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STRENGTH AND DURABILITY OF CONCRETE: EFFECTS OF CEMENT PASTE-AGGREGATE INTERFACES

PART I
THEORETICAL STUDY ON INFLUENCE OF INTERFACIAL TRANSITION ZONE ON PROPERTIES OF CONCRETE MATERIALS

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Indiana
Department
of Transportation

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16. Abstract

This research was based on a two-part basic research investigation studying the effects of cement paste-aggregate interfaces (or interfacial transition zones-ITZ) on strength and durability of concrete. Part I dealt with the theoretical study and Part II dealt with the experimental.

Part I, the theoretical part, illustrates the effect of ITZ on the concrete properties by assuming its elastic moduli to be varied continuously in the region. A four-phase composite model is employed and three functions are chosen to model the moduli variation in the ITZ. A theoretical solution for an n-layered spherical inclusion model is used to estimate the overall effective moduli of the modified four-phase model. The influence of material and geometric characteristics of the ITZ, as well as that of the aggregate on the overall effective moduli is investigated. The effects of three different moduli variations in ITZ on the overall moduli are compared. Their potential application is discussed. Finally, by comparing the prediction of the proposed models to a set of data on mortar, it is found that the elastic modulus at the interface is about 20-70% lower than that in the bulk paste for portland cement mortar, and 10-40% lower for silica fume mortar.

Part II, the experimental part, illustrates the relationship between the ITZ microstructure and the mechanical properties of the concrete. The mechanical properties studied included the dynamic modulus of elasticity, dynamic shear modulus, logarithmic decrement of damping, flexural tensile strength, and compressive strength. In addition, the effects of changing the water-to-cementitious material ratio by mass, aggregate type, volume fraction of aggregate, and silica fume substitution, on these properties were investigated. A criterion based on water quantity and the specific surface area of aggregate by mass in a mixture was developed to eliminate biased date from the analysis process. This criterion was used to detect mixing and compaction problems that may have resulted in erroneous values of mechanical properties of specimen. In order to realize the compaction condition of the fresh mixture, an index of compaction (called gross porosity) was introduced. The three-phase model of Hashin-Shtrikman bounds was employed, tested, and validated with the experimental data from this research. A modification of this model linked the theory of Hashin-Shtrikman bounds to the results of this research on dynamic moduli of the transition zone. A form of optimal water content is recommended. This optimal water content may be used for a mixture to gain its possibly highest moduli, strengths and density. Thus, the rule of the optimal water content may potentially be applied to optimize the mixture design for conventional and high-strength concrete with consideration of ITZ.

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1. Introduction

1.1 Concrete as a three-phase composite material

It has been recognized for many years that in cement composites (mortar, concrete, etc.) the microstructure of paste in the vicinity of an inclusion (sand particle, coarse aggregate, etc.) is significantly different from that of bulk cement paste. However, the interfacial effect has not received special attention until recent study on the role of silica fume (SF) in the high-performance concrete. Using scanning electron microscopy (SEM), Bentur and Cohen (1987) demonstrated that in mortar the paste located at 50 μm and less from the sand particle surface is a relatively porous region containing large calcium hydroxide crystals which have little cementitious properties, forming the weakest link in mortar. This particular region is termed as the interfacial transition zone (ITZ). While, with 15% condensed silica fume added, the ITZ has a homogeneous and dense microstructure, similar to that of the bulk paste. In the work of Cohen, Goldman and Chen (1993), an experimental program was successfully developed to verify the ability of silica fume to modify the microstructure of the interfacial transition zone (i.e. transition zone modification), and thus the significant influence of the interfacial transition zone on the overall behavior of mortar is exposed. These experimental events require that concrete materials should be considered as a three phase composite, consisting of inclusion, interfacial transition zone and bulk cement paste.

As for the inclusion problem, there is a considerable literature available in the field of composite mechanics. Hashin (1962) first proposed a two-phase concentric-spheres model and derived the bounds and expressions for its effective elastic moduli by an approximate method based on the variational theorems of the theory of elasticity. A more general bounds approach applicable to any macroscopically isotropic two-phase composite material was derived by Hashin and Shtrikman (1963). To calculate the effective properties, Christinsen and Lo (1979) centered their interest around macroscopically homogeneous, isotropic or transversely isotropic two phase material, and proposed a model referred to as the three-phase model. This model consists of the single composite sphere embedded in the infinite medium of unknown effective properties. A generalized self consistent method was formalized to derive the exact expression for the effective shear modulus of this model. Herve and Zaoui (1993) extended Christinsen-Lo's work, and obtained a general analytical solution for the effective bulk and shear modulus for an n-layered spherical inclusion model. This n-layer model will be employed in the present work and more details will be given later.

Zhao and Chen (1996a,1998a) proposed a dual-layer inclusion model to model concrete material as a three-phase composite material, and derived a close-form solution for the stress and displacement fields of this model. The influence of the ITZ was exclusively investigated (Zhao and Chen, 1996b). Then, the effective elastic moduli of this model was obtained (Zhao and Chen, 198b,c), and its practical application and the significance of the ITZ was discussed by Chen and Zhao (1995). Meanwhile, following Christensen and Lo's work, Ramesh, Sotelino and Chen (1995, 1996a,b) proposed a four-phase model for a three-phase composite, and derived the effective moduli for the four-phase model. Application of this model is also made to estimate the overall moduli of concrete or mortar. However, the ITZ in all these models is assumed to be isotropic and homogeneous with a lower elastic moduli than that of bulk cement paste. Thus there is a jump in material properties between the ITZ and the bulk cement paste.

1.2 Microstructure and micromechanical properties of ITZ

According to the work of Scrivener and Gartner (1988), there is a porosity gradient in the transition zone, with increasing porosity as one approaches the aggregate particle. When the computerized imagine analysis was used to quantify the porosity gradient in the transition zone, it shows that there is no clear demarcation between the ITZ and the bulk cement paste. Therefore, the material properties are not expected to have a jump between these two phases.

Some experimental tests indicate that the Young's modulus E is related to the porosity η by $E=C/\eta^a$, where a and C are material constants. Hence, it is believed that the mechanical properties in the ITZ are not uniform, but vary gradually as a function of distance from the inclusion, approaching continuously to that of the bulk cement paste.

To this end, Lutz and Monteiro (1995) propose a spherical model considering the variation of interface moduli. They assume the elastic moduli in the ITZ vary with radius according to a power law. The primary reason for such an assumption is because a closed-form expression for the overall effective bulk modulus of this model has been found by Lutz and Zimmerman (1996). Using this model, the predicted overall bulk modulus is compared to the data of Zimmerman et al. (1986), where mortar specimens were measured to obtain the effective bulk modulus. They came to the conclusion that the modulus in the ITZ is about 15-50% lower than that in the bulk cement paste. This conclusion is somewhat closer to the prediction of Cohen, Lee and Goldman (1995) for silica fume (SF) mortar, where the average elastic moduli is indicated about 12-52% less than that of SF paste, but does not match very well with that for Portland cement (PC) mortar, where 26-85% is indicated. This fact gives us an impression that the power law model, which by itself is a very localized damage model, can model the interface in SF mortar quite well, but may be improper to model the interface in PC mortar. The objective of this work is to propose several types of variation for the elastic moduli in the ITZ. Their effects on the overall moduli are investigated and the behavior of the proposed models is discussed for their applications to concrete materials.

In this work, the four-phase model proposed by Ramesh et al. (1995, 1996a,b) is modified by considering the variation of the elastic moduli through the interface layer. Three types of function (linear, quadratic and power law) are assumed to model the elastic moduli in the ITZ. A general analytical solution for an n-layered inclusion model obtained by Herve and Zaoui (1993) is utilized to predict the overall effective moduli of the modified four-phase model. Therefore, the influence of different types of variation for the interface moduli as well as other parameters of the proposed model on the overall behavior can be revealed. The inverse application of this procedure can be used to estimate the elastic moduli at the interface if the overall moduli are given. This serves as a quantitative and non-destructive means of estimating the properties in the

ITZ.

2. Effective moduli of the modified four-phase model

2.1 Modified four-phase model with variation of interface moduli

We propose a micromechanical model for concrete material based on the four-phase model by Ramesh et al. (1995, 1996a,b) as shown in Fig. 1. The aggregate particle is assumed to be spherical with radius a. Outside the aggregate, there is an interfacial transition zone from inner layer with radius a to outer layer with radius b. Beyond the ITZ, there is the bulk cement paste which limited in a sphere with radius c. This individual composite sphere is then embedded in an equivalent homogeneous medium. The material property for each constituent phase is also shown in Fig. 1. The aggregate and bulk cement paste are taken to be isotropic homogeneous with constant elastic moduli E_1 (k_1,μ_1,ν_1) and E_3 (k_3,μ_3,ν_3) respectively. The interfacial transition zone is considered as a radially-inhomogeneous region where the elastic modulus E_2 (k_2,μ_2,ν_2) is taken as a function of the distance t from the inclusion, with E_{20} $(k_{20},\mu_{20},\nu_{20})$ as the modulus at $t = 0$. In the present work, three types of function are proposed, i.e. linear, quadratic and power law. They are described below respectively.

1). Linear model:

$$E_2(t)=E_{20}+kt \tag{1}$$

Let $E_2(b-a)=E_3$, we have

$$k=\frac{E_3-E_{20}}{b-a} \tag{2}$$

The linear function is the simplest zero-order continuous model, where $E_2(t)$ does not smoothly approach E_3, i.e. $E_2^{/}(b-a) \neq E_3^{/}(b-a) = 0$.

2). Quadratic model:

$$E_2(t)=E_{20}+k_1t+k_2t^2 \tag{3}$$

Let $E_2(b-a)=E_3$ and $E_2^{/}(b-a)=0$, we have

$$k_1=\frac{2(E_3-E_{20})}{b-a},\ k_2=-\frac{E_3-E_{20}}{(b-a)^2} \tag{4}$$

The quadratic function is the simplest first-order continuous model, where $E_2(t)$ approaches E_3 smoothly.

3). Power law model

Following the work of Lutz and Monteiro (1995), we assume the moduli vary according to

$$E_2(t)=E_3-(E_3-E_{20})\left(\frac{a+t}{a}\right)^{-\beta} \tag{5}$$

in which $E_3-E_2(b-a)=1\%(E_3-E_{20})$, we have

$$\beta=\frac{\ln(100)}{\ln\left(\frac{b}{a}\right)} \tag{6}$$

$E_2(t)$ in this model can approach continuously and smoothly in an approximate manner, i.e. $E_2(b-a)\to E_3$, $E_2^{/}(b-a)\to 0$. Generally, β is around 8 for concrete materials and this model is actually representing the most localized damage zone among the three models and referred to as Power law model in the present work.

Herein, the unknown parameter for the three models reduces to the elastic modulus E_{20}.

Due to the incomplete process of hydration especially around the aggregate, E_{20} is expected to lie between 0 and E_3. We define E_{20}=(1-D)E_3, where D is a local damage parameter. When D=0, we have E_{20}=E_3, there is no damage in the ITZ or the model can be taken here as a two-phase composite model. The three proposed models will converge to the same one. When D = 1, we have E_{20} = 0, there is a complete damage around the aggregate. We will show later that there is a so-called "hole-effect" for this case.

2.2 Application of n-layered solution to present model

Herve and Zaoui (1993) obtained a general solution for the effective moduli of an n-layered spherical inclusion model, which can be utilized to approximate the proposed models stepwisely. The n-layered spherical inclusion model is shown in Fig. 2. Each phase is assumed to be homogeneous and isotropic with elastic constants (μ_i, v_i, k_i) representing shear modulus, Poisson's ratio and bulk modulus respectively for phase I, which lies within the shell limited by spheres with the radii R_{i-1} and R_i ($I \in [1, n+1]$, R_0=0, $R_{n+1} \to \infty$). They derived the elastic strain and stress fields subjected to hydrostatic pressure and pure shear conditions at infinity. Then, using the average strain method, which has been shown to be equivalent to Christinsen-Lo's energy condition, they obtained the effective bulk modulus k for the equivalent homogeneous medium as

$$k = \frac{3k_n R_n^3 Q_{11}^{(n-1)} - 4\mu_n Q_{21}^{(n-1)}}{3\left(R_n^3 Q_{11}^{(n-1)} + Q_{21}^{(n-1)}\right)} \tag{7}$$

where $Q_{ij}^{(n-1)}$ (I = 1,2; j = 1,2) are the elements of a 2×2 matrix $Q^{(n-1)}$ given by

$$Q^{(n-1)} = \prod_{j=1}^{n-1} N^{(j)} \tag{8}$$

The effective shear modulus can be expressed by the following second-order equation:

$$N^{(j)} = \frac{1}{3k_{j+1}+4\mu_{j+1}} \begin{bmatrix} 3k_j+4\mu_{j+1} & \frac{4}{R_j^3}\left(\mu_{j+1}-\mu_j\right) \\ 3\left(k_{j+1}-k_j\right)R_j^3 & 3k_{j+1}+4\mu_j \end{bmatrix} \tag{9}$$

$$A\left(\frac{\mu}{\mu_n}\right)^2 + B\left(\frac{\mu}{\mu_n}\right) + C = 0 \tag{10}$$

where A, B and C are constants in terms of the elastic moduli as well as the dimension parameters of each phase.

This general solution for $(n+1)$-phase model can be reduced to the classical two-phase model of Hashin (1962) and three-phase model of Christinsen and Lo (1979) or the recent four-phase model of Ramesh et al. (1995, 1996a,b) as well. With the bulk modulus and shear modulus known, one can easily obtain the commonly-used engineering parameters E and v as follow:

$$E = \frac{9k\mu}{9k+\mu}, \quad \nu = \frac{3k-2\mu}{6k+2\mu} \tag{11}$$

When the $(n+1)$-phase composite model is incorporated into the present models, we take phase 1 as the aggregate, phase n as the bulk cement, and leave phase 2 to phase $(n-1)$ with stepwisely increased constant moduli to model the interfacial transition zone. The overall effective moduli of the present model are then predicated from the solution of the n-layered inclusion model as that of the equivalent homogeneous medium — phase (n+1). Theoretically, the smooth variation of moduli can be modeled to any required accuracy by simply increasing n.

We should point out here that the proposed models as well as the *n*-layered inclusion model require constant ratios of *a*/*b* and *b*/*c* for each individual composite sphere, independent of its absolute size. This assumption requires particle size down to infinitesimal, while still having a volume filling configuration. These models would be expected to predict reasonable results for actual systems that have a rather fine gradation of sizes, but it should not be expected to provide reasonable results for systems containing single size particles at high concentrations (Christinsen, 1979). In other words, no overlap of the associated cement paste of any single aggregate is considered, i.e. the interaction effects between particles are not accounted for in the present models. It is important to keep this limitation in mind in estimating the possible error when applying these models to experimental data.

3. Numerical results

In this section, we shall provide some numerical results of the application of the *n*-layer solution to the modified four-phase models. Since the present work is limited to the stress level within elastic stage, for the sake of simplicity, we take $c=1$. The relationship between the volume fraction of each constituent component and the dimension size of each phase is the same as that of the four-phase model of Ramesh et al. (1995, 1996a,b). If we define v_1, v_2 and v_3 as the volume fraction of the aggregate, interface and bulk cement paste respectively, then we have $a=\sqrt[3]{v_1}, b=\sqrt[3]{v_1+v_2}$ and $v_1+v_2+v_3=1$.

In the following examples, we first investigate the sensitivity of the overall moduli to *n* for the three models, then the influence of material and geometric characteristics of the inclusion as well as that of the ITZ on the overall properties is studied. Finally, we apply the present models to the data of Zimmerman et al. (1986) and that of Cohen et al. (1995).

3.1 Sensitivity of overall moduli to different types of function

The three types of function have been briefly described in the preceding section. To investigate the sensitivity of the overall moduli predicted by these models to the number of $n_2 = (n\text{-}2)$, we choose the most critical damage case, i.e. $D = 1$, where the sensitivity can be greatly amplified. Taking volume fraction $v_1 = 0.5$, $v_2 = 0.3$ and Young's modulus $E_1 = 5.0$, $E_3 = 1.0$ with the Poisson's ratio $\nu_i = 0.3$ ($I = 1, 2, 3$) as the known parameters, we can obtain the effective Young's modulus as listed in Table 1 and shown in Fig. 3.

It is clear from Fig.3 that as n_2 increases, the three models predict smaller effective moduli. There is an obvious drop for the effective moduli when n_2 increases from 1 to 10, then turns to a rather slow decrease when n_2 increases beyond 10. The power law model exhibits a less sensitivity to n_2, with the predicated overall Young's modulus only 22% less when n_2 increases from 1 to 100. However, the linear model is very sensitive to n_2 , with the overall modulus up to 92% less. The quadratic model lies somewhere in between with a moderate sensitivity.

Table 1 The effective Young's modulus with different types of n_2 functions

n_2	1	2	3	4	5	10	15	20	50	100
Linear	1.474	1.273	1.180	1.122	1.083	0.979	0.930	0.899	0.817	0.768
Quadratic	1.804	1.626	1.539	1.483	1.444	1.336	1.282	1.248	1.153	1.094
Power law	2.024	1.969	1.932	1.906	1.886	1.826	1.793	1.771	1.705	1.660

It is also important to notice that power law model, which can be considered as the most localized damage model among the threes, always predicts the highest overall Young's modulus, and the linear model predicts the lowest value. This fact indicates that the less the damage zone, the higher the overall moduli. This observation agrees well with that of Zhao et al. (1994b) and Chen et al. (1995). In the following, we will take n_2 =100 unless otherwise indicated.

3.2 The effects of the interfacial transition zone

In the proposed models, the interfacial transition zone is controlled by two factors: the local damage parameter *D* and the volume fraction v_2. In this section, we shall study the influence of these two factors on the overall moduli. The other parameters are taken to be the same as those in Section 3.1.

3.2.1 The influence of local damage parameter *D*

In this study, we shall investigate the influence of the local damage parameter *D* on the Young's modulus. The predicted effective Young modulus with various *D* for the three models is listed in Table 2 and also plotted in Fig. 4a, where $v_2 = 0.3$.

Table 2 The effective Young's modulus with various D values

D	Linear	Quadratic	Power law
0.0	2.047	2.047	2.047
0.1	2.003	2.018	2.035
0.2	1.955	1.985	2.022
0.3	1.901	1.950	2.007
0.4	1.842	1.909	1.990
0.5	1.774	1.863	1.971
0.6	1.694	1.808	1.948
0.7	1.599	1.741	1.920
0.8	1.477	1.653	1.883
0.9	1.300	1.521	1.824
1.0	0.768	1.094	1.660

It is observed that as *D* increases from 0 to 0.9, the overall modulus decreases slowly, although the power law model always predicts the highest value. Then when D increases beyond 0.9 and up to 1.0, there is an obvious drop of the overall modulus, especially for the linear model. The less sensitivity of the power law model to the local damage factor D is due to the fact that the model itself has a extremely localized damage zone. This observation indicates that if the interface is extremely localized, its influence on the overall modulus is negligible.

Figure 4b shows the results of the four-phase model, which can be reproduced by the present models with $n_2 = 1$ (thus the term "four-phase model" in this paper specially refers to the triple-layered inclusion model with the interface considered to be homogeneous). It can be seen that there is no obvious drop of the overall modulus even when the local damage parameter D approaches its maximum value, as revealed by the present models. This observation implies that the four-phase model is not a suitable one for the seriously damaged case, i.e. $D > 0.9$.

3.2.2 The influence of the interface volume fraction

In this study, the effect of the interface thickness or the volume fraction, i.e. $v_2 = (b^3 - a^3)/c^3$, on the overall modulus of the model is addressed. The predicated effective Young's modulus is listed in Table 3 and also shown in Fig. 5a, where $D = 1$.

Table 3 The effective Young's modulus with various volume fraction v_2

v_2	Linear	Quadratic	Power law
0.0	2.047	2.047	2.047
0.1	1.219	1.515	1.870
0.2	0.933	1.257	1.749
0.3	0.768	1.094	1.660
0.4	0.654	0.976	1.589
0.5	0.568	0.885	1.532

It is observed that the overall modulus has a large reduction when the interface volume fraction increases from 0 to 0.2, as revealed by the linear and quadratic models, then turns to a moderate reduction for further increase of the interface volume fraction. Again, the power law model turns out to be less sensitive to the effects of the interface volume fraction due to its localized damage zone feature.

From the result of the four-phase model, plotted in Fig. 5b, it shows that the four-phase model cannot detect the obvious drop of the overall moduli if the interface fraction increases from 0 to 0.2, and always predicts a much higher value for the overall modulus than that from the present models if we take an arithmatic average value for the interface modulus.

3.3 The effects of aggregate type

It is well known that the aggregate type has a considerable influence on the properties of concrete materials. In this section, we shall investigate the effects of the inclusion concentration v_1 and stiffness E_1 on the effective moduli of the proposed models. The other parameters are taken to be the same as those in Section 3.1.

3.3.1 The effect of aggregate concentration

The effect of aggregate concentration, i.e. $v_1 = a^3/c^3$, on the overall modulus of the proposed models is investigated here. The predicted effective Young's modulus is listed in Table 4 and also shown in Fig. 6a, where $E_1 = 5.0$.

Table 4 The effective Young's modulus with different aggregate concentration v_1

v_1	Linear	Quadratic	Power law
0.0	0.904	0.962	0.999
0.1	0.745	0.861	1.042
0.2	0.716	0.880	1.148
0.3	0.717	0.930	1.286
0.4	0.736	1.002	1.455
0.5	0.768	1.094	1.660
0.6	0.811	1.206	1.906
0.7	0.866	1.341	2.201

In the case of $D = 1$, there is a totally damaged zone around the aggregate, the increase of the aggregate concentration has almost no contribution to the overall moduli as revealed by the linear and quadratic models. However, the power law model fails to demonstrate this phenomenon clearly. Similarly, the prediction of the four-phase model, as shown in Fig. 6b, cannot reveal this special phenomenon.

3.3.2 The effects of aggregate stiffness

In this study, we first investigate the effects of aggregate stiffness with $D = 0.5$. Fig. 7a shows the influence of the aggregate stiffness E_1 increasing from 0 to $10E_3$. It can be seen that the prediction from the three models agree very well for this case.

However, when we take the critical case, i.e. $D = 1$, the difference of these models is revealed as shown in Fig. 7b with the results listed in Table 5. For this case there is a complete damage around the aggregate, the increase of the aggregate stiffness has no obvious effects on the overall moduli except that this increase is made from zero to that of the bulk cement paste E_3. This phenomenon is the so-called "hole-effects" and revealed quite well by the linear and quadratic models, where two horizontal lines are predicted. The power law model again fails in this special case.

Table 5 The effective Young's modulus with various aggregate stiffness E_I

E_I	Linear	Quadratic	Power law
0.0	0.187	0.222	0.265
1.0	0.585	0.734	0.912
2.0	0.680	0.906	1.226
3.0	0.725	0.998	1.423
4.0	0.751	1.055	1.559
5.0	0.768	1.094	1.660
6.0	0.780	1.122	1.737
7.0	0.789	1.144	1.798
8.0	0.796	1.161	1.848
9.0	0.801	1.174	1.890
10.0	0.806	1.186	1.925

From Fig. 7c where the prediction from the four-phase model is plotted, we can see that the four-phase model cannot reveal the hole-effect very well.

3.4 Application to the data of Zimmerman et al. (1986)

In the previous sections, we have studied the behavior of the proposed models for predicting the overall moduli in terms of that of the constituent components. In this section, we shall apply these models to the experimental data reported by Zimmerman et al. (1986).

In their work, acoustic wave measurements were performed on mortar specimens with 0-60% sand concentration. The effective bulk modulus can be found from the measured wavespeeds using the relationship established in their work. However, these experimental results are found to lie below the Hashin-Shtrikman lower bound, which can be reproduced by the proposed models with $D = 0$. Since the Hashin-Shtrikman bounds are developed for a two-phase composite material, such violation can be accepted by considering concrete material to be of a three-phase composite material (Nilsen and Monteiro, 1993).

In order to rationalize the test fact, Lutz and Monteiro (1995) apply the power law model to these data, using the closed-form expression obtained by Lutz and Zimmerman (1996) to predict the bulk modulus. Their application leads to the conclusion that the modulus at the interface is about 15-50% lower than in bulk cement paste (i.e. $D = 0.15\text{-}0.5$).

The sand grains used in the mortar specimens tested by Zimmerman et al. (1986) had radii of about 50 μm, according to the SEM photographs. While it is always difficult to determine the volume fraction v_2 for ITZ, not to mention that there is no information about the interfacial transition zone reported in their work at that time. To be consistent to the assumption made in the proposed models, the volume fraction v_2 for the ITZ, in the present models should be chosen not exceeding 0.4, since the aggregate concentration v_1 will approach 0.6. The material parameters for the mortar specimens are listed in Table 6 for reference.

Table 6 Material parameters of mortar specimens tested by Zimmerman et al. (1986)

Material	k (GPa)	μ (GPa)	E (GPa)	ν	ρ (kg/m^3)
Sand inclusion	44.0	37.0	86.7	0.17	2700
Cement paste	20.8	11.3	28.7	0.27	2120

In Fig. 8a-l, we present the prediction of the proposed three models for the effective bulk modulus as functions of the aggregate concentration v_1 for various values of D, with v_2 ranging from 0.1 to 0.4. By comparing the experimental results of Zimmerman et al. (1986), we can locate the range as well as the best fitting value for the local damage parameter D. They are summarized in Table 7, where the best fitting values are indicated in parentheses.

Table 7 The estimated local damage parameter D based on Zimmerman et al. tests (1986)

v_2	Linear	Quadratic	Power law
0.1	0.3 - 0.9 (0.8)	0.5 - 0.95 (0.9)	0.8 - 1.0+ (1.0+)
0.2	0.2 - 0.75 (0.55)	0.3 - 0.8 (0.7)	0.6 - 1.0+ (0.95)
0.3	0.1 - 0.6 (0.4)	0.2 - 0.8 (0.6)	0.5 - 1.0 (0.9)
0.4	0.1 - 0.5 (0.35)	0.2 - 0.7 (0.5)	0.4 - 0.95 (0.85)

From Table 7, we can see that the linear model predicts the smallest value for local damage parameter *D*, the quadratic predicts a moderate value, while the power law model predicts the largest value. In other words, a more localized interface layer can bear a more serious local damage to remain a same overll moduli, as demonstrated in Fig. 9.

The obviously different prediction between the present power law model and that in Lutz-Monteiro's work might be due to the choice of the interface thickness. In their work, $a = 50$ μm, $b\text{-}a = 40$ μm. Such a choice must lead to the overlap of the associated cement paste of each particle when applied to the case with sand concentration up to 0.6. Since the work of Lutz and Zimmerman (1996) is not yet accessible to the authors, we cannot verify if such a choice meets the requirement of their model. In the present work, we prefer to keep the consistency in the proposed models and leave the error to the difference between the model and the actual case.

3.5 Application to the data of Cohen et al. (1995)

In the previous section, the proposed models are applied to predict the effective bulk modulus, In this study, we shall apply the models to predict the effective elastic modulus and to compare it with the data of Cohen et al. (1995).

In their work, the average values of the dynamic moduli at the interface are estimated using the logarithmic mixture rule. By assuming the interface volume fractions, it indicates that the elastic modulus at interface is 26-85% less than that of bulk cement paste for PC mortar, and 12-52% less for SF mortar.

The volume fraction for the sand particles is kept to be constant at 37%. The dynamic modulus of elasticity is measured for PC mortar (4.2×10^6 psi), PC paste (2.7×10^6 psi), SF mortar (4.4×10^6 psi), SF paste (2.5×10^6 psi) and sand particle (14×10^6 psi). In this study, we take the Poisson's ratio $\nu = 0.17$ for sand particle, $\nu = 0.27$ for PC paste and $\nu = 0.37$ for SF paste.

In Fig. 10, we present the prediction of the proposed three models for the effective elastic modulus as functions of the local damage parameter D, with the interface fraction $v_2 = 0.54$ for PC mortar and $v_2 = 0.1$ for SF mortar. By comparing the experimental results, i.e. 4.2×10^6 psi for PC mortar and 4.4×10^6 psi for SF mortar, we can determine the local damage parameters for the three models as listed in Table 8.

From the prediction of the proposed models, we can conclude that the elastic modulus at the interface is 23-69% lower than that of bulk paste for PC mortar, and 12-36% lower for SF mortar. This conclusion agrees reasonably well with that of Cohen et al. (1995), in which the logarithmic mixture rule is applied to estimate the average elastic modulus at the interface. We can also see that silica fume in mortar reduces the interface volume fraction and the extent of damage (thus the lower of the local damage parameter). These experimental events can be explained here by the proposed models.

Table 8 The estimated local damage parameter D based on Cohen et al. tests (1995)

	Linear	Quadratic	Power law
PC mortar	0.23	0.33	0.69
SF mortar	0.12	0.17	0.36

4. Conclusions and discussions

In this work, we have proposed three models to represent different variation of the interface moduli. Taking the advantage of the available n-layer solution and incorporating it into the present models, the behavior of the three models is investigated and compared. The effects of the interface revealed by these models can be used to explain some experimental events. Some characteristics of this investigation are summarized as follows:

1) The more sensitivity of the linear model to the number n_2 than the power law model indicates that when the thickness of the interface layer is appreciable, the effects of the variation of the elastic moduli within the ITZ become important, otherwise large error may rise.

2) With the same initial damage parameter D, the power law model always predicts the highest overall modulus and the linear model predicts the lowest value. On the other hand, if the overall modulus is given, the power law model predicts the most serious initial damage at the interface while the linear and quadratic models predict a moderate initial damage around the aggregate.

3) When the damage parameter D increases from 0 to 0.9, the overall modulus decreases slowly in all three models. When D increases beyond 0.9 and up to 1.0, there is an obvious drop of the overall modulus, especially for the linear model. The less sensitivity of the power law model, which by itself a localized damage model, to the damage parameter D implies that if the interface layer is extremely thin, its influence on the overall modulus is negligible.

4) When D = 1, which means a complete damage around the aggregate, the increase of aggregate concentration and stiffness has almost no contribution to the overall modulus, the so-called hole-effect. This phenomenon is revealed quite

well by the linear and quadratic models, while the power law model fails in this special case.

5) When comparing the prediction of the proposed model to the experimental data by Cohen, Lee and Goldman (1995), we find that the elastic modulus at the interface is about 20-70% lower than that in bulk paste for Portland cement, and 10-40% lower for silica fume cement. This procedure serves as a quantitative and non-destructive means of estimating the properties in the interfacial transition zone.

From the present work, we have realized that much more remains to be learned about the microstructure of the concrete materials, especially the interfacial transition zone, in order to propose a reasonable micromechanical model, which can in turn help to predict the overall mechanical properties.

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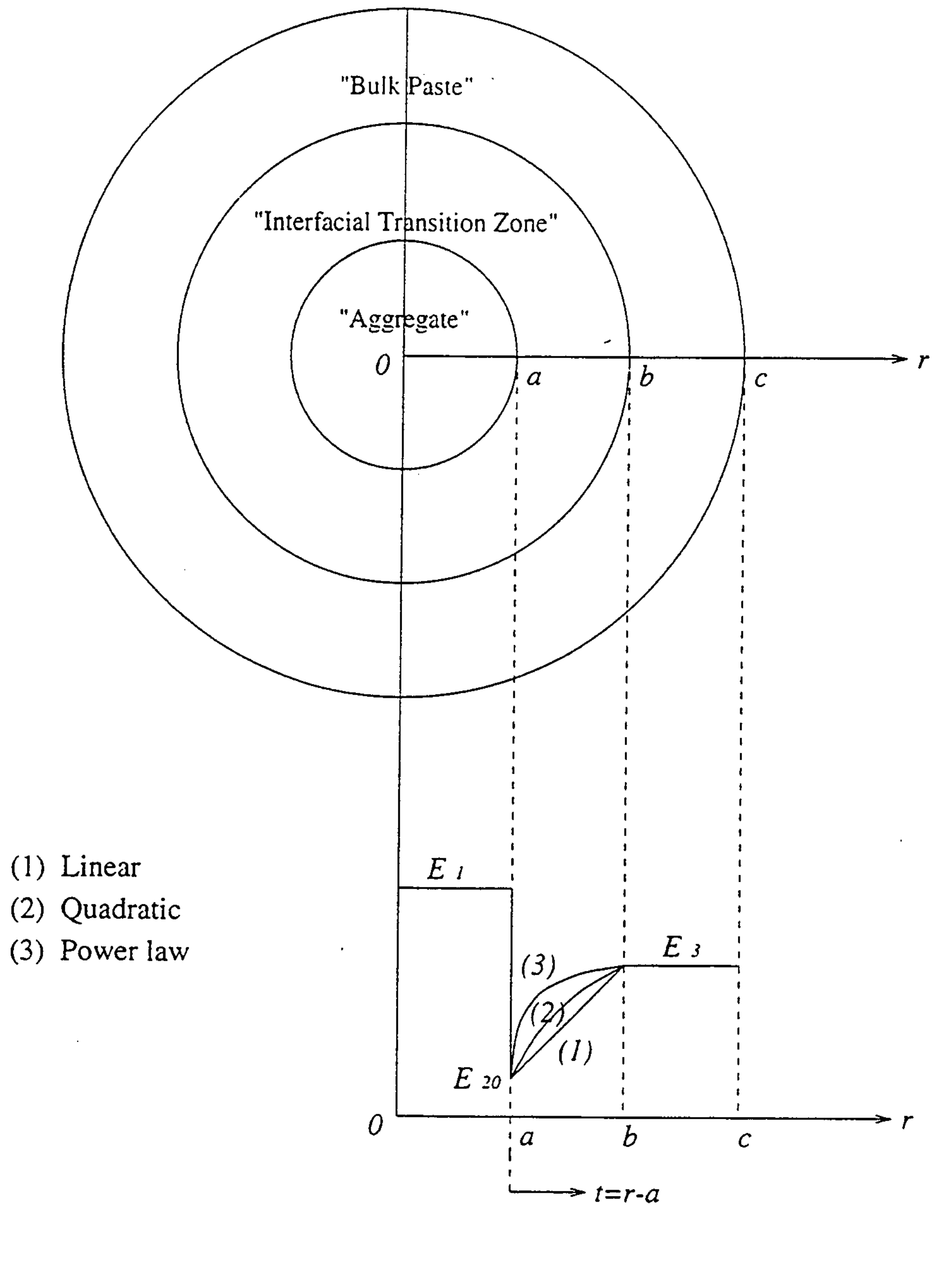


Figure 1 Modified four-phase composite model

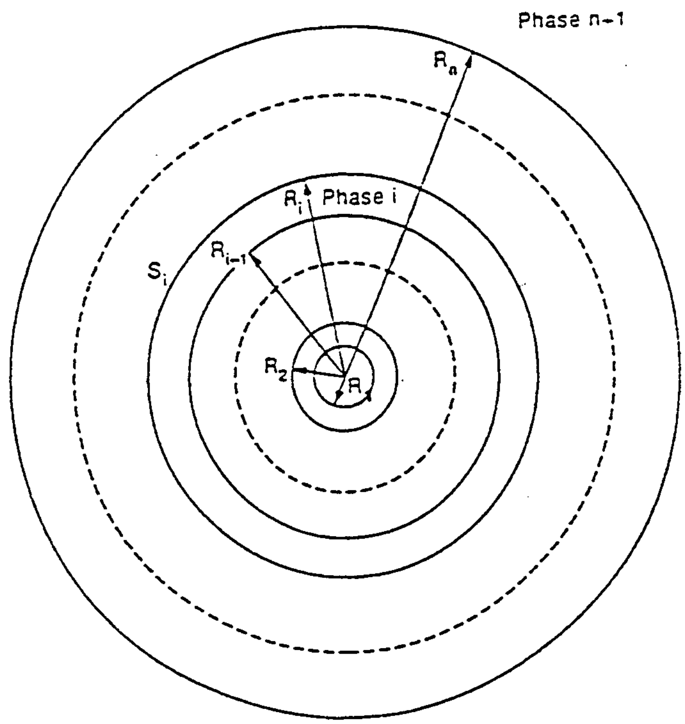


Figure 2 *n*-layered spherical inclusion model (Herve and Zaoui, 1993)

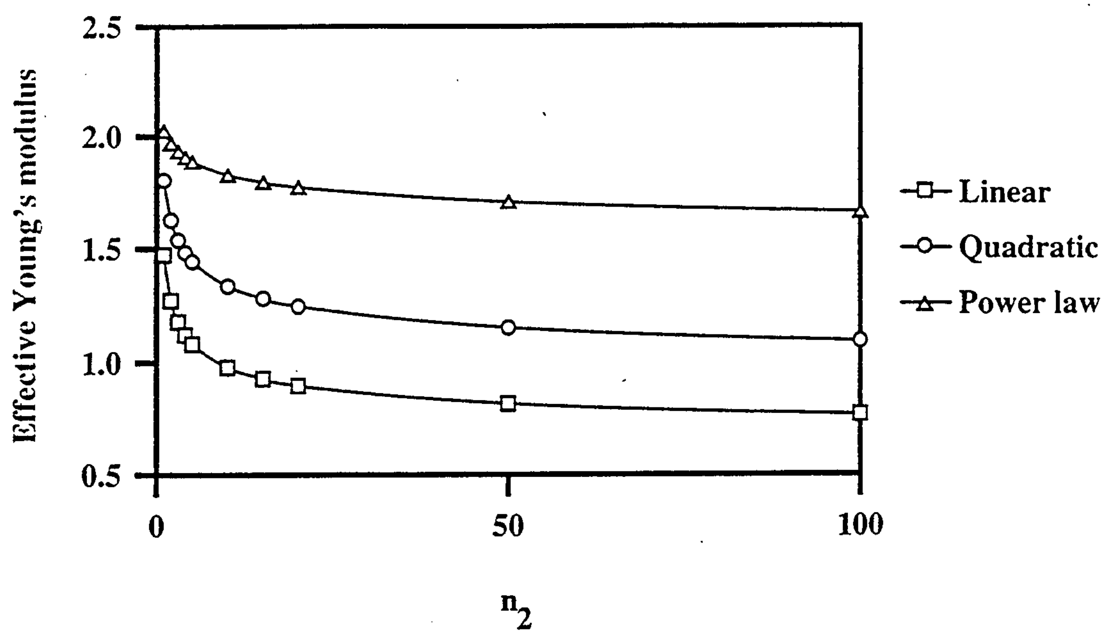
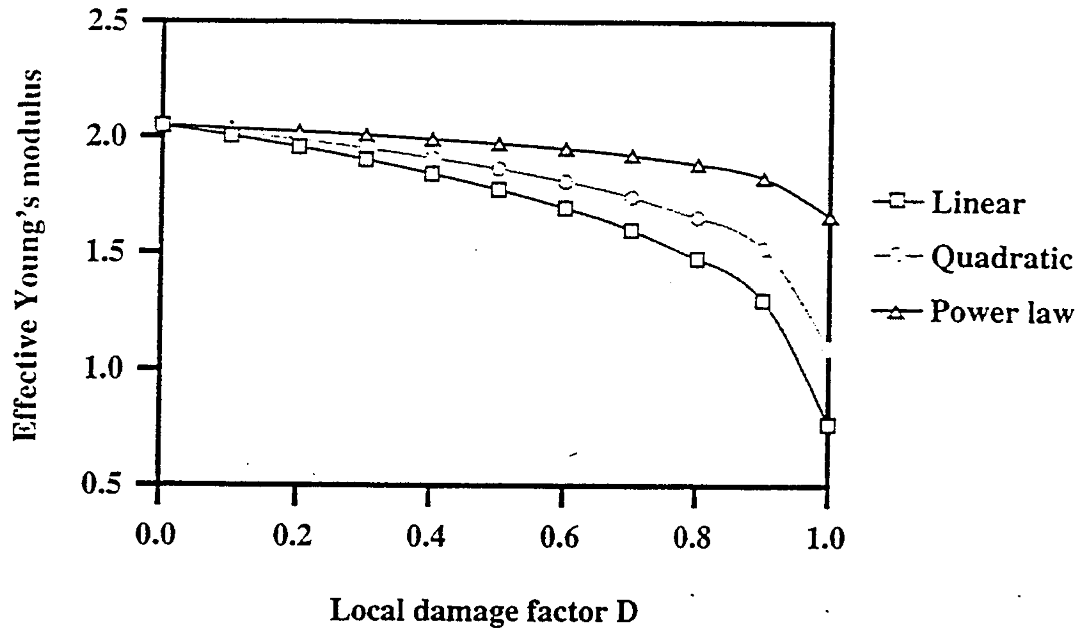
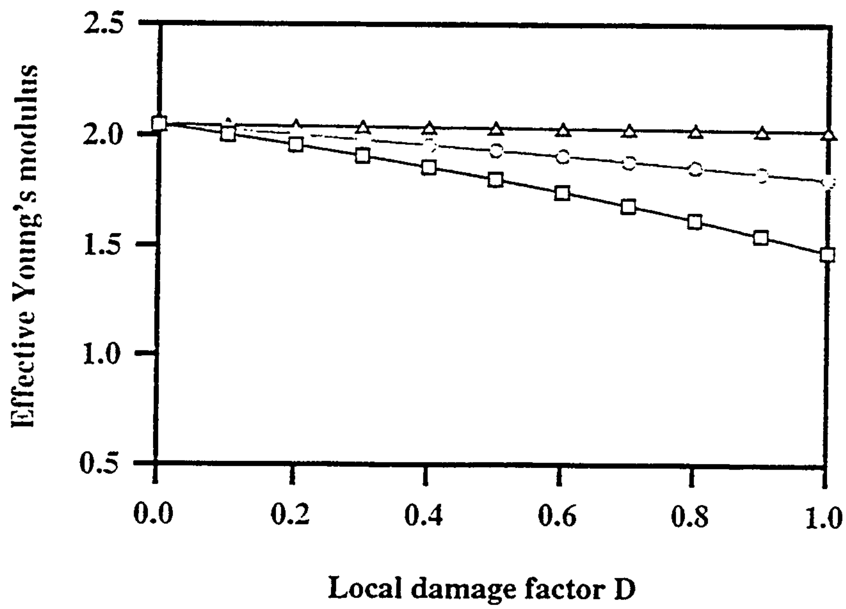


Figure 3 Influence of n_2

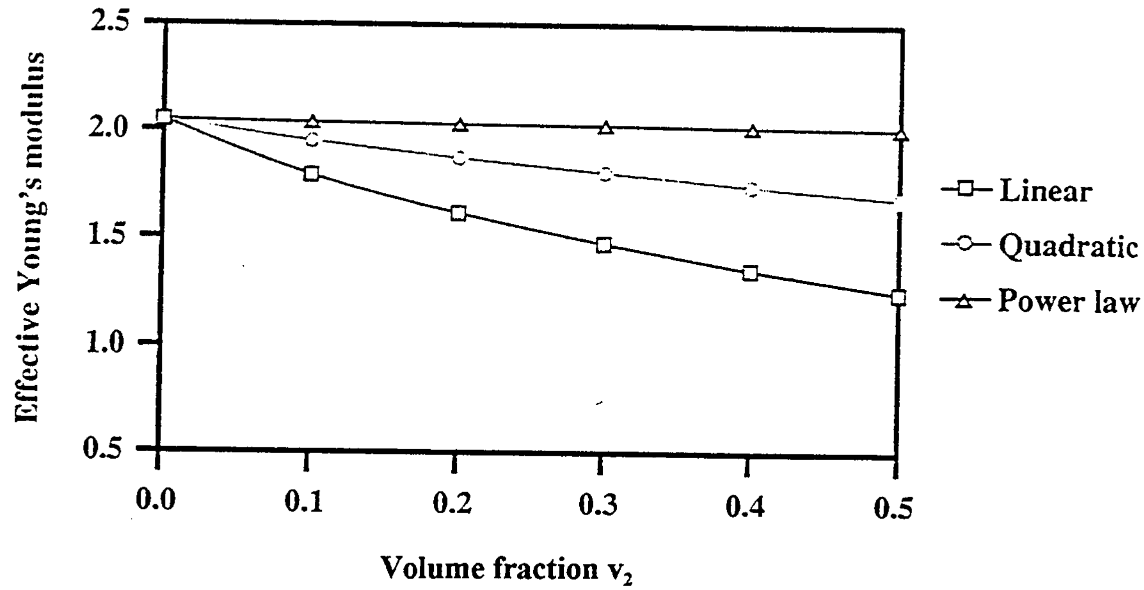


(a) $n_2 = 100$

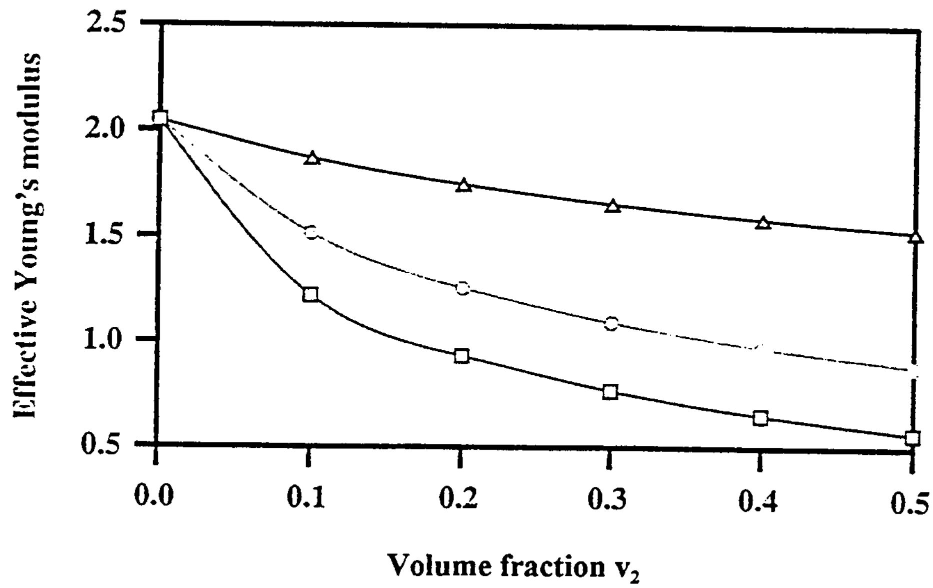


(b) $n_2 = 1$

Figure 4 Effects of local damage factor D

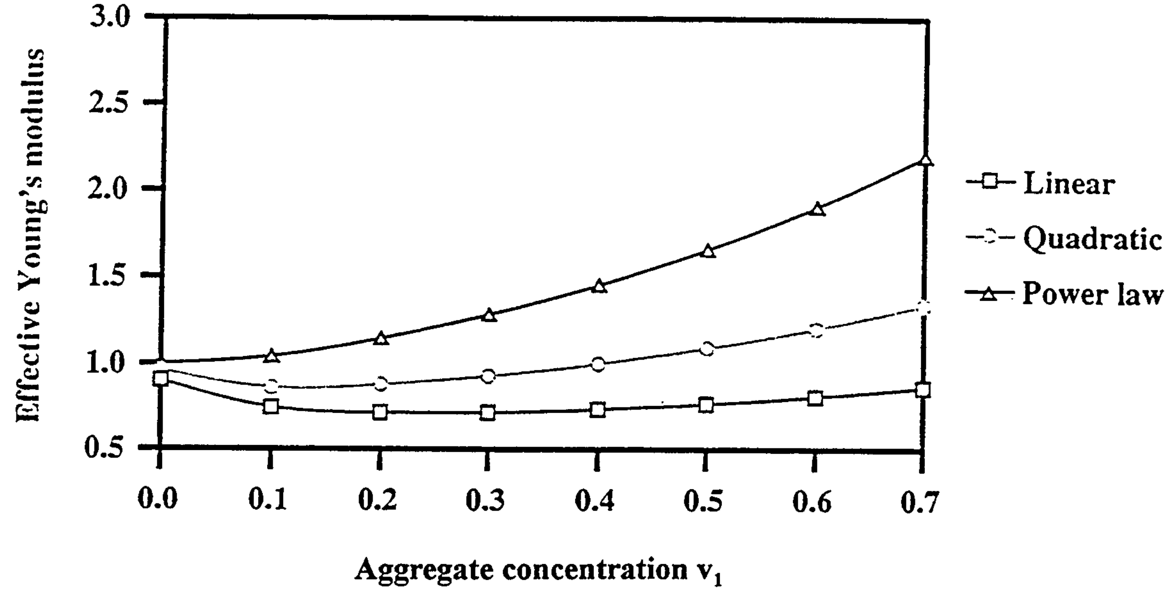


(a) $n_2 = 100$

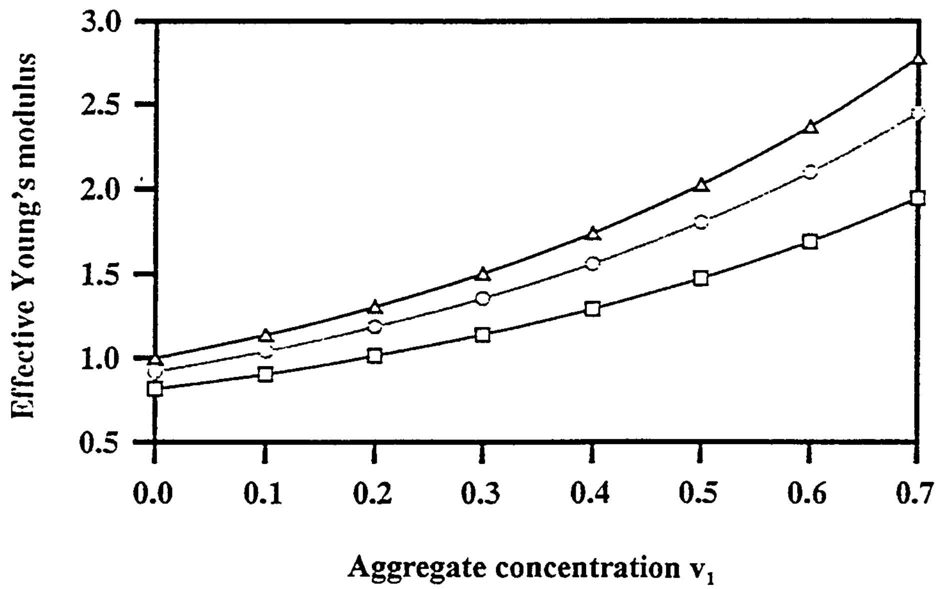


(b) $n_2 = 1$

Figure 5 Effects of the volume fraction v_2

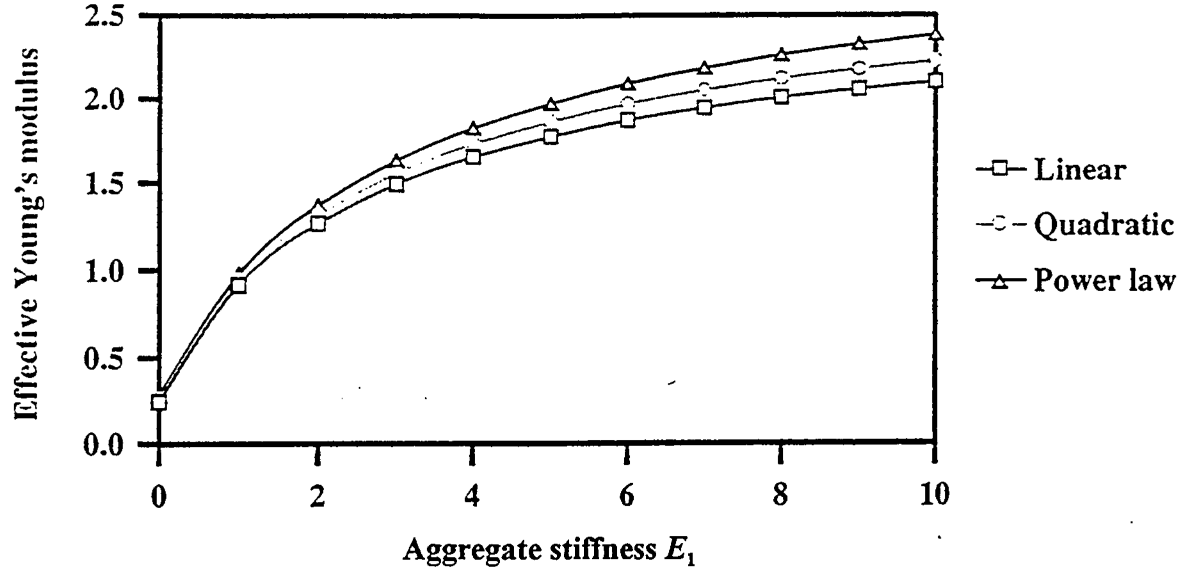


(a) $n_2 = 100$

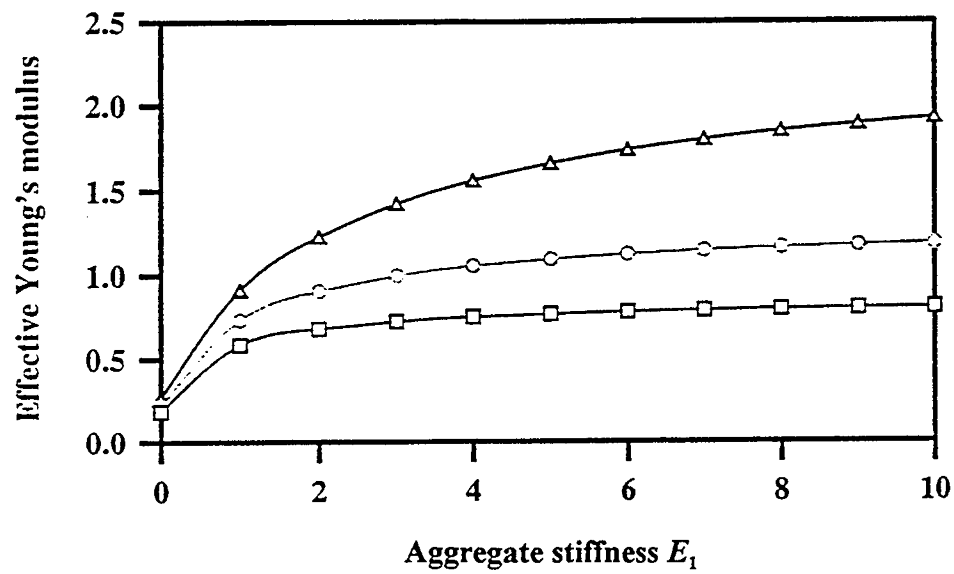


(b) $n_2 = 1$

Figure 6 Effects of the aggregate concentration v_1

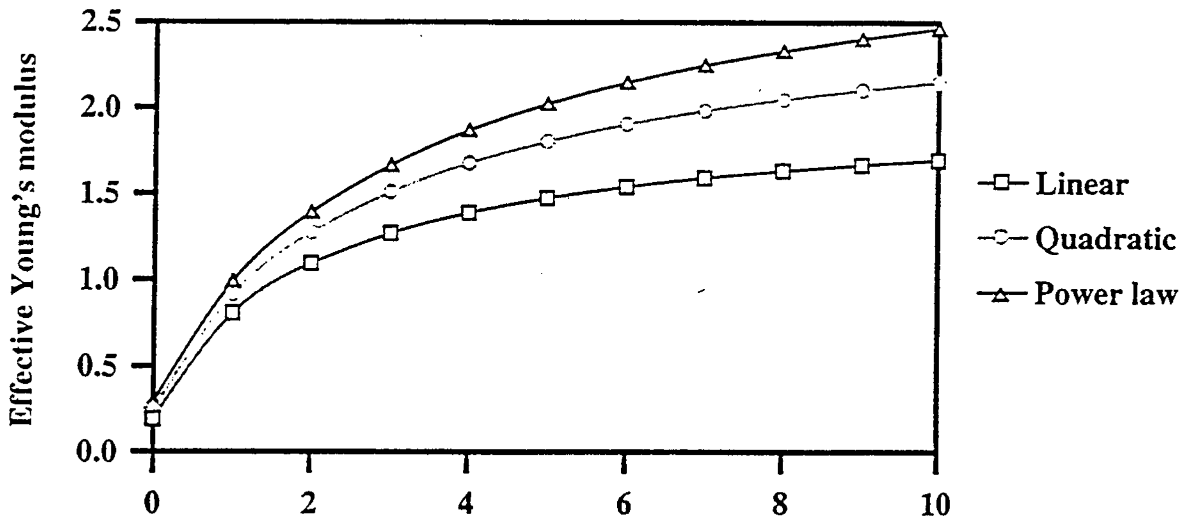


(a) $D = 0.5$, $n_2 = 100$



(b) $D = 1.0$, $n_2 = 100$

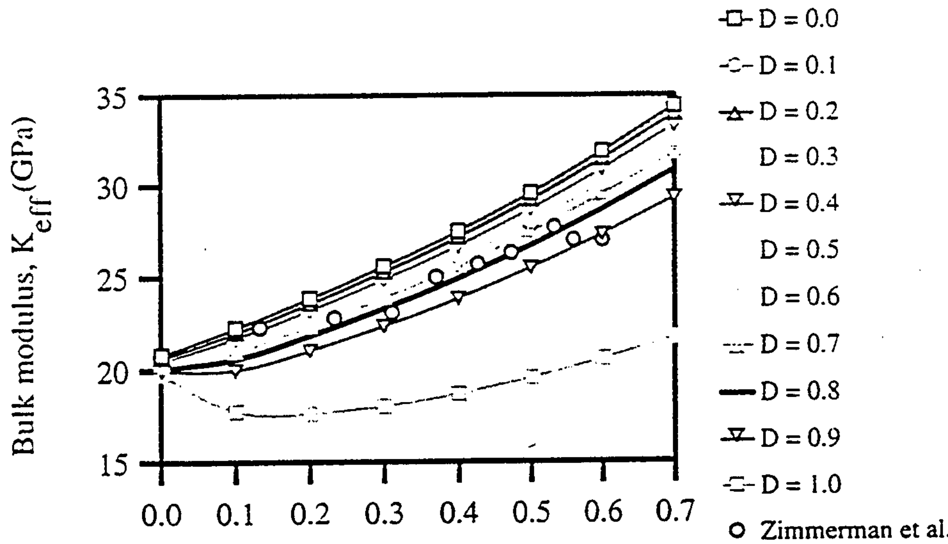
Figure 7 Effect of the aggregate stiffness E_1



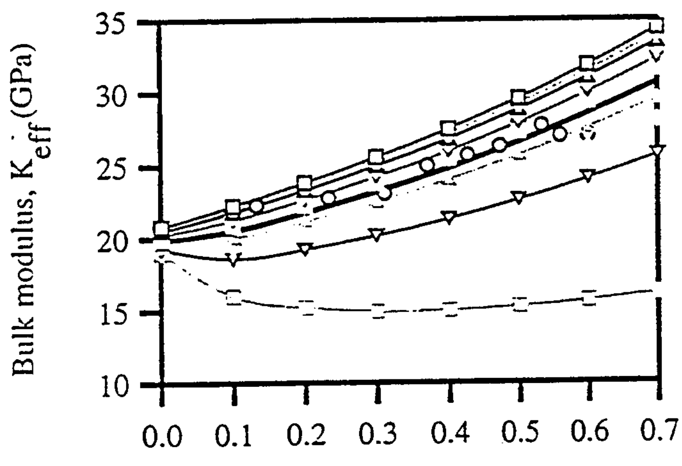
Aggregate stiffness E_1

(c) $D = 1.0$, $n_2 = 1$

Figure 7 (continued)

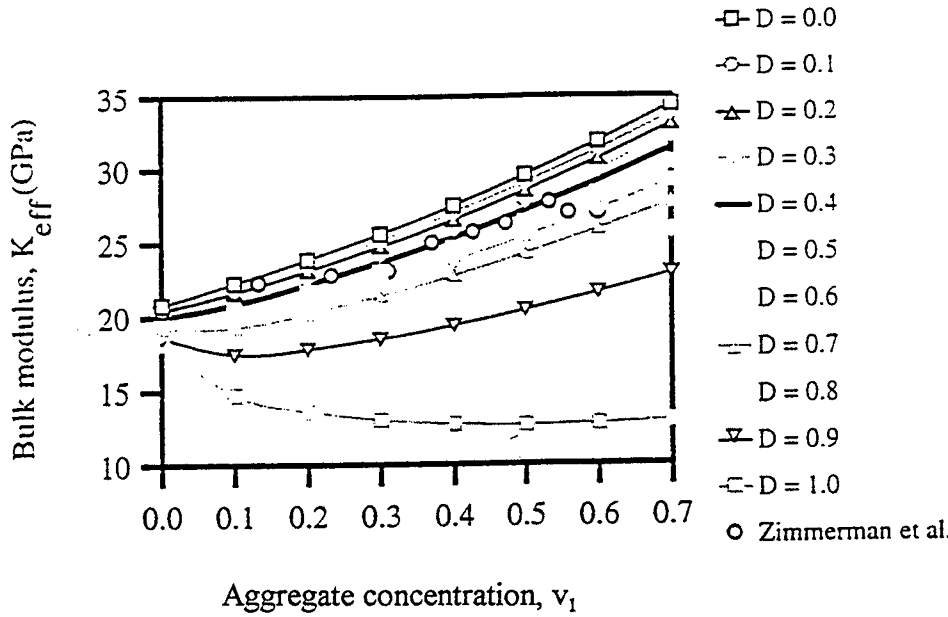


(a) Linear model ($v_2 = 0.1$)

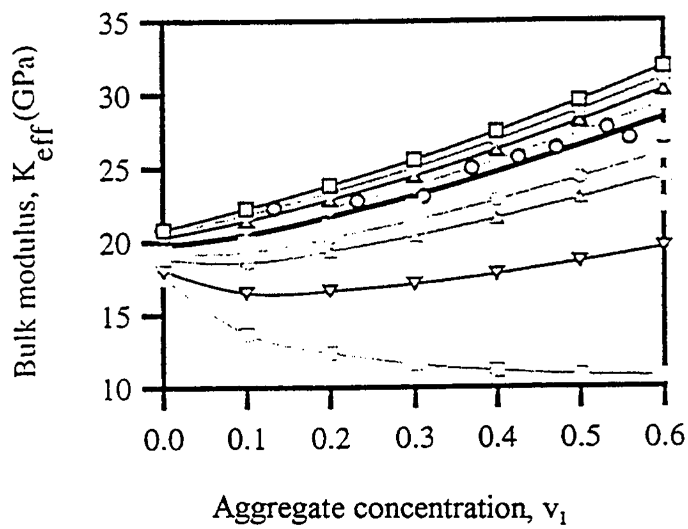


(b) Linear model ($v_2 = 0.2$)

Figure 8 Comparison with the test results by Zimmerman et al. (1986)

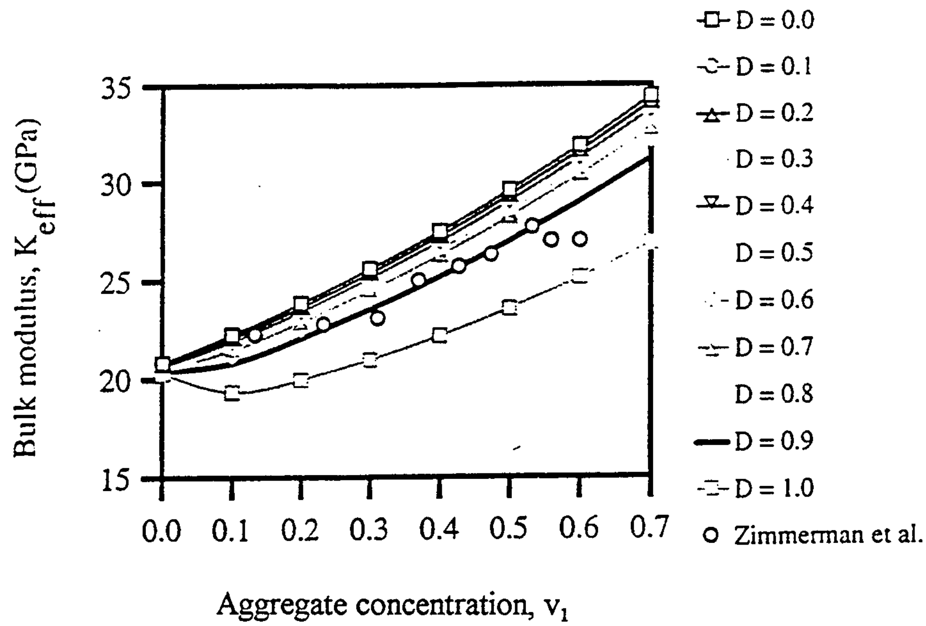


(c) Linear model ($v_2 = 0.3$)

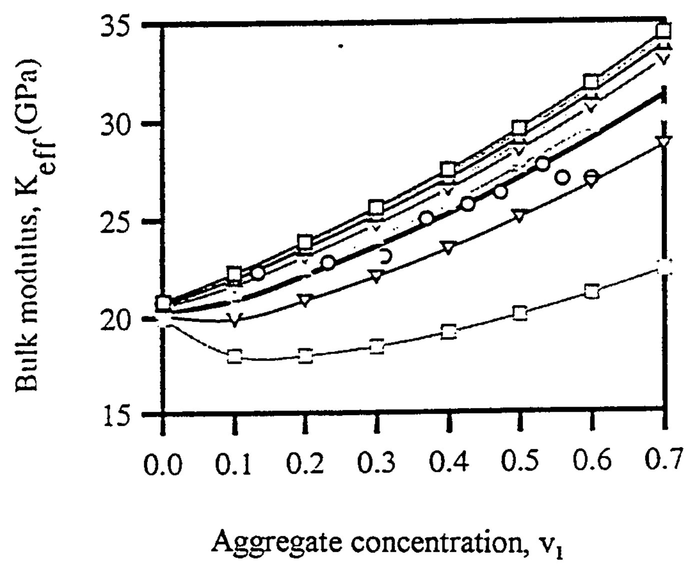


(d) Linear model ($v_2 = 0.4$)

Figure 8 (continued)

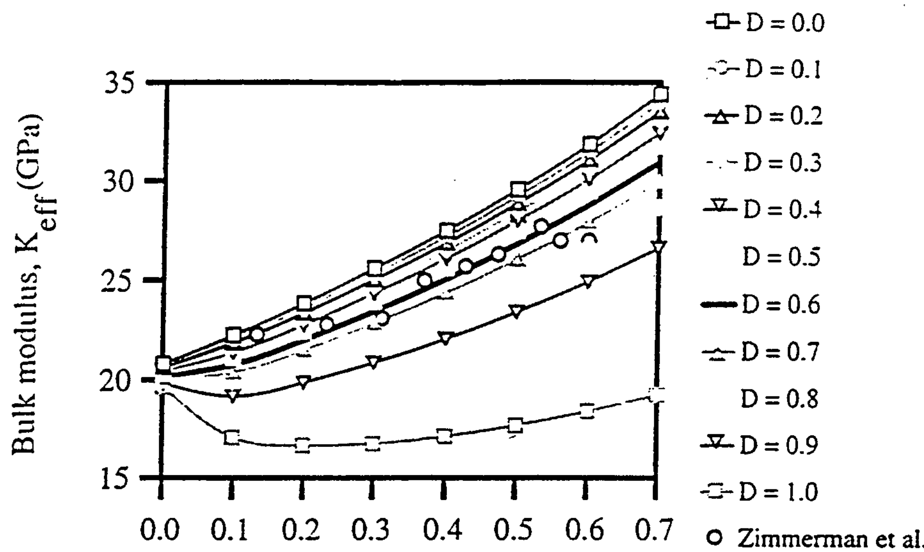


(e) Quadratic model ($v_2 = 0.1$)



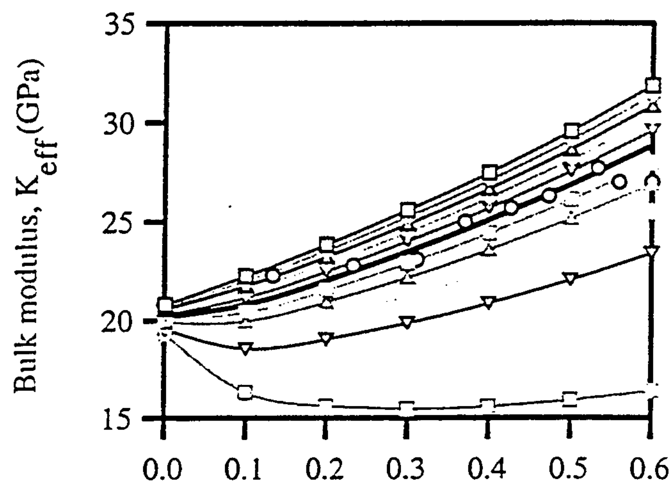
(f) Quadratic model ($v_2 = 0.2$)

Figure 8 (continued)



Aggregate concentration, v_1

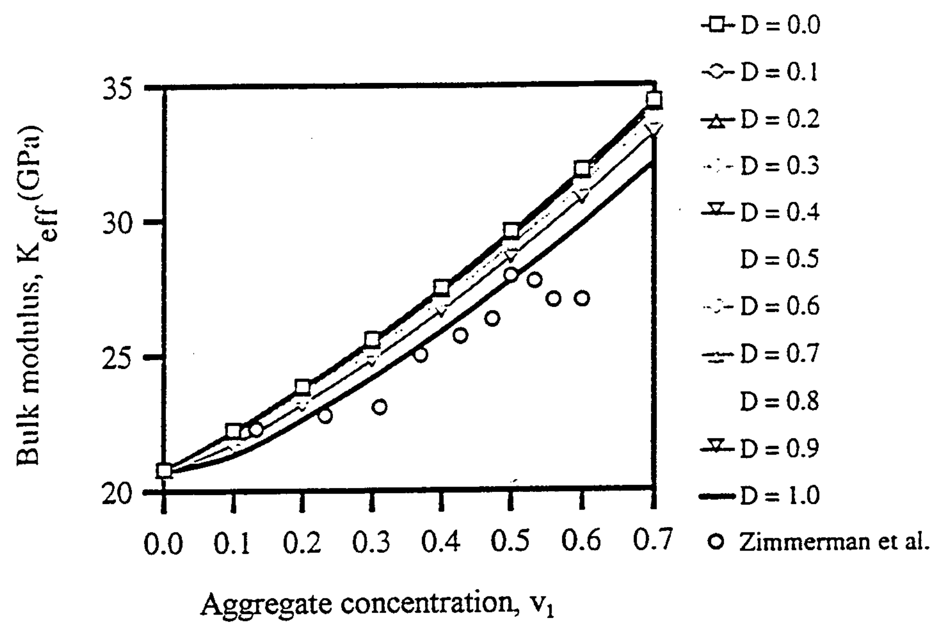
(g) Quadratic model ($v_2 = 0.3$)



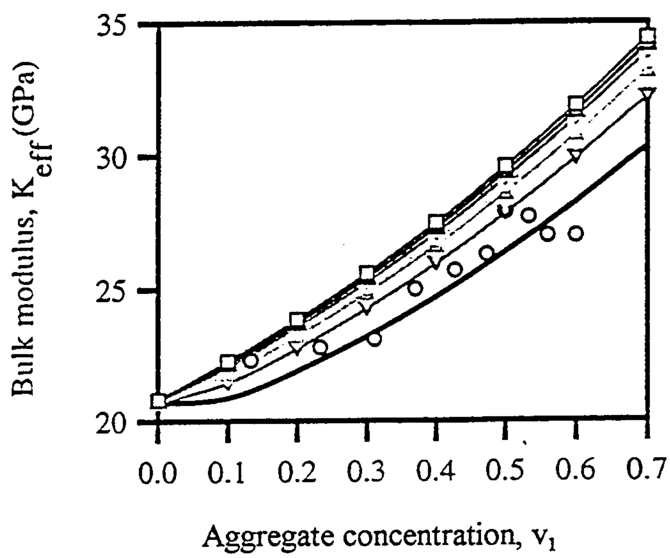
Aggregate concentration, v_1

(h) Quadratic model ($v_2 = 0.4$)

Figure 8 (continued)

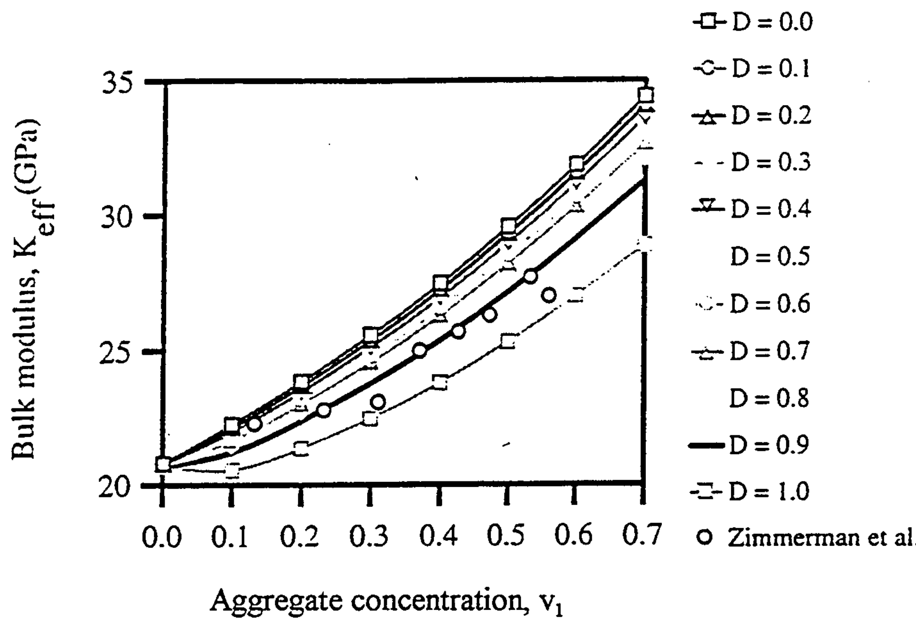


(i) Power law model ($v_2 = 0.1$)

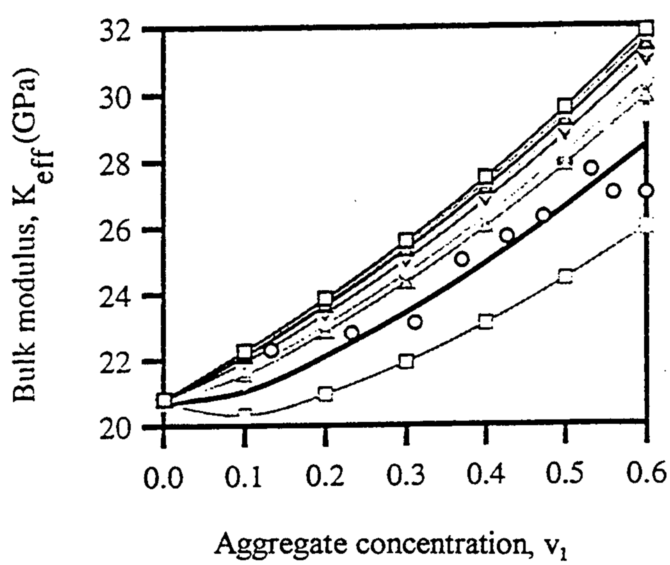


(j) Power law model ($v_2 = 0.2$)

Figure 8 (continued)



(k) Power law model ($v_2 = 0.3$)



(l) Power law model ($v_2 = 0.4$)

Figure 8 (continued)

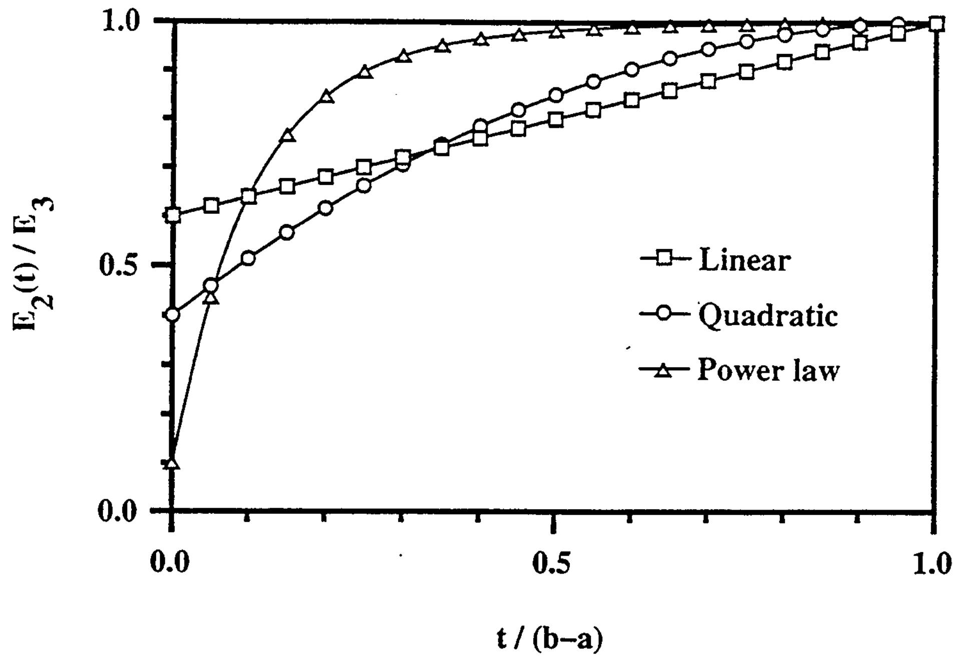
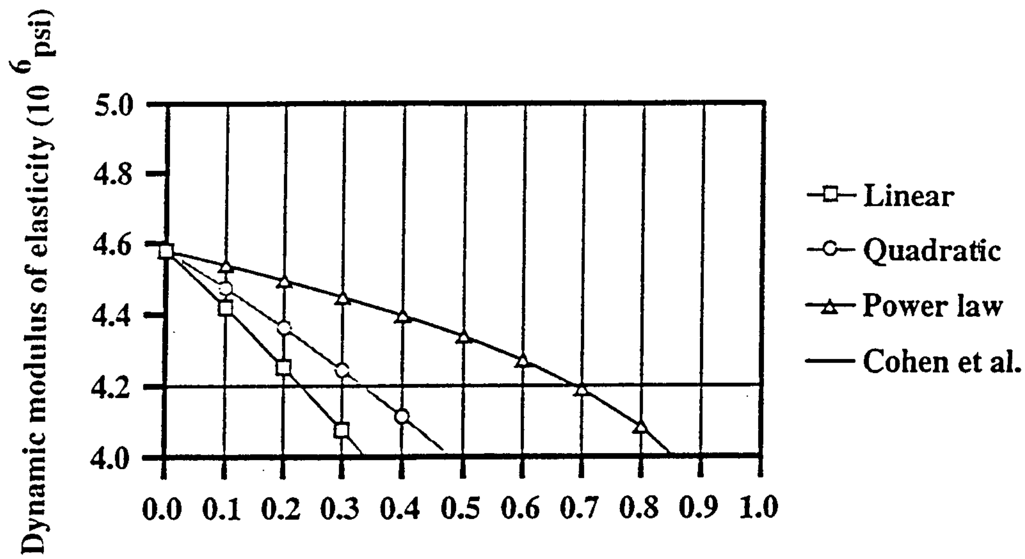
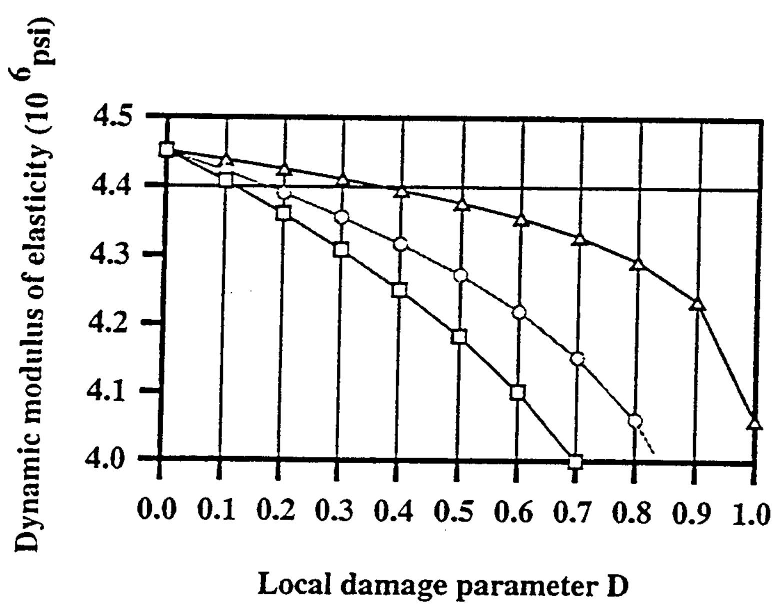


Figure 9 Comparison of different models



(a) Prediction for PC mortar



(b) Prediction for SF mortar

Figure 10 Comparison with the test results by Cohen et al. (1995)