Experimental Investigations in Nonlinear Elastic Behavior of Subgrade Soils under Traffic-induced Loading

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16. Abstract

This paper concerns experimental investigations of elastic nonlinearity of subgrade soils under vehicle-induced repeated loading. Two groups of soil samples (A8 and HA) were used to conduct triaxial shear tests with repeated loading. Investigations in this paper have included three aspects. First, a new nonlinear elastic model is introduced. In this model, shear modulus μ or resilient modulus M_r is assumed to be a function of the recoverable strain and the number of repetition N. The new model has distinguished itself from those introduced by other investigators, especially in studies of the shear and resilient modulus. Second, the shear modulus μ or resilient modulus M_r introduced in this paper not only allows one to study how shear and resilient modulus change with recoverable strain but also gives an opportunity to investigate effect of repetitions on the resilient modulus. An introduced function $f_1(N)$ relates the shear modulus to the effect of repeated loading. Introducing such a function actually opens a door for one to conduct further investigations in behavior of material fatigue and creeping because such a model is featured as a nonlinear relation of stress-strain-N (or stress-strain-time). Third, three constitutive parameters k_1, k_2 and k_3 are calibrated through results gained from triaxial tests performed in the laboratory. It is found that when parameter k_1 and k_3 are negligibly small, this model reduces to a linear elastic model. A nonlinear relation between normalized deviatoric stress and recoverable strain is discussed. Effect of shear stress level on the elastic nonlinearity has been studied as well.

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Introduction

Performance of pavement is a key factor in highway performance analysis. In fact, the top priority for improving the nation's highways is to focus on the quality of the roadway surface. Performance of pavement will largely rely on the mechanical behavior of subgrade soil layers. In order to increase the capacity of transportation of highway systems, it is recommended currently worldwide to use the higher payload of trucks to improve road productivity. The higher payload reduces both the total number of vehicles in operations and the transport cost that takes about 10% of the Gross National Product annually. The higher payload, however, induces a higher level of shear stress within pavement and subgrade layers. The higher shear stress level not only imposes the quality of the performance of road surface but also impacts on interaction between pavement and subgrade soil layers. As a result, dynamic response of subgrade materials under cyclic loading due to vehicles plays a key role in roadbed design and analysis. Therefore, it is important to enhance our understanding of dynamic behavior of subgrade soils under vehicle-induced loading at a higher level of shear stress.

Currently, a linear elastic relation between axial deviatoric stress and axial recoverable strain of subgrade soils is employed in pavement design. Namely, constant elastic parameters such as Young's modulus, shear modulus, resilient modulus, etc., are assumed as constants and applied to engineering. However, soils under external forces do not behave linearly, especially when deformation is not very small. To simulate the stress-strain relationship, a nonlinear elastic relation is introduced as a new model. The linear relation between elastic stress and strain is a special case of this new model so that when deformation is very small due to a small shear stress level, stress-strain relation can be adequately described. This new constitutive law considers the effects of soil deformation and strength weakening under repeated loading. Studies are focused on the nonlinear relation between shear stress and shear strain in the present research.

Resilient modulus, an alternative form of shear modulus, is our main concern in this investigation. Resilient modulus of soils is a required parameter in pavement design and analysis (*1*). Resilient modulus is defined as the ratio of axial deviatoric stress σ_d to axial recoverable strain and is assumed to be a constant in most design-related applications. Namely, linear elastic relationship is assumed, which means the resilient modulus is constant. From physical nonlinearity of soil materials, it is known that relations between stress and strain are not linear, especially when shear stress increases and causes a larger deformation within the layers of the roadbed. Many previous investigations have been conducted by other investigators (*2-7*). Former studies concentrated on investigating how resilient modulus are affected by deviatoric stress and confining pressure, but less research effort has been focused on studying characteristics of resilient modulus related to both the axial recoverable strain and the number of repetitions N (*2-7*). Under some circumstances, mechanical response (i.e., strength-failure, stress-strain, etc.) is dominated by deformation within a roadbed system. It is associated with features of traffic-induced repeated loading such as frequency (intensity of vehicles), magnitude (payload weight of vehicles), repetitions (lifetime of roads), etc. In the new model introduced in this study, therefore, resilient modulus M_r is actually suggested to be a function of the recoverable strain and the vehicle-induced loading. Thus resilient modulus not only allows one to study how M_r changes with recoverable strain but also gives an opportunity to investigate the effects of strength weakening on the resilient modulus due to repeated loading. Moreover, this nonlinear relation of stress-strain-N or stress-strain-time allows further investigations on time-related mechanical characteristics of geomaterial such as fatigue and creeping that play a central role in the mechanism of road rutting and cracking, which are factors that significantly affect the performance of road pavement systems.

This is the first attempt to introduce the concept of shear stress level in investigations of soil nonlinearity. In this study, the shear stress level is defined as the ratio of dynamic deviatoric stress to static uni-axial strength (static stress at the point of failure). The static uni-axial strength is determined under the undrained and unconsolidated test conditions (UU condition). This static strength represents the maximum potential of resistance of subgrade soils to external forces. The shear stress level represents the utilized portion of potential resistance of the maximum static strength to resist external force. The shear stress level (or ratio) is able to indicate possible damage to subgrade materials caused by the cyclic loading. By introducing the concept of shear stress level, the test results from different soil samples with variant strengths can be linked, and correlations of dynamic response can be analyzed (i.e., stress-strain relations, resilient modulus, etc.). In brief, experimental investigations are emphasized. The dynamic behavior of subgrade soil under vehicle-induced load is studied on the basis of the nonlinear characteristics of subgrade soils. Constitutive parameters are calibrated through laboratory facilities with a quasi-triaxial stress-strain system.

A Nonlinear Elastic Stress-strain Relation

If the stress-strain relation for subgrade soil is assumed to be elastic, according to theory of elasticity (*8*), one has the following expression:

$$\sigma_{ij} = E_{ijkl}\varepsilon_{kl}, \tag{1}$$

where σ_{ij} and ε_{kl} represent elastic stress and strain tensors that are the symmetric second order tensors. The term E_{ijkl} is a fourth order tensor of elasticity for the constitutive law and can be defined as a function of stress, strain, rate of strain, time, space, and temperature for cases of physical nonlinearity. From the symmetry of the stress and strain tensors and with an assumption of isotropic soil material, E_{ijkl}, a fourth order tensor in Equation 1, can be written as:

$$E_{ijkl} = (k - 2\mu/3)\delta_{ij}\delta_{kl} + \mu(\delta_{il}\delta_{jk} + \delta_{ik}\delta_{jl}), \tag{2}$$

where δ_{ij} is the Kronecker delta, κ and μ are the bulk and shear elastic parameters, respectively. It is known that for subgrade soils, the stress-strain relation is *not* linear. In other words, the elastic parameters κ and μ are not constant. In order to consider elastic nonlinearity between vehicle-induced shear stress and recoverable shear strain, shear modulus μ is assumed to be a function of both the number of repetitions and the second deviatoric invariant J^D_2 of strain [i.e., $\mu = \mu(N, J_2^D)$]. If it is further assumed that shear modulus μ can be expressed by the product of two functions, $f_1(N)$ and $f_2(J_2^D)$, then a form of $\mu(N, J_2^D)$ is given by:

$$\mu = \mu(N, J_2^D) = f_1(N) f_2(J_2^D), \tag{3}$$

where J^D_2 is a function of shear or deviatoric stress in three dimensional space and is defined as $(\varepsilon^D_{ij}\,\varepsilon^D_{ij})/2$ (*8*). The term ε^D_{ij} is an infinitesimal deviatoric strain, the second order tensor. The variable N is the number of repetitions of cyclic loading. The superscript D denotes a deviatoric variable. The two independent functions [$f_1(N)$ and $f_2(J_2^D)$] in Equation 3 mean that effects of these two variables (N and ε^D_{ij}) upon shear modulus μ can be decoupled. In other words, $f_1(N)$ and $f_2(J_2^D)$ do not affect each other though shear modulus is a function of both variables N and ε^D_{ij}. This assumption is verified with experimental results that are shown in a later section of this report.

Alternatively, Equation 1, can be written in the following two parts (*8*):

$$\sigma_{ij} = \sigma_{kk}\delta_{ij}/3 + \sigma_{ij}^D = \kappa\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij}^D, \quad (4)$$

where $\sigma_{kk}/3$ is the spherical or mean stress that is associated with volume strain ε_{kk} in a form (k = 1, 2 and 3) (*8*):

$$\sigma_{kk}/3 = \kappa\varepsilon_{kk}. \quad (5a)$$

σ^D_{ij} is the second order tensor of deviatoric or shear stress that is related to deviatoric strain tensor ε^D_{ij} by:

$$\sigma_{ij}^D = 2\mu\varepsilon_{ij}^D \quad (5b)$$

For convenience of further investigations, Equations 5a and 5b are written in terms of invariants of stress and strain as follows (*8*):

$$I_1 = 3\kappa J_1, \qquad \sqrt{I_2^D} = 2\mu\sqrt{J_2^D}, \quad (6a, b)$$

where I_1 (= σ_{kk}) and I_2^D (= $\sigma_{ij}^D\sigma_{ij}^D/2$) are invariants of the first spherical stress and the second deviatoric stress individually (*8*). The terms J_1 (= ε_{kk}) and J_2^D represent the first spherical and the second deviatoric strain tensor invariants. Normally, laboratory studies are conducted in a quasi-triaxial space. For the special case of a quasi-triaxial system and in the principal stress and strain spaces, one has principal stresses $\sigma_1 \neq \sigma_2 = \sigma_3$ and principal strains $\varepsilon_3 \neq \varepsilon_2 = \varepsilon_3$. If one keeps the definitions of $I_1 = \sigma_{kk}$ and $I_2^D = \sigma_{ij}^D \sigma_{ij}^D/2$ in mind, then stress-strain relations of Equations 6a and 6b are respectively reduced to:

$$(\sigma_1 + 2\sigma_3)/3 = \kappa(\varepsilon_1 + 2\varepsilon_3)\,, \qquad (\sigma_1 - \sigma_3) = 2\mu(\varepsilon_1 - \varepsilon_3)\,. \tag{7a, b}$$

Equation 7a and 7b are relations of volume stress-strain and shear stress-strain under a quasi-triaxial system. In the present paper, Equation 7b is our main concern. Namely, the investigation of the relation between deviatoric stress $\sigma_1 - \sigma_3$ and deviatoric strain $\varepsilon_1 - \varepsilon_3$ is emphasized. The bulk modulus κ can be evaluated by using shear modulus μ and introducing the Poisson's ratio υ. According to theory of elasticity (*8*), ε_1 is associated with ε_3 by Poisson's ratio (i.e., $\upsilon = - \varepsilon_3/\varepsilon_1$). If Poisson's ratio is assumed to be negligibly small when compared to unit, then Equation 7b is allowed to be replaced with $\sigma_d = 2\mu\varepsilon_d$ where σ_d and ε_d denote deviatoric cyclic stress and recoverable strain along the axial direction, respectively. In the case that Poisson's ratio is not negligibly small, Equation 7b can also be written in terms of σ_d and ε_d [i.e., $\sigma_d = 2\mu'\varepsilon_d$, where $\mu' = \mu(1+\upsilon)$]. Such a relation of $\sigma_d = 2\mu\varepsilon_d$ can be conveniently applied to a quasi-triaxial apparatus. The notation μ instead of μ' is to be used in the present paper to indicate an average (or constant). Equation 7b shows how the shear modulus μ is related to the resilient modulus M_r. In fact, the resilient modulus is an alternative form of shear modulus. Since the resilient modulus is an important input parameter in highway pavement design, the discussion of shear modulus or resilient modulus is emphasized in the later sections. Poisson's ratio is assumed. Further investigations are to calibrate constitutive parameters from experimental results.

Experimental Investigations in the Laboratory

Samples preparation and basic properties

Two groups of subgrade soil samples (A8 and HA) are received from the Soil/Aggregate Laboratory at the Geotechnical Exploration Division at Maryland State Highway Administration, Maryland Department of Transportation. Theses subgrade soil samples were originally collected from two sites in Howard County, Maryland. The main physical properties of the samples from two sites are listed in Table 1. Sample preparation for testing the resilient modulus follows the code of AASHTO T292-91.

Equipment and test conditions

Experimental investigations have been conducted at the Soil/Aggregate Laboratory at the Geotechnical Exploration Division of Maryland State Highway Administration, Maryland Department of Transportation. Samples were tested with two models of triaxial apparatus (RMT HX100 and RMT HX1000) that were specially designed for resilient modulus by the Structure Behavior Laboratory Equipment Inc. Samples were first subjected to a conditioning period with a zero confining pressure. Then the samples were subjected to undrained and unconfined conditions (UU conditions). The magnitude of the repeated loading increases with a constant number of repetitions in each sequence. More information on the testing conditions is listed in Table 2.

Parameter calibration and test data

Based on the nonlinear elastic Equation 7b for shear stress and strain, a simplified form in quasi-triaxial space is suggested as:

$$\sigma_d = 2\mu(N, \varepsilon_d)\varepsilon_d, \tag{8}$$

where μ in Equation 2 is assumed to have the following expression:

$$2\mu(N, \varepsilon_d) = k_2 N^{k_3} \varepsilon_d^{k_1}, \tag{9}$$

in which the terms k_1, k_2, and k_3 are constitutive parameters, k_2 has a dimension of kPa, and k_1 and k_3 are dimensionless. For simplicity, k_1 is assumed to be a constant in the present paper though in a more general case k_1 can be a function of N as well. In fact, keeping Equation 3 in mind, one can find that functions of $f_1(N)$ and $f_2(J^D{}_2)$ have been introduced as the following two forms:

$$f_1(N_i) = N_i^{k_3}, \qquad f_2(\varepsilon_d) = k_2 \varepsilon_d^{k_1} / 2, \qquad (10a.\ b)$$

where N_i is the number of repeated loading in each test. Parameters k_1, k_2, and k_3 are calibrated from experimental results. The procedure of calibration for constitutive parameters involves two steps. The first step is to find k_1 and $k_2N^{k_3}$ from each stress-strain curve. The calibrated parameters k_1 and $k_2N_i^{k3}$ are listed in Table 3 in which R^2 is determined from a regression of test data. The test results are drawn along with the calculated curve represented in Figure 1 and Figure 2 in which the solid curves are calculated with the calibrated constitutive parameters listed in Table 3.

The second step is to find k_2 and k_3 using the set of data $k_2N_i^{k3}$ and N_i in Table 3. The calibrated parameters k_2 and k_3 are given in Table 4. Furthermore, in order to analyze the test results without considering the effect of repetitions, the deviatoric stress σ_d is divided by $f_1(N)$ (i.e., σ_d / N^{k3}). From this, one can draw the relation σ_d/N^{k3} against ε_d. In Figures 3 and 4, the calculated solid curves are drawn along the points of test data in each figure. It is interesting to note that samples A8 and HA both become a single curve after the deviatoric stress has been normalized by the factor of $f_1(N)$. Moreover, sample HA has a smaller k_1 and shows an almost linear relation in Figure 4. In contrast, sample A8 has a larger value of k_1, and illustrates a more observable nonlinear curve. For a case of $k_1 = 0$, the curves in both Figures 3 and 4 become straight lines. In other words, the nonlinear relation Equation 7b reduces to a linear one: $\sigma_d = k_2\varepsilon_d$. Figures 1 and 2 show that parameter k_3 determines how fast the σ_d - ε_d curve decreases with an increasing number of N of repetitions. For a specialized case of a negligibly small value of k_1, there is no effect of repetitions on the stress-strain relation.

Discussion and Analysis of Results

Shear modulus μ as a function of ε_d and N

It is worthwhile to discuss the following facts illustrated in Figures 1 and 2. First, deviatoric (shear) cyclic stress and recoverable deviatoric (shear) strain (i.e., σ_d and ε_d) have a nonlinear elastic relation. In fact, a linear assumption can only be applicable within a very small range of strain. The assumption of elastic linearity for subgrade soils under repeated loading might not be applicable for a larger change in recoverable strain. Due to a heavier payload, a higher vehicle-induced shear stress level is yielded within the subgrade layers and causes a larger shear strain. Second, physical nonlinearity is a function of recoverable shear strain. Nonlinearity increases when elastic strain increases. This fact demonstrates that shear modulus μ decreases when the recoverable strain increases. Under increasing repetitions of repeated loading, subgrade soil material is softened with increase of recoverable strain when the value of k_1 in Equation 9 is smaller than zero. Third, the nonlinearity is also related to loading repetitions that weaken strength of subgrade soils. From Figures 1 and 2, the stress-strain relation subjected to a larger number of repetitions has a lower stress-strain curve. Namely, a lower value of deviatoric stress is obtained from a given recoverable strain with a small number of repetitions N because of the damage to soil structure caused by repetitions of repeated loading. Finally, in order to verify the assumption introduced in Equation 3, the deviatoric stress σ_d is divided by $f_1(N)$. The test data in Figures 1 and 2 are well converged along the calculated single curve σ_d /N^{k_3} - ε_d in Figures 3 and 4. The relations σ_d /N^{k_3} against ε_d represent the curves without the effect of cyclic loading. This fact suggests that $f_1(N)$ is independent from $f_2(\varepsilon_d)$. In other words, the deviatoric stress normalized by the function $f_1(N)$ will not change with the repetition N any more. Accordingly, modified deviatoric stress and recoverable strain ε_d have a relation as follows:

$$\sigma^*_d = 2\mu^*\varepsilon_d, \qquad (11)$$

where $\sigma^*_d = \sigma_d /N^{k_3}$ and $\mu^*(\varepsilon_d) = k_2\, \varepsilon_d^{\,k_1}$. From a practical point of view, one can also take advantage of the decoupled effect of N and ε_d on the deviatoric stress. For instance, since

Equation (11) shows a single curve in Figures 3 and 4 and is converted from a family of curves in Figure1 or Figure 2, it is more convenient to apply this relation to pavement design and analysis.

From Equation 11, it is evident that the values of parameter k_1 affect the nonlinearity of stress-strain relationship. Soil dynamic stress-strain relations with larger absolute values of k_1 show curves with higher nonlinearity than those curves with smaller absolute values of k_1. When k_1 equals zero, Equation 11 reduces to a linear relation because of a constant μ^*. The test results or calibrated constitutive parameters listed in Table 3 indicate support the conclusion made above. Namely, the sample group A8 has a larger absolute value k_1 than HA and illustrates higher nonlinearity than the sample group HA. Parameter k_3 is related to the shear stress level that is discussed later in this report.

Bulk modulus κ and Poisson's ratio υ

From the theory of elasticity (*8*), the bulk modulus κ can be written in terms of the Poisson's ratio υ and the shear modulus μ in the following expression:

$$\kappa = 2\mu(1+\upsilon)/3(1-2\upsilon). \qquad (12)$$

The bulk modulus κ can, therefore, be investigated through shear modulus and Poisson's ratio. The shear modulus is given by Equation 8. The values of Poisson's ratio υ for the two tested sample groups are evaluated from a static unit-axial shear test in Figure 5. According to the curves in Figure 5, the Poisson's ratio υ is negligibly small when compared to unit if recoverable strain ε_d changes between 0 to 1.75% for sample A8 and from 0 to 1.2% for sample HA. Thus the elastic parameter κ in Equation 12 reduces to $\kappa=2\mu/3$, and Equation 7b becomes $\sigma_d = 2\mu\varepsilon_d$. For a case of larger value of ε_d that is out of the ranges mentioned above, an average constant value of υ is suggested for simplicity.

Resilient modulus as a function of ε_d and N

From previous discussion, it is known that the definition of the resilient modulus M_r is a simplified form of Equation 6b for a special case of quasi-triaxial space. With this simplification (i.e., principal stress $\sigma_2 = \sigma_3$ and principal strain $\varepsilon_2 = \varepsilon_3$), resilient modulus M_r ($= \sigma_d/\varepsilon_d$) equals 2μ. The discussion of nonlinearity for shear modulus μ in the previous section, therefore, is also applicable for resilient modulus M_r. If one recalls Equations 8 and 9, resilient modulus M_r has an expression in the present paper:

$$M_r \stackrel{\Delta}{=} \sigma_d / \varepsilon_d = k_2 N^{k_3} \varepsilon_d^{k_1}, \tag{13}$$

in which the resilient modulus M_r is defined as a function of loading number N and recoverable strain ε_d. The equal sign with a delta above in Equation 13 represents *equal by definition.* Equation 13 clearly states that resilient modulus M_r decreases with increase of repetitions and recoverable strain ε_d when constitutive parameters k_3 and k_1 are both less than zero. Thus a proper value of resilient modulus M_r chosen for roadbed design should be examined along with allowable deformation and the number N of repetitions. A family of curves M_r versus ε_d with different N has been dawn in Figures 6 and 7 based on Equation 13 and parameters calibrated from test data in Tables 3 and 4.

In previous investigations (*2-7*), resilient modulus is usually related to deviatoric stress (i.e., M_r - σ_d) or confining pressure (i.e., M_r - σ_0). In contrast, Equation 13 states that the resilient modulus changes with shear strain and repetitions of repeated loading (i.e., resilient modulus is expressed as a function of strain and repetition: M_r - ε_d - N). Under some circumstances, the information of deformation or strain within a pavement system is more significantly important than shear stress because the shear resistance of subgrade relies on deformation of the subgrade soils. Both results from the static and dynamic shear tests (see Figures 1, 2, and 6) indicate that soil increases its capacity of shear resistance with increase of shear strain. Furthermore, at the same time, resilient modulus M_r also decreases with increase of receptions N (See Figures 6 and 7). Therefore, without knowing the information of resilient modulus changing with shear strain and loading repetitions, the chosen value of resilient modulus is most likely to be larger, and causes negative effects on the pavement design and analysis.

From Table 3 and 4, if one takes the average value of k_1 for each group, the resilient modulus for the two groups of samples is given as the following two expressions:

$$\text{Group A8:}\quad M_r = 627 N^{-0.45} \varepsilon_d^{-0.1} \text{ (Kpa)} \tag{14}$$

$$\text{Group HA: } M_r = 580N^{-0.1}\varepsilon_d^{-0.24} \text{ (Kpa)} \quad (15)$$

The above expressions show the fact that resilient modulus is not a constant. In this research report, the resilient modulus changes with the repetition number N and deviatoric strain ε_d.

Shear stress level and nonlinearity

From the results, it is evident that parameter k_1 controls the nonlinearity due to soil skeletal deformation, while the parameter k_3 dominates the nonlinearity due to the cyclic shearing effect. The former illustrates the reduction of shear elastic modulus when displacement between soil particles increases, and the latter shows the fact that structural damage is caused by repeated shearing stress. The following discussion will center on what causes the difference between the two sample groups. In particular, the reason why group HA demonstrates less nonlinearity than the sample group A8 will be discussed.

Shear stress level affects soil nonlinearity (*9*). The shear stress level is defined as the ratio of repeated deviatoric stress σ_d to static deviatoric stress at failure or UU strength σ_{uu} (i.e., $R = \sigma_d / \sigma_{uu}$). For a case of small values of σ_d / σ_{uu}, the curve of stress-strain-N of a sample depicts reduced nonlinearity. For instance, the group of sample A8 (A8-4 and A8-5) has a lower value of UU strength σ_{uu} (478 kPa) than that (800 kPa) of the group of sample HA (HA7 and HA8) in Figure 8. For a given σ_d, the former shows a higher nonlinearity than the latter does. In order to avoid the diversity of test results, the soil samples, even those in the same group, should be grouped according to their σ_{uu} value. If the stress level is defined by a modified ratio $R' = \sigma^*_d / \sigma_{uu}$, where σ^*_d is a normalized deviatoric stress and equals $\sigma_d/f_1(N)$, the relation of ratio σ^*_d / σ_{uu} at $\varepsilon_d = 0.3\%$ against parameter $-k_1$ is drawn in Figure 9. Since parameter k_1 is a nonlinear factor, the curve in Figure 9 illustrates that the shear stress level R increases with increase of the nonlinear factor $-k_1$. For instance, the sample group HA subjected to a lower value of R' indicates less nonlinearity with a smaller value of $-k_1$ when compared to the sample group A8 that is subjected to a higher value of R' with a larger value of $-k_1$. Hence, shear stress level decides the pattern of stress-strain-time because a higher value of R' causes more damage to soil structure of samples than lower one does.

Summary and Conclusions

Based on results discussed in this paper, the following conclusions are drawn:

1. A new nonlinear elastic model has been introduced in this paper. The relationship between shear stress and shear strain is emphasized. In this model, the nonlinearity is associated with deviatoric strain and repetition of cyclic loading. Since the introduced model can be considered as a stress-strain-time model, it is a powerful tool to predict the performance of pavement in subgrade soil conditions, taking changes in time into account. A simplified case in a quasi-triaxial space has been discussed for convenience of further experimental investigations in constitutive parameters.

2. In this model, there are three constitutive parameters (k_1, k_2 and k_3) that can be easily calibrated from results gained from triaxial test experimental investigations. The parameters k_1 and k_3 respectively carry information of the nonlinearity due to soil deformation and structural damage. When constitutive parameters k_1 and k_3 are negligibly small, the nonlinear elastic model reduces a linear one. Test results are based on two groups of subgrade samples collected from different sites.

3. A new expression of shear modulus μ or resilient modulus M_r is introduced as a function of recoverable strain and the number of repetitions. Such an expression [i.e., $\mu = \mu(N, \varepsilon_d)$ or $M_r = M_r(N, \varepsilon_d)$] provides a powerful tool for analysis of M_r in pavement design, especially when information concerning deformation and repetitions is known. Furthermore, due to the feature of the stress-strain-time relation, the door is open for further research in characteristics of fatigue and creeping for subgrade materials.

4. The effect of shear stress level is introduced and discussed in this paper. Shear stress affects elastic nonlinearity. Results indicate that soil samples subjected to a lower shear stress level represent less nonlinearity than those subjected to a higher shear stress level. In other words, samples subjected to a higher shear stress level produce a larger deformation that reduces resilient modulus, as shown in Equation 13.

5. Finally, it should be pointed out that the research results from this investigation not only can be applied to designs of pavement roadbed, but also can be used to other projects in highway

systems. For example, nonlinear behavior of soil plays a role in bridge foundations or slopes in highway systems.

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Table 1

Physical Properties of Subgrade Samples

Sample Name	Optimum Water Content (%)	$\gamma_{soil\ wet}$ (10^{-3} kN/m^3)	Plastic Limit (%)	Liquid Limit (%)	Classification (AASHTO)
A8-4	32.4	18.52	29.0	77.0	A-7-5(32)
A8-5	32.4	18.52	29.0	77.0	A-7-5(32)
A8-7	31.2	18.42	29.0	77.0	A-7-5(32)
A8-8	29.5	18.42	29.0	77.0	A-7-5(32)
HA-7	14.0	16.66	5.0	30.0	A4
HA-8	14.0	16.66	5.0	30.0	A4

Table 2

Test Conditions

Sample No.	Cycles No. in Sequence	Load Time (s)	Cycle Time (s)	Sitting Load (kPa)	Loading Wave Type
A8-4	100	0.1	1.0	6.9	Havesine
A8-5	700	0.1	1.0	6.9	Havesine
HA-7	150	0.1	1.0	6.9	Havesine
HA-8	20	0.1	1.0	6.9	Havesine

Table 3

Parameter k_1

Test No.	k_1	$k_2 N_i^{k3}$(kPa)	N_i	R^2
A8-4	-0.41	392	100	0.99
A8-5	-0.48	323	700	0.98
HA-7	-0.14	358	150	0.99
HA-8	-0.071	572	20	0.99

Table 4

Parameters k_2 and k_3

Sample Group	k_2 (kPa)	k_3	R^2
A8	626.8	-0.101	0.99
HA	580.6	-0.244	0.99

Figure 1

Deviatoric stress versus recoverable strain

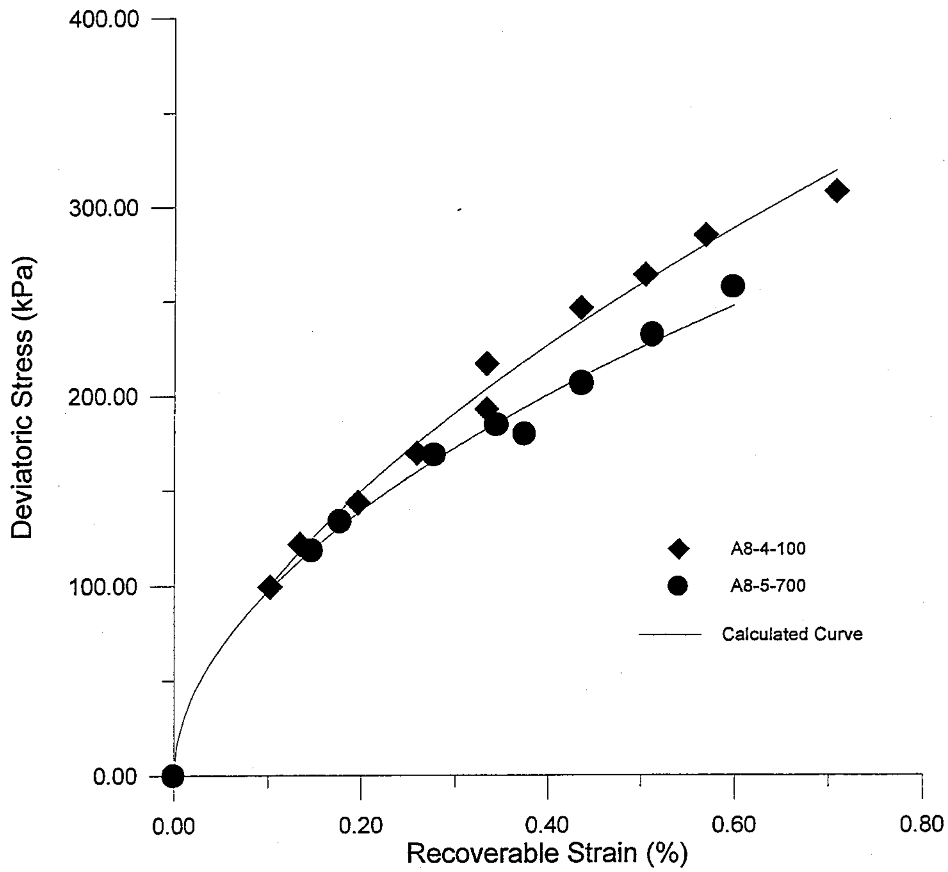
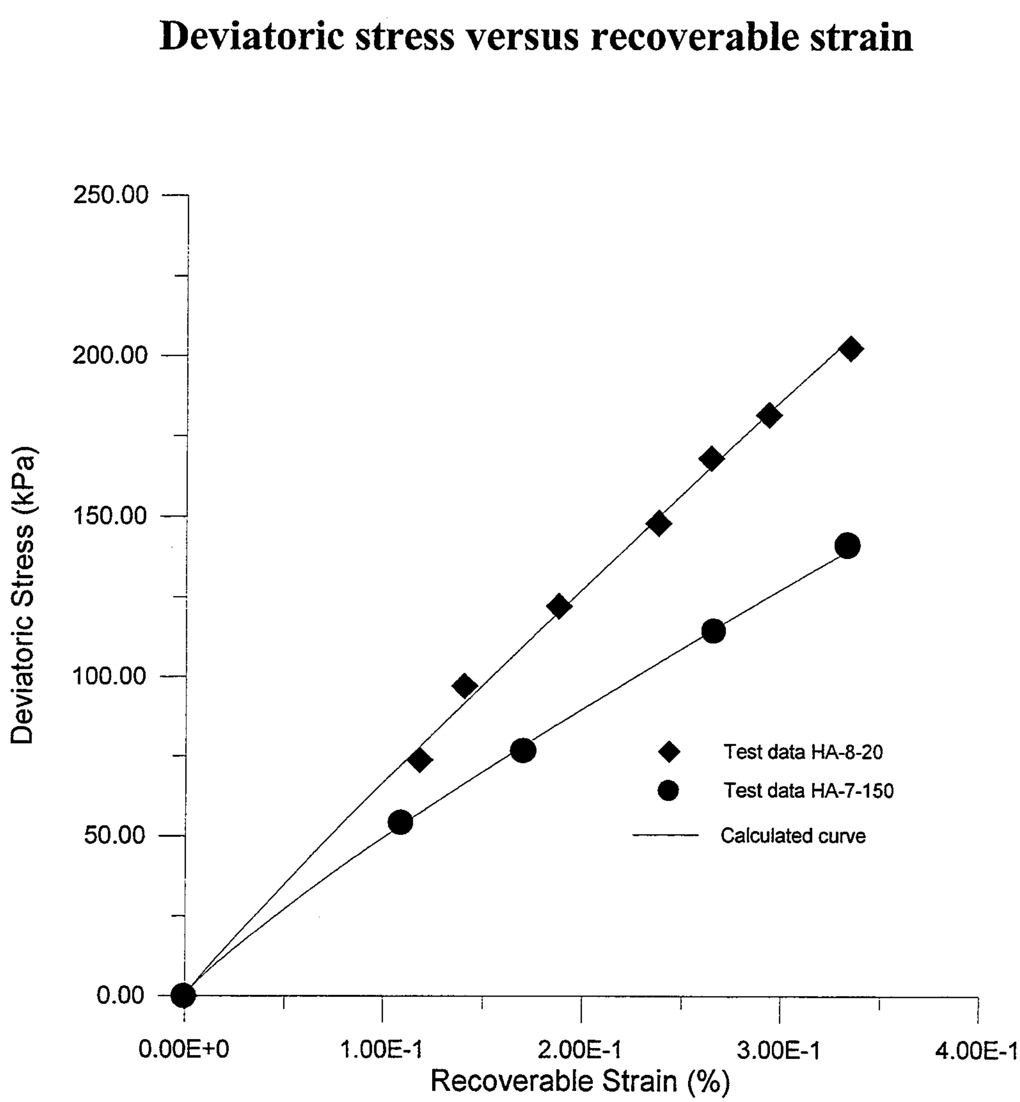


Figure 2



Deviatoric stress versus recoverable strain
250.00
200.00
150.00
100.00
50.00
0.00
Deviatoric Stress (kPa)
0.00E+0
1.00E-1
2.00E-1
3.00E-1
4.00E-1
Recoverable Strain (%)
Test data HA-8-20
Test data HA-7-150
Calculated curve

Figure 3

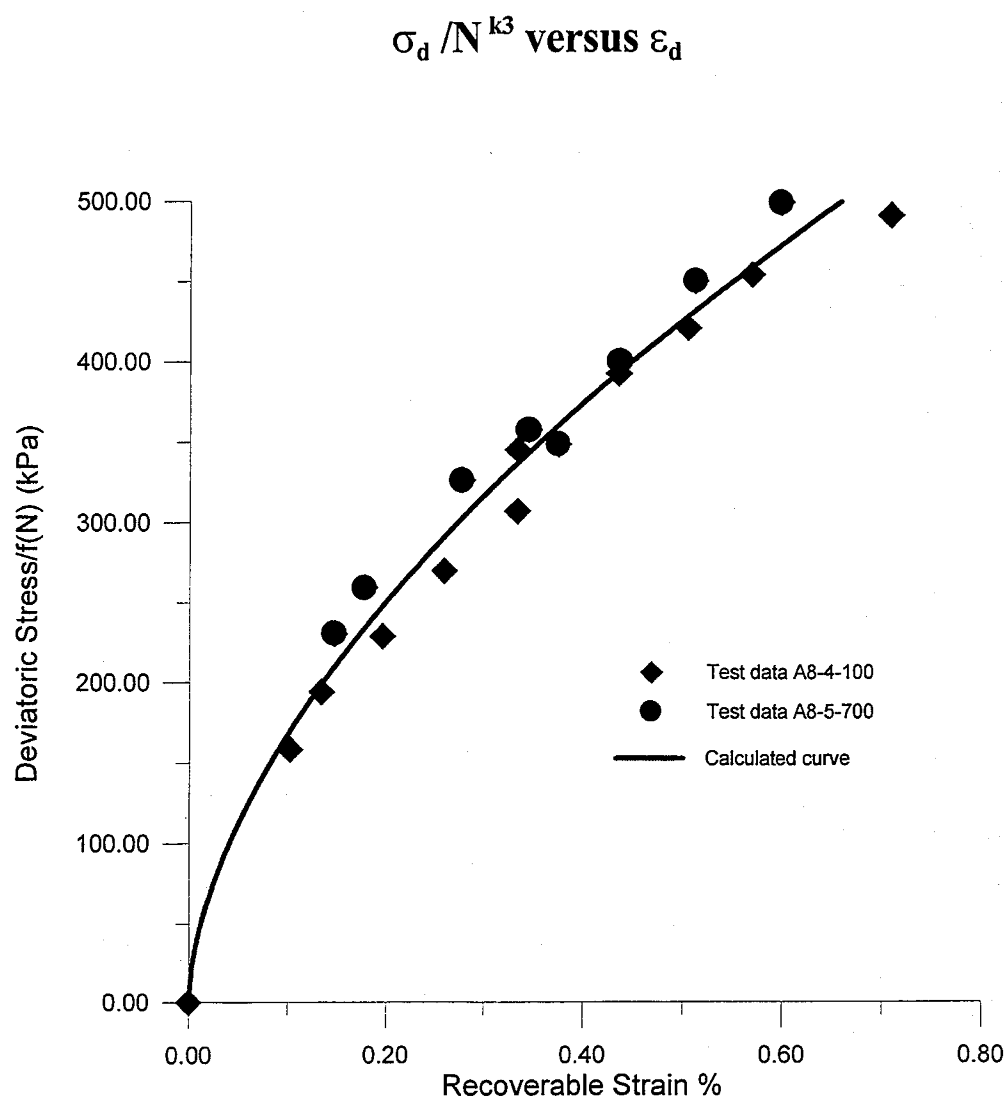
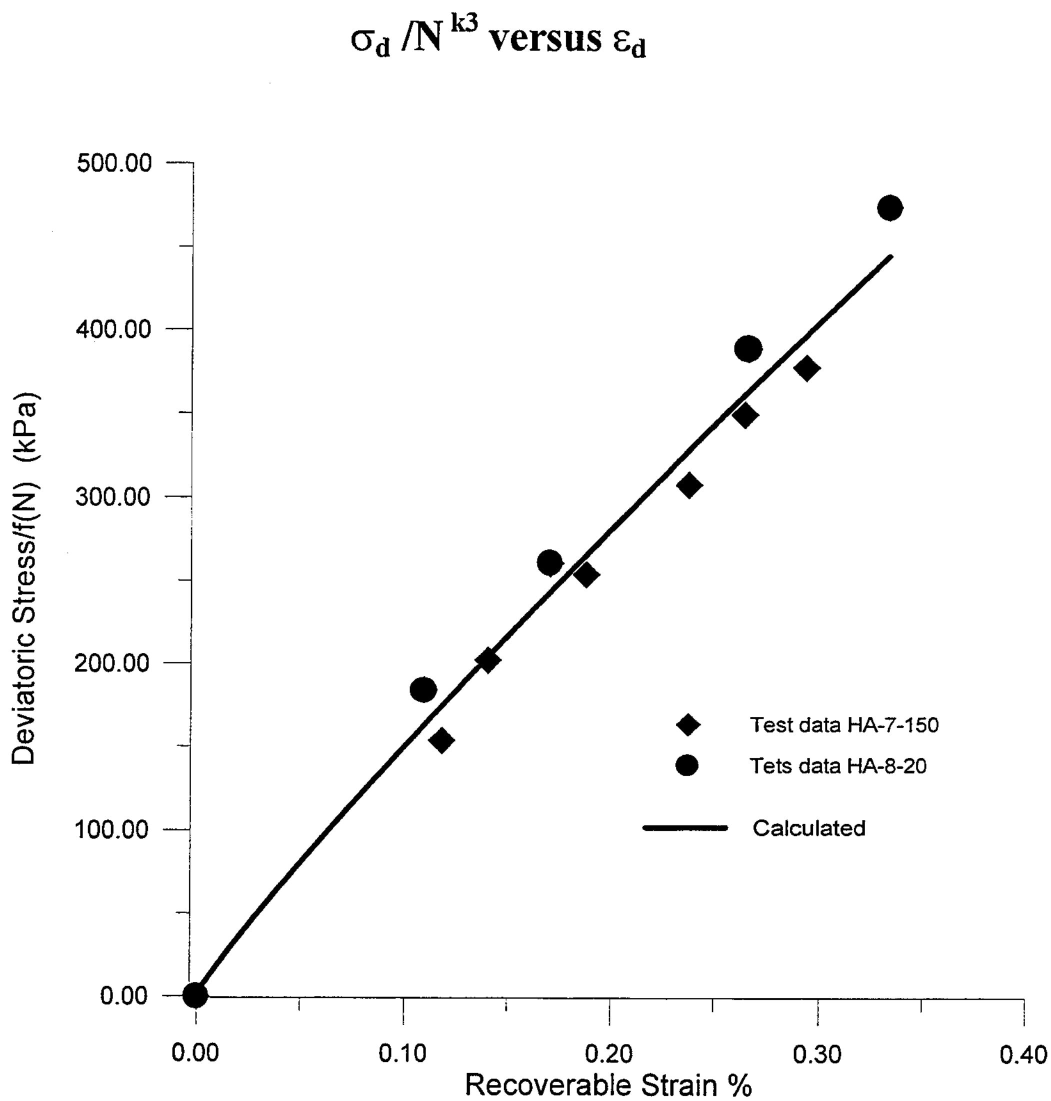

σ_d /N ^{k3} versus ε_d
500.00
400.00
300.00
200.00
100.00
0.00
Deviatoric Stress/f(N) (kPa)
0.00
0.20
0.40
0.60
0.80
Recoverable Strain %
Test data A8-4-100
Test data A8-5-700
Calculated curve

Figure 4



σ_d / N^{k3} versus ε_d
500.00
400.00
300.00
200.00
100.00
0.00
Deviatoric Stress/f(N) (kPa)
0.00
0.10
0.20
0.30
0.40
Recoverable Strain %
Test data HA-7-150
Tets data HA-8-20
Calculated

Figure 5

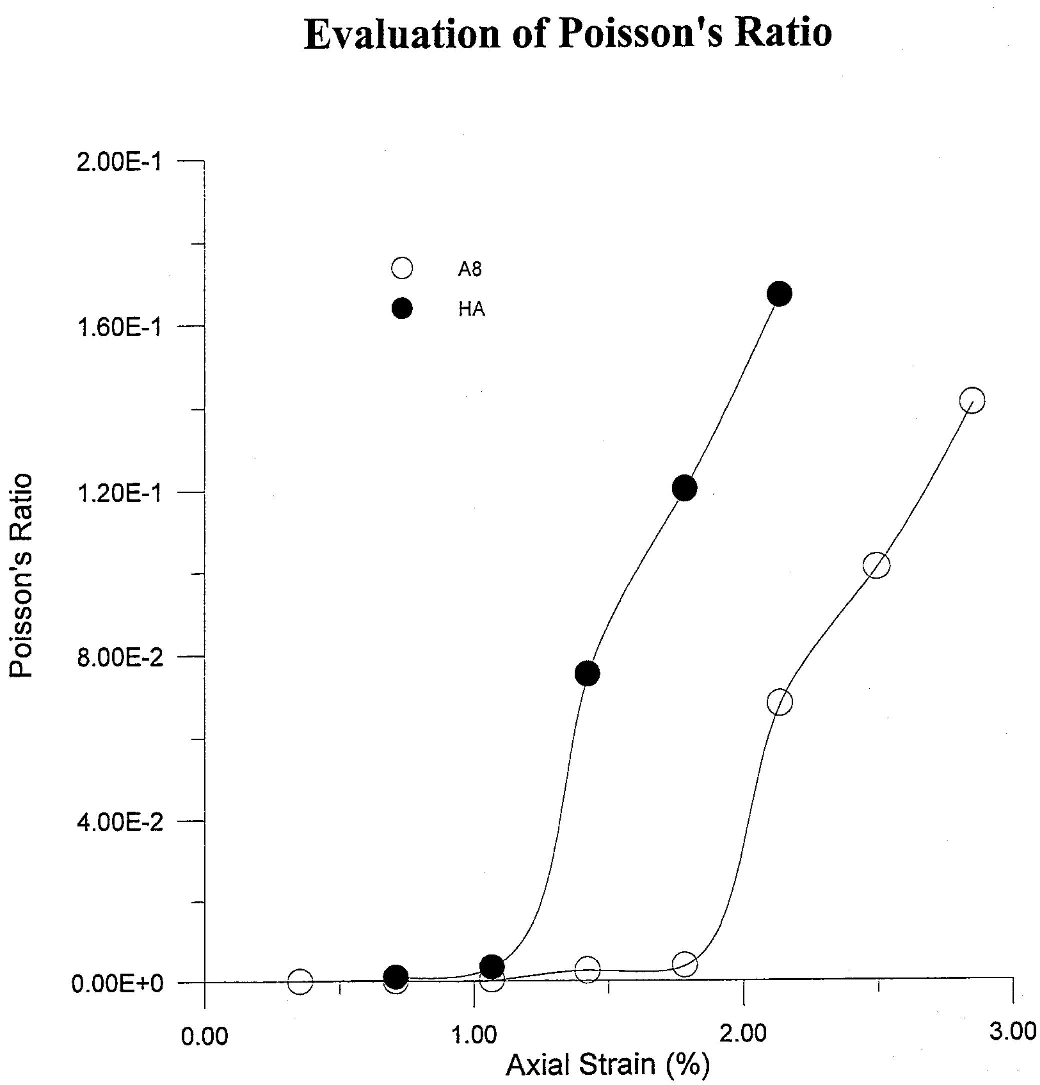

Evaluation of Poisson's Ratio
2.00E-1
1.60E-1
1.20E-1
8.00E-2
4.00E-2
0.00E+0
Poisson's Ratio
A8
HA
0.00
1.00
2.00
3.00
Axial Strain (%)

Figure 6

Relations of M_r - ε_d - N for Sample Group A8

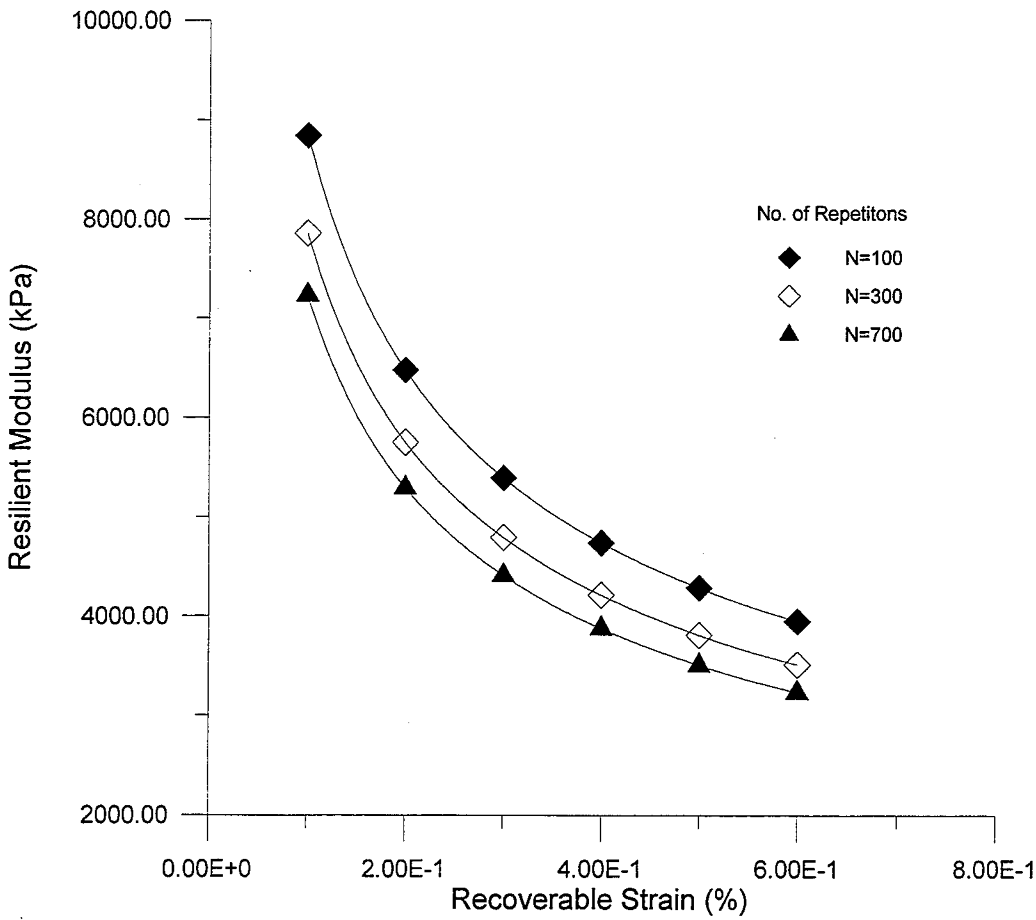


Figure 7

Relations of M_r - ε_d - N for Sample Group HA

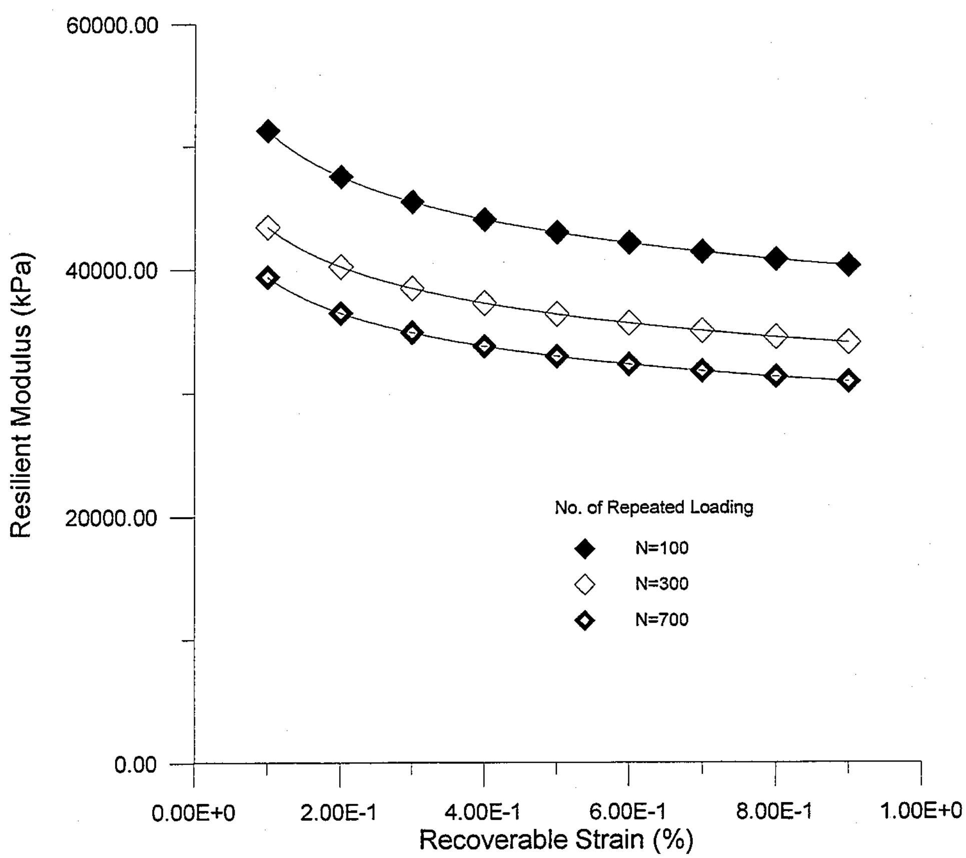


Figure 8

Deviatoric stress versus axial strain

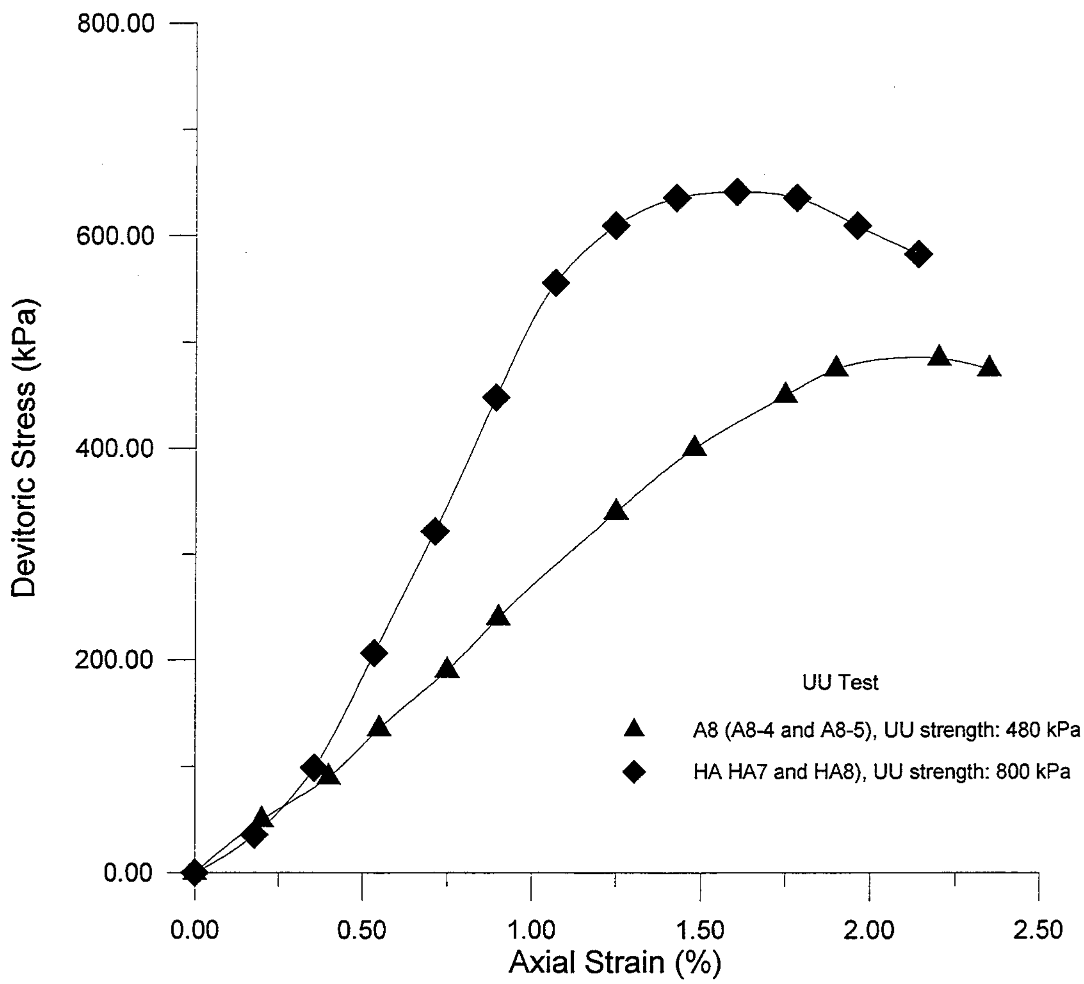
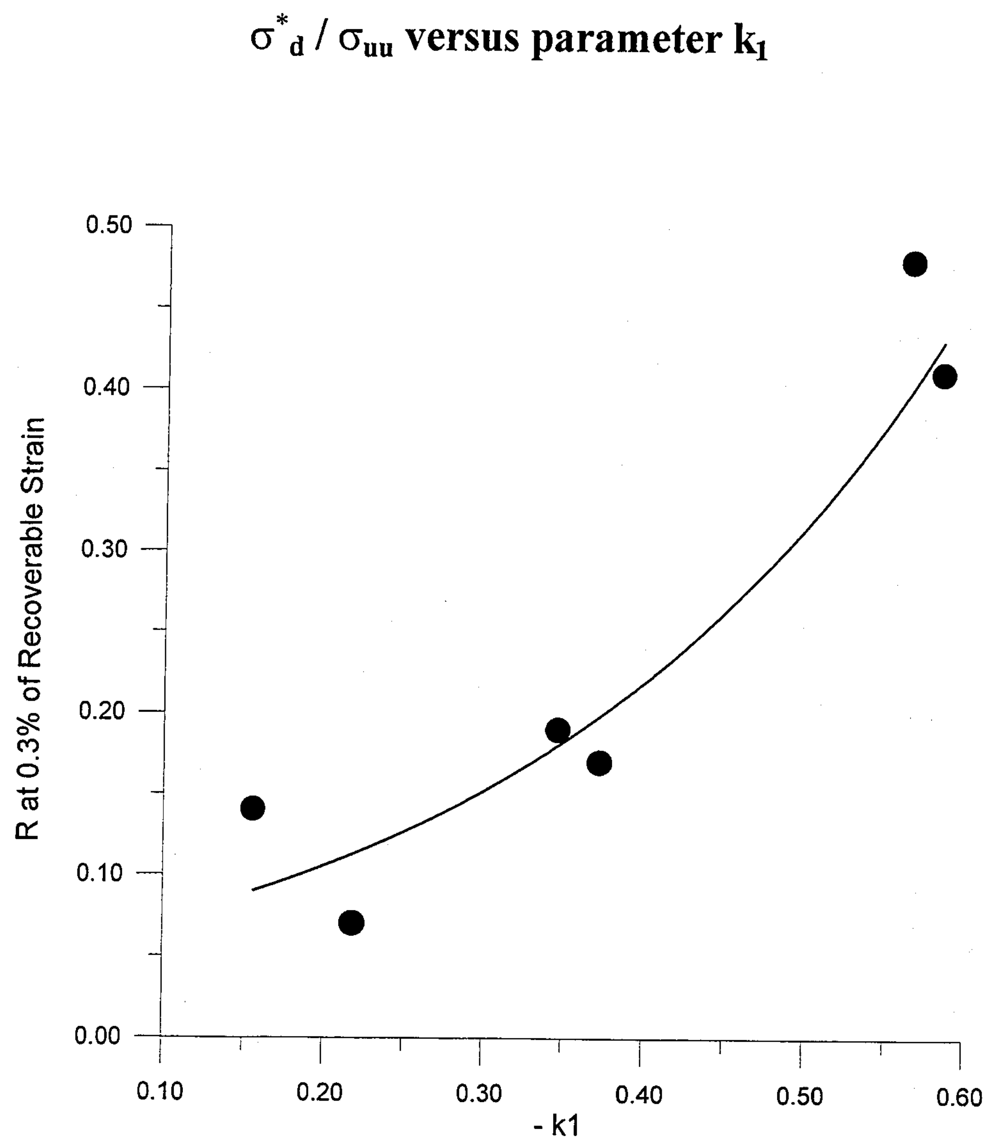


Figure 9


σ^*_d / σ_{uu} versus parameter k_1
0.50
0.40
0.30
0.20
0.10
0.00
R at 0.3% of Recoverable Strain
0.10
0.20
0.30
0.40
0.50
0.60
- k1