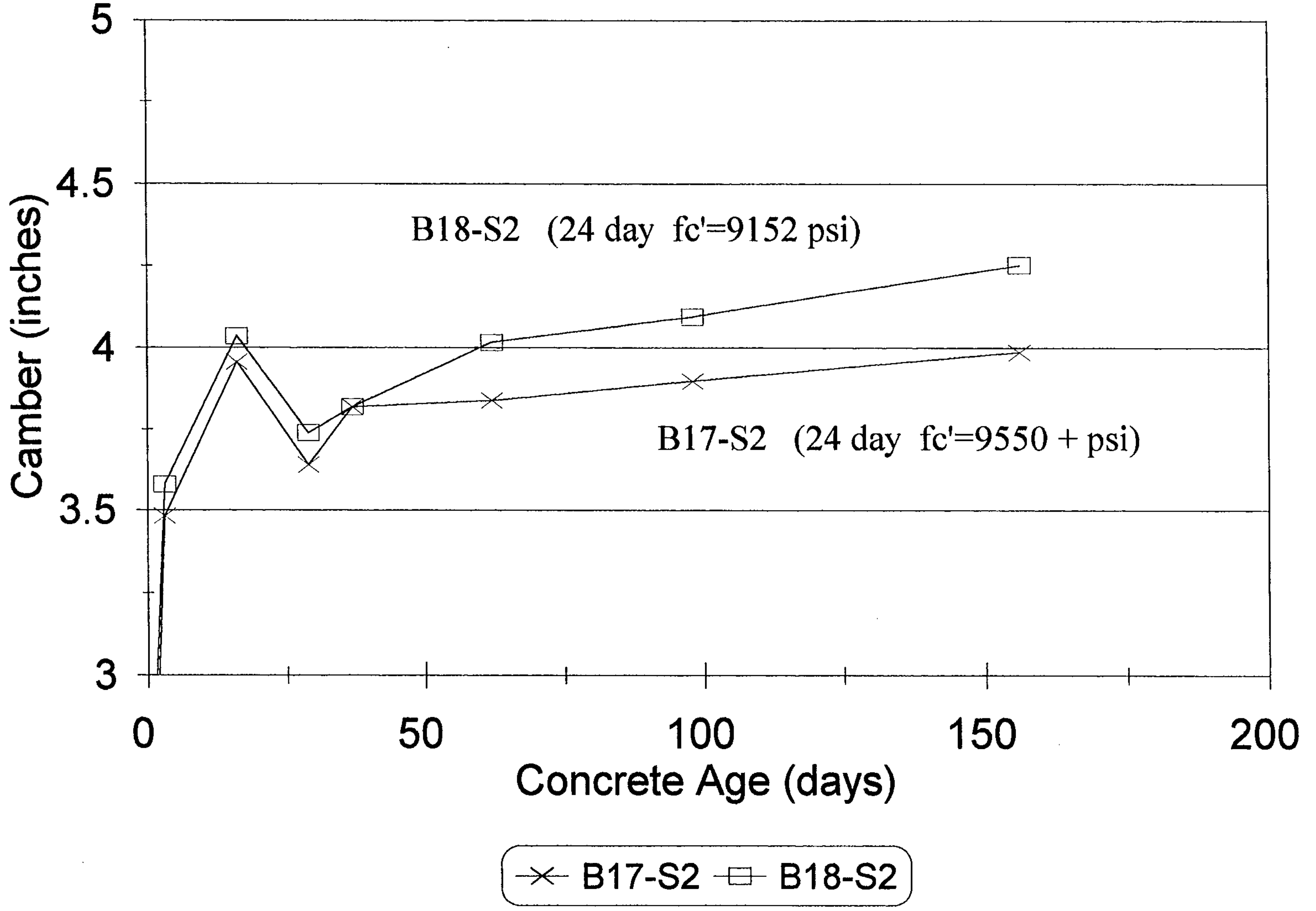


Figure 4.14- Comparison of Sensitivity
(Bridge 19045, Girder B18-S2)

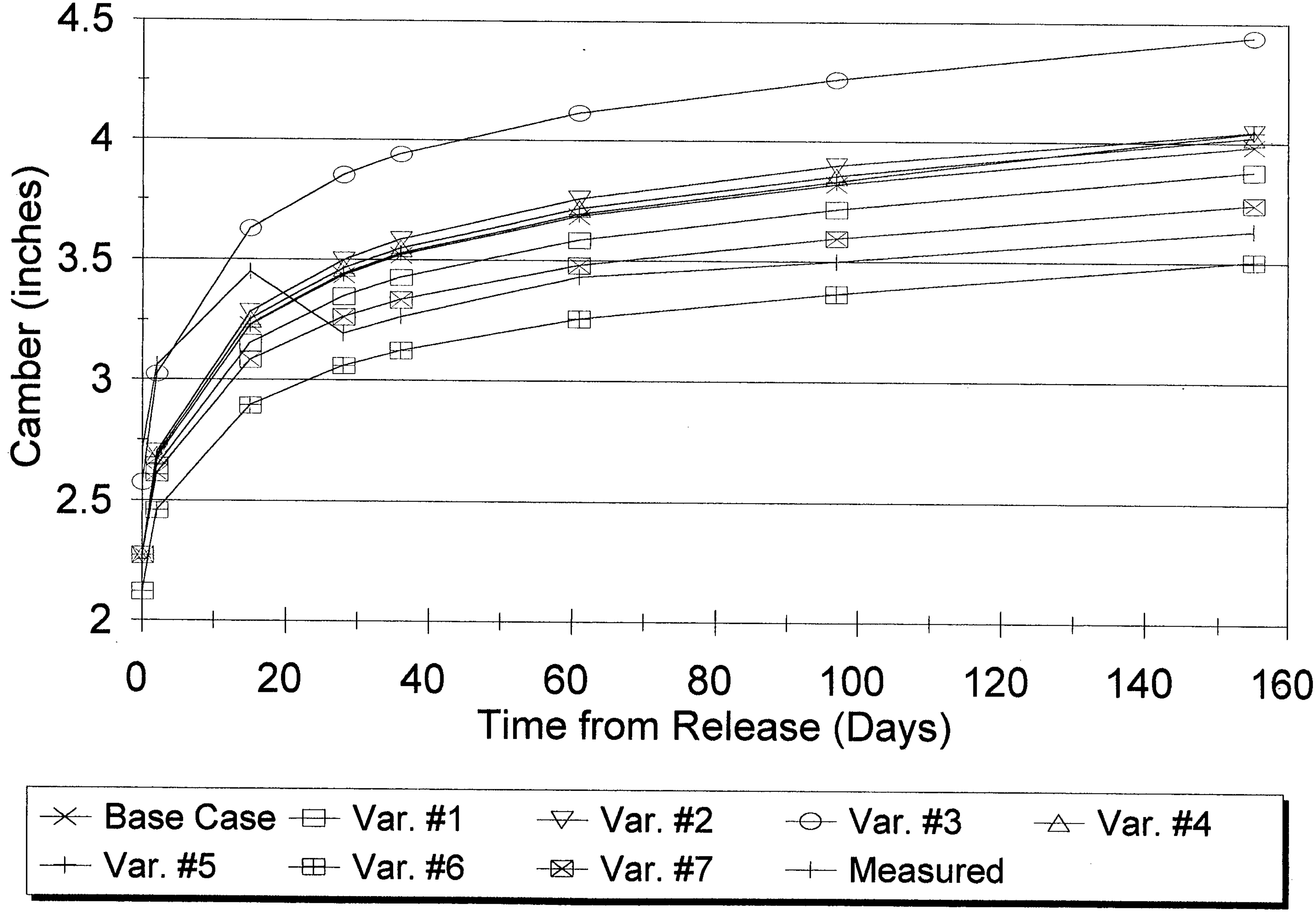


Figure 5.1 Thermal Camber Over 1 Day

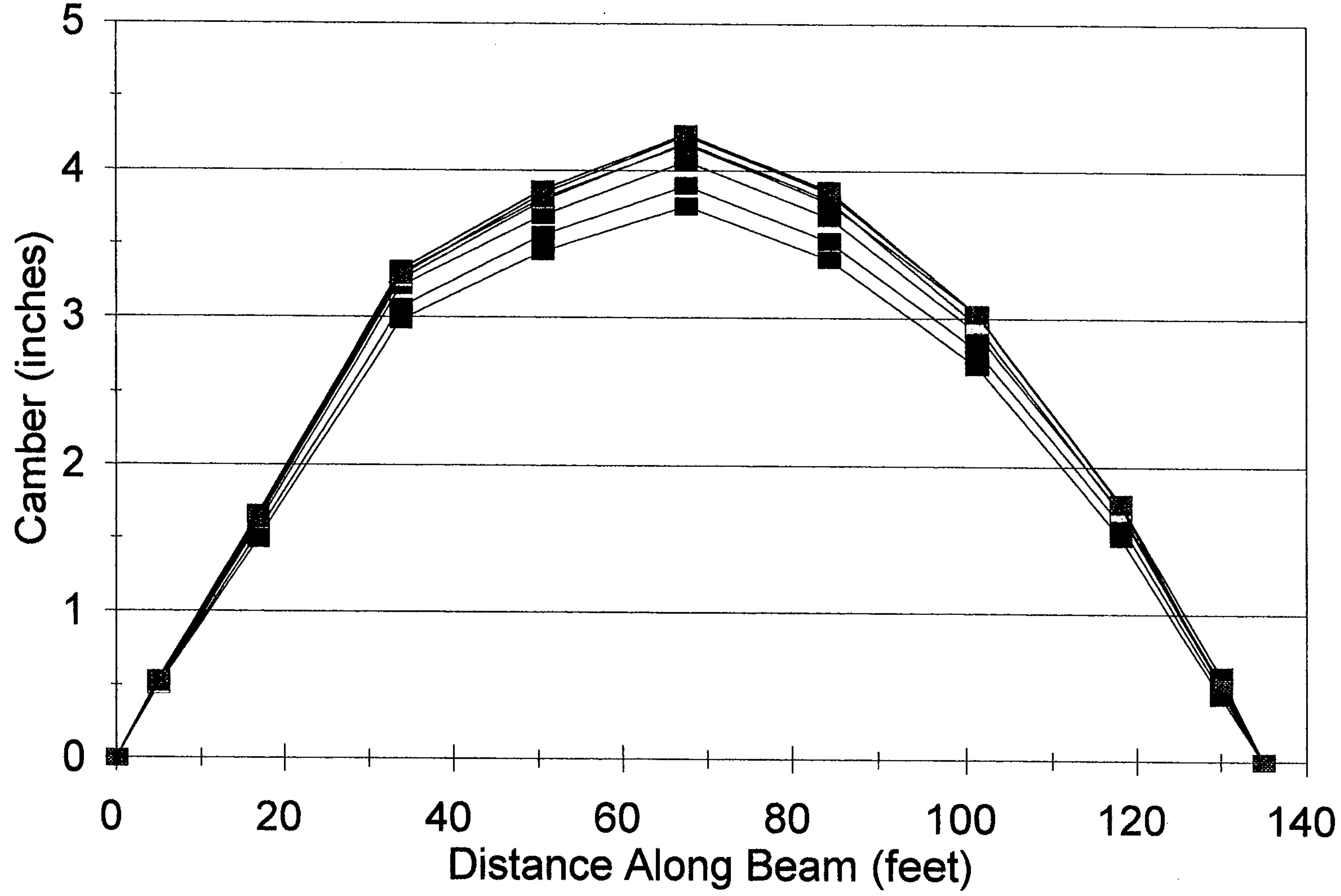


Figure 5.2 - Thermal Camber Over 1 Day

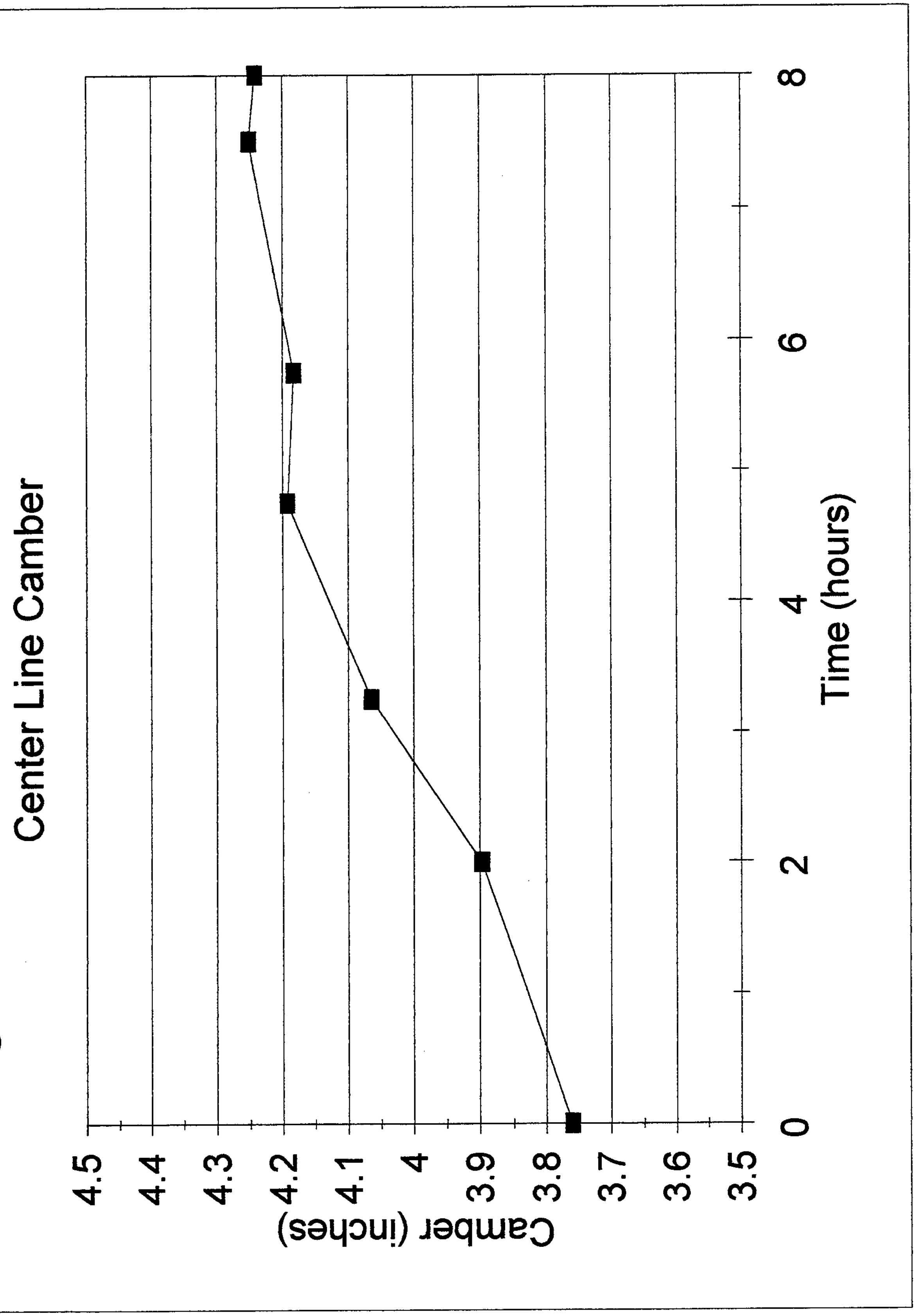


Figure 5.3 - Temperature Change

(from 6:30 to 2:30 on 6/17/95)

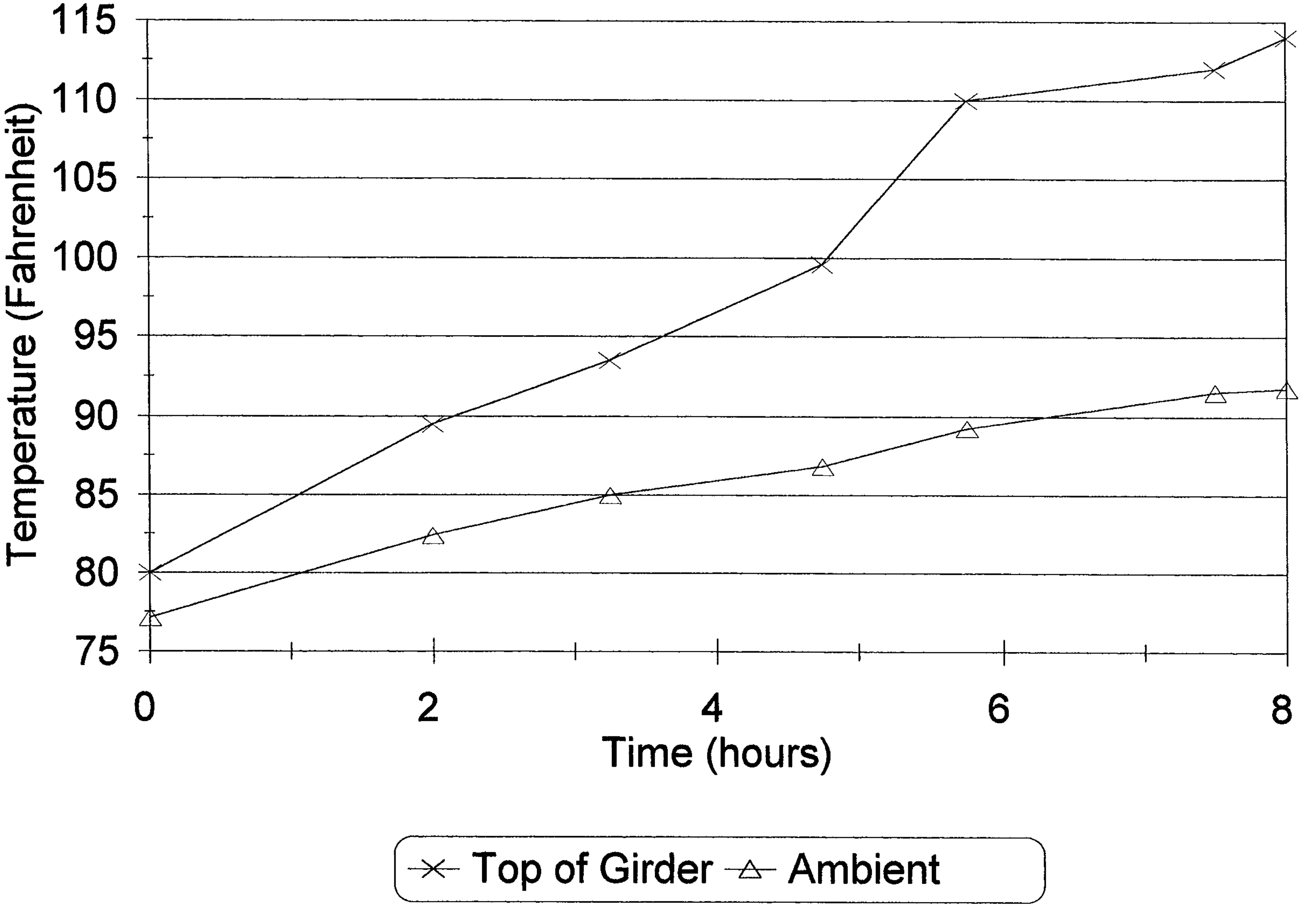
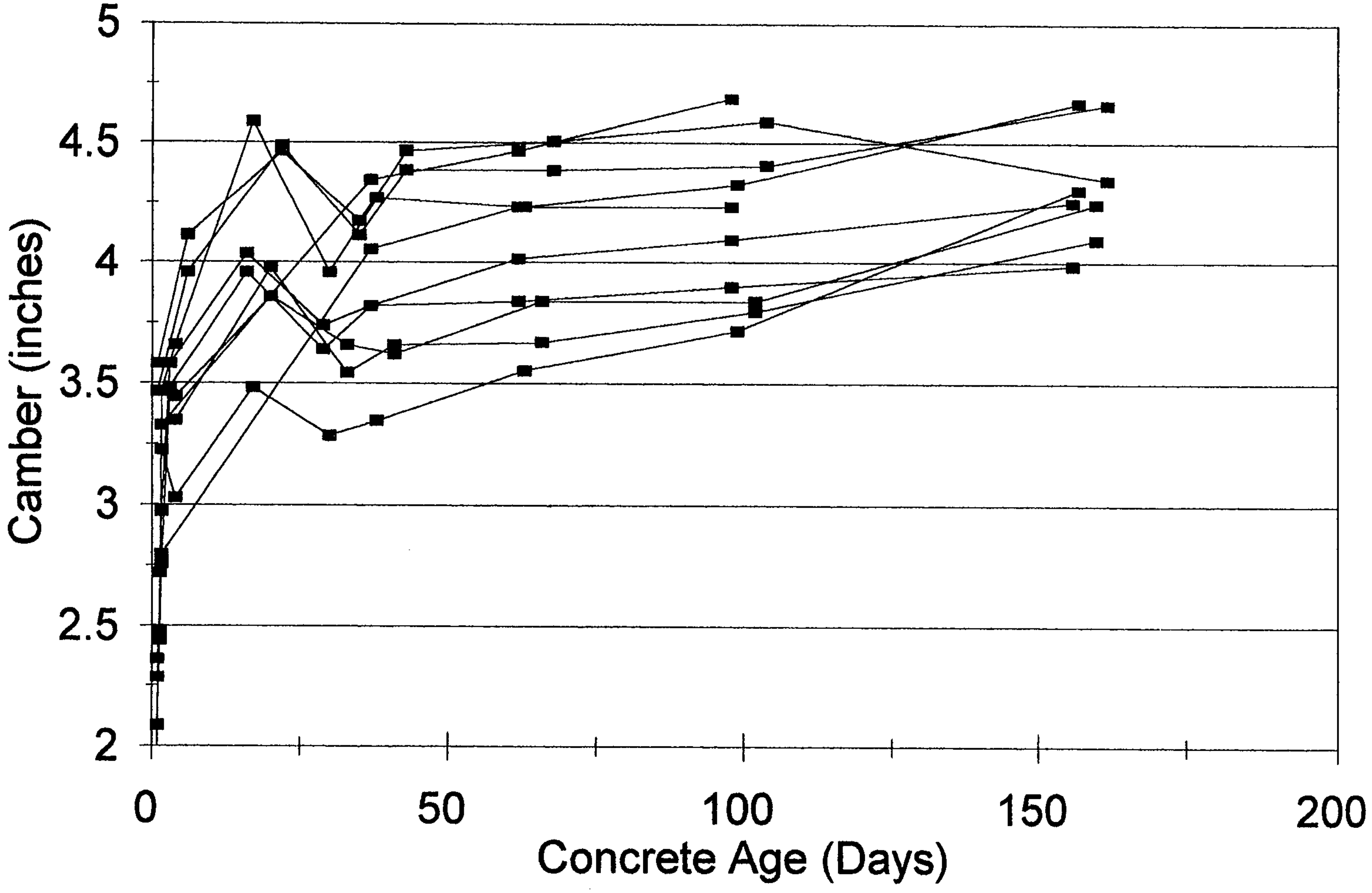


Fig. 6.1 - Measured Camber Comparisons

B11-S2 through B18-S2



APPENDIX A

Short-term Sensitivity Analysis Spreadsheets

PCI Initial Camber Design Sensitivity Analysis

Bridge # : 19045
Girder# : B18-S2
Girder type: 72-139

	Base Case	Variation #1	Variation #2	Variation #3	Variation #4	Variation #5	Variation #6	Variation #7	Variation #8	Variation #9 ***	Variation #10	Variation #11	Variation #12	Variation #13	Measured #14	Measured #15
(Some input data is linked)	Using Base data **	Changing steel to 29000	Eci being based on fci' (5800 psi)	Eci using ACI 318 eq (8.5.1) (fc'=6800)	Changing initial prestress by 5%	Changing from Tran. I @ CL to I gross	Changing Concrete weight by 5 pcf incr.	Changing Draping points by 1'	Changing length of beam to 1' shorter	Changing support condition to storage conditions	Using the actual concrete strengths	Consider Constant Thermal Expansion (20 deg.)	Consider Gradient Thermal Expansion (20-0 deg.)	Constant Thermal Expan. in top flange (20 deg.)	Actual Data coll. after pick-up & set down	Actual Data coll. after moving to temp storage
Input Data																
** Number of Draped Strands =	12	12	12	12	12	12	12	12	12	12	12	12	12	12		
** Number of Straight Strands =	42	42	42	42	42	42	42	42	42	42	42	42	42	42		
Modulus of Elasticity for steel (kip/in^2) =	28500	29000	28500	28500	28500	28500	28500	28500	28500	28500	28500	28500	28500	28500		
Ultimate Strength of Strand (Kip/in^2) =	270	270	270	270	270	270	270	270	270	270	270	270	270	270		
Area of One Strand (Inch^2) =	0.153	0.153	0.153	0.153	0.153	0.153	0.153	0.153	0.153	0.153	0.153	0.153	0.153	0.153		
Initial Strand Prestress @ seating (* fpu) =	0.75	0.75	0.75	0.75	0.8	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75		
Eccent. for draped st @ CL - - from bottom (inches)=	15	15	15	15	15	15	15	15	15	15	15	15	15	15		
Eccent. for draped st @ end - - from bottom (inches)=	62	62	62	62	62	62	62	62	62	62	62	62	62	62		
Eccent. for straight strands - - from bottom (inches)=	4.57	4.57	4.57	4.57	4.57	4.57	4.57	4.57	4.57	4.57	4.57	4.57	4.57	4.57		
Average eccentricity @ CL - - from bottom (inches)	6.89	6.89	6.89	6.89	6.89	6.89	6.89	6.89	6.89	6.89	6.89	6.89	6.89	6.89		
** Transformed Moment of Inertia of the Sect (inches^4)=	607551	608315	611880	597795	607551	547927	607551	607551	607551	607551	600159	607551	607551	607551		
Concrete Strength initial (lbs / in^2) =	5800	5800	5800	5800	5800	5800	5800	5800	5800	5800	6113.4	5800	5800	5800		
Concrete 28-day Strength (lbs / in^2)=	6800	6800	6800	6800	6800	6800	6800	6800	6800	6800	9070	6800	6800	6800		
Modulus of Elasticity for Concrete (kip/ in^2) =	4298	4298	4046	4999	4298	4298	4298	4298	4298	4298	4809	4298	4298	4298		
Weight of the concrete (Kcf) =	0.15	0.15	0.15	0.15	0.15	0.15	0.155	0.15	0.15	0.15	0.15	0.15	0.15	0.15		
** Center of gravity at CL - - from bottom (inches) =	34.76	34.73	34.70	34.89	34.76	35.60	34.76	34.76	34.76	34.76	34.86	34.76	34.76	34.76		
Area of the Cross Section (in^2) =	786	786	786	786	786	786	786	786	786	786	786	786	786	786		
Length of the unsupported beam (feet) =	137.50	137.50	137.50	137.50	137.50	137.50	137.50	137.50	136.50	137.50	137.50	137.5165	137.50	137.50		
Location of draping points from ends (feet) =	62.75	62.75	62.75	62.75	62.75	62.75	62.75	63.75	62.75	62.75	62.75	62.75	62.75	62.75		
Calculated Data								Constant thermal expansion of length (feet)				0.0165				
fcs	-2.78	-2.77	-2.76	-2.80	-3.03	-2.96	-2.74	-2.78	-2.79	-2.78	-2.80	-2.78	-2.78	-2.78		
Initial losses due to Elastic Shortening (%)=	9.09	9.23	9.61	7.89	9.31	9.71	8.97	9.09	9.14	9.09	8.18	9.09	9.09	9.09		
Straight Strand Force (Po (kips)) =	1183	1181	1176	1199	1259	1175	1185	1183	1182	1183	1195	1183	1183	1183		
Drapped Strand Force (Po(kips)) =	338	337	336	342	360	336	338	338	338	338	341	338	338	338		
Length of the unsupported span (inches) =	1650	1650	1650	1650	1650	1650	1650	1650	1638	1650	1650	1650.198	1650	1650		
Length to draping point (inches) =	753	753	753	753	753	753	753	765	753	753	753	753	753	753		
Actual Ecc. for straight strands - - e(c) (inches) =	30.19	30.16	30.13	30.32	30.19	31.03	30.19	30.19	30.19	30.19	30.29	30.19	30.19	30.19		
Actual Ecc. for draped strands @ end - - e(e) (inch) =	-27.24	-27.27	-27.30	-27.11	-27.24	-26.40	-27.24	-27.24	-27.24	-27.24	-27.14	-27.24	-27.24	-27.24		
Diff in Ecc. for dr. strands @ end & CL - - e' (inch) =	47	47	47	47	47	47	47	47	47	47	47	47	47	47		
Weight of the concrete (Kips/inch) =	0.0682	0.0682	0.0682	0.0682	0.0682	0.0682	0.0705	0.0682	0.0682	0.0682	0.0682	0.0682	0.0682	0.0682		
Camber due to the Straight strands (inches) =	4.65	4.64	4.87	4.14	4.95	5.27	4.66	4.65	4.58	4.65 *	4.27	4.65	4.65	4.65		
Camber due to the Draped strands (inches) =	0.30	0.29	0.31	0.27	0.31	0.37	0.30	0.28	0.28	0.30 *	0.27	0.30	0.30	0.30		
Gravity deflection component. (inches) =	-2.52	-2.52	-2.66	-2.20	-2.52	-2.80	-2.61	-2.52	-2.45	-2.27	-2.28	-2.52	-2.52	-2.52		
Thermal Expansion Gradient camber (inches)=													0.679	0.036		

	Base Case	Variation #1	Variation #2	Variation #3	Variation #4	Variation #5	Variation #6	Variation #7	Variation #8	Variation #9	Variation #10	Variation #11	Variation #12	Variation #13	Measured #14	Measured #15
Total Camber for the member (inches) =	2.43	2.41	2.52	2.20	2.74	2.84	2.35	2.41	2.42	2.68	2.26	2.43	3.11	2.46	2.72	2.97

2.53
(av. release & pick-up camber for this girder type.)

Base Data Includes the following assumptions:

- Ec is based on 28 day strength for initial camber. (fc' = 6800 psi)
  (Based on the work of Ali)
- Ec is based on 40000*(fc')^0.5 + 1000000 from (ACI 363 report for high strength concretes).
- Moment of Inertia of the beam is based on a Transformed sections analysis.

***Changing support conditions
reflects the actual suport conditions
in the field during storage of the beam.

supports are located @ approximately:
- 4.9' from left end

- Supports in storage are at center of end plates.
- 3.65' from right end
- Initial Jacking force = 0.75*fpu.
- Weight of the concrete is 150 pcf.
- Harping or draping points are located according to the shop drawings.
- Modulus of elasticity for strands is 28.5 x 10^3 ksi

Sensitivity of the Changes (Percentage)

(Percentages Based on the assumptions made in the Base analysis)

Steel Density		Variation #1	Variation #2	Variation #3	Variation #4	Variation #5	Variation #6	Variation #7	Variation #8	Variation #9	Variation #10	Variation #11	Variation #12	Variation #13	Measured #14	Measured #15
	Girder Type	Changing steel to 29000	Eci based on fci'	Eci using ACI 318 eq (8.5.1)	Changing initial jacking ratio by 5%	Changing from Tran I @ CL to I gross	Changing Concrete weight by 5 pcf incr.	Changing Draping points by 1'	Changing length of beam to 1' shorter	Changing support condition to storage conditions	Using the actual concrete strengths	Consider Constant Thermal Expansion (20 deg.)	Consider Gradient Thermal Expansion (20-0 deg	Constant Thermal Expan. in top flange (20 deg.)	Actual Data coll. after pick-up & set down	Actual D. after moving to temp storage
0.01051	72-139 (BR#19045)	-0.68	3.74	-9.31	13.07	16.92	-3.20	-0.76	-0.42	10.56	-6.93	0.01	27.95	1.50	12.04	22.34
0.00981	45M-95 (BR#27112)	-0.62	4.65	-10.12	10.99	14.11	-2.16	-1.02	-1.21	Data missing	-7.96	0.01	28.13	2.76	Data missing	Data missing
0.00685	45M-67 (BR#49535)	-0.38	5.35	-13.65	9.00	10.48	-1.13	-1.24	-2.50	Data missing	-3.63	0.02	28.28	2.79	-14.66	23.39

In order to get the actual spread of Initial Camber results for these girders (BR# **19045**), we would need the following in actual material properties:

Initial Camber off of the bed. (inches)		**Required initial prestress (* fpu)	**Required strength of concret (psi)		
2.44		75.19	6657		
2.72		79.61	4012		

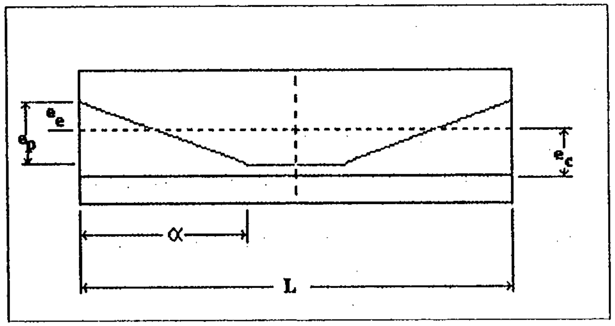
** Note: All other parameters are the same as the Basic Case with c initial prestress or strength varying seperately.

In order to get the actual spread of Initial Camber results for these girders (BR# **49535**), we would need the following in actual material properties:

Initial Camber after pick up & set down (inches)		**Required initial prestress (* fpu)	**Required strength of concret (psi)		
0.75		62.29	13572		
0.91		71.43	7493		

** Note: All other parameters are the same as the Basic Case with c initial prestress or strength varying seperately.

** Changing the Transformed I @ CL to the Transformed I @ the End varies the camber up by 3.55%



PCI Initial Camber Design Sensitivity Analysis

Bridge # : 27112
Girder# : B2-S2-3
Girder ty| 45M-95

(Some input data is linked)

	Base Case	Variation #1	Variation #2	Variation #3	Variation #4	Variation #5	Variation #6	Variation #7	Variation #8	Variation #9 ***	Variation #10	Variation #11	Variation #12	Variation #13
	Using Base data **	Changing steel to 29000	Eci being based on fci' (5750 psi)	Eci using ACI 318 eq (8.5.1) (fc'=6860)	Changing initial prestress by 5%	Changing from Tran. I @ CL to I gross	Changing Concrete weight by 5 pcf incr.	Changing Draping points by 1'	Changing length of beam to 1' shorter	Changing support condition to storage conditions	Using the actual concrete strengths	Consider Constant Thermal Expansion (20 deg.)	Consider Gradient Thermal Expansion (20-0 deg.	Constant Thermal Expan. in top flange (20 deg.)
Input Data														
** Number of Draped Strands =	10	10	10	10	10	10	10	10	10		10	10	10	10
** Number of Straight Strands =	30	30	30	30	30	30	30	30	30		30	30	30	30
Modulus of Elasticity for steel (kip/in^2) =	28500	29000	28500	28500	28500	28500	28500	28500	28500		28500	28500	28500	28500
Ultimate Strength of Strand (Kip/in^2) =	270	270	270	270	270	270	270	270	270		270	270	270	270
Area of One Strand (Inch^2) =	0.153	0.153	0.153	0.153	0.153	0.153	0.153	0.153	0.153		0.153	0.153	0.153	0.153
Initial Strand Prestress @ seating (* fpu) =	0.75	0.75	0.75	0.75	0.8	0.75	0.75	0.75	0.75		0.75	0.75	0.75	0.75
Eccent. for draped st @ CL - - from bottom (inches)=	7	7	7	7	7	7	7	7	7		7	7	7	7
Eccent. for draped st @ end - - from bottom (inches)=	36	36	36	36	36	36	36	36	36		36	36	36	36
Eccent. for straight strands - - from bottom (inches)=	4	4	4	4	4	4	4	4	4		4	4	4	4
Average eccentricity @ CL - - from bottom (inches)	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	No Support Information Available	4.75	4.75	4.75	4.75
** Transformed Moment of Inertia of the Sect (inches^4)=	182978	183188	184279	180334	182978	167048	182978	182978	182978		180883	182978	182978	182978
Concrete Strength initial (lbs / in^2) =	5750	5750	5750	5750	5750	5750	5750	5750	5750		6989.24	5750	5750	5750
Concrete 28-day Strength (lbs / in^2)=	6860	6860	6860	6860	6860	6860	6860	6860	6860		9290	6860	6860	6860
Modulus of Elasticity for Concrete (kip/ in^2) =	4313	4313	4033	5021	4313	4313	4313	4313	4313		4855	4313	4313	4313
Weight of the concrete (Kcf) =	0.15	0.15	0.15	0.15	0.15	0.15	0.155	0.15	0.15		0.15	0.15	0.15	0.15
** Center of gravity at CL - - from bottom (inches) =	21.86	21.84	21.83	21.94	21.86	22.34	21.86	21.86	21.86		21.92	21.86	21.86	21.86
Area of the Cross Section (in^2) =	624	624	624	624	624	624	624	624	624		624	624	624	624
Length of the unsupported beam (feet) =	93.29	93.29	93.29	93.29	93.29	93.29	93.29	93.29	92.29		93.29	93.30	93.29	93.29
Location of draping points from ends (feet) =	40.65	40.65	40.65	40.65	40.65	40.65	40.65	41.65	40.65		40.65	40.65	40.65	40.65
Calculated Data														
Stress in the Concrete at the Steel level (fcs) =	-2.78	-2.77	-2.77	-2.81	-3.02	-2.96	-2.75	-2.78	-2.80		-2.80	-2.78	-2.78	-2.78
Initial losses due to Elastic Shortening (%)=	9.07	9.21	9.65	7.87	9.23	9.66	8.98	9.07	9.12		8.12	9.07	9.07	9.07
Straight Strand Force (Po (kips)) =	845	844	840	856	900	840	846	845	845		854	845	845	845
Drapped Strand Force (Po(kips)) =	282	281	280	285	300	280	282	282	282		285	282	282	282
Length of the unsupported span (inches) =	1119.5	1119.5	1119.5	1119.5	1119.5	1119.5	1119.5	1119.5	1107.5		1119.5	1119.634	1119.5	1119.5
Length to draping point (inches) =	487.75	487.75	487.75	487.75	487.75	487.75	487.75	499.75	487.75		487.75	487.75	487.75	487.75
Actual Ecc. for straight strands - - e(c) (inches) =	17.86	17.84	17.83	17.94	17.86	18.34	17.86	17.86	17.86		17.92	17.86	17.86	17.86
Actual Ecc. for draped strands @ end - - e(e) (inch) =	-14.14	-14.16	-14.17	-14.06	-14.14	-13.66	-14.14	-14.14	-14.14		-14.08	-14.14	-14.14	-14.14
Diff in Ecc. for dr. strands @ end & CL - - e' (inch) =	29	29	29	29	29	29	29	29	29		29	29	29	29
Weight of the concrete (Kips/inch) =	0.0542	0.0542	0.0542	0.0542	0.0542	0.0542	0.0560	0.0542	0.0542		0.0542	0.0542	0.0542	0.0542
Camber due to the Straight strands (inches) =	2.9968	2.9850	3.1560	2.6580	3.1909	3.3486	2.9997	2.9968	2.9312		2.7299	2.9976	2.9968	2.9968
Camber due to the Draped strands (inches) =	0.42	0.42	0.44	0.38	0.45	0.49	0.42	0.40	0.40	No Support Information Available	0.38	0.42	0.42	0.42
Gravity deflection component. (inches) =	-1.40	-1.40	-1.49	-1.22	-1.40	-1.54	-1.45	-1.40	-1.34		-1.26	-1.40	-1.40	-1.40
Thermal Expansion Gradient camber (inches)=													0.566	0.055
	Base Case	Variation #1	Variation #2	Variation #3	Variation #4	Variation #5	Variation #6	Variation #7	Variation #8	Variation #9	Variation #10	Variation #11	Variation #12	Variation #13
Total Camber for the member (inches) =	2.01	2.00	2.11	1.81	2.24	2.30	1.97	1.99	1.99		1.85	2.01	2.58	2.07

Constant thermal expansion of length (feet) = 0.011195

Base Data Includes the following assumptions:

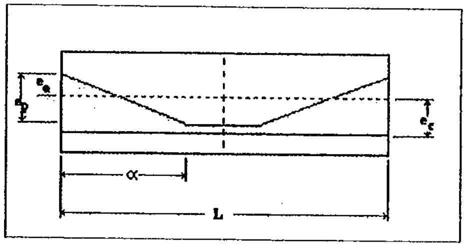
- Ec is based on 28 day strength for initial camber. (fc' = 6860 psi)
  (Based on the work of Ali)
- Ec is based on 40000*(fc')^0.5 + 1000000 from (ACI 363 report for high strength concretes).
- Moment of Inertia of the beam is based on a Transformed sections analysis.

***Changing support conditions
reflects the actual suport conditions
in the field during storage of the beam.

supports are located @ approximately:

- Supports in storage are at center of end plates.
- Initial Jacking force = 0.75*fpu.
- Weight of the concrete is 150 pcf.
- Harping or draping points are located according to the shop drawings.
- Modulus of elasticity for strands is 28.5 x 10^3 ksi

(There was no support information available)
(for this set of members)



PCI Initial Camber Design Sensitivity Analysis

Bridge # : 49535
Girder# : 4-B3
Girder ty 45M-67 (epoxy coated steel reinforcement)

	Base Case	Variation #1	Variation #2	Variation #3	Variation #4	Variation #5	Variation #6	Variation #7	Variation #8	Variation #9	Variation #10	Variation #11	Variation #12	Variation #13	Measured #14	Measured #15
(Some input data is linked)	Using Base data **	Changing steel to 29000	Eci being based on fci' (5000 psi)	Eci using ACI 318 eq (8.5.1) (fc'=6000)	Changing initial prestress by 5%	Changing from Tran. I @ CL to I gross	Changing Concrete weight by 5 pcf incr.	Changing Draping points by 1'	Changing length of beam to 1' shorter	Changing support condition to storage conditions	Using the actual concrete strengths	Consider Constant Thermal Expansion (20 deg.)	Consider Gradient Thermal Expansion (20-0 deg.	Constant Thermal Expan. in top flange (20 deg.)	Actual Data coll. after pick-up & set down	Actual Data coll. after moving to temp storage
Input Data																
** Number of Draped Strands =	8	8	8	8	8	8	8	8	8		8	8	8	8		
** Number of Straight Strands =	20	20	20	20	20	20	20	20	20		20	20	20	20		
Modulus of Elasticity for steel (kip/in^2) =	28500	29000	28500	28500	28500	28500	28500	28500	28500		28500	28500	28500	28500		
Ultimate Strength of Strand (Kip/in^2) =	270	270	270	270	270	270	270	270	270		270	270	270	270		
Area of One Strand (Inch^2) =	0.153	0.153	0.153	0.153	0.153	0.153	0.153	0.153	0.153		0.153	0.153	0.153	0.153		
Initial Strand Prestress @ seating (* fpu) =	0.75	0.75	0.75	0.75	0.8	0.75	0.75	0.75	0.75		0.75	0.75	0.75	0.75		
Eccent. for draped st @ CL - - from bottom (inches)=	6	6	6	6	6	6	6	6	6		6	6	6	6		
Eccent. for draped st @ end - - from bottom (inches)=	37	37	37	37	37	37	37	37	37		37	37	37	37		
Eccent. for straight strands - - from bottom (inches)=	3	3	3	3	3	3	3	3	3		3	3	3	3		
Average eccentricity @ CL - - from bottom (inches)	3.86	3.86	3.86	3.86	3.86	3.86	3.86	3.86	3.86	No Support Information Available	3.86	3.86	3.86	3.86		
** Transformed Moment of Inertia of the Sect (inches^4)=	181792	181967	183002	178888	181792	167048	181792	181792	181792		181003	181792	181792	181792		
Concrete Strength Initial (lbs / in^2) =	5000	5000	5000	5000	5000	5000	5000	5000	5000		6186.08	5000	5000	5000		
Concrete 28-day Strength (lbs / in^2)=	6000	6000	6000	6000	6000	6000	6000	6000	6000		6790	6000	6000	6000		
Modulus of Elasticity for Concrete (kip/ in^2) =	4098	4098	3828	4933	4098	4098	4098	4098	4098		4296	4098	4098	4098		
Weight of the concrete (Kcf) =	0.155	0.155	0.155	0.155	0.155	0.155	0.16	0.155	0.155		0.155	0.155	0.155	0.155		
** Center of gravity at CL - - from bottom (inches) =	22.08	22.07	22.06	22.13	22.08	22.34	22.08	22.08	22.08		22.09	22.08	22.08	22.08		
Area of the Cross Section (in^2) =	624	624	624	624	624	624	624	624	624		624	624	624	624		
Length of the unsupported beam (feet) =	65.00	65.00	65.00	65.00	65.00	65.00	65.00	65.00	64.00		65.00	65.01	65.00	65.00		
Location of draping points from ends (feet) =	26.00	26.00	26.00	26.00	26.00	26.00	26.00	27.00	26.00		26.00	26.00	26.00	26.00		
Calculated Data																
Constant thermal expansion of length (feet) =												0.0078				
fcs	-2.25	-2.25	-2.24	-2.27	-2.43	-2.38	-2.24	-2.25	-2.26		-2.26	-2.25	-2.25	-2.25		
Initial losses due to Elastic Shortening (%)=	7.73	7.86	8.24	6.49	7.82	8.16	7.68	7.73	7.77		7.39	7.73	7.73	7.73		
Straight Strand Force (Po (kips)) =	572	571	569	579	609	569	572	572	571		574	572	572	572		
Drapped Strand Force (Po(kips)) =	229	228	227	232	244	228	229	229	229		230	229	229	229		
Length of the unsupported span (inches) =	780	780	780	780	780	780	780	780	768		780	780.0936	780	780		
Length to draping point (inches) =	312	312	312	312	312	312	312	324	312		312	312	312	312		
Actual Ecc. for straight strands - - e(c) (inches) =	19.08	19.07	19.06	19.13	19.08	19.34	19.08	19.08	19.08		19.09	19.08	19.08	19.08		
Actual Ecc. for draped strands @ end - - e(e) (inch) =	-14.92	-14.93	-14.94	-14.87	-14.92	-14.66	-14.92	-14.92	-14.92		-14.91	-14.92	-14.92	-14.92		
Diff In Ecc. for dr. strands @ end & CL - - e' (inch) =	31	31	31	31	31	31	31	31	31		31	31	31	31		
Weight of the concrete (Kips/inch) =	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06		0.06	0.06	0.06	0.06		
Camber due to the Straight strands (inches) =	1.11	1.11	1.18	0.96	1.19	1.22	1.11	1.11	1.08	No Support Information Available	1.07	1.11	1.11	1.11		
Camber due to the Draped strands (inches) =	0.22	0.22	0.23	0.19	0.24	0.25	0.22	0.21	0.21		0.21	0.22	0.22	0.22		
Gravity deflection component. (inches) =	-0.36	-0.36	-0.39	-0.31	-0.36	-0.39	-0.37	-0.36	-0.34		-0.35	-0.36	-0.36	-0.36		
Thermal Expansion Gradient camber (inches)=													0.275	0.027		

	Base Case	Variation #1	Variation #2	Variation #3	Variation #4	Variation #5	Variation #6	Variation #7	Variation #8	Variation #9	Variation #10	Variation #11	Variation #12	Variation #13	Measured #14	Measured #15
Total Camber for the member (inches) =	0.97	0.97	1.02	0.84	1.06	1.07	0.96	0.96	0.95		0.94	0.97	1.25	1.00	0.83	1.20

0.83
(av. release & pick-up camber for this girder type.)

Base Data includes the following assumptions:

- Ec is based on 28 day strength for initial camber. (fc' = 6000 psi)
  (Based on the work of Ali)
- Ec is based on 40000*(fc')^0.5 + 1000000 from (ACI 363 report for high strength concretes).
- Moment of Inertia of the beam is based on a Transformed sections analysis.
- Supports in storage are at center of end plates.

***Changing support conditions
reflects the actual suport conditions
in the field during storage of the beam.

supports are located @ approximately:
- 4.9' from left end
- 3.65' from right end

- Initial Jacking force = 0.75*fpu.
- Weight of the concrete is 150 pcf.
- Harping or draping points are located according to the shop drawings.

APPENDIX B

Short-term Analysis Spreadsheets

PCI Short Term Camber Analysis

Bridge # : 19045

Girder type: 72-139

(Some input data is linked)

****Bolded** values in the table below are estimated concrete strengths

Input Data

		B11-S2	B12-S2	B13-S2	B14-S2	B15-S2	B16-S2	B17-S2	B18-S2	B19-S2	B20-S2
**	Number of Draped Strands =	12	12	12	12	12	12	12	12	12	12
**	Number of Straight Strands =	42	42	42	42	42	42	42	42	42	42
	Modulus of Elasticity for steel (kip/in^2) =	28500	28500	28500	28500	28500	28500	28500	28500	28500	28500
	Ultimate Strength of Strand (Kip/in^2) =	270	270	270	270	270	270	270	270	270	270
	Area of One Strand (Inch^2) =	0.153	0.153	0.153	0.153	0.153	0.153	0.153	0.153	0.153	0.153
	Initial Strand Prestress @ seating (* fpu) =	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75
	Eccent. for draped st @ CL - - from bottom (inches)=	15	15	15	15	15	15	15	15	15	15
	Eccent. for draped st @ end - - from bottom (inches)=	62	62	62	62	62	62	62	62	62	62
	Eccent. for straight strands - - from bottom (inches)=	4.57	4.57	4.57	4.57	4.57	4.57	4.57	4.57	4.57	4.57
	Average eccentricity @ CL - - from bottom (inches)	6.89	6.89	6.89	6.89	6.89	6.89	6.89	6.89	6.89	6.89
***	Transformed Moment of Inertia of the Sect (inches^4)=	600350	600051	599283	598897	601774	598897	597081	600159	601774	601021
	Concrete Strength initial (lbs / in^2) =	6113	6369	8090	7943	5603	6663	6918	6113	5747	6699
	Concrete 28-day Strength (lbs / in^2)=	**9000**	9110	9400	9550	**8500**	9550	**10300**	9070	**8500**	8760
	Modulus of Elasticity for Concrete (kip/ in^2) =	4795	4818	4878	4909	4688	4909	5060	4809	4688	4744
	Weight of the concrete (Kcf) =	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
***	Center of gravity at CL - - from bottom (inches) =	34.85	34.86	34.87	34.87	34.82	34.87	34.90	34.86	34.82	34.85
	Area of the Cross Section (in^2) =	786	786	786	786	786	786	786	786	786	786
	Length of the unsupported beam (feet) =	137.50	137.50	137.50	137.50	137.50	137.50	137.50	137.50	137.50	137.50
	Location of draping points from ends (feet) =	62.75	62.75	62.75	62.75	62.75	62.75	62.75	62.75	62.75	62.75

Calculated Data

fcs	-2.80	-2.80	-2.80	-2.80	-2.79	-2.80	-2.81	-2.80	-2.79	-2.79
Initial losses due to Elastic Shortening (%)=	8.21	8.17	8.08	8.03	8.38	8.03	7.80	8.18	8.38	8.29
Straight Strand Force (Po (kips)) =	1194	1195	1196	1197	1192	1197	1200	1195	1192	1193
Drapped Strand Force (Po(kips)) =	341	341	342	342	341	342	343	341	341	341
Length of the unsupported span (inches) =	1650	1650	1650	1650	1650	1650	1650	1650	1650	1650
Length to draping point (inches) =	753	753	753	753	753	753	753	753	753	753
Actual Ecc. for straight strands - - e(c) (inches) =	30.28	30.29	30.30	30.30	30.25	30.30	30.33	30.29	30.25	30.28
Actual Ecc. for draped strands @ end - - e(e) (inch) =	-27.15	-27.14	-27.13	-27.13	-27.18	-27.13	-27.10	-27.14	-27.18	-27.15
Diff in Ecc. for dr. strands @ end & CL - - e' (inch) =	47	47	47	47	47	47	47	47	47	47
Weight of the concrete (Kips/inch) =	0.0682	0.0682	0.0682	0.0682	0.0682	0.0682	0.0682	0.0682	0.0682	0.0682
Camber due to the Straight strands (inches) =	4.28	4.26	4.22	4.20	4.35	4.20	4.10	4.27	4.35	4.31
Camber due to the Draped strands (inches) =	0.27	0.27	0.27	0.27	0.28	0.27	0.26	0.27	0.28	0.28
Total Camber due to strands (inches) =	4.55	4.53	4.49	4.47	4.63	4.47	4.36	4.54	4.63	4.59
Gravity deflection component. (inches) =	-2.29	-2.28	-2.25	-2.24	-2.33	-2.24	-2.18	-2.28	-2.33	-2.31
	B11-S2	B12-S2	B13-S2	B14-S2	B15-S2	B16-S2	B17-S2	B18-S2	B19-S2	B20-S2
Total Camber for the member (inches) =	2.26	2.26	2.24	2.23	2.29	2.23	2.18	2.26	2.29	2.28
Measured Camber for the member (inches)=					2.44		2.48	2.72	2.48	
% Difference					-5.95		-11.94	-16.93	-7.47	

Long Term Camber of the Member

PCI Method (Uses table factors)

		B11-S2	B12-S2	B13-S2	B14-S2	B15-S2	B16-S2	B17-S2	B18-S2	B19-S2	B20-S2
At erection of structure: (30-60 days)	Factors										
Prestress Camber:	1.8	8.19	8.16	8.08	8.04	8.33	8.04	7.85	8.17	8.33	8.26
Self weight Deflection:	1.85	-4.23	-4.21	-4.17	-4.14	-4.32	-4.14	-4.03	-4.22	-4.32	-4.27
Total Camber		3.96	3.95	3.92	3.90	4.01	3.90	3.82	3.95	4.01	3.99
Final Camber:	Factors										
Prestress Camber:	2.45	11.15	11.11	11.00	10.95	11.34	10.95	10.69	11.12	11.34	11.25
Self weight Deflection:	2.7	-6.18	-6.15	-6.08	-6.05	-6.30	-6.05	-5.89	-6.16	-6.30	-6.24
Total Camber		4.97	4.96	4.92	4.90	5.04	4.90	4.81	4.97	5.04	5.01

(Note: only the self weight is being considered here & support conditions are not being considered)

	Time (days)										
	0	2.26	2.26	2.24	2.23	2.29	2.23	2.18	2.26	2.29	2.28
For Plotting	45	3.96	3.95	3.92	3.90	4.01	3.90	3.82	3.95	4.01	3.99
	730 (2 year)	4.97	4.96	4.92	4.90	5.04	4.90	4.81	4.97	5.04	5.01

PCI Short Term Camber Analysis

Bridge # : 27112
Girder# : B2-S2-3
Girder ty| 45M-95

****Bolded** values in the table below are estimated concrete strengths

(Some input data is linked)

Input Data

		B2-S2-1	B2-S2-2	B2-S2-3	B2-S2-4	B2-S2-5	B2-S2-6	B2-S2-7	B2-S2-8	B2-S2-9	B2-S2-10	B2-S2-11	B2-S2-12	B2-S2-13	B2-S2-14	B2-S2-15	B2-S2-26
**	Number of Draped Strands =	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
**	Number of Straight Strands =	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30
	Modulus of Elasticity for steel (kip/in^2) =	28500	28500	28500	28500	28500	28500	28500	28500	28500	28500	28500	28500	28500	28500	28500	28500
	Ultimate Strength of Strand (Kip/in^2) =	270	270	270	270	270	270	270	270	270	270	270	270	270	270	270	270
	Area of One Strand (Inch^2) =	0.153	0.153	0.153	0.153	0.153	0.153	0.153	0.153	0.153	0.153	0.153	0.153	0.153	0.153	0.153	0.153
	Initial Strand Prestress @ seating (* fpu) =	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75
	Eccent. for draped st @ CL - - from bottom (inches)=	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	Eccent. for draped st @ end - - from bottom (inches)=	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36
	Eccent. for straight strands - - from bottom (inches)=	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
	Average eccentricity @ CL - - from bottom (inches)	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75	4.75
**	Transformed Moment of Inertia of the Sect (inches^4)=	181168	180883	180883	180984	180736	181035	180736	181168	181206	181168	181206	181290	180841	180841	181168	180421
	Concrete Strength initial (lbs / in^2) =	6660	6953	6989	6843	6367	6550	5526	5928	6331	6489	5857	6369	6589	6260	6004	6552
	Concrete 28-day Strength (lbs / in^2)=	**8900**	**9290**	9290	**9150**	**9500**	9080	**9500**	**8900**	**8850**	**8900**	**8850**	8740	9350	**9350**	**8900**	9970
	Modulus of Elasticity for Concrete (kip/ in^2) =	4774	4855	4855	4826	4899	4812	4899	4774	4763	4774	4763	4740	4868	4868	4774	4994
	Weight of the concrete (Kcf) =	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
**	Center of gravity at CL - - from bottom (inches) =	21.91	21.92	21.92	21.92	21.93	21.92	21.93	21.91	21.91	21.91	21.91	21.91	21.92	21.92	21.91	21.93
	Area of the Cross Section (in^2) =	624	624	624	624	624	624	624	624	624	624	624	624	624	624	624	624
	Length of the unsupported beam (feet) =	93.29	93.29	93.29	93.29	93.29	93.29	93.29	93.29	93.29	93.29	93.29	93.29	93.29	93.29	93.29	93.29
	Location of draping points from ends (feet) =	40.65	40.65	40.65	40.65	40.65	40.65	40.65	40.65	40.65	40.65	40.65	40.65	40.65	40.65	40.65	40.65

Calculated Data

	B2-S2-1	B2-S2-2	B2-S2-3	B2-S2-4	B2-S2-5	B2-S2-6	B2-S2-7	B2-S2-8	B2-S2-9	B2-S2-10	B2-S2-11	B2-S2-12	B2-S2-13	B2-S2-14	B2-S2-15	B2-S2-26
Stress in the Concrete at the Steel level (fcs) =	-2.80	-2.80	-2.80	-2.80	-2.80	-2.80	-2.80	-2.80	-2.80	-2.80	-2.80	-2.80	-2.80	-2.80	-2.80	-2.80
Initial losses due to Elastic Shortening (%)=	8.25	8.12	8.12	8.16	8.05	8.19	8.05	8.25	8.26	8.25	8.26	8.30	8.10	8.10	8.25	7.90
Straight Strand Force (Po (kips)) =	853	854	854	854	855	853	855	853	853	853	853	852	854	854	853	856
Drapped Strand Force (Po(kips)) =	284	285	285	285	285	284	285	284	284	284	284	284	285	285	284	285
Length of the unsupported span (inches) =	1119.5	1119.5	1119.5	1119.5	1119.5	1119.5	1119.5	1119.5	1119.5	1119.5	1119.5	1119.5	1119.5	1119.5	1119.5	1119.5
Length to draping point (inches) =	487.75	487.75	487.75	487.75	487.75	487.75	487.75	487.75	487.75	487.75	487.75	487.75	487.75	487.75	487.75	487.75
Actual Ecc. for straight strands - - e(c) (inches) =	17.91	17.92	17.92	17.92	17.93	17.92	17.93	17.91	17.91	17.91	17.91	17.91	17.92	17.92	17.91	17.93
Actual Ecc. for draped strands @ end - - e(e) (inch) =	-14.09	-14.08	-14.08	-14.08	-14.07	-14.08	-14.07	-14.09	-14.09	-14.09	-14.09	-14.09	-14.08	-14.08	-14.09	-14.07
Diff in Ecc. for dr. strands @ end & CL - - e' (inch) =	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29
Weight of the concrete (Kips/inch) =	0.0542	0.0542	0.0542	0.0542	0.0542	0.0542	0.0542	0.0542	0.0542	0.0542	0.0542	0.0542	0.0542	0.0542	0.0542	0.0542
Camber due to the Straight strands (inches) =	2.7669	2.7299	2.7299	2.7435	2.7114	2.7504	2.7114	2.7669	2.7719	2.7669	2.7719	2.7832	2.7242	2.7242	2.7669	2.6686
Camber due to the Draped strands (inches) =	0.39	0.38	0.38	0.39	0.38	0.39	0.38	0.39	0.39	0.39	0.39	0.39	0.38	0.38	0.39	0.38
Total Camber due to strands (inches) =	3.16	3.11	3.11	3.13	3.09	3.14	3.09	3.16	3.16	3.16	3.16	3.18	3.11	3.11	3.16	3.05
Gravity deflection component. (inches) =	-1.28	-1.26	-1.26	-1.27	-1.25	-1.27	-1.25	-1.28	-1.28	-1.28	-1.28	-1.29	-1.26	-1.26	-1.28	-1.23

	B2-S2-1	B2-S2-2	B2-S2-3	B2-S2-4	B2-S2-5	B2-S2-6	B2-S2-7	B2-S2-8	B2-S2-9	B2-S2-10	B2-S2-11	B2-S2-12	B2-S2-13	B2-S2-14	B2-S2-15	B2-S2-26	Average
Total Camber for the member (inches) =	1.88	1.85	1.85	1.86	1.84	1.87	1.84	1.88	1.88	1.88	1.88	1.89	1.85	1.85	1.88	1.82	
Measured Camber for the member (inches)=	1.37	1.37	1.37	1.37	1.50	1.37	1.63	1.63	1.37	1.50	1.50	1.50	1.63	1.50	1.50	1.75	
												(off because friction losses were not accounted for)					
% Difference	36.92	35.29	35.29	35.91	22.85	36.23	13.05	15.08	37.15	25.05	25.26	25.73	13.49	23.32	25.05	3.75	25.59

Long Term Camber of the Member

PCI Method (Uses table factors)

			B2-S2-1	B2-S2-2	B2-S2-3	B2-S2-4	B2-S2-5	B2-S2-6	B2-S2-7	B2-S2-8	B2-S2-9	B2-S2-10	B2-S2-11	B2-S2-12	B2-S2-13	B2-S2-14	B2-S2-15	B2-S2-26
At erection of structure: (30-60 d		Factors																
Prestress Camber:		1.8	5.68	5.61	5.61	5.63	5.57	5.65	5.57	5.68	5.69	5.68	5.69	5.72	5.59	5.59	5.68	5.48
Self weight Deflectio		1.85	-2.37	-2.33	-2.33	-2.35	-2.31	-2.35	-2.31	-2.37	-2.37	-2.37	-2.37	-2.39	-2.33	-2.33	-2.37	-2.27
Total Camber			3.31	3.27	3.27	3.29	3.25	3.30	3.25	3.31	3.32	3.31	3.32	3.33	3.27	3.27	3.31	3.21
Final Camber:		Factors																
Prestress Camber:		2.45	7.73	7.63	7.63	7.67	7.58	7.69	7.58	7.73	7.75	7.73	7.75	7.78	7.62	7.62	7.73	7.46
Self weight Deflectio		2.7	-3.46	-3.41	-3.41	-3.42	-3.38	-3.43	-3.38	-3.46	-3.47	-3.46	-3.47	-3.48	-3.40	-3.40	-3.46	-3.32
Total Camber			4.28	4.23	4.23	4.24	4.20	4.25	4.20	4.28	4.28	4.28	4.28	4.30	4.22	4.22	4.28	4.14

(Note: only the self weight is being considered here & support conditions are not being considered)

	Time (days)																	
	0		1.88	1.85	1.85	1.86	1.84	1.87	1.84	1.88	1.88	1.88	1.88	1.89	1.85	1.85	1.88	1.82
For Plotting	45		3.31	3.27	3.27	3.29	3.25	3.30	3.25	3.31	3.32	3.31	3.32	3.33	3.27	3.27	3.31	3.21
	730 (2 year)		4.28	4.23	4.23	4.24	4.20	4.25	4.20	4.28	4.28	4.28	4.28	4.30	4.22	4.22	4.28	4.14

PCI Short Term Camber Analysis

Bridge # : 4516

Girder ty| 45M-95

(Some input data is linked)

****Bolded** values in the table below are estimated concrete strengths

Input Data

		B2-S1-1	B2-S1-2	B2-S1-3	B2-S1-4	B2-S2-5
**	Number of Draped Strands =	16	16	16	16	16
**	Number of Straight Strands =	34	34	34	34	34
	Modulus of Elasticity for steel (kip/in^2) =	28500	28500	28500	28500	28500
	Ultimate Strength of Strand (Kip/in^2) =	270	270	270	270	270
	Area of One Strand (Inch^2) =	0.153	0.153	0.153	0.153	0.153
	Initial Strand Prestress @ seating (* fpu) =	0.75	0.75	0.75	0.75	0.75
	Eccent. for draped st @ CL - - from bottom (inches)=	11	11	11	11	11
	Eccent. for draped st @ end - - from bottom (inches)=	35	35	35	35	35
	Eccent. for straight strands - - from bottom (inches)=	4.12	4.12	4.12	4.12	4.12
	Average eccentricity @ CL - - from bottom (inches)	6.32	6.32	6.32	6.32	6.32
**	Transformed Moment of Inertia of the Sect (inches^4)=	181808	181808	181732	181166	182208
	Concrete Strength initial (lbs / in^2) =	6333	6333	7578	8346	7321
	Concrete 28-day Strength (lbs / in^2)=	**9200**	**9200**	**9300**	**10100**	8690
	Modulus of Elasticity for Concrete (kip/ in^2) =	4837	4837	4857	5020	4729
	Weight of the concrete (Kcf) =	0.15	0.15	0.15	0.15	0.15
**	Center of gravity at CL - - from bottom (inches) =	21.82	21.82	21.82	21.84	21.80
	Area of the Cross Section (in^2) =	624	624	624	624	624
	Length of the unsupported beam (feet) =	93.00	93.00	93.00	93.00	93.00
	Location of draping points from ends (feet) =	39.50	39.50	39.50	39.50	39.50

Calculated Data

	B2-S1-1	B2-S1-2	B2-S1-3	B2-S1-4	B2-S2-5
fcs	-3.36	-3.36	-3.36	-3.37	-3.35
Initial losses due to Elastic Shortening (%)=	9.77	9.77	9.73	9.43	9.97
Straight Strand Force (Po (kips)) =	950	950	951	954	948
Drapped Strand Force (Po(kips)) =	447	447	447	449	446
Length of the unsupported span (inches) =	1116	1116	1116	1116	1116
Length to draping point (inches) =	474	474	474	474	474
Actual Ecc. for straight strands - - e(c) (inches) =	17.70	17.70	17.70	17.72	17.68
Actual Ecc. for draped strands @ end - - e(e) (inch) =	-13.18	-13.18	-13.18	-13.16	-13.20
Diff in Ecc. for dr. strands @ end & CL - - e' (inch) =	24	24	24	24	24
Weight of the concrete (Kips/inch) =	0.05	0.05	0.05	0.05	0.05
Camber due to the Straight strands (inches) =	2.98	2.98	2.97	2.89	3.03
Camber due to the Draped strands (inches) =	0.40	0.40	0.40	0.39	0.41
Total Camber due to strands (inches) =	3.38	3.38	3.37	3.28	3.43
Gravity deflection component. (inches) =	-1.24	-1.24	-1.24	-1.20	-1.27

	B2-S1-1	B2-S1-2	B2-S1-3	B2-S1-4	B2-S2-5	
Total Camber for the member (inches) =	2.13	2.13	2.13	2.08	2.17	
Measured Camber for the member (inches)=	1.75	1.75	1.87	1.75	1.63	(off because friction l
% Difference	21.95	21.95	13.76	18.88	19.13	Average

Long Term Camber of the Member

PCI Method (Uses table factors)

	Factors	B2-S1-1	B2-S1-2	B2-S1-3	B2-S1-4	B2-S2-5
At erection of structure: (30-60 d						
Prestress Camber:	1.8	6.08	6.08	6.06	5.91	6.18
Self weight Deflectio	1.85	-2.30	-2.30	-2.29	-2.23	-2.35
Total Camber		3.78	3.78	3.77	3.68	3.83
Final Camber:	Factors					
Prestress Camber:	2.45	8.28	8.28	8.25	8.04	8.42
Self weight Deflectio	2.7	-3.36	-3.36	-3.35	-3.25	-3.43
Total Camber		4.92	4.92	4.90	4.80	4.99

(Note: only the self weight is being considered here & support conditions are not being considered)

	Time (days)					
For Plotting	0	2.13	2.13	2.13	2.08	2.17
	45	3.78	3.78	3.77	3.68	3.83
	730 (2 year)	4.92	4.92	4.90	4.80	4.99

PCI Short Term Camber Analysis

Bridge # : 49535

Girder ty 45M-67 (epoxy coated steel reinforcement)

(Some input data is linked)

****Bolded** values in the table below are estimated concrete strengths

Input Data

		1-B1	2-B2	3-B2	4-B3
**	Number of Draped Strands =	8	8	8	8
**	Number of Straight Strands =	20	20	20	20
	Modulus of Elasticity for steel (kip/in^2) =	28500	28500	28500	28500
	Ultimate Strength of Strand (Kip/in^2) =	270	270	270	270
	Area of One Strand (Inch^2) =	0.153	0.153	0.153	0.153
	Initial Strand Prestress @ seating (* fpu) =	0.75	0.75	0.75	0.75
	Eccent. for draped st @ CL - - from bottom (inches)=	6	6	6	6
	Eccent. for draped st @ end - - from bottom (inches)=	37	37	37	37
	Eccent. for straight strands - - from bottom (inches)=	3	3	3	3
	Average eccentricity @ CL - - from bottom (inches)	3.86	3.86	3.86	3.86
**	Transformed Moment of Inertia of the Sect (inches^4)=	179775	180637	180323	181003
	Concrete Strength initial (lbs / in^2) =	5491	6296	7394.96	6186
	Concrete 28-day Strength (lbs / in^2)=	**8300**	**7200**	**7600**	6790
	Modulus of Elasticity for Concrete (kip/ in^2) =	4644	4394	4487	4296
	Weight of the concrete (Kcf) =	0.155	0.155	0.155	0.155
**	Center of gravity at CL - - from bottom (inches) =	22.12	22.10	22.10	22.09
	Area of the Cross Section (in^2) =	624	624	624	624
	Length of the unsupported beam (feet) =	65.00	65.00	65.00	65.00
	Location of draping points from ends (feet) =	26.00	26.00	26.00	26.00

Calculated Data

	1-B1	2-B2	3-B2	4-B3	
fcs	-2.27	-2.26	-2.26	-2.26	
Initial losses due to Elastic Shortening (%)=	6.87	7.24	7.09	7.39	
Straight Strand Force (Po (kips)) =	577	575	576	574	
Drapped Strand Force (Po(kips)) =	231	230	230	230	
Length of the unsupported span (inches) =	780	780	780	780	
Length to draping point (inches) =	312	312	312	312	
Actual Ecc. for straight strands - - e(c) (inches) =	19.12	19.10	19.10	19.09	
Actual Ecc. for draped strands @ end - - e(e) (inch) =	-14.88	-14.90	-14.90	-14.91	
Diff in Ecc. for dr. strands @ end & CL - - e' (inch) =	31	31	31	31	
Weight of the concrete (Kips/inch) =	0.06	0.06	0.06	0.06	
Camber due to the Straight strands (inches) =	1.01	1.05	1.03	1.07	
Camber due to the Draped strands (inches) =	0.20	0.21	0.21	0.21	
Total Camber due to strands (inches) =	1.20	1.26	1.24	1.28	
Gravity deflection component. (inches) =	-0.32	-0.34	-0.33	-0.35	
	1-B1	2-B2	3-B2	4-B3	
Total Camber for the member (inches) =	0.88	0.92	0.91	0.94	
Measured Camber for the member (inches)=	0.75	0.83	0.91	0.83	
% Difference	17.57	10.96	-0.50	12.92	10.24 Average

Long Term Camber of the Member

PCI Method (Uses table factors)

	Factors	1-B1	2-B2	3-B2	4-B3
At erection of structure: (30-60 d					
Prestress Camber:	1.8	2.17	2.27	2.23	2.31
Self weight Deflectio	1.85	-0.60	-0.63	-0.62	-0.64
Total Camber		1.57	1.64	1.61	1.67
Final Camber:					
Prestress Camber:	2.45	2.95	3.09	3.04	3.15
Self weight Deflectio	2.7	-0.87	-0.92	-0.90	-0.94
Total Camber		2.08	2.17	2.13	2.21

(Note: only the self weight is being considered here & support conditions are not being considered)

	Time (days)				
	0	0.88	0.92	0.91	0.94
For Plotting	45	1.57	1.64	1.61	1.67
	730 (2 year)	2.08	2.17	2.13	2.21

PCI Short Term Camber Analysis

Bridge # : 27624

Girder tyı 81M-139

(Some input data is linked)

****Bolded** values in the table below are estimated concrete strengths

Input Data

	B8S5N-366	B7S5N-365	B10S5N-36	B9S5N-367
** Number of Draped Strands =	16	16	16	16
** Number of Straight Strands =	42	42	42	42
Modulus of Elasticity for steel (kip/in^2) =	28500	28500	28500	28500
Ultimate Strength of Strand (Kip/in^2) =	270	270	270	270
Area of One Strand (Inch^2) =	0.153	0.153	0.153	0.153
Initial Strand Prestress @ seating (* fpu) =	0.75	0.75	0.75	0.75
Eccent. for draped st @ CL - - from bottom (inches)=	17	17	17	17
Eccent. for draped st @ end - - from bottom (inches)=	69	69	69	69
Eccent. for straight strands - - from bottom (inches)=	4.57	4.57	4.57	4.57
Average eccentricity @ CL - - from bottom (inches)	8.00	8.00	8.00	8.00
** Transformed Moment of Inertia of the Sect (inches^4)=	801450	797492	796619	796162
Concrete Strength initial (lbs / in^2) =	6681	6443	6369	6516
Concrete 28-day Strength (lbs / in^2)=	7240	**8300**	8560	**8700**
Modulus of Elasticity for Concrete (kip/ in^2) =	4404	4644	4701	4731
Weight of the concrete (Kcf) =	0.15	0.15	0.15	0.15
** Center of gravity at CL - - from bottom (inches) =	38.90	38.97	38.98	38.99
Area of the Cross Section (in^2) =	840	840	840	840
Length of the unsupported beam (feet) =	137.13	137.13	137.13	137.13
Location of draping points from ends (feet) =	54.73	54.73	54.73	54.73

Calculated Data

fcs	-2.90	-2.91	-2.91	-2.92
Initial losses due to Elastic Shortening (%)=	9.27	8.83	8.72	8.67
Straight Strand Force (Po (kips)) =	1181	1186	1188	1188
Drapped Strand Force (Po(kips)) =	450	452	452	453
Length of the unsupported span (inches) =	1645.5	1645.5	1645.5	1645.5
Length to draping point (inches) =	656.75	656.75	656.75	656.75
Actual Ecc. for straight strands - - e(c) (inches) =	34.33	34.40	34.41	34.42
Actual Ecc. for draped strands @ end - - e(e) (inch) =	-30.10	-30.03	-30.02	-30.01
Diff in Ecc. for dr. strands @ end & CL - - e' (inch) =	52	52	52	52
Weight of the concrete (Kips/inch) =	0.07	0.07	0.07	0.07
Camber due to the Straight strands (inches) =	3.89	3.73	3.69	3.68
Camber due to the Draped strands (inches) =	0.47	0.45	0.45	0.45
Total Camber due to strands (inches) =	4.36	4.18	4.14	4.12
Gravity deflection component. (inches) =	-1.97	-1.88	-1.86	-1.85

	B8S5N-366	B7S5N-365	B10S5N-36	B9S5N-367	
Total Camber for the member (inches) =	2.38	2.30	2.28	2.27	
Measured Camber for the member (inches)=	2.36	2.24	2.24	2.56	
% Difference	0.97	2.75	1.89	-11.22	-1.40 Average

Long Term Camber of the Member

PCI Method (Uses table factors)

At erection of structure: (30-60 d	Factors	B8S5N-366	B7S5N-365	B10S5N-36	B9S5N-367
Prestress Camber:	1.8	7.84	7.53	7.45	7.42
Self weight Deflectio	1.85	-3.65	-3.48	-3.44	-3.42
Total Camber		4.19	4.05	4.02	4.00
Final Camber:	Factors				
Prestress Camber:	2.45	10.67	10.24	10.15	10.10
Self weight Deflectio	2.7	-5.33	-5.07	-5.02	-4.99
Total Camber		5.35	5.17	5.13	5.11

(Note: only the self weight is being considered here & support conditions are not being considered)

	Time (days)				
For Plotting	0	2.38	2.30	2.28	2.27
	45	4.19	4.05	4.02	4.00
	730 (2 year)	5.35	5.17	5.13	5.11

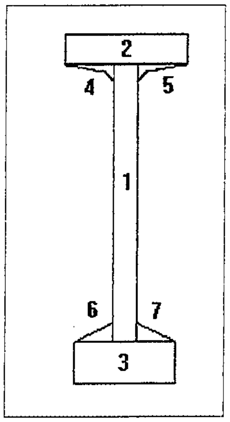
APPENDIX C

Moment of Inertia Calculations

Cross-Sectional Moment of Inertia Calculation

Bridge# 19045

Beam Type: 72



	Section #	Height (inches)	Width (inches)	Area (in^2)	Location of C.G. from base (inches)	Area*CG	Moment of I (x) (Section) (in^4)	A*d^2 (in^4)
	1	58.5	6	351	36.75	12899.25	100100.81	462.76
	2	6	30	180	69.00	12420.00	540.00	200779.38
	3	7.5	26	195	3.75	731.25	914.06	197834.51
Approx	4	2.5	10	12.5	65.17	814.58	4.34	10926.03
Approx	5	2.5	10	12.5	65.17	814.58	4.34	10926.03
	6	3.5	10	17.5	8.67	151.67	11.91	12696.26
	7	3.5	10	17.5	8.67	151.67	11.91	12696.26
	Summation			786		27983.00	101587.38	446321.23

Sect CG= 35.60

Moment of Inertia
Total = 547908.61 in^4

Transformed Moment to include the steel contribution

29000 ksi
28500 ksi
4298.48 ksi
6.75
6.63

6800 psi

@ Center Line

Strd Area: 0.153 inch^2

	# of Strnd	Area (in^2)	Trns. Area (in^2)	Location of C.G. from base (inches)	Area*CG	Moment of I (x) (Section) (in^4)	A*d^2 (in^4)	
Drped Str	12	1.836	10.34	15.00	155.06	0.00	4035.36	
Strt Strds	42	6.426	36.18	4.57	165.34	0.00	32971.15	
Regular Re-Bar		3.160	18.16	69.63	1264.3302	0.00	22076.32	Section #
							1392.91	1
Summation			64.68	24.50	1584.73		211053.70	2
							187490.67	3
			Sect CG=	34.76			11558.65	4
							11558.65	5
							11913.18	6
							11913.18	7
							505963.78	Sum

Transformed Moment of Inertia
Totl @ CL = 607551.15 in^4

@ End of beam

Strd Area: 0.153 inch^2

	# of Strnd	Area (in^2)	Trns. Area (in^2)	Location of C.G. from base (inches)	Area*CG	Moment of I (x) (Section) (in^4)	A*d^2 (in^4)	
Drped Str	12	1.836	10.34	62.00	640.90	0.00	7353.21	
Strt Strds	42	6.426	36.18	4.57	165.34	0.00	34230.52	
Regular Re-Bar		3.160	18.16	69.63	1264.3302	0.00	21359.02	Section #
							708.71	1
Summation			64.68	32.01	2070.57		204072.03	2
							194460.98	3
			Sect CG=	35.33			11128.55	4
							11128.55	5
							12440.44	6
							12440.44	7
							509322.45	Sum

Transformed Moment of Inertia
Totl @ End = 610909.82 in^4

Transformed Moment of Inertia
Averaged = 609230.49 in^4

APPENDIX D

Significance of Shear Deflection Calculation

Look at considering the case of Bridge 49535 to check that shear deflections are not important. This member has the smallest span to depth ratio and thus the largest shear to bending deformation ratio.

$\text{kip} := 1000\cdot\text{lb}$ $\quad \text{ksi} := \frac{\text{kip}}{\text{in}^2}$ $\quad A_c := 624\cdot\text{in}^2$ $\quad t_w := 6\cdot\text{in}$

$w := 0.05597\cdot\frac{\text{kip}}{\text{in}}$ $\quad E := 4098\cdot\text{ksi}$ $\quad L := 65\cdot\text{ft}$ $\quad I := 181792\cdot\text{in}^4$

$M := -w\cdot x\cdot\frac{x}{2} + w\cdot\frac{L}{2}\cdot x$ $\quad M := E\cdot I\cdot\phi$ $\quad h_w := \left(45 - \frac{6}{2} - \frac{7.5}{2}\right)\cdot\text{in}$

Bending Displacements are then:

$$y = \frac{1}{E\cdot I}\cdot\left[\int\int\left(-w\cdot x\cdot\frac{x}{2} + w\cdot\frac{L}{2}\cdot x\right)dx\,dx\right]$$

$$y = \frac{1}{E\cdot I}\cdot\left[\int\left(\frac{-1}{6}\cdot w\cdot x^3 + \frac{1}{4}\cdot w\cdot L\cdot x^2 + C1\right)dx\right]$$

$$y = \left(\frac{-1}{24}\cdot w\cdot x^4 + \frac{1}{12}\cdot w\cdot L\cdot x^3 + C1\cdot x + C2\right)\cdot\frac{1}{E\cdot I}$$

@ x=0; y=0 -- Thus, $C2 := 0$

@ x=L; y=0 -- Thus, $C1 := \frac{-1}{24}\cdot w\cdot L^3$

Which results in bending deflection of:

$$y = \left(\frac{-1}{24}\cdot w\cdot x^4 + \frac{1}{12}\cdot w\cdot L\cdot x^3 - \frac{1}{24}\cdot w\cdot L^3\cdot x\right)\cdot\frac{1}{E\cdot I}$$

Where the following values are:

w = distributed loading
E = Modulus of elasticity
L = length of the beam
x = distance along the beam
I = Moment of Inertia of section

If we look at the midspan bending displacements, the equation becomes:

$$y(L/2)= \left[\frac{-1}{24}\cdot w\cdot\left(\frac{L}{2}\right)^4+\frac{1}{12}\cdot w\cdot L\cdot\left(\frac{L}{2}\right)^3-\frac{1}{24}\cdot w\cdot L^3\cdot\left(\frac{L}{2}\right)\right]\cdot\frac{1}{E\cdot I}$$

$$y(L/2)= \frac{-5}{384}\cdot w\cdot\frac{L^4}{(E\cdot I)}$$

The Slope will be:

$$\theta := \frac{-1}{6}\cdot w\cdot x^3+\frac{1}{4}\cdot w\cdot L\cdot x^2+C1$$

$$\theta := \frac{-1}{6}\cdot w\cdot x^3+\frac{1}{4}\cdot w\cdot L\cdot x^2+\left(\frac{1}{24}\cdot w\cdot L^3-\frac{1}{12}\cdot w\cdot L\cdot L^2\right)$$

$$\theta := \frac{-1}{6}\cdot w\cdot x^3+\frac{1}{4}\cdot w\cdot L\cdot x^2-\frac{1}{24}\cdot w\cdot L^3$$

If we look at rotation at midspan, the equation becomes:

$$\theta := \frac{-1}{6}\cdot w\cdot\left(\frac{L}{2}\right)^3+\frac{1}{4}\cdot w\cdot L\cdot\left(\frac{L}{2}\right)^2-\frac{1}{24}\cdot w\cdot L^3$$

$$\theta := 0$$

If we look at rotation at the ends, the equation becomes:

$$\theta := \frac{-1}{6}\cdot w\cdot(0\cdot in)^3+\frac{1}{4}\cdot w\cdot L\cdot(0\cdot in)^2-\frac{1}{24}\cdot w\cdot L^3$$

$$\theta := \frac{-1}{24}\cdot w\cdot L^3$$

Looking at Shear Displacements in the sections

$\nu := 0.2$ See PCI 1-23

$$G := \frac{E}{2\cdot(1+\nu)} \qquad G := \frac{E}{2\cdot(1+0.2)}$$

$$f := \frac{A_c}{h_w\cdot t_w} \qquad V := w\cdot\frac{L}{2}-w\cdot x$$

Deflection due to shear is then:

$$\Delta_s := \int_{0 \cdot in}^{\frac{L}{2}} -f \cdot \left(\frac{V}{G \cdot A_c}\right) dx$$ From: "Matrix Analysis of Framed Structures - 3rd edition"
by: William Weaver Jr & James M. Gere

Where the following values are:
V = Shear force
G = Shear Modulus of Elasticity
Ac = Area of the concrete cross section
f = Shape factor for the section
ν = Poisson's Ratio (transverse to axial stain)

$$\Delta_s := \int_{0 \cdot in}^{\frac{L}{2}} \left(\frac{-A_c}{h_w \cdot t_w}\right) \cdot \left[\frac{w \cdot \frac{L}{2} - w \cdot x}{\left[\frac{E}{2 \cdot (1 + 0.2)}\right] \cdot A_c}\right] dx$$

$$\Delta_s := \int_{0 \cdot in}^{\frac{L}{2}} -1.2 \cdot w \cdot \frac{(L - 2. \cdot x)}{\left[h_w \cdot \left(t_w \cdot E\right)\right]} dx \qquad \Delta_s := -.3 \cdot w \cdot \frac{L^2}{\left[h_w \cdot \left(t_w \cdot E\right)\right]}$$

$$\Delta_s = -0.011 \cdot in$$

Camber due to the Prestress: (from sensitivity base case data)

$$\Delta_p := 1.33 \cdot in$$

The deflection due to the bending is equal to:

$$\Delta_m := \frac{-5}{384} \cdot w \cdot \frac{L^4}{(E \cdot I)} \qquad \Delta_m = -0.362 \cdot in$$

Percentage is very minor when compared to the camber effects at initial conditions.

$$\%Diff := \frac{\Delta_s}{\Delta_m + \Delta_s + \Delta_p} \cdot 100 \qquad \%Diff = -1.135$$

APPENDIX E

Approximate Time Step Method Calculations

Approximate Time Step Method for Bridge 19045.

Section Properties for the calculations (input data):

$kip := 1000 \cdot lb$

$h := 72 \cdot in$ $\quad RH := 72$

$L := 137.5 \cdot ft$ $\quad f_{pu} := 270000 \cdot \frac{lb}{in^2}$ $\quad ksi := \frac{kip}{in^2}$

$W_{SD} := \frac{lb}{ft} \cdot 0$ $\quad f_{pi} := f_{pu} \cdot 0.75$

$\lambda := 1$

$W_D := 0.0682 \cdot \frac{kip}{in}$ $\quad f_{py} := 0.9 \cdot f_{pu}$

$Surf := (30 + 26 + (6 + 10.11 + 2.83 + 51.5 + 10.59 + 7.5) \cdot 2) \cdot in$

$Surf = 233.06 \cdot in$

$y_{str} := 4.57 \cdot in$ $\quad y_{drp\_CL} := 15 \cdot in$ $\quad y_{drp\_END} := 62 \cdot in$

$Num_{str} := 42$ $\quad Num_{drp} := 12$

$Num := Num_{str} + Num_{drp}$

$A_{ps} := Num \cdot 0.1531 \cdot in^2$ $\quad A_{ps} = 8.2674 \cdot in^2$

$A_s := 4 \cdot 0.79 \cdot in^2$ $\quad min\_thick := 6.5inches$

$$y_{ps\_CL} := \frac{Num_{str} \cdot (y_{str}) + Num_{drp} \cdot (y_{drp\_CL})}{Num} \qquad y_{ps\_CL} = 6.89 \cdot in$$

$$y_{ps\_END} := \frac{Num_{str} \cdot (y_{str}) + Num_{drp} \cdot (y_{drp\_END})}{Num} \qquad y_{ps\_END} = 17.33222 \cdot in$$

Specific Data for each of the individual members of Bridge 19045, take from data found in Appendix B.

Data from:

B11-S2
B12-S2
B13-S2
B14-S2
B15-S2
B16-S2
B17-S2
B18-S2
B19-S2
B20-S2

$$I_t := \begin{bmatrix} 600350 \\ 600051 \\ 599283 \\ 598897 \\ 601774 \\ 598897 \\ 597081 \\ 600159 \\ 601774 \\ 601021 \end{bmatrix} \cdot in^4 \qquad CG := \begin{bmatrix} 34.85 \\ 34.86 \\ 34.87 \\ 34.87 \\ 34.82 \\ 34.87 \\ 34.90 \\ 34.86 \\ 34.82 \\ 34.85 \end{bmatrix} \cdot in \qquad f_{cprime} := \begin{bmatrix} 9000 \\ 9110 \\ 9400 \\ 9550 \\ 8500 \\ 9550 \\ 10300 \\ 9070 \\ 8500 \\ 8760 \end{bmatrix} \cdot \frac{lb}{in^2}$$

$e_{c\_CL} := CG - y_{ps\_CL}$ $\quad e_{c\_CL_5} = 27.98222 \cdot in$

(All members have approximately this same eccentricity.)

$e_{c\_END} := CG - y_{ps\_END}$

Now consider the concrete properties of the section:

$A_g := 786 \cdot in^2$ $\quad z := 0..9 \quad n := 8 \quad x := 0..n$

$$E_{c_z} := 40000 \cdot \sqrt{f_{cprime_z}} \cdot \sqrt{\frac{lb}{in^2}} + 1000000 \cdot \frac{lb}{in^2}$$

$weight := \frac{W_D}{A_g}$ $\quad weight = 149.93588 \cdot \frac{lb}{ft^3}$

Results for:

	$E_c =$ 0
B11-S2	0: 4794.733
B12-S2	1: 4817.853
B13-S2	2: 4878.144
B14-S2	3: 4908.964
B15-S2	4: 4687.818
B16-S2	5: 4908.964
B17-S2	6: 5059.557
B18-S2	7: 4809.462
B19-S2	8: 4687.818
B20-S2	9: 4743.795

$$E_c = \begin{array}{|c|c|}\hline & 0 \\ \hline 0 & 4794.733 \\ \hline 1 & 4817.853 \\ \hline 2 & 4878.144 \\ \hline 3 & 4908.964 \\ \hline 4 & 4687.818 \\ \hline 5 & 4908.964 \\ \hline 6 & 5059.557 \\ \hline 7 & 4809.462 \\ \hline 8 & 4687.818 \\ \hline 9 & 4743.795 \\ \hline \end{array} \cdot \frac{kip}{in^2}$$

Mix Design Properties:

$fines := 1534 \cdot lb \quad Agg := 1578 \cdot lb \quad \%fineagg := \frac{fines}{(Agg + fines)} \cdot 100 \quad \%fineagg = 49.29306$

$w\_c\_ratio := 0.31$

$cement := 750 \cdot \frac{lb}{yd^3}$

$slump := 5.5 \cdot in$

Other Calculated Properties:

$E_{ps} := 28500 \cdot \frac{kip}{in^2} \quad E_s := 29000 \cdot \frac{kip}{in^2} \quad n1_z := \frac{E_{ps}}{E_{c_z}} \quad n2_z := \frac{E_s}{E_{c_z}}$

$M_{SDc} := \frac{W_{SD} \cdot L^2}{8} \quad M_{Dc} := \frac{W_D \cdot L^2}{8}$

Input the Initial Deflection values: (from spread sheet)

Input for:	Initial Camber with Initial Elastic Losses	Initial Self Weight Deflection	Initial Superimposed DL Deflection
	$\Delta_{pso} :=$ ·in	$\Delta_{Di} :=$ ·in	$\Delta_{SD} :=$ ·in
B11-S2	4.55	-2.29	0
B12-S2	4.53	-2.28	0
B13-S2	4.49	-2.25	0
B14-S2	4.47	-2.24	0
B15-S2	4.63	-2.33	0
B16-S2	4.47	-2.24	0
B17-S2	4.36	-2.18	0
B18-S2	4.54	-2.28	0
B19-S2	4.63	-2.33	0
B20-S2	4.59	-2.31	0

Input the time to be considered for each of the members:

(Time at which the camber is considered corresponds to when the field data was collected on each of the individual members.)

Time to be looked at from release to final set of measured data.

$$t := \begin{array}{cccccccccc} B11 & B12 & B13 & B14 & B15 & B16 & B17 & B18 & B19 & B20 \\ \left[\begin{array}{cccccccccc} 1 & 1 & 3 & 3 & 1 & 1 & 1 & 1 & 1 & {}^{**}1 \\ 6 & 6 & 4 & 4 & 4 & 4 & 3 & 3 & 14 & 14 \\ 22 & 22 & 20 & 20 & 17 & 17 & 16 & 16 & 27 & 27 \\ 35 & 35 & 33 & 33 & 30 & 30 & 29 & 29 & 35 & 35 \\ 43 & 43 & 41 & 41 & 38 & 38 & 37 & 37 & 60 & 60 \\ 68 & 68 & 66 & 66 & 63 & 63 & 62 & 62 & 96 & 96 \\ 104 & 104 & 102 & 102 & 99 & 99 & 98 & 98 & 154 & 154 \\ 162 & 162 & 160 & 160 & 157 & 157 & 156 & 156 & 500 & 500 \\ 10000 & 10000 & 10000 & 10000 & 10000 & 10000 & 10000 & 10000 & 10000 & 10000 \end{array}\right] \end{array}$$

** Note: First row of time matrix needs to be the age of the concrete at loading of the member. (At release for only dead loading).

At Service Loads for the section, determine the losses by various methods

$$A_t := A_g + A_{ps}\cdot(n1 - 1) + A_s\cdot(n2 - 1)$$

$$P_i := f_{pi}\cdot\left(A_{ps}\right)$$

Stress in concrete at the level of the steel

$$f_{cs_z} := -0.9\cdot\left[\frac{P_i}{A_{t_z}} + \frac{P_i}{I_{t_z}}\cdot\left(e_{c\_CL_z}\right)^2\right] + \frac{M_{Dc}\cdot e_{c\_CL_z}}{I_{t_z}}$$

$$f_{cs} = \begin{array}{|c|c|} \hline & 0 \\ \hline 0 & -2.66905 \\ \hline 1 & -2.6712 \\ \hline 2 & -2.67514 \\ \hline 3 & -2.6766 \\ \hline 4 & -2.66063 \\ \hline 5 & -2.6766 \\ \hline 6 & -2.6866 \\ \hline 7 & -2.67079 \\ \hline 8 & -2.66063 \\ \hline 9 & -2.66651 \\ \hline \end{array} \cdot\frac{kip}{in^2}$$

(Mitchell uses the transformed section properties in calculating the stresses in the section - relating to the strains (pg. 14O))

A) PCI Total Prestress Losses Estimation

1. Elastic Shortening

$$K_{es} := 1$$

$$\varepsilon_{ES_z} := -K_{es}\cdot\frac{1}{E_{c_z}}\cdot f_{cs_z}$$

2. Creep

$K_{CR} := 2$ Assume NWC not sand-light weight

$$f_{csd_z} := \frac{M_{SDc}\cdot e_{c\_CL_z}}{I_{t_z}} \qquad f_{csd_1} = 0\cdot\frac{kip}{in^2}$$

$$CR_z := K_{CR} \cdot \left(\frac{E_{ps}}{E_{c_z}} \right) \cdot \left(-f_{cs_z} - f_{csd_z} \right)$$

3. Shrinkage

$$V\_S := \frac{A_g}{Surf} \qquad V\_S = 3.37252 \cdot in$$

$$SH := 8.2 \cdot 10^{-6} \cdot \left(1 - 0.06 \cdot \frac{V\_S}{in} \right) \cdot (100 - RH) \cdot E_{ps}$$

$$SH = 5.21949 \cdot \frac{kip}{in^2}$$

4. Relaxation

$$K_{re} := 5000 \cdot \frac{lb}{in^2} \qquad J := 0.04 \qquad C := 1.0$$

$$RE := \left[K_{re} - J \cdot \left(CR + SH + \varepsilon_{ES} \cdot E_{ps} \right) \right] \cdot C$$

Loss Data for the Individual Members:

Results for:
B11-S2
B12-S2
B13-S2
B14-S2
B15-S2
B16-S2
B17-S2
B18-S2
B19-S2
B20-S2

$\varepsilon_{ES} =$

	0
0	0.000557
1	0.000554
2	0.000548
3	0.000545
4	0.000568
5	0.000545
6	0.000531
7	0.000555
8	0.000568
9	0.000562

$CR =$

	0
0	31.72978
1	31.60298
2	31.25836
3	31.07914
4	32.35103
5	31.07914
6	30.26677
7	31.65325
8	32.35103
9	32.04001

$\cdot \frac{kip}{in^2}$

$RE =$

	0
0	2.88743
1	2.89504
2	2.91572
3	2.92647
4	2.85016
5	2.92647
6	2.97521
7	2.89203
8	2.85016
9	2.86882

$\cdot \frac{kip}{in^2}$

5. Total Losses

$$Loss := RE + CR + SH + \varepsilon_{ES} \cdot E_{ps} \qquad \%loss := \frac{Loss}{f_{pi}} \cdot 100 \qquad f_{pe} := f_{pi} - Loss$$

Results for:
B11-S2
B12-S2
B13-S2
B14-S2
B15-S2
B16-S2
B17-S2
B18-S2
B19-S2
B20-S2

$Loss =$

	0
0	55.7016
1	55.519
2	55.02275
3	54.76468
4	56.5962
5	54.76468
6	53.59486
7	55.5914
8	56.5962
9	56.14833

$\cdot \frac{kip}{in^2}$

$\%loss =$

	0
0	27.50696
1	27.41679
2	27.17173
3	27.04428
4	27.94874
5	27.04428
6	26.4666
7	27.45254
8	27.94874
9	27.72757

$f_{pe} =$

	0
0	146.7984
1	146.981
2	147.47725
3	147.73532
4	145.9038
5	147.73532
6	148.90514
7	146.9086
8	145.9038
9	146.35167

$\cdot \frac{kip}{in^2}$

B) AASHTO Total Prestress Losses Estimation

1. Elastic Shortening Losses are calculated the same way as in PCI method

$$ES := \varepsilon_{ES} \cdot E_{ps}$$

2. Creep

Assume NWC not sand-light weight

$$f_{cds_z} := \frac{M_{SDc} \cdot e_{c\_CL_z}}{I_{t_z}} \qquad f_{cds_1} = 0 \cdot \frac{kip}{in^2}$$

$$CR_{c_z} := \left[\left(-f_{cs_z} \right) \cdot 12 - \left(f_{cds_z} \right) \cdot 7 \right]$$

$CR_c =$ $\cdot \frac{kip}{in^2}$

	0
0	32.0286
1	32.05442
2	32.10164
3	32.11924
4	31.92753
5	32.11924
6	32.23925
7	32.0495
8	31.92753
9	31.99815

3. Shrinkage

$$SH\_2 := (17000 - 150 \cdot RH) \cdot \frac{lb}{in^2} \qquad SH\_2 = 6.2 \cdot \frac{kip}{in^2}$$

4. Relaxation

$$CR_s := 5000 \cdot \frac{lb}{in^2} - 0.1 \cdot ES - 0.05 \cdot \left(SH + CR_c \right)$$

$CR_s =$ $\cdot \frac{kip}{in^2}$

	0
0	1.55111
1	1.55616
2	1.57103
3	1.57911
4	1.5251
5	1.57911
6	1.61372
7	1.55389
8	1.5251
9	1.53712

5. Total Losses

$$Loss := CR_c + CR_s + SH\_2 + ES \qquad \%loss := \frac{Loss}{f_{pi}} \cdot 100 \qquad f_{pe} := f_{pi} - Loss$$

Results for:

B11-S2
B12-S2
B13-S2
B14-S2
B15-S2
B16-S2
B17-S2
B18-S2
B19-S2
B20-S2

$Loss =$ $\cdot \frac{kip}{in^2}$

	0
0	55.6446
1	55.61207
2	55.50184
3	55.43792
4	55.82814
5	55.43792
6	55.18636
7	55.63001
8	55.82814
9	55.75527

$\%loss =$

	0
0	27.47881
1	27.46275
2	27.40832
3	27.37675
4	27.56945
5	27.37675
6	27.25252
7	27.47161
8	27.56945
9	27.53347

$f_{pe} =$ $\cdot \frac{kip}{in^2}$

	0
0	146.8554
1	146.88793
2	146.99816
3	147.06208
4	146.67186
5	147.06208
6	147.31364
7	146.86999
8	146.67186
9	146.74473

C) The PCI and the AASHTO methods for computing ultimate losses seem to be fairly close to one another. However, in order to do an approximate time step analysis of the section, we need to calculate losses at time increments. The AASHTO code does not seem to address methods for determining the time step losses of prestressing force.

ACI 435R-95 does address this issue and so losses were using methods outlined in this report as well as in the report by ACI Committee 209.

Elastic Shortening will be treated by applying it as a constant to the final time dependent loss since it occurs at the release of the member.

1. Creep Losses in the section (using ACI 209R-92 & ACI 435-95)

$C_u := 2.35$ (From ACI test data the ultimate creep ranges from 2-4 depending on the particular concrete mix. The average value is 2.35.)

Now apply Creep Correction Factors in order to characterize the creep for a particular member.

a.) Loading Age factor: $t_{la} := 1$

$\gamma_{la} := 1.13\cdot\left(t_{la}\right)^{-0.094}$ $\gamma_{la} := 1$ (Loaded before 1-3 days)

b.) Ambient Relative Humidity factor:

$\gamma_{lambda} := \text{if}(RH > 40, 1.27 - 0.0067\cdot RH, 1)$ $\gamma_{lambda} = 0.7876$

c.) Volume to Surface Factor:

$$\gamma_h := \frac{2}{3}\cdot\left[1 + 1.13\cdot\left(\exp\left(-0.54\cdot\frac{V\_S}{in}\right)\right)\right] \qquad \gamma_h = 0.78858$$

d.) Temperature factor:

$\gamma_{temp} := 1$

e.) Composition factors:

$$\gamma_{slump} := 0.82 + 0.067\cdot\frac{slump}{in} \qquad \gamma_{slump} = 1.1885$$

$\gamma_{fineagg} := 0.88 + 0.0024\cdot\%fineagg$ $\gamma_{fineagg} = 0.9983$

$\gamma_{comp} := \gamma_{slump}\cdot\gamma_{fineagg}$ $\gamma_{comp} = 1.18648$

(Air Content is not a problem
- not air entrained concrete)

f.) Total factors will be:

$\gamma_{c\_tot} := \gamma_{comp}\cdot\gamma_{temp}\cdot\gamma_h\cdot\gamma_{lambda}\cdot\gamma_{la}$ $\gamma_{c\_tot} = 0.73691$

$CRult := \left(\gamma_{c\_tot}\right)\cdot C_u$ $CRult = 1.73174$

$$C_{x,z} := \frac{\left(t_{x,z} - t_{0,z}\right)^{0.6}}{10 + \left(t_{x,z} - t_{0,z}\right)^{0.6}}\cdot CRult$$

Thus, the resulting creep for the section would be:

$$CR\_3_{x,z} := \left(C_{x,z}\right)\cdot\left(-f_{cs_z}\right)\cdot\left(\frac{E_{ps}}{E_{c_z}}\right)$$

$CR\_3 =$

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	5.7	5.7	2.5	2.4	4.5	4.4	3.4	3.6	8.9	8.8
2	10.5	10.5	9.6	9.5	9.7	9.3	8.8	9.2	11.6	11.5
3	12.5	12.4	11.8	11.7	12	11.6	11.1	11.6	12.7	12.6
4	13.3	13.3	12.7	12.6	13.1	12.5	12.1	12.7	15	14.9
5	15.2	15.2	14.8	14.7	15.2	14.6	14.2	14.8	17	16.8
6	17	16.9	16.6	16.5	17.1	16.4	16	16.7	18.8	18.6
7	18.6	18.6	18.3	18.2	18.9	18.1	17.6	18.5	22.6	22.4
8	26.4	26.3	26	25.9	26.9	25.9	25.2	26.4	26.9	26.7

$\cdot\frac{kip}{in^2}$

2. Shrinkage Component of losses

$$\varepsilon_{sh\_u} := 780\cdot 10^{-6}$$

Now apply Shrinkage Correction Factors in order to characterize the creep value (ACI 209R-92 & ACI 435R-95).

a.) Ambient Relative Humidity factor:

$$\gamma_{s\_lam} := \text{if}(RH \ge 40, 1.4 - 0.01\cdot RH, 1)$$

$$\gamma_{s\_lambda} := \text{if}\left(RH > 80, 3.0 - 0.03\cdot RH, \gamma_{s\_lam}\right) \quad \gamma_{s\_lambda} = 0.68$$

b.) Volume to Surface Factor:

$$\gamma_{s\_h} := 1.2\cdot\left(\exp\left(-0.12\cdot\frac{V\_S}{in}\right)\right) \qquad \gamma_{s\_h} = 0.80061$$

c.) Temperature factor:

$$\gamma_{s\_temp} := 1$$

d.) Composition factors :

$$\gamma_{s\_slump} := 0.89 + 0.041\cdot\frac{slump}{in} \qquad \gamma_{s\_slump} = 1.1155$$

$$\gamma_{s\_fineagg} := \text{if}(\%fineagg > 50, 0.9 + 0.002\cdot\%fineagg, 0.3 + 0.014\cdot\%fineagg)$$

$$\gamma_{s\_fineagg} = 0.9901$$

$$\gamma_{s\_cem} := 0.75 + 0.00036\cdot\frac{cement}{\left(\frac{lb}{yd^3}\right)} \qquad \gamma_{s\_cem} = 1.02$$

(Air Content is not a problem
- not air entrained concrete)

$$\gamma_{s\_comp} := \gamma_{s\_slump}\cdot\gamma_{s\_fineagg}\cdot\gamma_{s\_cem} \qquad \gamma_{s\_comp} = 1.12655$$

e.) Total factors will be:

$$\gamma_{s\_tot} := \gamma_{s\_comp} \cdot \gamma_{s\_temp} \cdot \gamma_{s\_h} \cdot \gamma_{s\_lambda} \qquad \gamma_{s\_tot} = 0.61331$$

$$\varepsilon_{sh_{x,z}} := \left[\frac{\left(t_{x,z} - t_{0,z}\right)}{55 + \left(t_{x,z} - t_{0,z}\right)} \right] \cdot \left(\varepsilon_{sh\_u}\right) \cdot \gamma_{s\_tot}$$

Thus the loss of prestress due to the shrinkage at various time steps is equal to:

$$SH\_3_{x,z} := \varepsilon_{sh_{x,z}} \cdot E_{ps}$$

For Girders B11-B20

For Various Time steps

$SH\_3 =$ $\cdot \frac{kip}{in^2}$

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	1.14	1.14	0.24	0.24	0.71	0.71	0.48	0.48
2	3.77	3.77	3.22	3.22	3.07	3.07	2.92	2.92
3	5.21	5.21	4.81	4.81	4.71	4.71	4.6	4.6
4	5.9	5.9	5.57	5.57	5.48	5.48	5.39	5.39
5	7.49	7.49	7.28	7.28	7.22	7.22	7.17	7.17
6	8.89	8.89	8.76	8.76	8.73	8.73	8.7	8.7
7	10.16	10.16	10.1	10.1	10.08	10.08	10.06	10.06
8	13.56	13.56	13.56	13.56	13.56	13.56	13.56	13.56

3. Reduced Relaxation losses for the section (considering relaxation to begin at 18 hours after stressing, approximately at the time of casting).

In order to account for the reduced relaxation due to the Initial losses, we need to find the intrinsic relaxation and multiply it by a reduction factor in order to get the reduced relaxation.(ACI 209R-92 section 3.7.)

The intrinsic relaxation, relaxation of strand in a constant tensile lab test, can be found using the formula below:

$$f_{sr_{x,z}} := f_{pi} \cdot \left[\frac{\log\left(24 \cdot t_{x,z}\right)}{45} \cdot \left(\frac{f_{pi}}{f_{py}} - 0.55 \right) \right]$$ (from table 3.7.1 ACI 209R-92)

For Girders B11-B20

For Various Time steps

$f_{sr} =$ $\cdot \frac{kip}{in^2}$

	0	1	2	3	4	5	6	7	8	9
0	1.76	1.76	2.37	2.37	1.76	1.76	1.76	1.76	1.76	1.76
1	2.75	2.75	2.53	2.53	2.53	2.53	2.37	2.37	3.22	3.22
2	3.47	3.47	3.42	3.42	3.33	3.33	3.3	3.3	3.58	3.58
3	3.73	3.73	3.7	3.7	3.64	3.64	3.62	3.62	3.73	3.73
4	3.84	3.84	3.82	3.82	3.77	3.77	3.76	3.76	4.03	4.03
5	4.1	4.1	4.08	4.08	4.05	4.05	4.05	4.05	4.29	4.29
6	4.33	4.33	4.32	4.32	4.3	4.3	4.3	4.3	4.55	4.55
7	4.58	4.58	4.57	4.57	4.56	4.56	4.56	4.56	5.2	5.2
8	6.86	6.86	6.86	6.86	6.86	6.86	6.86	6.86	6.86	6.86

In order to account for a situation similar to that in the member itself, a reduction factor needs to be applied. This factor accounts for the interdependence of the relaxation, creep, and shrinkage in the member by looking at the total losses up to this point.

$$\text{fsi\_fsy} := \text{if}\left(\frac{f_{pi}}{f_{py}} < 0.8, \frac{f_{pi}}{f_{py}}, 0.8\right) \qquad \text{fsi\_fsy\_ratio} := \text{if}(\text{fsi\_fsy} > 0.5, \text{fsi\_fsy}, 0.5)$$

Since determining the reduced relaxation is an iterative process, we need to begin with an estimated set of data for the % of total losses that there is

$$\lambda 1 := \begin{bmatrix} 0.93774 & 0.93743 & 1.25804 & 1.25746 & 0.93926 & 0.93615 & 0.93416 & 0.93755 & 0.93926 & 0.9385 \\ 4.99945 & 4.98603 & 2.74059 & 2.73203 & 4.06539 & 3.95979 & 3.28545 & 3.37925 & 7.6327 & 7.58244 \\ 9.16383 & 9.13924 & 8.3585 & 8.32649 & 8.3197 & 8.09709 & 7.74642 & 7.98212 & 10.0883 & 10.02302 \\ 11.00083 & 10.97178 & 10.41987 & 10.38058 & 10.51251 & 10.23601 & 9.92853 & 10.22521 & 11.14347 & 11.072 \\ 11.84784 & 11.81663 & 11.34355 & 11.30111 & 11.48395 & 11.18438 & 10.88768 & 11.21014 & 13.39973 & 13.31496 \\ 13.74478 & 13.70911 & 13.37265 & 13.32319 & 13.60313 & 13.25262 & 12.96585 & 13.34478 & 15.31191 & 15.21622 \\ 15.42853 & 15.3889 & 15.13662 & 15.08125 & 15.43216 & 15.03901 & 14.75116 & 15.17707 & 17.06323 & 16.95688 \\ 17.02364 & 16.97998 & 16.78031 & 16.71934 & 17.13013 & 16.69557 & 16.39904 & 16.86912 & 20.36124 & 20.23449 \\ 23.43928 & 23.37817 & 23.21224 & 23.12609 & 23.73922 & 23.12616 & 22.73794 & 23.40239 & 23.73922 & 23.58894 \end{bmatrix}$$

$$\omega_{x,z} := \frac{\lambda 1_{x,z}}{100} - \frac{f_{sr_{x,z}}}{f_{py}}$$

$\lambda 1$ = Percentage of strand stress loss to that point in time (excluding initial losses at release). Since we do not know this value yet because we do not know the relaxation losses, an **iteration** must be done in order to determine these values.

ω = A parameter that accounts for reducing relaxation by considering the intrinsic relaxation and the losses up to that point.

For Girders B11-B20

For Various Time steps

$\omega =$

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0.04	0.04	0.02	0.02	0.03	0.03	0.02	0.02	0.06	0.06
2	0.08	0.08	0.07	0.07	0.07	0.07	0.06	0.07	0.09	0.09
3	0.09	0.09	0.09	0.09	0.09	0.09	0.08	0.09	0.1	0.1
4	0.1	0.1	0.1	0.1	0.1	0.1	0.09	0.1	0.12	0.12
5	0.12	0.12	0.12	0.12	0.12	0.12	0.11	0.12	0.14	0.13
6	0.14	0.14	0.13	0.13	0.14	0.13	0.13	0.13	0.15	0.15
7	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.18	0.18
8	0.21	0.21	0.2	0.2	0.21	0.2	0.2	0.21	0.21	0.21

Knowing the previous terms, we can determine the relaxation reduction coefficient and apply it to the intrinsic relaxation for the members.
(Table 3.7.2 - ACI 209R - 92)

$$x1 := 0..6 \qquad y1 := 0..7$$

$$\text{ReductTable} := \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0.547 & 0.729 & 0.798 & 0.835 & 0.857 & 0.872 \\ 0 & 0.289 & 0.516 & 0.627 & 0.689 & 0.729 & 0.756 \\ 0 & 0.172 & 0.361 & 0.486 & 0.564 & 0.615 & 0.652 \\ 0 & 0.099 & 0.262 & 0.375 & 0.458 & 0.516 & 0.55 \\ 0 & 0.013 & 0.150 & 0.238 & 0.305 & 0.361 & 0.406 \\ 0 & 0.000 & 0.077 & 0.159 & 0.216 & 0.262 & 0.300 \\ 0 & 0.000 & 0.029 & 0.102 & 0.157 & 0.197 & 0.230 \end{bmatrix} \qquad \text{fsi\_fsy\_range} := \begin{bmatrix} 0.5 \\ 0.55 \\ 0.6 \\ 0.65 \\ 0.7 \\ 0.75 \\ 0.8 \end{bmatrix}$$

$$\text{Red}_{0_{x1}} := \left(\text{ReductTable}^{T}\right)_{x1,0} \qquad \text{Red}_{0.05_{x1}} := \left(\text{ReductTable}^{T}\right)_{x1,1}$$

$$\text{Red}_{0.1_{x1}} := \left(\text{ReductTable}^{T}\right)_{x1,2} \qquad \text{Red}_{0.15_{x1}} := \left(\text{ReductTable}^{T}\right)_{x1,3}$$

$$\text{Red}_{0.2_{x1}} := \left(\text{ReductTable}^{T}\right)_{x1,4} \qquad \text{Red}_{0.3_{x1}} := \left(\text{ReductTable}^{T}\right)_{x1,5}$$

$$\text{Red}_{0.4_{x1}} := \left(\text{ReductTable}^{T}\right)_{x1,6} \qquad \text{Red}_{0.5_{x1}} := \left(\text{ReductTable}^{T}\right)_{x1,7}$$

ψ_{range} reduction ranges are dependent on the ratio of fpi/fpy (initial/yield prestress strand stress) (See Table 3.7.2 ACI 209)

$$\psi_{\text{range}_0} := \text{linterp}\left(\text{fsi\_fsy\_range}, \text{Red}_{0}, \text{fsi\_fsy\_ratio}\right)$$

$$\psi_{\text{range}_1} := \text{linterp}\left(\text{fsi\_fsy\_range}, \text{Red}_{0.05}, \text{fsi\_fsy\_ratio}\right)$$

$$\psi_{\text{range}_2} := \text{linterp}\left(\text{fsi\_fsy\_range}, \text{Red}_{0.1}, \text{fsi\_fsy\_ratio}\right)$$

$$\psi_{\text{range}_3} := \text{linterp}\left(\text{fsi\_fsy\_range}, \text{Red}_{0.15}, \text{fsi\_fsy\_ratio}\right)$$

$$\psi_{\text{range}_4} := \text{linterp}\left(\text{fsi\_fsy\_range}, \text{Red}_{0.2}, \text{fsi\_fsy\_ratio}\right)$$

$$\psi_{\text{range}_5} := \text{linterp}\left(\text{fsi\_fsy\_range}, \text{Red}_{0.3}, \text{fsi\_fsy\_ratio}\right)$$

$$\psi_{\text{range}_6} := \text{linterp}\left(\text{fsi\_fsy\_range}, \text{Red}_{0.4}, \text{fsi\_fsy\_ratio}\right)$$

$$\psi_{\text{range}_7} := \text{linterp}\left(\text{fsi\_fsy\_range}, \text{Red}_{0.5}, \text{fsi\_fsy\_ratio}\right)$$

$$\omega_{\text{range}} := \begin{bmatrix} 0 \\ 0.05 \\ 0.1 \\ 0.15 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \end{bmatrix}$$

$$\psi_{x,z} := \text{linterp}\left(\omega_{\text{range}}, \psi_{\text{range}}, \omega_{x,z}\right)$$

Thus, the final Relaxation Reduction Values for Bridge 19045 are

For Girders B11-B20

For Various Time steps

$\psi =$

	0	1	2	3	4	5	6	7
0	0.995	0.995	0.993	0.993	0.994	0.995	0.995	0.995
1	0.901	0.901	0.956	0.957	0.923	0.925	0.941	0.938
2	0.809	0.809	0.827	0.827	0.827	0.832	0.84	0.834
3	0.768	0.769	0.782	0.782	0.779	0.785	0.792	0.785
4	0.75	0.751	0.761	0.762	0.758	0.765	0.771	0.764
5	0.713	0.714	0.721	0.722	0.716	0.723	0.729	0.721
6	0.68	0.681	0.686	0.687	0.68	0.688	0.694	0.685
7	0.649	0.65	0.654	0.655	0.647	0.656	0.662	0.652
8	0.549	0.55	0.552	0.553	0.544	0.553	0.56	0.549

The Reduced relaxation is then,

$$REL\_3_{x,z} := f_{sr_{x,z}} \cdot \psi_{x,z}$$

For Girders B11-B20

For Various Time steps

$REL\_3 =$

	0	1	2	3	4	5	6	7
0	1.75	1.75	2.351	2.351	1.75	1.75	1.75	1.75
1	2.479	2.48	2.417	2.418	2.332	2.338	2.228	2.222
2	2.807	2.809	2.826	2.829	2.752	2.769	2.767	2.749
3	2.865	2.867	2.888	2.892	2.838	2.861	2.871	2.846
4	2.884	2.886	2.905	2.909	2.859	2.885	2.899	2.871
5	2.921	2.924	2.941	2.945	2.902	2.931	2.949	2.917
6	2.946	2.95	2.965	2.97	2.926	2.961	2.983	2.945
7	2.972	2.976	2.989	2.995	2.951	2.99	3.016	2.971
8	3.763	3.77	3.787	3.796	3.732	3.796	3.839	3.767

$\cdot \frac{kip}{in^2}$

Thus the final losses for the section can be determined by combining the loss terms:

$$Losses\_3_{x,z} := REL\_3_{x,z} + SH\_3_{x,z} + CR\_3_{x,z} + ES_z$$

For Girders B11-B20

For Various Time steps

$Losses\_3 =$

	0	1	2	3	4	5	6	7	8	9
0	17.6	17.6	18	17.9	17.9	17.3	16.9	17.6	17.9	17.8
1	25.2	25.1	20.8	20.6	23.8	22.9	21.3	22.1	30.4	30.2
2	33	32.9	31.2	31.1	31.7	30.7	29.6	30.7	35	34.7
3	36.4	36.3	35.1	34.9	35.8	34.7	33.7	34.9	36.9	36.7
4	38	37.9	36.8	36.7	37.6	36.4	35.5	36.8	41.1	40.8
5	41.5	41.4	40.6	40.4	41.5	40.3	39.4	40.7	44.7	44.4
6	44.7	44.5	43.9	43.7	44.9	43.7	42.8	44.2	48	47.6
7	47.6	47.5	47	46.8	48.1	46.8	45.9	47.3	54.1	53.8
8	59.6	59.4	59	58.8	60.4	58.8	57.7	59.5	60.4	60

$\cdot \frac{kip}{in^2}$

$f_{po} := f_{pi} - ES$

$P_{e_{x,z}} := P_i - Losses\_3_{x,z} \cdot A_{ps}$

Results for:
B11-S2
B12-S2
B13-S2
B14-S2
B15-S2
B16-S2
B17-S2
B18-S2
B19-S2
B20-S2

$f_{po} =$

	0
0	186.63511
1	186.69851
2	186.87082
3	186.96043
4	186.32448
5	186.96043
6	187.36662
7	186.67337
8	186.32448
9	186.48

$\cdot \frac{kip}{in^2}$

Now determine if another Iteration for λ1 needs to be performed:

$P_o := f_{po} \cdot A_{ps}$ $\Delta P_{x,z} := P_{o_z} - P_{e_{x,z}}$

For Girders B11-B20

For Various Time steps

$\Delta P =$

	0	1	2	3	4	5	6	7
0	14.5	14.5	19.4	19.4	14.5	14.5	14.5	14.5
1	77.1	77	42.3	42.2	62.6	61.2	50.9	52.2
2	141.4	141.1	129.1	128.7	128.2	125.2	120	123.2
3	169.7	169.3	161	160.4	161.9	158.2	153.8	157.8
4	182.8	182.4	175.2	174.7	176.9	172.9	168.7	173
5	212.1	211.6	206.6	205.9	209.5	204.8	200.8	205.9
6	238.1	237.5	233.8	233.1	237.7	232.5	228.5	234.2
7	262.7	262.1	259.2	258.4	263.9	258.1	254	260.3
8	361.7	360.8	358.6	357.4	365.7	357.4	352.2	361.2

$\cdot kip$

Compare this set of values to the initial guess of the percentage of total losses:

$\lambda 1_{x,z} := \frac{\Delta P_{x,z}}{P_{o_z}} \cdot 100$

For Girders B11-B20

For Various Time steps

$\lambda 1 =$

	0	1	2	3	4	5	6	7
0	0.94	0.94	1.26	1.26	0.94	0.94	0.93	0.94
1	5	4.99	2.74	2.73	4.07	3.96	3.29	3.38
2	9.16	9.14	8.36	8.33	8.32	8.1	7.75	7.98
3	11	10.97	10.42	10.38	10.51	10.24	9.93	10.23
4	11.85	11.82	11.34	11.3	11.48	11.18	10.89	11.21
5	13.74	13.71	13.37	13.32	13.6	13.25	12.97	13.34
6	15.43	15.39	15.14	15.08	15.43	15.04	14.75	15.18
7	17.02	16.98	16.78	16.72	17.13	16.7	16.4	16.87
8	23.44	23.38	23.21	23.13	23.74	23.13	22.74	23.4

D.) Approximate Approach to deflection (Brandon's model)

For Girders B11-B20

For Various
Time steps

$\lambda a_{x,z} \quad 1 \quad \left(\frac{\Delta P_{x,z}}{P_{o_z}}\right)\cdot 2$

$\lambda a =$

	0	1	2	3	4	5	6	7
0	0.995	0.995	0.994	0.994	0.995	0.995	0.995	0.995
1	0.975	0.975	0.986	0.986	0.98	0.98	0.984	0.983
2	0.954	0.954	0.958	0.958	0.958	0.96	0.961	0.96
3	0.945	0.945	0.948	0.948	0.947	0.949	0.95	0.949
4	0.941	0.941	0.943	0.943	0.943	0.944	0.946	0.944
5	0.931	0.931	0.933	0.933	0.932	0.934	0.935	0.933
6	0.923	0.923	0.924	0.925	0.923	0.925	0.926	0.924
7	0.915	0.915	0.916	0.916	0.914	0.917	0.918	0.916
8	0.883	0.883	0.884	0.884	0.881	0.884	0.886	0.883

$\Delta_{SD1_{x,z}} \quad 0\cdot in$

$K_{a_{x,z}} \quad 1.13\cdot\left(t_{x,z}\right)^{0.095}$

$A_{sprime} \quad 0\cdot in^2$

Asprime is steel on the same side of the centroid as the pretensioned steel. (See ACI 209R-92, sec 3.5)

$k_r \quad \frac{1}{\left(1 \quad \frac{A_{sprime}}{A_{ps}}\right)}$

$k_r = 1$

If As'/Aps <= 2.0 then kr can be used, for all practical purposes kr will be 1 if the nonprestressed steel is minimal.

For non-composite beams, Branson's model can be reduced down to the following equation for our beams. (Nawy, 423)

$$\delta_{x,z} \quad \left(1 \quad \frac{\Delta P_{x,z}}{P_{o_z}} \quad \lambda a_{x,z}\cdot k_r\cdot C_{x,z}\right)\cdot\Delta_{pso_z} \quad \left(k_r\cdot C_{x,z} \quad 1\right)\cdot\Delta_{Di_z} \quad \left(K_{a_{x,z}}\cdot k_r\cdot C_{x,z} \quad 1\right)\cdot\Delta_{SD1_{x,z}}$$

For Girders B11-B20

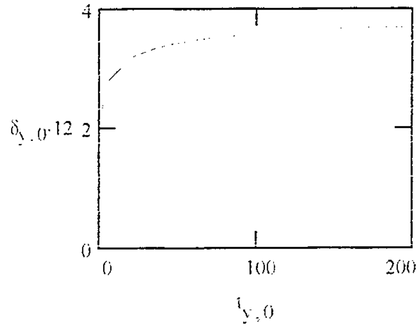
For Various
Time steps

$\delta =$

	0	1	2	3	4	5	6	7	8	9
0	2.22	2.21	2.18	2.17	2.26	2.19	2.14	2.22	2.26	2.24
1	2.81	2.79	2.46	2.45	2.73	2.65	2.52	2.6	3.12	3.09
2	3.2	3.19	3.12	3.11	3.18	3.09	3.02	3.11	3.31	3.29
3	3.34	3.32	3.28	3.27	3.34	3.26	3.19	3.29	3.39	3.36
4	3.39	3.38	3.35	3.33	3.41	3.33	3.26	3.36	3.53	3.5
5	3.51	3.49	3.47	3.46	3.54	3.46	3.39	3.49	3.63	3.61
6	3.6	3.59	3.57	3.56	3.64	3.56	3.5	3.59	3.73	3.7
7	3.69	3.67	3.66	3.65	3.73	3.65	3.59	3.68	3.91	3.89
8	4.07	4.06	4.06	4.05	4.12	4.05	3.99	4.08	4.12	4.09

$\cdot in$

$y \quad 0..8$



APPENDIX F

Example of Input/Output File for the Program "CRACK"

```
Bridge 19045, Girder B18-S2 - Using Amin Ghali's.
  2   1   2                    NLCR,NCTYP,NSTYP
 21   6   2   1                 NSEC,NCL,NPSL,NSL
  1650.0   2   0                 AL,INTEG,ISPT
   37.14     0.0                 DO,SRM
 20      0.05                   NC,DX
  1   1   0    30.0    30.0      J=1,(ICTYP(J,K),K=1,2),BT(J),BB(J)
  1  21     6.0     6.0    6.0  NSTRT,NEND,DL,DC,DR
  2   1   0    30.0    10.0      J=2,(ICTYP(J,K),K=1,2),BT(J),BB(J)
  1  21     1.5     1.5    1.5  NSTRT,NEND,DL,DC,DR
  3   1   0    10.0     6.0     J=3,(ICTYP(J,K),K=1,2),BT(J),BB(J)
  1  21     2.0     2.0    2.0  NSTRT,NEND,DL,DC,DR
  4   1   0     6.0     6.0     J=4,(ICTYP(J,K),K=1,2),BT(J),BB(J)
  1  21    51.5    51.5   51.5  NSTRT,NEND,DL,DC,DR
  5   1   0     6.0    26.0     J=5,(ICTYP(J,K),K=1,2),BT(J),BB(J)
  1  21     3.5     3.5    3.5  NSTRT,NEND,DL,DC,DR
  6   1   0    26.0    26.0     J=6,(ICTYP(J,K),K=1,2),BT(J),BB(J)
  1  21     7.5     7.5    7.5  NSTRT,NEND,DL,DC,DR
  1   1   0   341.0  1.836    0.0  J,(IPSTYP(J,K),K=1,2),PL,APSL,DUCTL
  1  10    10.0    33.5     57.0  NSTRT,NEND,DPSL,DPSC,DPSR
 10  12    57.0    57.0     57.0  NSTRT,NEND,DPSL,DPSC,DPSR
 12  21    57.0    33.5     10.0  NSTRT,NEND,DPSL,DPSC,DPSR
 99   0     0.0     0.0      0.0
  2   1   0  1195.0  6.426    0.0  J,(IPSTYP(J,K),K=1,2),PL,APSL,DUCTL
  1  21    67.43   67.43    67.43  NSTRT,NEND,DPSL,DPSC,DPSR
 99   0     0.0     0.0      0.0
  1   2   3.1600    2.37         J=1,INSTYP(J),ASL(J),DSL(J)
  1  21                        NSTRT,NEND
  1  28500.0   0.0   1.0          I=1,ES(I),FPTK(I),BETA1(I)
  2  29000.0   0.0   1.0          I=1,ES(I),FPTK(I),BETA1(I)
  1     1.0                    ITPND,BETA2
  1   4809.0     0.714           I,EC(I),FCT(I)
   0.0000                       CJI(I,1)
   0.7325   -10.E-30              CHI(I),S(I)
  1     -1.750                  I,REL
 99      0.0
  1  21     0.0     0.0     0.0  NSTRT,NEND,ANL,ANC,ANR
  1  11     0.0  17414.4  23219.2  NSTRT,NEND,AML,AMC,AMR
 11  21  23219.2  17414.4    0.0  NSTRT,NEND,AML,AMC,AMR
  1     0.5                    ITPND,BETA2
  1   4809.0     0.714           I,EC(I),FCT(I)
   1.1661                       CJI(I,1)
   1.1661                       CJI(I,2)
   0.7526   -35.E-5               CHI(I),S(I)
  1     -1.221                  I,REL
 99      0.0
  1  21     0.0     0.0     0.0  NSTRT,NEND,ANL,ANC,ANR
  1  21     0.0     0.0     0.0  NSTRT,NEND,AML,AMC,AMR
Bridge 19045, Girder B18-S2 - Using Amin Ghali's.
  6   1   2                    NLCR,NCTYP,NSTYP
 21   6   2   1                 NSEC,NCL,NPSL,NSL
  1650.0   2   0                 AL,INTEG,ISPT
   37.14     0.0                 DO,SRM
 20      0.05                   NC,DX
```

```
 1  1  0    30.0    30.0      J=1,(ICTYP(J,K),K=1,2),BT(J),BB(J)
 1  21     6.0     6.0     6.0  NSTRT,NEND,DL,DC,DR
 2  1  0    30.0    10.0      J=2,(ICTYP(J,K),K=1,2),BT(J),BB(J)
 1  21     1.5     1.5     1.5  NSTRT,NEND,DL,DC,DR
 3  1  0    10.0     6.0      J=3,(ICTYP(J,K),K=1,2),BT(J),BB(J)
 1  21     2.0     2.0     2.0  NSTRT,NEND,DL,DC,DR
 4  1  0     6.0     6.0      J=4,(ICTYP(J,K),K=1,2),BT(J),BB(J)
 1  21    51.5    51.5    51.5  NSTRT,NEND,DL,DC,DR
 5  1  0     6.0    26.0      J=5,(ICTYP(J,K),K=1,2),BT(J),BB(J)
 1  21     3.5     3.5     3.5  NSTRT,NEND,DL,DC,DR
 6  1  0    26.0    26.0      J=6,(ICTYP(J,K),K=1,2),BT(J),BB(J)
 1  21     7.5     7.5     7.5  NSTRT,NEND,DL,DC,DR
 1  1  0   341.0  1.836    0.0  J,(IPSTYP(J,K),K=1,2),PL,APSL,DUCTL
 1  10    10.0    33.5    57.0  NSTRT,NEND,DPSL,DPSC,DPSR
10  12    57.0    57.0    57.0  NSTRT,NEND,DPSL,DPSC,DPSR
12  21    57.0    33.5    10.0  NSTRT,NEND,DPSL,DPSC,DPSR
99  0      0.0     0.0     0.0
 2  1  0  1195.0  6.426    0.0  J,(IPSTYP(J,K),K=1,2),PL,APSL,DUCTL
 1  21   67.43   67.43   67.43  NSTRT,NEND,DPSL,DPSC,DPSR
99  0      0.0     0.0     0.0
 1  2  3.1600    2.37           J=1,INSTYP(J),ASL(J),DSL(J)
 1  21                          NSTRT,NEND
 1  28500.0  0.0  1.0           I=1,ES(I),FPTK(I),BETA1(I)
 2  29000.0  0.0  1.0           I=1,ES(I),FPTK(I),BETA1(I)
 1     1.0                      ITPND,BETA2
 1   4809.0    0.714            I,EC(I),FCT(I)
   0.2279                       CJI(I,1)
   0.7325   -17.E-6             CHI(I),S(I)
 1    -2.222                    I,REL
99     0.0
 1  21     0.0     0.0     0.0  NSTRT,NEND,ANL,ANC,ANR
 1  11     0.0  17414.4  23219.2  NSTRT,NEND,AML,AMC,AMR
11  21  23219.2  17414.4    0.0  NSTRT,NEND,AML,AMC,AMR
 1     0.5                      ITPND,BETA2
 1   4809.0    0.714            I,EC(I),FCT(I)
   0.5832                       CJI(I,1)
   0.5832                       CJI(I,2)
   0.7369   -86.E-6             CHI(I),S(I)
 1    -0.527                    I,REL
99     0.0
 1  21     0.0     0.0     0.0  NSTRT,NEND,ANL,ANC,ANR
 1  21     0.0     0.0     0.0  NSTRT,NEND,AML,AMC,AMR
 1     0.5                      ITPND,BETA2
 1   4809.0    0.714            I,EC(I),FCT(I)
   0.7356                       CJI(I,1)
   0.7356                       CJI(I,2)
   0.7356                       CJI(I,3)
   0.7425   -59.E-6             CHI(I),S(I)
 1    -0.098                    I,REL
99     0.0
 1  21     0.0     0.0     0.0  NSTRT,NEND,ANL,ANC,ANR
 1  21     0.0     0.0     0.0  NSTRT,NEND,AML,AMC,AMR
 1     0.5                      ITPND,BETA2
 1   4809.0    0.714            I,EC(I),FCT(I)
```

```
    0.8000                   CJI(I,1)
    0.8000                   CJI(I,2)
    0.8000                   CJI(I,3)
    0.8000                   CJI(I,4)
    0.7448   -28.E-6            CHI(I),S(I)
 1     -0.025                I,REL
99      0.0
 1  21     0.0      0.0      0.0  NSTRT,NEND,ANL,ANC,ANR
 1  21     0.0      0.0      0.0  NSTRT,NEND,AML,AMC,AMR
 1      0.5                 ITPND,BETA2
 1   4809.0      0.714          I,EC(I),FCT(I)
    0.9367                   CJI(I,1)
    0.9367                   CJI(I,2)
    0.9367                   CJI(I,3)
    0.9367                   CJI(I,4)
    0.9367                   CJI(I,5)
    0.7496   -62.E-6            CHI(I),S(I)
 1     -0.045                I,REL
99      0.0
 1  21     0.0      0.0      0.0  NSTRT,NEND,ANL,ANC,ANR
 1  21     0.0      0.0      0.0  NSTRT,NEND,AML,AMC,AMR
 1      0.5                 ITPND,BETA2
 1   4809.0      0.714          I,EC(I),FCT(I)
    1.0543                   CJI(I,1)
    1.0543                   CJI(I,2)
    1.0543                   CJI(I,3)
    1.0543                   CJI(I,4)
    1.0543                   CJI(I,5)
    1.0543                   CJI(I,6)
    0.7539   -54.E-6            CHI(I),S(I)
 1     -0.028                I,REL
99      0.0
 1  21     0.0      0.0      0.0  NSTRT,NEND,ANL,ANC,ANR
 1  21     0.0      0.0      0.0  NSTRT,NEND,AML,AMC,AMR
```

```
****************************************************************************
* ************************************************************************ *
**                                  **
**                                  **
**       ANALYSIS OF PRESTRESSED AND NONPRESTRESSED          **
**          REINFORCED CONCRETE MEMBERS              **
**                BY                 **
**             MAMDOUH M. EL-BADRY              **
**             THE UNIVERSITY OF CALGARY              **
**             RELEASE 1.0 JANUARY, 1985             **
**             VERSION 2.0  FEBRUARY, 1991             **
**                                  **
**                                  **
**    ******  ******   ****   ******  **  **      **
**    **   *  **   *  ******  **   *  **  **      **
**    **     **  *  **   ** **     ** **      **
**    **     ******  **   ** **     ***       **
**    **     **  *  ********  **     ** **      **
**    **     **  **  **  **  **    ** **      **
   **       **    *  **   **  **   **  **    *  **  **       **
**     ******  **   **  **   **  ******  **   **     **
**                                  **
**                                  **
**    REFERENCE - GHALI, A. AND ELBADRY, M., USERS' MANUAL      **
**          AND COMPUTER PROGRAM "CRACK", RESEARCH        **
**          REPORT NO. CE85-1, DEPT. OF CIVIL ENGG.       **
**          THE UNIVERSITY OF CALGARY, JANUARY 1985,      **
**          (REVISED FEBRUARY, 1991).            **
**                                  **
**                                  **
* ************************************************************************ *
****************************************************************************
1
```

```
******************************************************************************
             *                                    *
             *              CONDITIONS OF RELEASE OF              *
             *                PROGRAMS FROM THE               *
             *          STRUCTURAL ENGINEERING SOFTWARE LIBRARY          *
             *            DEPARTMENT OF CIVIL ENGINEERING           *
             *            THE UNIVERSITY OF CALGARY, CANADA          *
             *                                    *
             *                                    *
             *                                    *
             *  1. THE SOFTWARE IS RELEASED TO THE RECEIVING PARTY ONLY, AND MAY NOT    *
             *     BE PASSED ON TO OTHER PARTIES WITHOUT WRITTEN PERMISSION OF THE     *
             *     DEPARTMENT OF CIVIL ENGINEERING, THE UNIVERSITY OF CALGARY.       *
             *                                    *
             *  2. THE SOFTWARE IS NOT GUARANTEED TO BE FREE OF ERRORS.           *
             *                                    *
             *  3. THE AUTHORS SHALL BE ACKNOWLEDGED IN ANY PUBLISHED REFERENCE TO THE   *
             *     SOFTWARE, OR RESULTS DERIVED FROM IT.            *
             *                                    *
             ******************************************************************************
```

```
1            *****************
             *       *
             * PROBLEM NO.  1 *     Bridge 19045. Girder B18-S2 - Using Amin Ghali's.
             *       *
             *****************


      TOTAL NUMBER OF LOAD STAGES              =       2
      TOTAL NUMBER OF CONCRETE TYPES             =       1
      TOTAL NUMBER OF STEEL TYPES             =       2

      TOTAL NUMBER OF SECTIONS              =      21
      TOTAL NUMBER OF CONCRETE LAYERS            =       6
      TOTAL NUMBER OF PRESTRESS STEEL LAYERS        =       2
      TOTAL NUMBER OF NON-PRESTRESSED STEEL LAYERS    =       1

      TOTAL LENGTH OF MEMBER, L              =   1650.000

      INTEGRATION SCHEME FOR DISPLACEMENT CALCULATION =        2
           0  FOR DISPLACEMENTS NOT REQUIRED
           1  FOR STRAIGHT LINE BET. TWO POINTS
           2  FOR A PARABOLA BET. THREE POINTS.

      TYPE OF SUPPORTS AT THE TWO ENDS OF THE MEMBER  =        0
           0  FOR SIMPLE SUPPORTS AT THE TWO ENDS
           1  FOR LEFT END FIXED AND RIGHT END FREE
           2  FOR LEFT END FREE AND RIGHT END FIXED.

      DEPTH OF REFERENCE POINT FROM TOP FIBRE, DO    =    37.140
      AVERAGE CRACK SPACING, Srm              =      .000


      TOTAL STORAGE REQUIRED FOR ARRAY {A}, N23       =      1081
      TOTAL STORAGE REQUIRED FOR ARRAY {B}, M34       =      2983
      TOTAL STORAGE REQUIRED FOR ARRAY {NA}, NT5       =        60




0                           ********************************
                            * PROPERTIES OF DIFFERENT LAYERS *
                            ********************************

      CONCRETE LAYERS :
      ================

      LAYER    CONCRETE   LAYER    TOP     BOTTOM
      NUMBER     TYPE      TYPE    WIDTH    WIDTH
      ======     ====      ====    =====    =====

       1         1        0     30.000    30.000
       2         1        0     30.000    10.000
       3         1        0     10.000     6.000
       4         1        0      6.000     6.000
       5         1        0      6.000    26.000
       6         1        0     26.000    26.000

      LAYER TYPE :  0  =  CAST-IN-SITU
                    1  =  PRECAST.
```

```
PRESTRESS STEEL LAYERS :
======================

LAYER   STEEL   METHOD OF      PRESTRESS    AREA OF    AREA OF
NUMBER  TYPE   PRESTRESSING     FORCE        STEEL      DUCT
======  ====   ============     =====        =====      ====

  1      1       0        341.000    1.836000     .000000
  2      1       0       1195.000    6.426000     .000000

METHOD OF PRESTRESSING : 0 = PRETENSIONING
                        >0 = POST-TENSIONING (THE NUMBER INDICATES THE
                             LOAD STAGE AT WHICH THE LAYER IS TENSIONED)

NON-PRESTRESSED STEEL LAYERS :
=============================

LAYER   STEEL   AREA OF
NUMBER  TYPE     STEEL
======  ====    =====

  1      2     3.16000


0                   ************************************************
                    * PROPERTIES OF DIFFERENT LAYERS IN EACH SECTION *
                    ************************************************


SECTION  DISTANCE X                      CONCRETE                      PRESTRESS     NON-PRESTRESSED
NUMBER   FROM LEFT END                    LAYERS                      STEEL LAYERS    STEEL LAYERS
======   ============                     ======                      ===========     ===========

          LAYER   DEPTH   DEPTH AT     AREA        FIRST       MOMENT    LAYER   DEPTH  LAYER   DEPTH
          NUMBER  AT TOP  BOTTOM       ====        MOMENT    OF INERTIA  NUMBER  =====  NUMBER
=====
          ======  ======  ======              ======     ==========  ======       ======


  1   .000000E+00   1     .000    6.000  .180000E+03  -.614520E+04  .210337E+06  1   10.000  1    2.370
                    2    6.000    7.500  .300000E+02  -.915450E+03  .279401E+05  2   67.430
                    3    7.500    9.500  .160000E+02  -.459573E+03  .132057E+05
                    4    9.500   61.000  .309000E+03  -.584010E+03  .693992E+05
                    5   61.000   64.500  .560000E+02   .145458E+04  .378317E+05
                    6   64.500   72.000  .195000E+03   .606645E+04  .189641E+06


  2   .500000E-01   1     .000    6.000  .180000E+03  -.614520E+04  .210337E+06  1   15.222  1    2.370
                    2    6.000    7.500  .300000E+02  -.915450E+03  .279401E+05  2   67.430
                    3    7.500    9.500  .160000E+02  -.459573E+03  .132057E+05
                    4    9.500   61.000  .309000E+03  -.584010E+03  .693992E+05
                    5   61.000   64.500  .560000E+02   .145458E+04  .378317E+05
                    6   64.500   72.000  .195000E+03   .606645E+04  .189641E+06


  3   .100000E+00   1     .000    6.000  .180000E+03  -.614520E+04  .210337E+06  1   20.444  1    2.370
                    2    6.000    7.500  .300000E+02  -.915450E+03  .279401E+05  2   67.430
                    3    7.500    9.500  .160000E+02  -.459573E+03  .132057E+05
                    4    9.500   61.000  .309000E+03  -.584010E+03  .693992E+05
                    5   61.000   64.500  .560000E+02   .145458E+04  .378317E+05
                    6   64.500   72.000  .195000E+03   .606645E+04  .189641E+06


  4   .150000E+00   1     .000    6.000  .180000E+03  -.614520E+04  .210337E+06  1   25.667  1    2.370
                    2    6.000    7.500  .300000E+02  -.915450E+03  .279401E+05  2   67.430
                    3    7.500    9.500  .160000E+02  -.459573E+03  .132057E+05
```

```
                4     9.500   61.000   .309000E+03   -.584010E+03    .693992E+05
                5    61.000   64.500   .560000E+02    .145458E+04    .378317E+05
                6    64.500   72.000   .195000E+03    .606645E+04    .189641E+06


  5   .200000E+00   1      .000    6.000   .180000E+03   -.614520E+04    .210337E+06   1     30.889    1      2.370
                2     6.000    7.500   .300000E+02   -.915450E+03    .279401E+05   2     67.430
                3     7.500    9.500   .160000E+02   -.459573E+03    .132057E+05
                4     9.500   61.000   .309000E+03   -.584010E+03    .693992E+05
                5    61.000   64.500   .560000E+02    .145458E+04    .378317E+05
                6    64.500   72.000   .195000E+03    .606645E+04    .189641E+06


  6   .250000E+00   1      .000    6.000   .180000E+03   -.614520E+04    .210337E+06   1     36.111    1      2.370
                2     6.000    7.500   .300000E+02   -.915450E+03    .279401E+05   2     67.430
                3     7.500    9.500   .160000E+02   -.459573E+03    .132057E+05
                4     9.500   61.000   .309000E+03   -.584010E+03    .693992E+05
                5    61.000   64.500   .560000E+02    .145458E+04    .378317E+05
                6    64.500   72.000   .195000E+03    .606645E+04    .189641E+06


  7   .300000E+00   1      .000    6.000   .180000E+03   -.614520E+04    .210337E+06   1     41.333    1      2.370
                2     6.000    7.500   .300000E+02   -.915450E+03    .279401E+05   2     67.430
                3     7.500    9.500   .160000E+02   -.459573E+03    .132057E+05
                4     9.500   61.000   .309000E+03   -.584010E+03    .693992E+05
                5    61.000   64.500   .560000E+02    .145458E+04    .378317E+05
                6    64.500   72.000   .195000E+03    .606645E+04    .189641E+06


  8   .350000E+00   1      .000    6.000   .180000E+03   -.614520E+04    .210337E+06   1     46.556    1      2.370
                2     6.000    7.500   .300000E+02   -.915450E+03    .279401E+05   2     67.430
                3     7.500    9.500   .160000E+02   -.459573E+03    .132057E+05
                4     9.500   61.000   .309000E+03   -.584010E+03    .693992E+05
                5    61.000   64.500   .560000E+02    .145458E+04    .378317E+05
                6    64.500   72.000   .195000E+03    .606645E+04    .189641E+06


  9   .400000E+00   1      .000    6.000   .180000E+03   -.614520E+04    .210337E+06   1     51.778    1      2.370
                2     6.000    7.500   .300000E+02   -.915450E+03    .279401E+05   2     67.430
                3     7.500    9.500   .160000E+02   -.459573E+03    .132057E+05
                4     9.500   61.000   .309000E+03   -.584010E+03    .693992E+05
                5    61.000   64.500   .560000E+02    .145458E+04    .378317E+05
                6    64.500   72.000   .195000E+03    .606645E+04    .189641E+06


 10   .450000E+00   1      .000    6.000   .180000E+03   -.614520E+04    .210337E+06   1     57.000    1      2.370
                2     6.000    7.500   .300000E+02   -.915450E+03    .279401E+05   2     67.430
                3     7.500    9.500   .160000E+02   -.459573E+03    .132057E+05
                4     9.500   61.000   .309000E+03   -.584010E+03    .693992E+05
                5    61.000   64.500   .560000E+02    .145458E+04    .378317E+05
                6    64.500   72.000   .195000E+03    .606645E+04    .189641E+06


 11   .500000E+00   1      .000    6.000   .180000E+03   -.614520E+04    .210337E+06   1     57.000    1      2.370
                2     6.000    7.500   .300000E+02   -.915450E+03    .279401E+05   2     67.430
                3     7.500    9.500   .160000E+02   -.459573E+03    .132057E+05
                4     9.500   61.000   .309000E+03   -.584010E+03    .693992E+05
                5    61.000   64.500   .560000E+02    .145458E+04    .378317E+05
                6    64.500   72.000   .195000E+03    .606645E+04    .189641E+06


 12   .550000E+00   1      .000    6.000   .180000E+03   -.614520E+04    .210337E+06   1     57.000    1      2.370
                2     6.000    7.500   .300000E+02   -.915450E+03    .279401E+05   2     67.430
                3     7.500    9.500   .160000E+02   -.459573E+03    .132057E+05
                4     9.500   61.000   .309000E+03   -.584010E+03    .693992E+05
                5    61.000   64.500   .560000E+02    .145458E+04    .378317E+05
                6    64.500   72.000   .195000E+03    .606645E+04    .189641E+06
```

```
13  .600000E+00  1     .000   6.000  .180000E+03  -.614520E+04  .210337E+06  1   51.778  1   2.370
                 2    6.000   7.500  .300000E+02  -.915450E+03  .279401E+05  2   67.430
                 3    7.500   9.500  .160000E+02  -.459573E+03  .132057E+05
                 4    9.500  61.000  .309000E+03  -.584010E+03  .693992E+05
                 5   61.000  64.500  .560000E+02   .145458E+04  .378317E+05
                 6   64.500  72.000  .195000E+03   .606645E+04  .189641E+06


14  .650000E+00  1     .000   6.000  .180000E+03  -.614520E+04  .210337E+06  1   46.556  1   2.370
                 2    6.000   7.500  .300000E+02  -.915450E+03  .279401E+05  2   67.430
                 3    7.500   9.500  .160000E+02  -.459573E+03  .132057E+05
                 4    9.500  61.000  .309000E+03  -.584010E+03  .693992E+05
                 5   61.000  64.500  .560000E+02   .145458E+04  .378317E+05
                 6   64.500  72.000  .195000E+03   .606645E+04  .189641E+06


15  .700000E+00  1     .000   6.000  .180000E+03  -.614520E+04  .210337E+06  1   41.333  1   2.370
                 2    6.000   7.500  .300000E+02  -.915450E+03  .279401E+05  2   67.430
                 3    7.500   9.500  .160000E+02  -.459573E+03  .132057E+05
                 4    9.500  61.000  .309000E+03  -.584010E+03  .693992E+05
                 5   61.000  64.500  .560000E+02   .145458E+04  .378317E+05
                 6   64.500  72.000  .195000E+03   .606645E+04  .189641E+06


16  .750000E+00  1     .000   6.000  .180000E+03  -.614520E+04  .210337E+06  1   36.111  1   2.370
                 2    6.000   7.500  .300000E+02  -.915450E+03  .279401E+05  2   67.430
                 3    7.500   9.500  .160000E+02  -.459573E+03  .132057E+05
                 4    9.500  61.000  .309000E+03  -.584010E+03  .693992E+05
                 5   61.000  64.500  .560000E+02   .145458E+04  .378317E+05
                 6   64.500  72.000  .195000E+03   .606645E+04  .189641E+06


17  .800000E+00  1     .000   6.000  .180000E+03  -.614520E+04  .210337E+06  1   30.889  1   2.370
                 2    6.000   7.500  .300000E+02  -.915450E+03  .279401E+05  2   67.430
                 3    7.500   9.500  .160000E+02  -.459573E+03  .132057E+05
                 4    9.500  61.000  .309000E+03  -.584010E+03  .693992E+05
                 5   61.000  64.500  .560000E+02   .145458E+04  .378317E+05
                 6   64.500  72.000  .195000E+03   .606645E+04  .189641E+06


18  .850000E+00  1     .000   6.000  .180000E+03  -.614520E+04  .210337E+06  1   25.667  1   2.370
                 2    6.000   7.500  .300000E+02  -.915450E+03  .279401E+05  2   67.430
                 3    7.500   9.500  .160000E+02  -.459573E+03  .132057E+05
                 4    9.500  61.000  .309000E+03  -.584010E+03  .693992E+05
                 5   61.000  64.500  .560000E+02   .145458E+04  .378317E+05
                 6   64.500  72.000  .195000E+03   .606645E+04  .189641E+06


19  .900000E+00  1     .000   6.000  .180000E+03  -.614520E+04  .210337E+06  1   20.444  1   2.370
                 2    6.000   7.500  .300000E+02  -.915450E+03  .279401E+05  2   67.430
                 3    7.500   9.500  .160000E+02  -.459573E+03  .132057E+05
                 4    9.500  61.000  .309000E+03  -.584010E+03  .693992E+05
                 5   61.000  64.500  .560000E+02   .145458E+04  .378317E+05
                 6   64.500  72.000  .195000E+03   .606645E+04  .189641E+06


20  .950000E+00  1     .000   6.000  .180000E+03  -.614520E+04  .210337E+06  1   15.222  1   2.370
                 2    6.000   7.500  .300000E+02  -.915450E+03  .279401E+05  2   67.430
                 3    7.500   9.500  .160000E+02  -.459573E+03  .132057E+05
                 4    9.500  61.000  .309000E+03  -.584010E+03  .693992E+05
                 5   61.000  64.500  .560000E+02   .145458E+04  .378317E+05
                 6   64.500  72.000  .195000E+03   .606645E+04  .189641E+06


21  .100000E+01  1     .000   6.000  .180000E+03  -.614520E+04  .210337E+06  1   10.000  1   2.370
```

```
             2     6.000    7.500   .300000E+02   -.915450E+03    .279401E+05   2       67.430
             3     7.500    9.500   .160000E+02   -.459573E+03    .132057E+05
             4     9.500   61.000   .309000E+03   -.584010E+03    .693992E+05
             5    61.000   64.500   .560000E+02    .145458E+04    .378317E+05
             6    64.500   72.000   .195000E+03    .606645E+04    .189641E+06


1                        ********************************
                         *                              *
                         * RESULTS FOR INTERVAL NO.   1 *
                         *                              *
                         ********************************



        INDICATOR FOR TIME-DEPENDENT EFFECTS   =   1
           0  FOR ANALYSIS NOT REQUIRED
           1  FOR ANALYSIS REQUIRED.

        COEFFICIENT FOR LOAD TYPE, BETA2       = 1.0
           1.0  FOR FIRST LOADING
           0.5  FOR SUSTAINED OR
                REPEATED LOADING.



        ********************************
        * TABLE OF CONCRETE PROPERTIES *
        ********************************

        TYPE       ECI       CJI   CHI       S         ECBAR      FCT
        ====       ===       ===   ===       =         =====      ===

         1      .48090E+04   .000  .733  -.10000E-28   .48090E+04    .714



        *****************************
        * TABLE OF STEEL PROPERTIES *
        *****************************

        TYPE    ES         FPTK        BETA1        REL
        ====    ==         ====        =====        ===

         1   .285000E+05   .000000E+00     1.0    -.175000E+01
         2   .290000E+05   .000000E+00     1.0     .000000E+00


        DISPLACEMENTS :
        ===============

        ELONGATION                   =  -.622347E+00
        ROTATION AT THE LEFT END     =  -.581941E-02
        ROTATION AT THE RIGHT END    =   .581941E-02
        CENTRAL DEFLECTION           =  -.226836E+01
        MAXIMUM DEFLECTION           =  -.226836E+01
          AT SECTION NO.                  11
1                        ********************************
                         *                              *
                         * RESULTS FOR INTERVAL NO.   2 *
                         *                              *
                         ********************************
```

```
INDICATOR FOR TIME-DEPENDENT EFFECTS  =  1
   0  FOR ANALYSIS NOT REQUIRED
   1  FOR ANALYSIS REQUIRED.

COEFFICIENT FOR LOAD TYPE, BETA2      =  .5
   1.0  FOR FIRST LOADING
   0.5  FOR SUSTAINED OR
        REPEATED LOADING.


********************************
* TABLE OF CONCRETE PROPERTIES *
********************************

TYPE      ECI       CJI   CHI      S         ECBAR      FCT
====      ===       ===   ===      =         =====      ===

  1    .48090E+04  1.166  .753  -.35000E-03  .25612E+04    .714


*****************************
* TABLE OF STEEL PROPERTIES *
*****************************

TYPE    ES         FPTK        BETA1      REL
====    ==         ====        =====      ===

  1   .285000E+05  .000000E+00    1.0    -.122100E+01
  2   .290000E+05  .000000E+00    1.0     .000000E+00

DISPLACEMENTS :
===============

ELONGATION                  =  -.172204E+01
ROTATION AT THE LEFT END    =  -.104449E-01
ROTATION AT THE RIGHT END   =   .104449E-01
CENTRAL DEFLECTION          =  -.398679E+01
MAXIMUM DEFLECTION          =  -.398679E+01
  AT SECTION NO.               11
```