



PB99-150070

**DOT/FAA/AR-99/14**

Office of Aviation Research  
Washington, D.C. 20591

# **An Evaluation of Methods to Separate Maneuver and Gust Load Factors From Measured Acceleration Time Histories**

April 1999

Final Report

This document is available to the U.S. public  
through the National Technical Information  
Service (NTIS), Springfield, Virginia 22161.



REPRODUCED BY: **NTIS**  
U.S. Department of Commerce  
National Technical Information Service  
Springfield, Virginia 22161

**U.S. Department of Transportation  
Federal Aviation Administration**

## **NOTICE**

This document is disseminated under the sponsorship of the U.S. Department of Transportation in the interest of information exchange. The United States Government assumes no liability for the contents or use thereof. The United States Government does not endorse products or manufacturers. Trade or manufacturer's names appear herein solely because they are considered essential to the objective of this report. This document does not constitute FAA certification policy. Consult your local FAA aircraft certification office as to its use.

This report is available at the Federal Aviation Administration William J. Hughes Technical Center's Full-Text Technical Reports page: [www.tc.faa.gov/its/act141/reportpage.html](http://www.tc.faa.gov/its/act141/reportpage.html) in Adobe Acrobat portable document format (PDF).

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |                                                      |                                                                                                                                                                |                  |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------|
| 1. Report No.<br>DOT/FAA/AR-99/14                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | 2. Government Accession No.                          | 3. Recipient's Catalog No.                                                                                                                                     |                  |
| 4. Title and Subtitle<br><br>AN EVALUATION OF METHODS TO SEPARATE MANEUVER AND GUST<br>LOAD FACTORS FROM MEASURED ACCELERATION TIME HISTORIES                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                      | 5. Report Date<br>April 1999                                                                                                                                   |                  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |                                                      | <br>PB99-150070<br>URD-TM-1998-00011                                        |                  |
| 7. Author(s)<br>John W. Rustenburg, Donald Skinn, and Daniel O. Tipps                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                      | 10. Work Unit No. (TRAIS)<br>RPD-510                                                                                                                           |                  |
| 9. Performing Organization Name and Address<br>University of Dayton Research Institute<br>Structural Integrity Division<br>300 College Park<br>Dayton, OH 45469-0120                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                      | 11. Contract or Grant No.<br>437-25-14                                                                                                                         |                  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |                                                      | 13. Type of Report and Period Covered<br>Final Report                                                                                                          |                  |
| 12. Sponsoring Agency Name and Address<br>U.S. Department of Transportation<br>Federal Aviation Administration<br>Office of Aviation Research<br>Washington, DC 20591                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                      | 14. Sponsoring Agency Code<br>ANM-100                                                                                                                          |                  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |                                                      |                                                                                                                                                                |                  |
| 15. Supplementary Notes<br>The Federal Aviation Administration William J. Hughes Technical Center COTR is Thomas DeFiore.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                      |                                                                                                                                                                |                  |
| 16. Abstract<br><br>The University of Dayton is supporting Federal Aviation Administration (FAA) research on the structural integrity requirements for the US commercial transport airplane fleet. The primary objective of this research is to support the FAA Airborne Data Monitoring Systems program by developing new and improved methods and criteria for processing and presenting large commercial transport airplane flight and ground loads usage data. The accelerations recorded in flight result from maneuver inputs initiated by the pilot and atmospheric turbulence. To determine the gust and maneuver load factor spectra from the recorded flight loads data, it is necessary to separate the gust and maneuver load factors. Various means to separate the accelerations due to pilot maneuvers and turbulence from measured acceleration time histories have been used. This report presents the results of a study to evaluate the validity and operational processing efficiency of three different methods for the separation of maneuvers and gusts from measured acceleration data obtained from Optical Quick Access Recorder (OQAR)-equipped commercial aircraft. Conclusions and recommendations for the use of a maneuver-gust separation method are also provided. |                                                      |                                                                                                                                                                |                  |
| 17. Key Words<br>Optical quick access recorder, Maneuver accelerations, Gust accelerations, Maneuver-gust separation criteria                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                      | 18. Distribution Statement<br>This document is available to the public through the National Technical Information Service (NTIS), Springfield, Virginia 22161. |                  |
| 19. Security Classif. (of this report)<br>Unclassified                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | 20. Security Classif. (of this page)<br>Unclassified | 21. No. of Pages<br>34                                                                                                                                         | 22. Price<br>N/A |



## PREFACE

This report presents the results from a study conducted to evaluate the validity of methods for the separation of maneuver and gust accelerations from measured flight load data. The Flight Systems Integrity Group of the Structural Integrity Division of the University of Dayton Research Institute (UDRI) performed this study under Contract 437-25-14 with the Iowa State University to support the FAA Airworthiness Assurance Center of Excellence. The Program Manager for the FAA was Mr. Thomas DeFiore of the FAA William J. Hughes Technical Center at Atlantic City International Airport, New Jersey. For UDRI, Mr. Daniel Tipps provided oversight direction for this study effort and coordination with the FAA. Mr. John Rustenburg directed the data processing effort, performed the data analysis, and prepared the report. Mr. Donald Skinn established data reduction criteria, developed the data reduction algorithms, and performed the data reduction.

PROTECTED UNDER INTERNATIONAL COPYRIGHT  
ALL RIGHTS RESERVED.  
NATIONAL TECHNICAL INFORMATION SERVICE  
U.S. DEPARTMENT OF COMMERCE



## TABLE OF CONTENTS

|                                                                 | Page |
|-----------------------------------------------------------------|------|
| EXECUTIVE SUMMARY                                               | ix   |
| 1. INTRODUCTION                                                 | 1    |
| 2. MANEUVER-GUST SEPARATION CRITERIA                            | 1    |
| 3. CRITERIA ANALYSIS AND EVALUATION                             | 2    |
| 3.1 Cycle Duration Maneuver-Gust Separation Criteria            | 2    |
| 3.2 FAA/NASA Maneuver-Gust Separation Criteria                  | 9    |
| 3.3 National Space Laboratory Maneuver-Gust Separation Criteria | 13   |
| 4. CONCLUSIONS AND RECOMMENDATIONS                              | 19   |
| 5. REFERENCES                                                   | 21   |
| Appendix—Maneuver Load Factor Calculations                      |      |

## LIST OF FIGURES

| Figure |                                                                                        | Page |
|--------|----------------------------------------------------------------------------------------|------|
| 1      | Application of 2-Second Rule for Maneuver-Gust Separation                              | 3    |
| 2      | Cumulative Occurrences of Vertical Load Factor per 1000 Hours                          | 4    |
| 3      | Altitude vs Characteristic Frequency, $N(0)$                                           | 6    |
| 4      | Range of Peak Cycle Durations for Two Aircraft                                         | 6    |
| 5      | Frequency Distribution of Gust Load Factors                                            | 7    |
| 6      | Frequency Distribution of Maneuver Load Factors                                        | 8    |
| 7      | Cumulative Frequency of Vertical Gust Acceleration, 2-Second Cycle Duration Criteria   | 8    |
| 8      | Cumulative Frequency of Maneuver Acceleration, 2-Second Cycle Duration Criteria        | 9    |
| 9      | Steady Level Banked Turn                                                               | 9    |
| 10     | Bank Angle and Load Factor Time Histories                                              | 10   |
| 11     | Cumulative Frequency of Gust Accelerations FAA/NASA Criteria                           | 11   |
| 12     | Cumulative Frequency of Maneuver Accelerations FAA/NASA Criteria                       | 12   |
| 13     | Comparison of FAA/NASA Identified Gust Acceleration and Pitching Maneuver Acceleration | 13   |
| 14     | Forces on Airplane During Steady Symmetrical Pull-Up                                   | 14   |
| 15     | Bank and Pitch Angle Time History                                                      | 15   |
| 16     | Acceleration Due to Bank Angle                                                         | 16   |
| 17     | Pitching Maneuver Time Histories                                                       | 17   |
| 18     | Comparison of Measured and Calculated Load Factor                                      | 18   |
| 19     | Derived Gust Acceleration Time History                                                 | 18   |



## LIST OF SYMBOLS

|                |                                                                            |
|----------------|----------------------------------------------------------------------------|
| $\bar{A}$      | aircraft gust response factor                                              |
| $A_R$          | aspect ratio $b^2/S$                                                       |
| $b$            | wing span                                                                  |
| $b_{1,2}$      | gust intensity parameters                                                  |
| $\bar{c}$      | wing mean geometric chord (ft)                                             |
| $C_{L_\alpha}$ | aircraft lift curve slope per radian                                       |
| $g$            | gravity constant, 32.17 ft/sec                                             |
| $k_0$          | reduced zero crossings frequency                                           |
| $L$            | airplane lift                                                              |
| $M$            | mach number                                                                |
| $n$            | load factor (g)                                                            |
| $N_y$          | average number of peaks per unit time exceeding a given response level $y$ |
| $N_0$          | average number of zero crossings per unit time with positive slope         |
| $P_{1,2}$      | proportion of time in turbulence                                           |
| $R$            | turning radius                                                             |
| $S$            | wing area (ft <sup>2</sup> )                                               |
| $U_{de}$       | derived gust velocity (ft/sec)                                             |
| $V$            | true airspeed                                                              |
| $W$            | gross weight (lbs)                                                         |
| $y$            | general response quantity                                                  |
| $\Lambda$      | sweep angle of quarter chord                                               |
| $\mu$          | airplane mass ratio, $\frac{2(W/S)}{\rho g \bar{c} C_{L_\alpha}}$          |
| $\rho$         | air density, slugs/ft <sup>3</sup> (at altitude)                           |
| $\rho_0$       | standard sea level air density, 0.0023769 slugs/ft <sup>3</sup>            |
| $\phi$         | bank angle                                                                 |
| $\gamma$       | flight path angle                                                          |
| $\theta$       | pitch angle                                                                |



## EXECUTIVE SUMMARY

The University of Dayton Research Institute (UDRI) is supporting Federal Aviation Administration (FAA) research on the structural integrity requirements for the US commercial transport airplane fleet. The primary objective of UDRI research is to support the FAA Airborne Data Monitoring Systems program by developing new and improved methods and criteria for processing and presenting large commercial transport airplane flight and ground loads usage data. The scope of activities involves (1) providing data processing and analysis support for airline commercial aircraft equipped with Optical Quick Access Recorder (OQAR) recorders, (2) interfacing with aircraft manufacturers and participating airlines to upgrade/update data presentation formats to be consistent with FAA policy and industry requirements, (3) developing algorithms and modifying data processing software to support additional aircraft types and new data formats, (4) researching and implementing new specialized data reduction and analysis procedures, and (5) researching and developing technology to support airline maintenance management and system health monitoring. This report presents the results of a study to evaluate the validity and operational processing efficiency of different methods for the separation of maneuvers and gusts from measured acceleration data obtained from Optical Quick Reference Access Recorder (OQAR)-equipped commercial aircraft. Conclusions and recommendations for the use of a maneuver-gust separation method are also provided.



## 1. INTRODUCTION.

Durability and damage tolerance analyses of an aircraft structure, either to validate an existing design or to establish the capability for a new design, requires separate maneuver and gust load spectra. Descriptions of these spectra are normally based on accelerations measured on operational aircraft during flight. However, the accelerations recorded in flight result from both maneuver inputs initiated by the pilot and atmospheric turbulence. To determine the gust and maneuver load factor spectra from recorded flight loads data, it is first necessary to separate the gust and maneuver load factors. Different methods to separate accelerations due to pilot maneuvers from those due to turbulence have been applied or proposed in the data reduction process by various researchers.

The University of Dayton Research Institute (UDRI) has studied the feasibility and impact of using three different maneuver-gust separation techniques to determine separate maneuver and gust spectra. The first of these methods has a history going back over 30 years, the second method has been used by UDRI for the past several years, and the third method has been proposed for future use. This report discusses the application of these three maneuver-gust separation techniques, the resulting maneuver and gust acceleration spectra, ease of application of each technique for the processing of large amounts of data on a production basis, and provides conclusions and recommendations.

## 2. MANEUVER-GUST SEPARATION CRITERIA.

With the advent of automated data processing, three basic methods have been used or are proposed to accomplish the separation of gust and maneuver accelerations from measured acceleration data. These three methods will be identified, discussed, and evaluated for applicability and ease of use in the processing of acceleration time history data.

The first criterion studied was the cycle duration separation method developed by the Wright Air Development Division (WADD). Under this criterion a 2-second time increment between zero crossings of the load factor trace is used to divide the data into gust and maneuver load factors. This criterion has been used successfully for the separation of gust and maneuver accelerations from measured accelerations obtained on military aircraft for many years. A similar more sophisticated technique employing low-pass and band-pass numerical filters set at cut off frequencies to isolate the gust and maneuver accelerations from measured acceleration time history data has been used by the National Aeronautics and Space Administration (NASA) Digital VGH program. An advantage of frequency filtering to separate gusts and maneuvers is the ability to isolate gusts regardless of whether they occur during a maneuver or not.

The 2-second criterion resulted from early reviews of measured data and studies of aircraft response to elevator motion that determined that for larger aircraft; essentially all of the maneuver load factor peaks can be expected to be counted if a time between zero crossings greater than 2 seconds is used. Thus in this criterion load factor peaks with positive or negative zero crossings less than 2 seconds, in other words, load factor peaks with time durations of less than 2 seconds are identified as gusts. The remaining load factor peaks with positive or negative zero crossings greater than 2 seconds or peak cycle durations greater than 2 seconds are assumed to be due to symmetric maneuvers. Studies were conducted to evaluate the applicability of the 2-

second time between zero crossings criterion based on gust response characteristics of modern commercial aircraft.

The second criteria studied were the joint Federal Aviation Administration and National Aeronautics and Space Administration (FAA/NASA) maneuver-gust separation criteria used by the University of Dayton Research Institute (UDRI) for identifying maneuver accelerations in measured acceleration time histories for the B-737 and MD-82/83 aircraft. These criteria were defined in the FAA/NASA Flight Loads Program System Requirements of reference 1. These criteria assume that all primary maneuver accelerations are the result of bank angle changes only and that all other accelerations are due to gusts. Thus, maneuver accelerations associated with pitch changes during takeoff to climb, climb to cruise, and approach to touchdown are not considered as maneuvers but are included in the gust accelerations. The criterion for identification as a maneuver acceleration peak is an acceleration associated with a turning maneuver as indicated by a change in heading and bank angle where the maneuver acceleration is determined from  $n_z = 1/\cos\phi$  where  $\phi$  is the bank angle. If the measured acceleration deviates from the calculated maneuver acceleration, the difference is attributed to gusts. Because turning maneuvers always result in positive accelerations no negative maneuver spectra are identified. Studies were conducted to show that acceleration peaks counted as gusts are in fact often the result of pitching maneuvers.

The third criteria studied were identical to that proposed by the National Aerospace Laboratory of the Netherlands (NLR) as defined in reference 2. These criteria specify consideration of maneuver accelerations as resulting from banked turns as well as pitching maneuvers. For banked turns a maneuver acceleration peak is an acceleration associated with a turning maneuver as indicated by a change in heading and bank angle and identical to the criteria discussed above. The criteria also specify that accelerations associated with pitching maneuvers at specific points in flight such as takeoff to initial climb, climb to cruise, and approach to touchdown are to be counted. Thus, application of these criteria provides both positive and negative maneuver spectra. However, reference 2 does not propose specific methods for identifying pitching maneuver-induced accelerations. Under the criteria, after accounting for accelerations resulting from turning and pitching maneuvers, all remaining acceleration peaks are assumed to be gust induced. Studies were conducted to evaluate the feasibility of deriving algorithms for the identification of pitching maneuver accelerations.

### 3. CRITERIA ANALYSIS AND EVALUATION.

#### 3.1 CYCLE DURATION MANEUVER-GUST SEPARATION CRITERION.

As discussed in section 2; for this criterion, the time between zero crossings of the load factor trace is used to divide the accelerations into maneuver and gust accelerations. Load factor peaks with zero crossings less than 2 seconds of time between successive peaks, i.e., frequencies higher than 0.5 cycle per second, are considered to be gust accelerations. The remaining load factor peaks with zero crossings greater than 2 seconds, i.e., frequencies slower than 0.5 cycle per second, are assumed to be due to symmetric maneuvers. Figure 1 presents an acceleration trace showing application of this criterion in defining maneuver and gust accelerations.

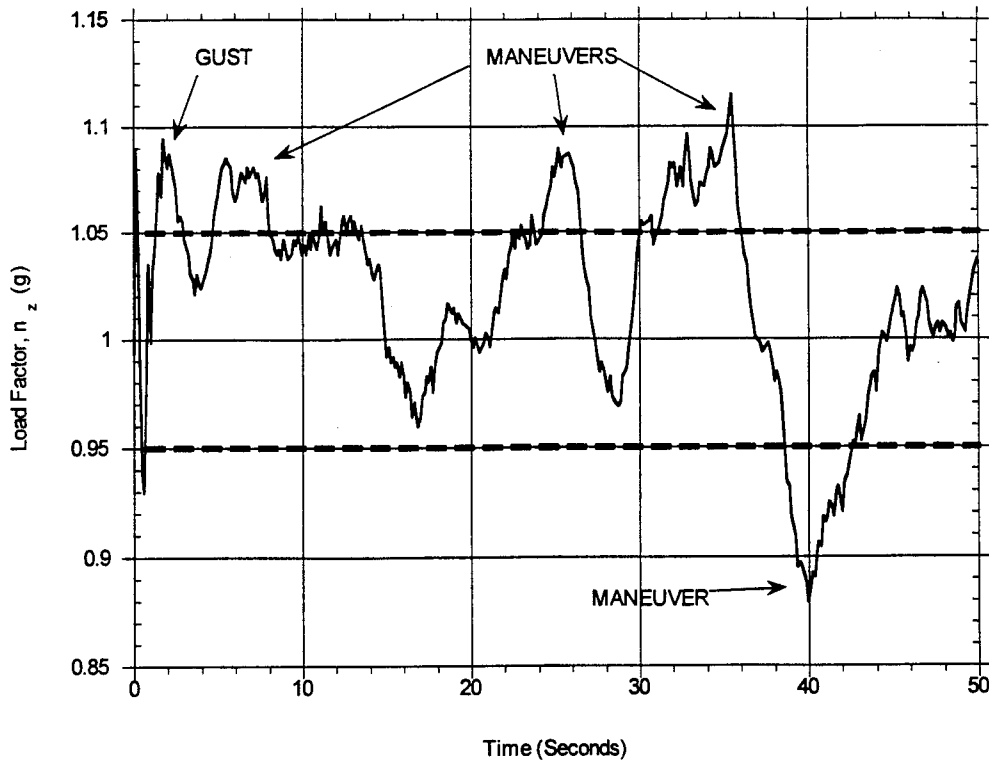


FIGURE 1. APPLICATION OF 2-SECOND RULE FOR MANEUVER-GUST SEPARATION

The time between zero crossings of the load factor due to gusts is influenced by the longitudinal response characteristics of the airplane. Without knowledge of the aerodynamic, structural, and flight control characteristics of the airplane these response characteristics cannot be evaluated analytically. However, they can be estimated indirectly from measured acceleration data and the validity of the 2-second rule can be evaluated.

Figure 2 presents the cumulative occurrences spectrum of measured accelerations. A theoretical representation of this spectrum can be derived by using the "Rice Equation" in reference 3. Using Rice's equation the statistical description of airplane response peak counts to atmospheric turbulence can be described by the following general equation.

$$N(\Delta n) = N_0 \left( P_1 e^{\frac{\Delta n}{\bar{A} b_1}} + P_2 e^{\frac{\Delta n}{\bar{A} b_2}} \right) \quad (1)$$

In equation 1 the response is described as resulting from two independent turbulence inputs. In the equation,  $\bar{A}$  reflects the airplane gust response sensitivity and  $b_1$  and  $b_2$  reflect gust intensity levels.  $P_1$  and  $P_2$  reflect the proportion of time in the two turbulence levels.  $N(\Delta n)$  describes the number of times per second that the response crosses a given level of acceleration  $\Delta n$ .  $N(0)$  is the number of times per second or frequency that the acceleration response crosses

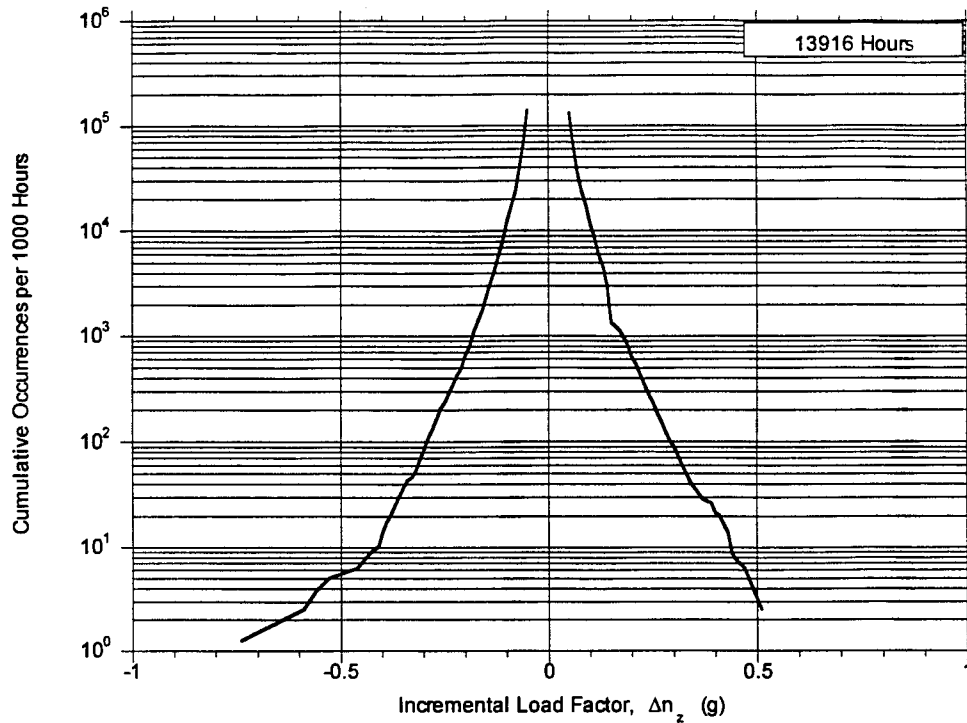


FIGURE 2. CUMULATIVE OCCURRENCES OF VERTICAL LOAD FACTOR PER 1000 HOURS

the mean or 1-g level with positive slope. The value of  $N(0)$  represents the characteristic frequency of the airplane response to turbulence inputs. The characteristic frequency of the acceleration data, and thus of the aircraft response, is defined as the extrapolated value at the mean or in the case of figure 2 the zero acceleration level.

The extrapolation used an exponential fit to the first two consecutive points of the spectra presented in figure 2. Extrapolation of the positive and negative data presented in figure 2 resulted in an average  $N(0)$  value of 1.21 cycles per second and a cycle duration of 0.82 second.

The value of  $N(0)$  can also be calculated directly. Unfortunately, the detailed calculation of  $N(0)$  is quite complex and again requires a detailed knowledge of the aircraft aerodynamic, flight control, and structural characteristics. However, a simple general expression for  $N_0$  was derived by Houbolt in reference 4 where

$$N(0) = \frac{V}{\pi c} * k_0 \quad (2)$$

$$k_0 = 0.496(\mu_g)^{-0.46} \quad (3)$$

and  $N(0)$  is in crossings per second.



$$\mu = \frac{2W}{\rho g \bar{c} C_{l_\alpha} S} \quad (4)$$

- $C_{l_\alpha}$  = aircraft lift curve slope per radian  
 $S$  = wing reference area (ft<sup>2</sup>)  
 $W$  = gross weight (lbs)  
 $\rho$  = air density, slug/ft<sup>3</sup> at pressure altitude (Hp)  
 $g$  = 32.17 ft/sec<sup>2</sup>  
 $\bar{c}$  = wing mean geometric chord (ft)

For this study the wing lift curve slope was obtained from the one-dimensional UDRI approximation given by

$$C_{l_\alpha} = \frac{2\pi A_r}{2 + \left( 4 + A_r^2 \beta^2 \left( 1 + \frac{\tan^2 \Lambda}{\beta^2} \right) \right)^{1/2}} \quad (5)$$

- $A_r = \frac{b^2}{S}$  = wing aspect ratio  
 $b$  = wing span  
 $\beta = \sqrt{1 - M^2}$   
 $C_{l_\alpha}$  = lift curve slope per radian or per degree  
 $\Lambda$  = quarter chord sweep angle  
 $M$  = Mach Number

Equation 5 provides an estimate of the wing lift curve slope. Airplane gust response calculations are based on the use of the airplane lift curve slope. Reference 5 suggests a factor of 1.15 to represent the average airplane to wing lift curve slope. Therefore, the wing lift curve slope values were multiplied by 1.15. Using the above relationships, the value of  $N(0)$  was calculated for the maximum and minimum speeds in the different phases for a number of flights. The results are shown in figure 3. As can be seen, the values of  $N(0)$  range from approximately 0.45 to 1.2 positive level crossings or cycles per second. In terms of peak length, this equates to from 2.2 seconds duration down to 0.83 second duration.

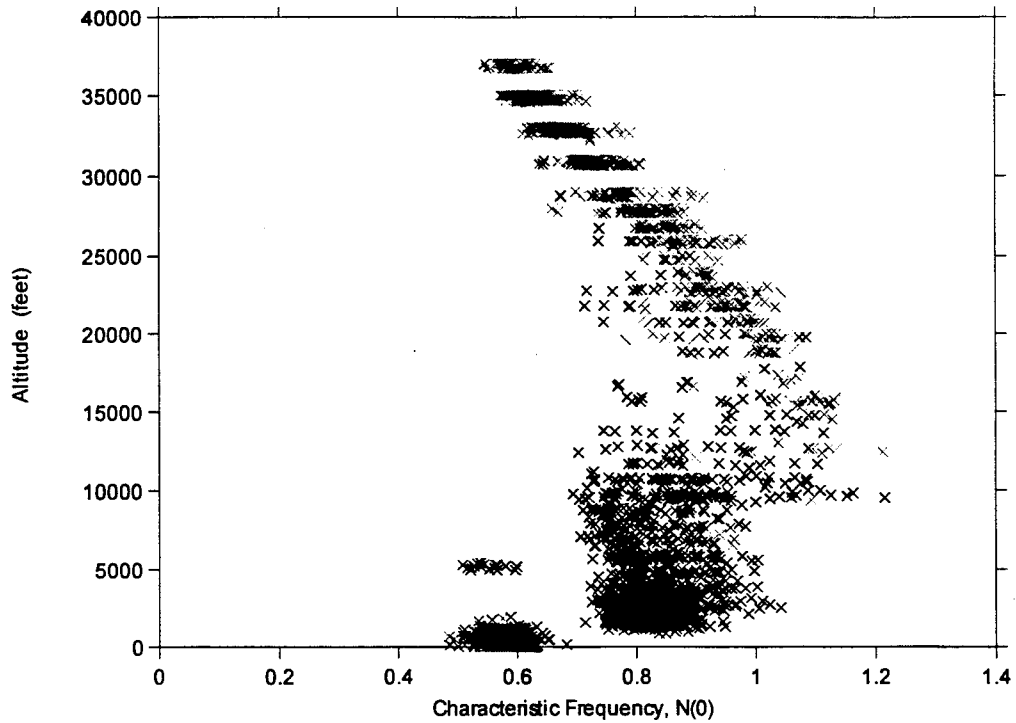


FIGURE 3. ALTITUDE VS CHARACTERISTIC FREQUENCY,  $N(0)$

Finally, a range of  $N(0)$  values was calculated for the B-737-400 and the MD-82/83 aircraft for all practicable gross weights within their flyable speed-altitude envelopes. Figure 4 presents the ranges of  $N(0)$  values for the B-737 and MD-82/83.

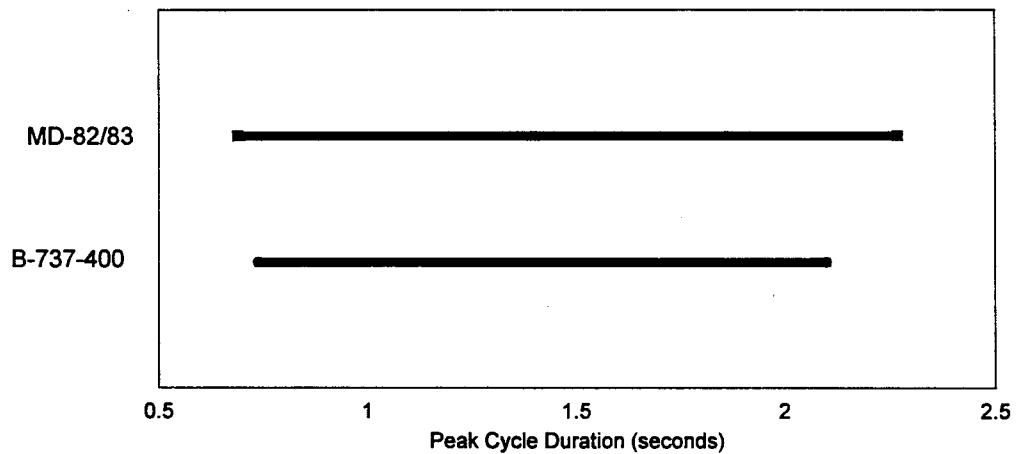


FIGURE 4. RANGE OF PEAK CYCLE DURATIONS FOR TWO AIRCRAFT

The  $N(0)$  ranges presented in figure 4 show that gust acceleration response data might in certain cases exhibit cycle duration greater than 2 seconds. How often this may occur is not known. Review of the  $N(0)$  values in figure 3 suggests that the longer durations may occur at lower altitudes where more turbulence can be expected. To study this effect in more detail, the acceleration data were processed using 2.0, 2.5, 3.0, and 4.0 seconds maneuver-gust separation criteria. Figures 5 and 6 present the results for the gust and maneuver accelerations, respectively, based on the total combined flight phases. As can be seen, the effect of changes in the duration of the peak cycle criteria is not large. However, for cycle durations greater than 2.0 seconds, the maneuver distribution shows a distinct drop-off in occurrences at  $-0.3$  g. Clearly the higher negative load factor pushover maneuvers are routinely longer than 2 seconds in duration and are now counted as gusts. There is no apparent explanation for such a sharp step in the distribution. Negative load factors are generally more uncomfortable to the passenger. It could be surmised that in order provide a more comfortable ride the pilot extends the duration of the negative maneuver rather than executing maneuvers at even higher negative load factors. Considering the points discussed above, it is concluded that a cycle duration criterion of 2.0 seconds for the separation of maneuvers and gusts is still a very viable approach.

Having determined the general applicability of the 2-second cycle duration for the cycle duration criterion, the rule was applied to the existing B-737-400 database. Separation of maneuvers and gusts by application of the 2-second rule to acceleration time histories resulted in the gust and maneuver acceleration spectra presented in figures 7 and 8 respectively. As can be seen, the negative and positive gust acceleration spectra are quite symmetrical, which is normally assumed to be the case.

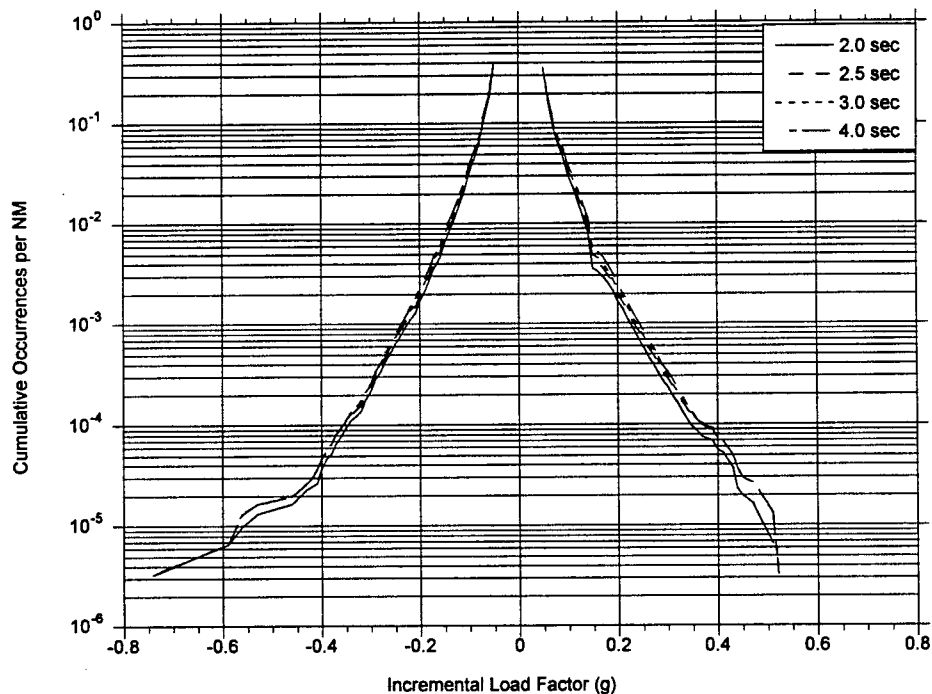


FIGURE 5. FREQUENCY DISTRIBUTION OF GUST LOAD FACTORS

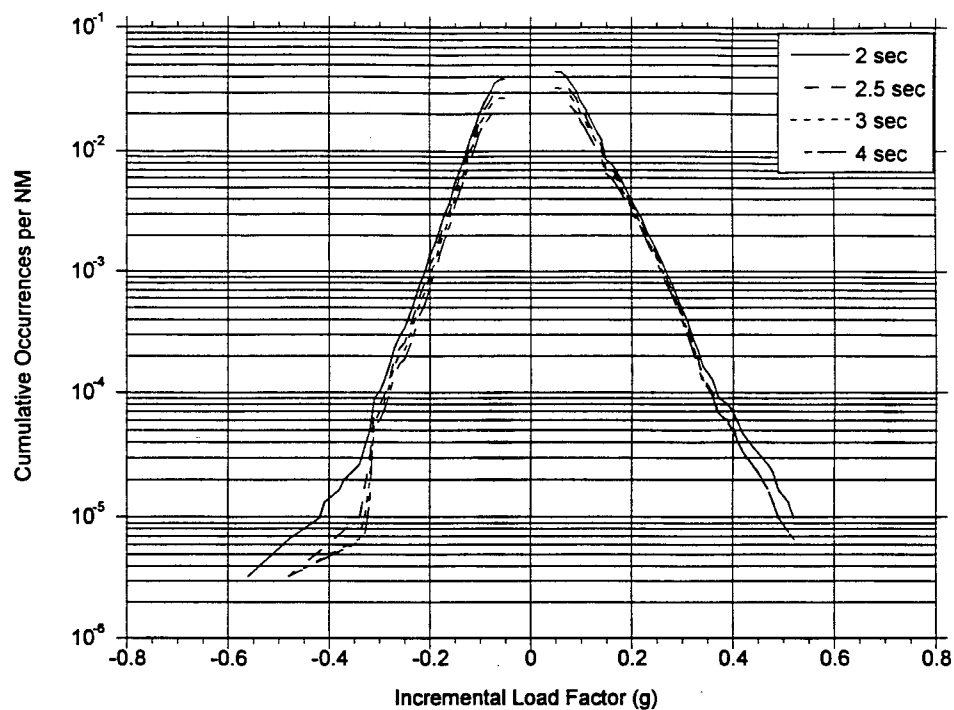


FIGURE 6. FREQUENCY DISTRIBUTION OF MANEUVER LOAD FACTORS

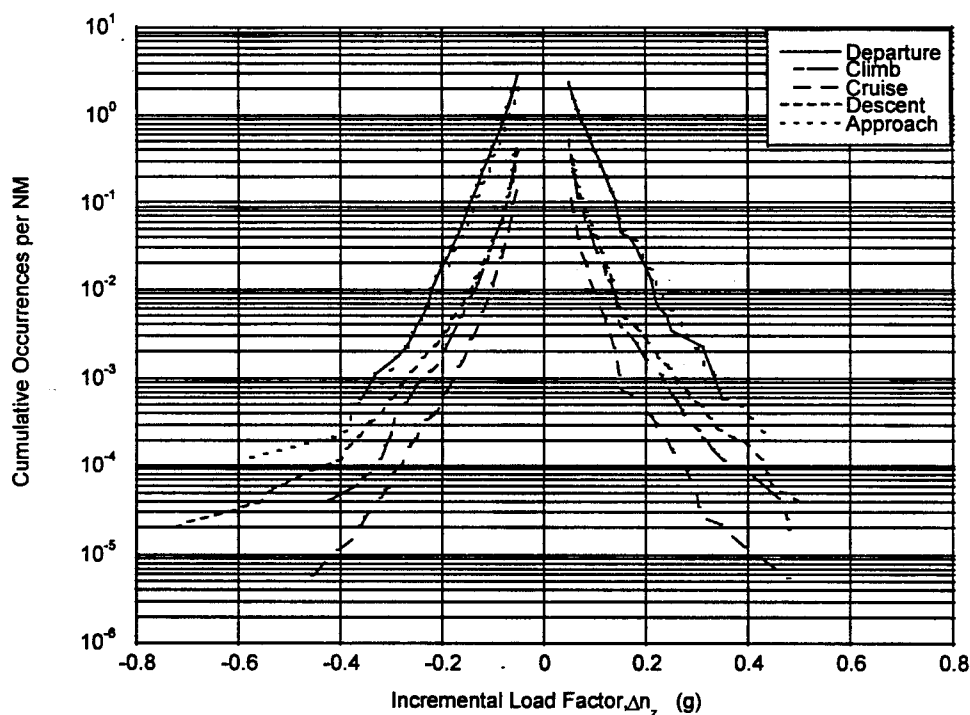


FIGURE 7. CUMULATIVE FREQUENCY OF VERTICAL GUST ACCELERATION, 2-SECOND CYCLE DURATION CRITERIA

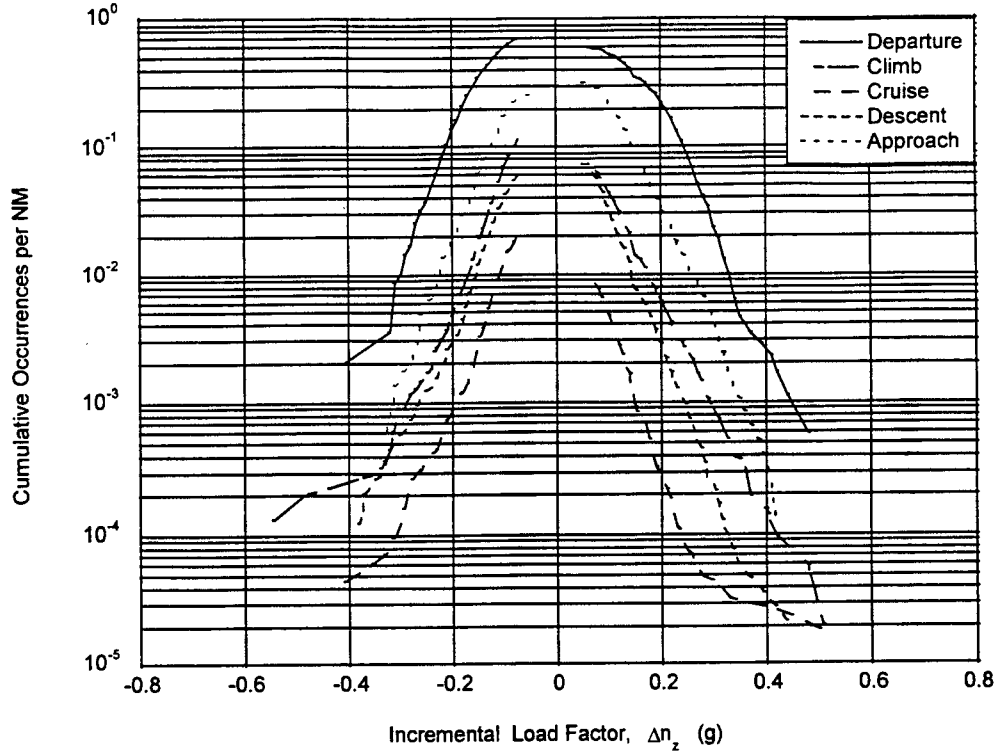


FIGURE 8. CUMULATIVE FREQUENCY OF MANEUVER ACCELERATION, 2-SECOND CYCLE DURATION CRITERIA

### 3.2 FAA/NASA MANEUVER-GUST SEPARATION CRITERIA.

The FAA/NASA criterion identifies a maneuver acceleration peak as any acceleration associated with a turning maneuver as indicated by a change in heading and bank angle. The maneuver acceleration is determined from  $n_z = 1/\cos\phi$ , where  $\phi$  is the bank angle. The acceleration is always positive and has a relatively long duration. The calculated acceleration is based on the assumption that the turning maneuver consists of a steady-state coordinated turn. In a coordinated turn, the airplane is tracing a circle in the horizontal plane. The forces on the airplane in a coordinated turn are as shown in figure 9. At any time the flight path has a radius of curvature  $R$  with a turning rate  $\dot{\phi}$ .

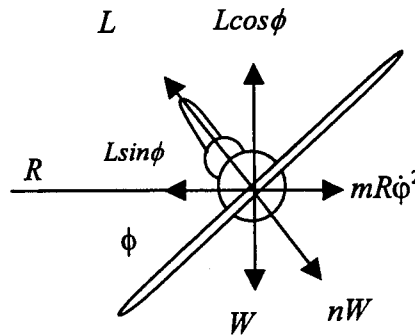


FIGURE 9. STEADY LEVEL BANKED TURN

$$mR\dot{\phi}^2 = mV\dot{\phi} \quad (6)$$

$$V^2 = R^2\dot{\phi}^2 \quad (7)$$

and

$$mR\dot{\phi}^2 = m\frac{V^2}{R} \quad (8)$$

From the forces shown in figure 9, the relationship between bank angle, load factor, turning radius, and turning rate can be derived.

$$n = \frac{L}{W} = \frac{W}{\cos \phi} \frac{1}{W} = \frac{V^2}{gR \sin \phi} = \frac{R\dot{\phi}^2}{g \sin \phi} \quad (9)$$

thus:

$$n = \frac{1}{\cos \phi} \quad (10)$$

In applying the FAA/NASA maneuver-gust separation criteria, the maneuver-induced acceleration is calculated using the above equation. The result is subtracted from the measured acceleration and the remainder is attributed to turbulence. Figure 10 presents a time history of a banked turn. The figure shows the bank angle and measured acceleration from a portion of a flight as well as the calculated maneuver acceleration and the acceleration attributed to gusts.

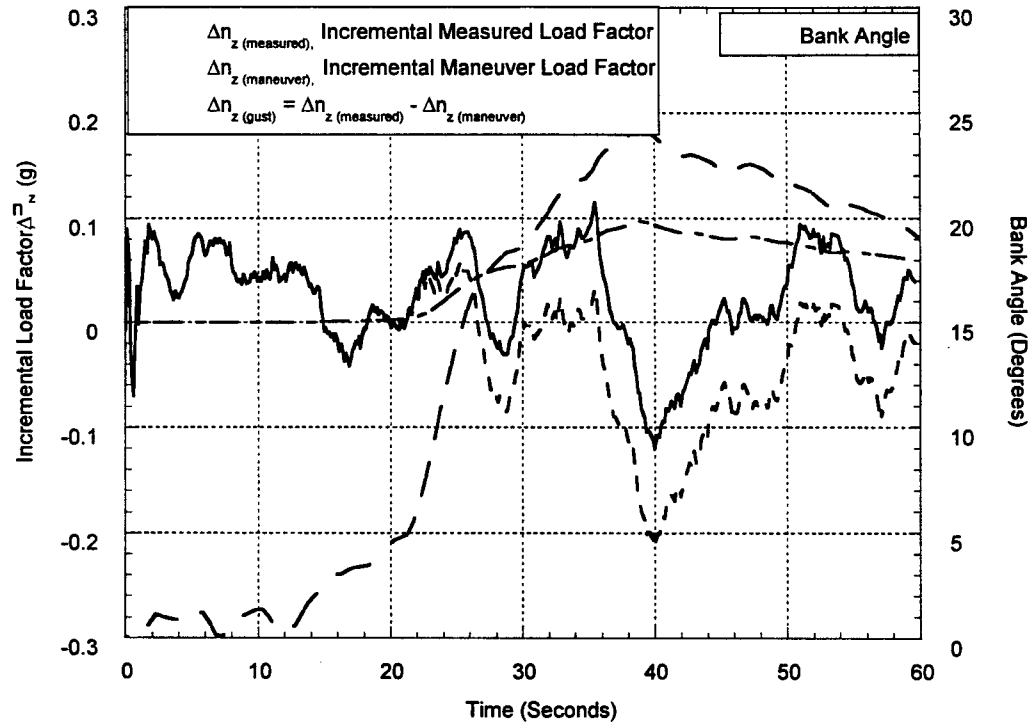


FIGURE 10. BANK ANGLE AND LOAD FACTOR TIME HISTORIES

Figures 11 and 12 present the cumulative distributions of maneuver and gust accelerations derived using this approach. However, there are two concerns when using this approach. The first and most important concern is the fact that the FAA/NASA criterion only considers banked turns as maneuvers. Any accelerations resulting from pitching maneuvers are counted as gust accelerations. As a consequence, the resulting maneuver load factors will be positive only, and the influence of pitch maneuvers that might result in negative maneuver load factors is not considered.

The second and less important concern is that when using the derived gust accelerations in the derivation of gust velocity, the gust velocities are counted as if they occurred in the vertical direction during straight and level flight. This introduces an error in the derived vertical gust velocities equal to the inverse of the cosine of the bank angle. The impact of this on the frequency distributions of vertical gusts has not been determined, but because of the relatively small bank angles normally flown in civil aviation flights, it is expected to be insignificant.

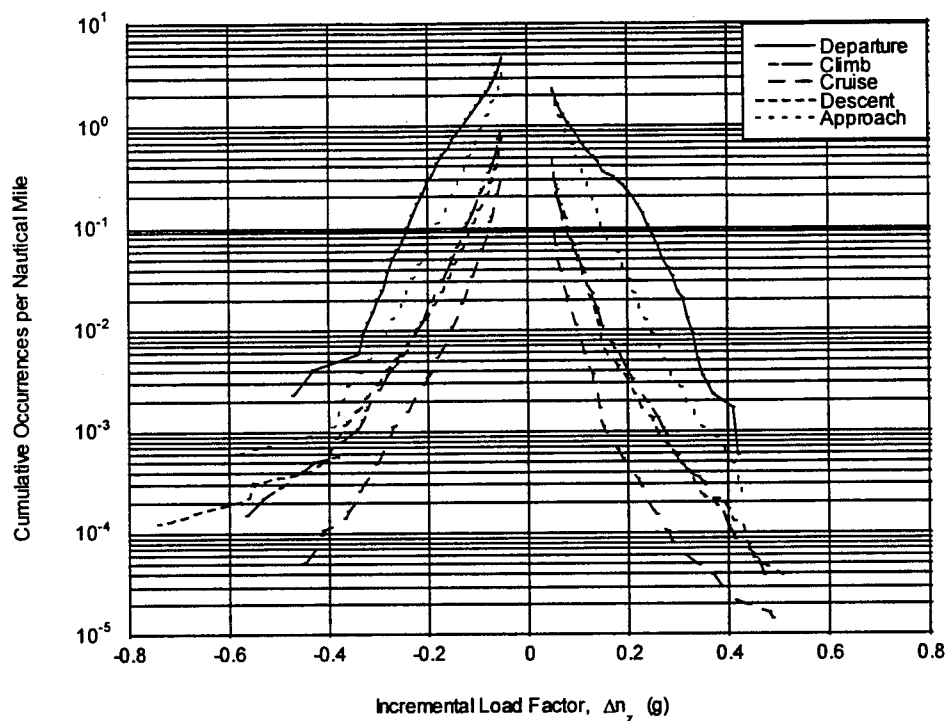


FIGURE 11. CUMULATIVE FREQUENCY OF GUST ACCELERATIONS  
FAA/NASA CRITERIA

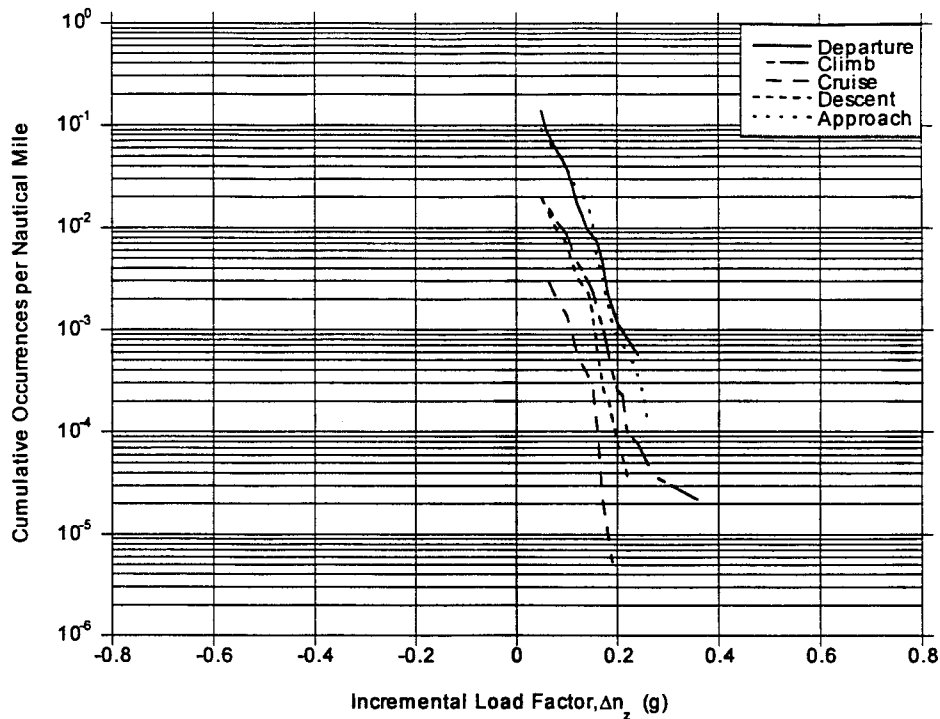


FIGURE 12. CUMULATIVE FREQUENCY OF MANEUVER ACCELERATIONS  
FAA/NASA CRITERIA

During the departure and approach phases of the flight, it is not unusual for banking maneuvers to occur simultaneously with pitching maneuvers. The relationship used to calculate the load factor in a banked turn assumes turning in a horizontal plane without changes in altitude and flight path angle. For turning maneuvers with associated flight path and altitude changes, the basic load factor relationship for banked turn maneuvers is no longer entirely correct. The accelerations from simultaneous turning and pitching maneuvers can be determined by combining the results from decoupled load factor calculations for turning and pitching. The procedure for determining the load factor resulting from simultaneous turning and pitching maneuvers is presented in the appendix. Figure 13 presents the time histories of the accelerations for a flight segment that exhibited simultaneous turning and pitching maneuvers. This is the same flight segment presented in figure 10. The acceleration associated with the pitching maneuver is compared with the FAA/NASA identified gust acceleration, i.e., the difference between the measured acceleration and the banking acceleration. It is clear that the pitching acceleration is the primary, if not the only, contributor to the difference between the measured and turning accelerations. Yet, it is this difference that is classified as gust accelerations in figure 2. The difference also represents the negative maneuver load factors that are not accounted for. The spikes shown at 56 seconds are not true acceleration pitching acceleration peaks but the result of integrating the pitch angle to obtain pitch velocity. This difficulty will be further addressed in the next section.

The major point that emerges here is the fact that the FAA/NASA method for identifying gust and maneuver accelerations from a continuous time history can inappropriately identify pitching



maneuver accelerations as gust-induced acceleration. This results in the loss of negative maneuver load factor spectra and an artificial increase in the number of gust accelerations.

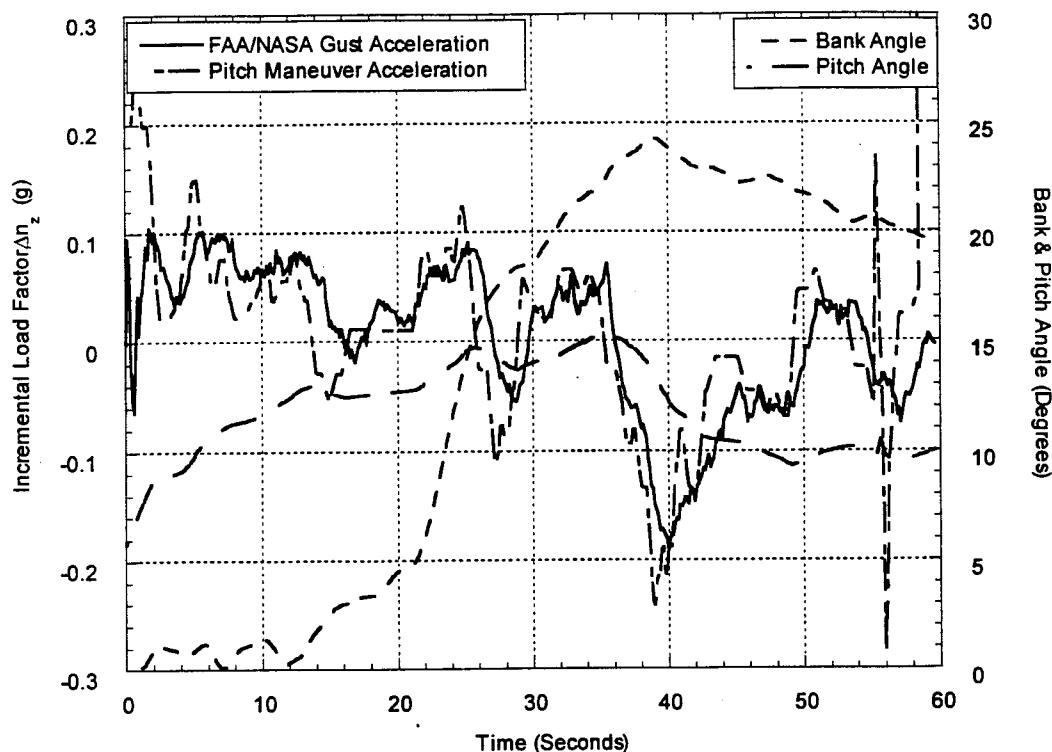


FIGURE 13. COMPARISON OF FAA/NASA IDENTIFIED GUST ACCELERATION AND PITCHING MANEUVER ACCELERATION

### 3.3 NATIONAL SPACE LABORATORY MANEUVER-GUST SEPARATION CRITERIA.

These criteria consider maneuver accelerations resulting from both turning and pitching maneuvers. The criteria for identification as a maneuver acceleration peak associated with a turning maneuver are identical to those used by the FAA/NASA maneuver-gust separation criteria discussed in the previous section. Thus the maneuver acceleration is determined from  $n_z = 1/\cos\phi$ , where  $\phi$  is the bank angle. Since these criteria are identical to those used by the FAA/NASA maneuver-gust separation criteria, they are subject to the same concerns expressed previously. The reader is referred to section 3.2 for a discussion of this approach. The criteria also suggest that accelerations associated with pitching maneuvers at specific points in flight such as takeoff to initial climb, climb to cruise, and approach to touchdown are to be counted. Unfortunately, no specific method for identifying or calculating pitching maneuver-induced accelerations is identified. For this study the criterion for identifying a pitching maneuver is the occurrence of simultaneous changes in altitude and pitch angle.

The acceleration peak associated with a pitching maneuver is based on changes in pitch angle where the acceleration is determined from  $n_z = 1 + V\dot{\theta}/g$  and  $\dot{\theta}$  is the rate of change in pitch angle. For pitching maneuvers such as a pull-up the airplane traces a curved path in the vertical

plane while maintaining its plane of symmetry. At any time the flight path has a radius of curvature  $R$  with a varying flight path angle  $\gamma$  does not equal zero. For this condition the forces on the airplane are as shown in figure 14.

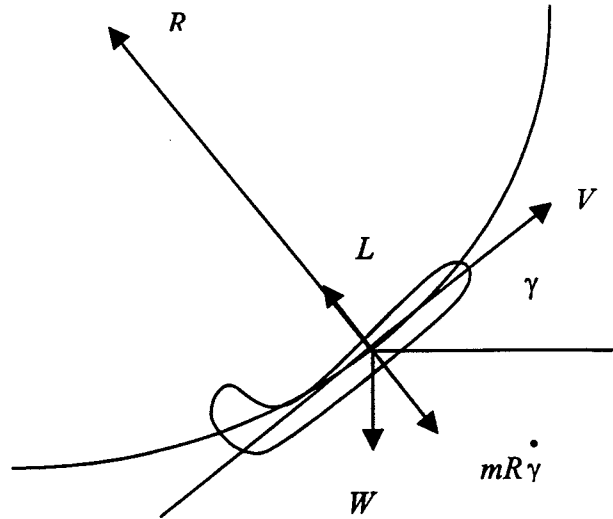


FIGURE 14. FORCES ON AIRPLANE DURING STEADY SYMMETRICAL PULL-UP

The centrifugal force on the airplane is  $mR\dot{\gamma}^2$  where  $\gamma = \theta - \alpha$  is the flight path angle.

The lift on the airplane is 
$$L = W \cos \gamma + mR\dot{\gamma}^2 \quad (11)$$

$$mR\dot{\gamma}^2 = mV\dot{\gamma} \quad (12)$$

thus

$$L = W \cos \gamma + mR\dot{\gamma} \quad (13)$$

since

$$L = nW \quad (14)$$

$$nW = W \cos \gamma + mV\dot{\gamma} \quad (15)$$

and

$$n = \cos \gamma + \frac{V\dot{\gamma}}{g} \quad (16)$$

but  $\dot{\gamma} = \dot{\theta} - \dot{\alpha}$  and for a steady symmetrical maneuver  $\dot{\alpha} = 0$

assuming

$$\cos \gamma = 1 \text{ then } n = 1 + \frac{V\dot{\theta}}{g} \quad (17)$$

Since pitch angle and velocity are measured parameters, the load factor at the center of gravity resulting from pitching maneuvers can be calculated for any time during the maneuver. This approach is valid for a symmetrical pull-up or pushdown maneuver. Most maneuvering occurs during departure and arrival phases and turning and pitching maneuvers are performed simultaneously. The two-dimensional approach discussed for symmetrical pitching maneuvers no longer holds. The appendix presents an approach to determine the accelerations from simultaneous turning and pitching maneuvers by combining the results from decoupled two-dimensional load factor calculations for turning and pitching. Figure 15 presents the time histories of simultaneous banking and pitching maneuvers.

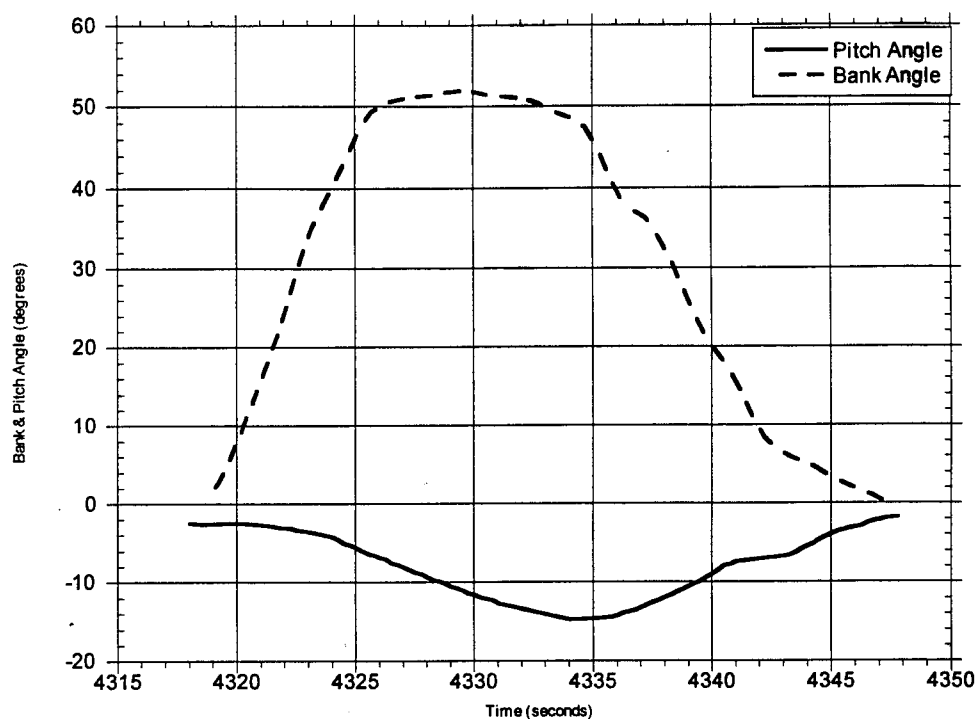


FIGURE 15. BANK AND PITCH ANGLE TIME HISTORIES

As can be seen, both the bank angle trace and the pitch angle trace appear quite smooth. However, it will be shown that the resulting measured and calculated acceleration traces do not exhibit this smoothness but are quite ragged. This is due to slight variations in the pitch angle during the pitch maneuvers. The calculation of turning accelerations is straightforward and uses the directly measured bank angle parameter. Figure 16 presents the acceleration trace for the banked turn of figure 15. The acceleration trace reflects the apparent smoothness of the bank angle variation during the turn.

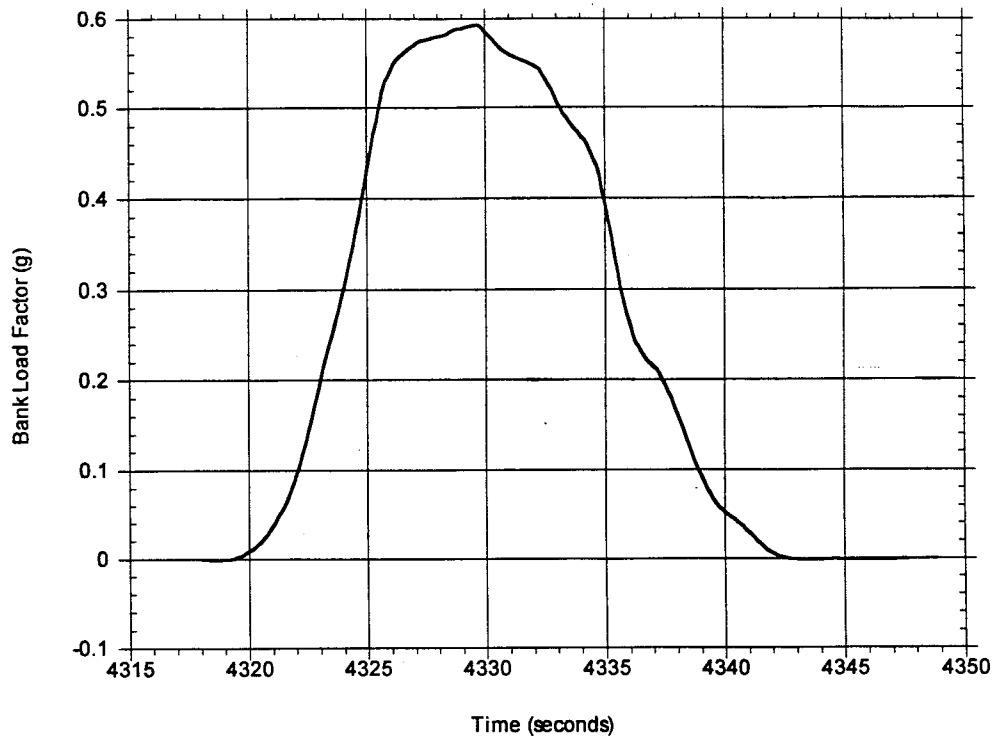


FIGURE 16. ACCELERATION DUE TO BANK ANGLE

The calculation of pitching acceleration requires knowledge of the pitching velocity. This is not a parameter directly available but must be derived from the changes of pitch angle over time. Since the change in pitch angle cannot readily be expressed as an analytical function, it is necessary to perform the necessary integration through a step-by-step computation. Generally, the successive pitch angle measurements do not exhibit continuously increasing or decreasing values for a pull-up or pushdown maneuver. For instance, while exhibiting an overall trend of increasing pitch angle measurements during a pull-up, individual measurements may at times be lower than a previous one. When integrating the time history in a step-by-step manner to obtain pitch velocity, such readings result in a momentary change of sign that translates into an acceleration peak. In comparison with most pitching maneuvers in the database, the pitching maneuver presented in figure 15 must be described as a smooth and simple pitching maneuver. Figure 17 presents the calculated accelerations for the pitching maneuver of figure 15. Figure 17 shows the calculated acceleration trace to be far from smooth and inconsistent with the apparent smoothness of the overall maneuver. The peaks result from the integration and are not picked up by the accelerometer. The smoothed curve provides a much improved acceleration trace.

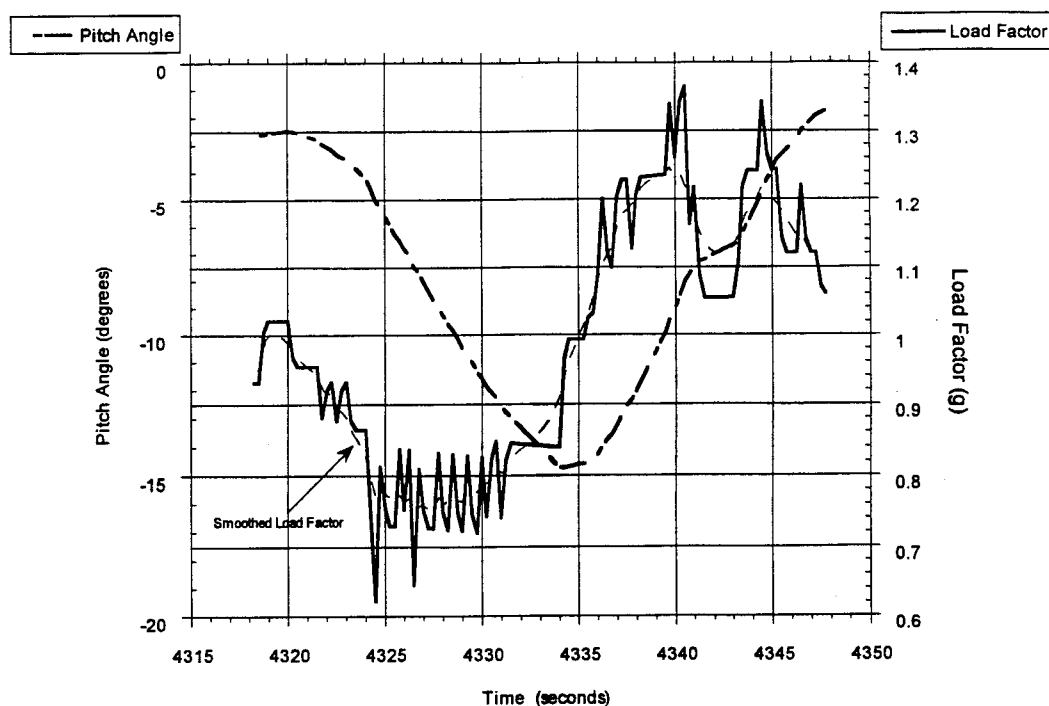


FIGURE 17. PITCHING MANEUVER TIME HISTORY

This exercise indicates that smoothing of the derived pitch velocity time history is essential if meaningful pitch accelerations are to be calculated or gust accelerations are to be derived from the difference between measured acceleration and calculated pitch acceleration. Figure 18 presents a comparison of the measured accelerations with the total calculated maneuver accelerations based on turning and pitching inputs using the approach presented in the appendix. The pitch maneuver accelerations were derived using a smoothed pitch velocity trace.

It is fair to say that the overall maneuver acceleration response time history can be calculated with a reasonable degree of accuracy from the available measurements even without accounting for the influences of pitch and roll accelerations on the sensor due to offset from the center of gravity. Admittedly these influences are small and may be neglected. Unfortunately, the manner in which the smoothing function is applied is rather subjective. The smoothing technique computes a moving average of the curve. The window size of the number of data points that are averaged will determine the smoothness of the final time history. Smaller smoothing windows allow the survival of nonexistent accelerations peaks, and larger smoothing windows will remove or alter the shape of legitimate peaks in the acceleration trace. Figure 19 presents the difference between the measured accelerations and the calculated maneuver accelerations shown in figure 18 and thus represents the derived gust acceleration time history as defined by the National Space Laboratory criteria.

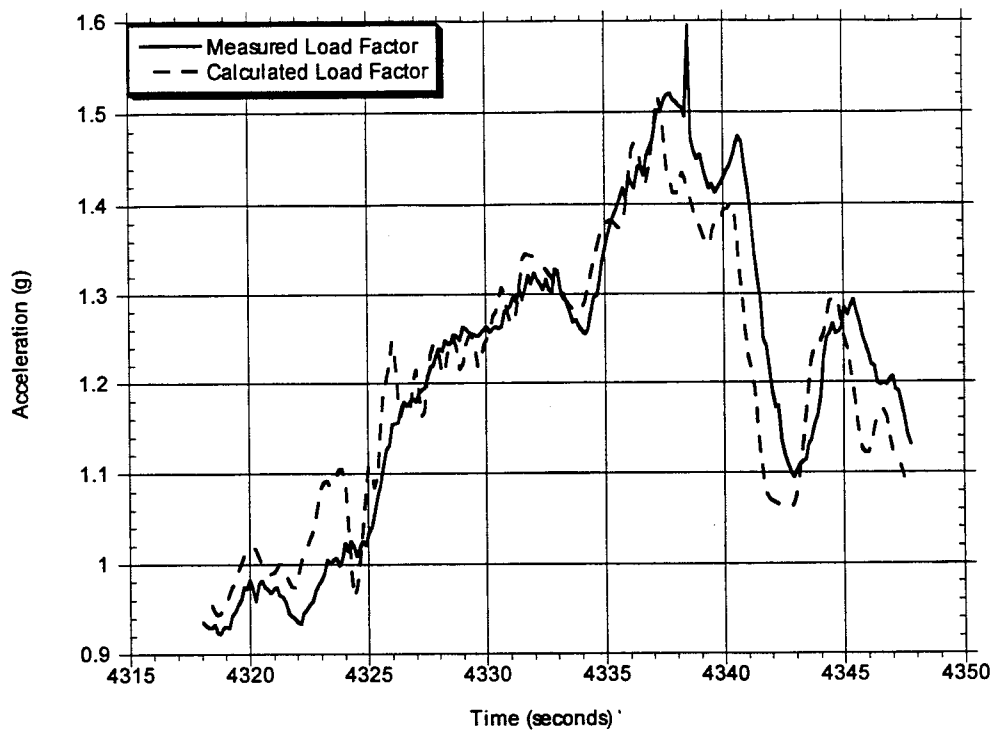


FIGURE 18. COMPARISON OF MEASURED AND CALCULATED LOAD FACTOR

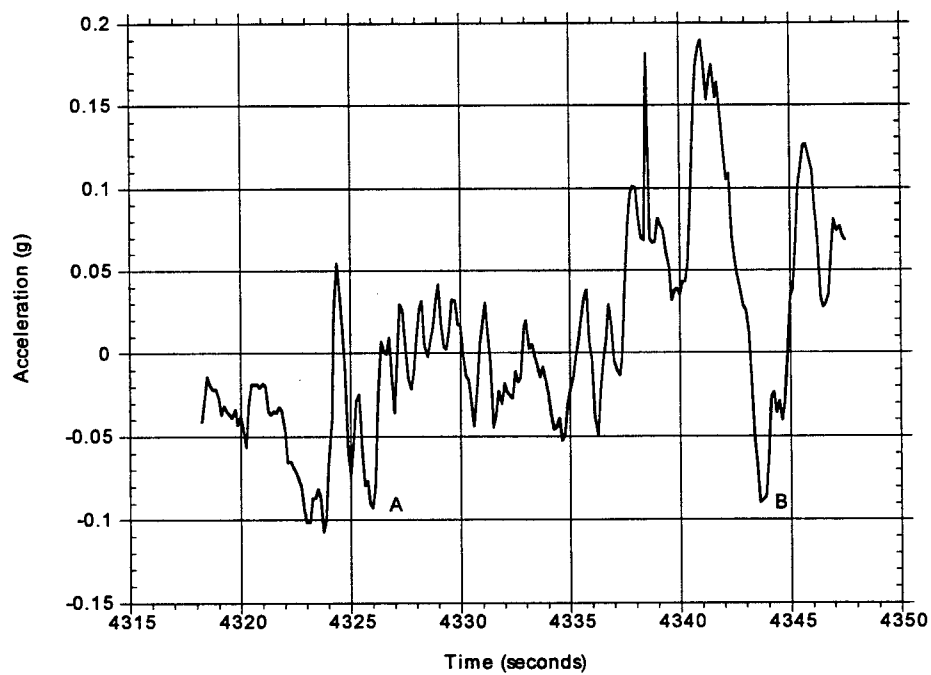


FIGURE 19. DERIVED GUST ACCELERATION TIME HISTORY

It has been shown that an average value of 1.21 cycles per second represents a reasonable description of the aircraft response to turbulence. When applying this standard to the gust acceleration time histories in figure 19, it is noted that only points A and B could be identified as gust peaks with any confidence. However, it is far from certain that any of the differences between the calculated maneuver acceleration response and the measured acceleration response can be arbitrarily attributed to gusts. It is much more reasonable to attribute much of the difference to the fidelity of the applied data smoothing and the inability of the mathematical approach based on steady-state flight conditions to properly account for changes in aerodynamic characteristics caused by perturbation from trimmed equilibrium conditions.

The questions raised so far are based on a case representing smooth banking and pitching maneuver inputs as shown in figure 15. The problem becomes more complicated for maneuvers such as shown in figure 10. It becomes very difficult to develop an algorithm that can incorporate a proper smoothing function and reliably determine the maneuver load factor associated with the pitching maneuver in a three-dimensional context, let alone, determine the value of any gust peaks occurring during the maneuver. The FAA/NASA procedure used by UDRI, which only considered the effect of banked turns, repeatedly required the manual examination of flight data because of unexplained large magnitude gust accelerations. As a rule, the values in question were those derived from the difference between calculated maneuver accelerations and measured accelerations and were judged to be in error. As more flight data were collected, the continual manual review of suspected acceleration values was becoming time consuming and impractical for long-term data processing operations. As shown, the addition of pitching maneuver inputs in the calculation of maneuver accelerations can only exacerbate the problem. The promised improvements of the approach for separation of maneuvers and gusts from measured accelerations appear to be mostly theoretical but unattainable when considered on a practical basis. Therefore, the application in the data reduction of large databases and for many different aircraft on a continuous basis is considered impractical.

#### 4. CONCLUSIONS AND RECOMMENDATIONS.

The accuracy of durability and damage tolerance analyses of an aircraft structure, either to validate an existing design or to establish capability for a new design, is enhanced by improved accuracy and reliability of the available maneuver and gust load spectra. The accuracy is improved by refinement of the maneuver-gust separation techniques used, while the reliability is improved by the reduction of larger amounts of measured flight loads data for different classes and types of airplanes. Improvements in accuracy of maneuver and gust spectra from measured data must be measured against the many other variables in the design process influencing the final structural integrity. Improvements in the reliability of the maneuver and gust spectra must be weighed against the ease or difficulty of processing large amounts of data. In short, the method used in separating maneuver and gust accelerations from measured acceleration time histories should (1) provide a good estimation of the frequency distribution of maneuvers and gust and (2) be general enough to allow the efficient processing of large amounts of data without requiring changes to account for differences in aircraft type and data parameters.

Maneuver-gust separation using a cycle duration criteria has been used for many years and is still being used, albeit on a more sophisticated basis through the application of band-pass filters to the data. This study indicates that aircraft gust response characteristics of the airplanes under

consideration are such that, practically speaking, all gust acceleration cycles will have durations of less than 2 seconds. The study also showed changes in the shape of the maneuver acceleration frequency distribution when cycle durations greater than 2 seconds were used. This strongly suggests a crossover between average gust and maneuver acceleration durations and is consistent with the expected gust acceleration cycle duration for the airplane of this study. The cycle duration rule is by far the simplest algorithm to implement and most efficient for use on large amounts of data. Since acceleration is a basic flight loads parameter measured on any aircraft, it can be used on all aircraft without alteration in the algorithm. Thus, the cycle duration rule still provides a reliable tool for separating maneuver and gust accelerations from a measured acceleration time history.

There is no doubt that the FAA/NASA method used by UDRI raises many doubts about its ability to provide an accurate description of the maneuver acceleration distributions. The method completely disregards any maneuver accelerations resulting from pull-up or pushdown maneuvers. The study showed that pitching maneuvers can contribute significant levels of positive and negative maneuver accelerations. These accelerations are considered as gust accelerations under the FAA/NASA/UDRI approach. The questionable magnitude of these so-called gusts resulting from pitching maneuvers repeatedly required manual review of the data, thus reducing the efficiency in the processing large amounts of data. As additional databases covering other aircraft become available, the need for manual reviews will become an increasing burden and reduce overall efficiency. Finally, when bank angle or heading measurement are unavailable, the measured accelerations become useless for obtaining maneuver and gust frequency distributions.

From a theoretical point of view, the method proposed by the National Aerospace Laboratory of the Netherlands (NLR) would be expected to provide the most realistic separation of maneuvers and gusts from measured acceleration time histories. Its approach as applied in this study is the most complete in that it uses time history data from a number of parameters to identify the occurrence of both turning and pitching maneuvers. Unfortunately, this is also its shortcoming. The sampling rates of the parameters needed to derive pitching maneuver accelerations are low, and the pitch angle traces show many perturbations for a single pull-up or pushdown maneuver. The study showed that this results in sharp spikes in the calculated pitch acceleration response that need to be removed through the use of smoothing techniques. Banking and pitching maneuvers often occur simultaneously resulting in three-dimensional aircraft motion. Combining the results of simple two-dimensional banking and pitch acceleration response calculations provides a reasonable but not accurate description of the total acceleration. As a result, implementation of a reliable algorithm to determine the maneuver accelerations is quite difficult and not very amenable to the efficient data reduction of large databases on a production basis. The problem of performing manual checks whenever unrealistic accelerations are reduced, as was experienced in the UDRI approach, would be further aggravated. The need for multiple parameters for its operation also make the approach subject to additional data loss. For instance, loss of any of the parameters required for the determination of acceleration due to either a banked turn or a pitching maneuver during a given flight invalidates the entire derived maneuver acceleration distribution for that flight. Finally, if a parameter required to identify the maneuvers is not available in the basic parameter list of a particular aircraft, different algorithms must be developed to obtain the required maneuver acceleration spectra. Thus, it is concluded



that although this approach appears the most logical for the separation of maneuvers and gusts, when considering all its ramifications discussed above, its use in data processing of large databases for various aircraft models and recording systems cannot be justified.

As a result of this study, it is recommended that the cycle duration rule be used in all future data reductions efforts. A cycle duration of 2.0 seconds as a criteria for separating maneuvers and gusts is suggested for use with B-737 and MD-82/83 acceleration data. If any concerns exist about its applicability to other aircraft, this study shows that a check of  $N(0)$  values using Houbolt's method should be sufficient to determine its validity or to establish a new cycle duration for the specific aircraft.

Reference 1 suggested development and evaluation of a maneuver-gust separation method similar to the frequency separation method used in the NASA DVGH program. This method would use low-pass and band-pass filtering techniques to separate the gust and maneuver inputs from measured data. It would also allow the separation of gusts occurring during maneuvers. Unfortunately, such development and subsequent evaluation were never conducted. Intuitively it is felt that this method might provide some improvement over the 2-second criterion.

## 5. REFERENCES.

1. Crabill, Norman L., "FAA/NASA Prototype Flight Loads Program Systems Requirements, B737-400 Aircraft," Eagle Aerospace Inc., Contract NAS1-19659, report draft dated November 1994.
2. de Jonge, B., "Reduction of Incremental Load Factor Acceleration Data to Gust Statistics," DOT/FAA/CT-94/57, August 1994.
3. Rice, S. O., "Mathematical Analysis of Random Noise," Bell Systems Technical Journal, Volume XXIII, No. 3, July 1944, pp 282-332, and Volume XXIV No. 1 January 1945, pp 46-156.
4. Houbolt, John C., "Status Review of Atmospheric Turbulence and Aircraft Response," AGARD Report No. 734, December 1987.
5. Hoblit, Frederic M., et al., "Development of a Power Spectral Gust Design Procedure for Civil Aircraft," Technical Report FAA-ADS-53, 1966.



## APPENDIX—MANEUVER LOAD FACTOR CALCULATIONS

### Airplane Coordinate System.

The airplane coordinate system shown in figure A-1 will be used in the subsequent discussion of maneuver load factor derivation for turning and pitching maneuvers.

$\alpha$  is the angle of attack—This is the angle between the airplane axis and the flight path.

$\gamma$  is the flight path angle—This is the angle between the horizontal and the flight path.

$\theta$  is the pitch attitude angle—This is the angle between the horizontal and the airplane axis.

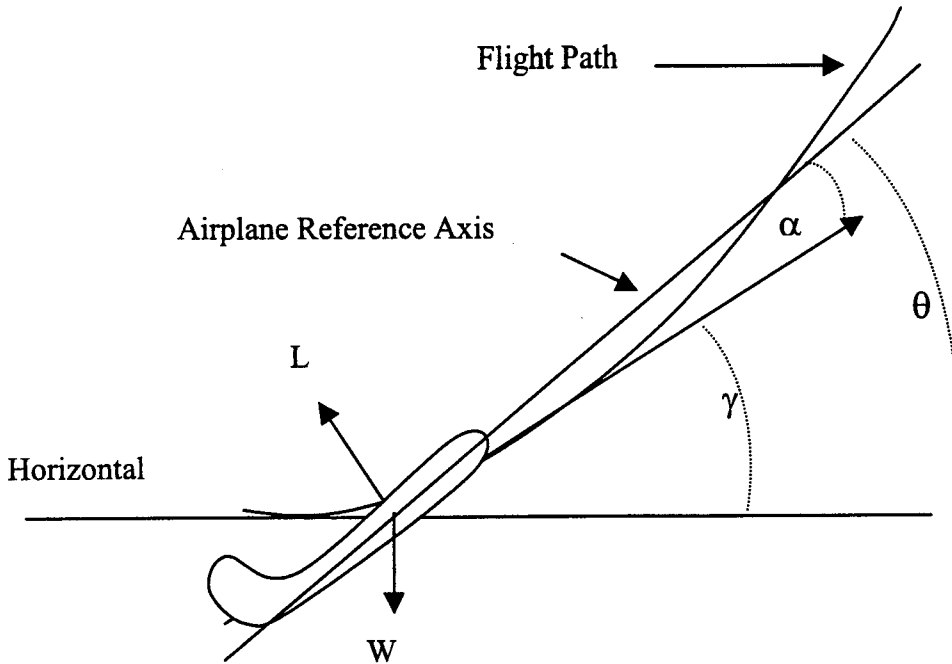


FIGURE A-1. AIRPLANE COORDINATES

### Turning Maneuver.

For a coordinated turn at constant speed and altitude, the airplane traces a circle in the horizontal plane with a radius of curvature  $R$ . For a constant altitude, the flight path angle  $\gamma$  equals 0. For such a condition, the forces on the airplane are as shown in figure A-2.

$$mR\dot{\phi}^2 = mV\dot{\phi}$$

$$V^2 = R^2\dot{\phi}^2$$

and

$$mR\dot{\phi}^2 = m\frac{V^2}{R}$$

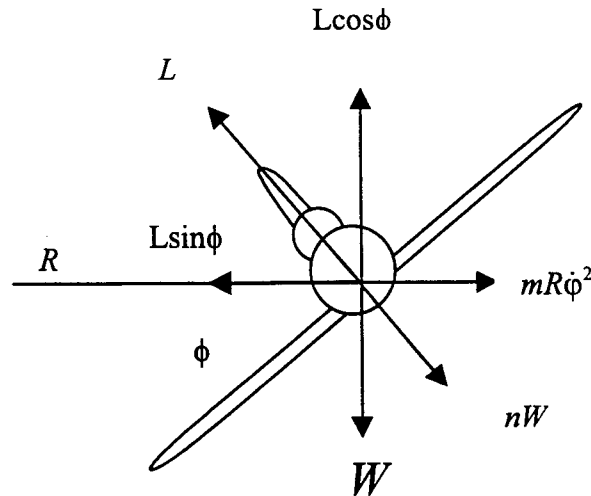


FIGURE A-2. STEADY LEVEL BANKED TURN

From the forces shown in figure A-2, the relationship between bank angle, load factor, turning radius, and turning rate can be derived.

$$n = \frac{L}{W} = \frac{1}{\cos \phi} = \frac{V^2}{gR \sin \phi} = \frac{R\dot{\phi}^2}{g \sin \phi}$$

thus

$$n = \frac{1}{\cos \phi}$$

This relationship is valid for a banked turn in a horizontal plane. In ascending or descending turns, this relationship is no longer entirely correct. Many banked turns include changes in altitude or flight path angle. Because of the difficulty of visualizing a three-dimensional geometry of a climbing or descending turn, figure A-3 presents a simple two-dimensional geometrical relationship for a banked turn with a change in flight path.

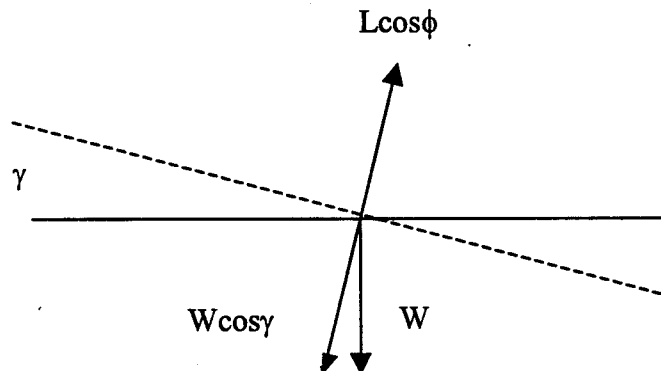


FIGURE A-3. FORCES OF A BANKED TURN DURING CLIMB

The lift  $L \cos \phi$  resulting from the bank angle is now normal to the flight path angle  $\gamma$  and  $L \cos \phi = W \cos \gamma$ .

and 
$$n = \frac{L}{W} = \frac{\cos \gamma}{\cos \phi}$$

The flight path angle  $\gamma = \theta - \alpha \cong \theta - \frac{w}{V}$

For our purposes it will be assumed that  $\frac{w}{V}$  is small and that the pitch angle equals the flight path angle and  $n = \frac{\cos \theta}{\cos \phi}$

#### Pitching Maneuver.

The pitching maneuvers represent steady symmetrical pull-ups with the airplane in equilibrium with zero pitching acceleration. Under these conditions the airplane is tracing a circle in the vertical plane at constant pitching velocity. At any time the flight path has a radius of curvature  $R$  with a varying flight path angle,  $\dot{\gamma} \neq 0$ . The forces on the airplane are as shown in figure A-4.

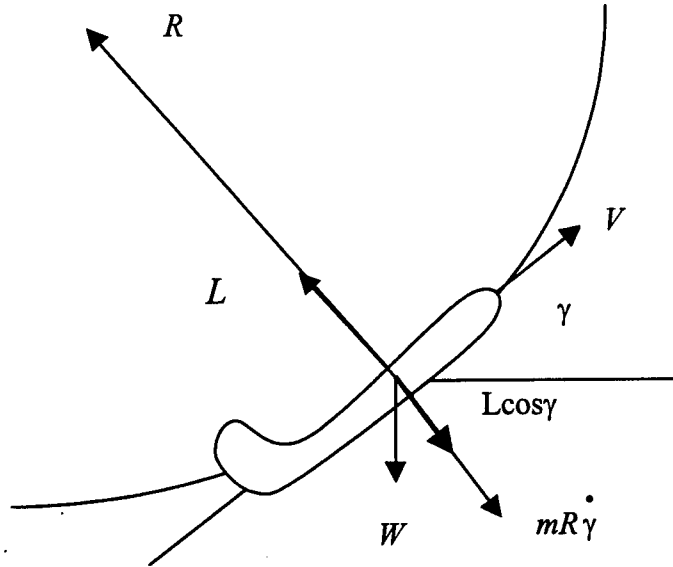


FIGURE A-4. FORCES ON AIRPLANE DURING STEADY SYMMETRICAL PULL-UP

The centrifugal force on the airplane is

$$mR\dot{\gamma}^2$$

Where  $\dot{\gamma}$  is the rate of change in flight path angle.

The lift on the airplane is

$$L = W \cos \gamma + mR\dot{\gamma}^2$$

$$mR\dot{\gamma}^2 = mV\dot{\gamma} = \frac{W}{g}V\dot{\gamma}$$

Thus

$$L = W \cos \gamma + mV\dot{\gamma}$$

since

$$L = nW$$

and

$$nW = W \cos \gamma + mV\dot{\gamma}$$

$$n = \cos \gamma + \frac{V\dot{\gamma}}{g}$$

This relationship is valid for a pitching maneuver in a vertical plane, that is, without a bank angle. Many pitching maneuvers include banked turns. In pitching maneuvers that exhibit bank angles, the effects of the bank angle need to be accounted for. Because of the difficulty of visualizing a three-dimensional geometry of a simultaneous pitch and bank condition, figure A-5 presents a simple two-dimensional geometrical relationship for a pitching maneuver while in a banked turn.

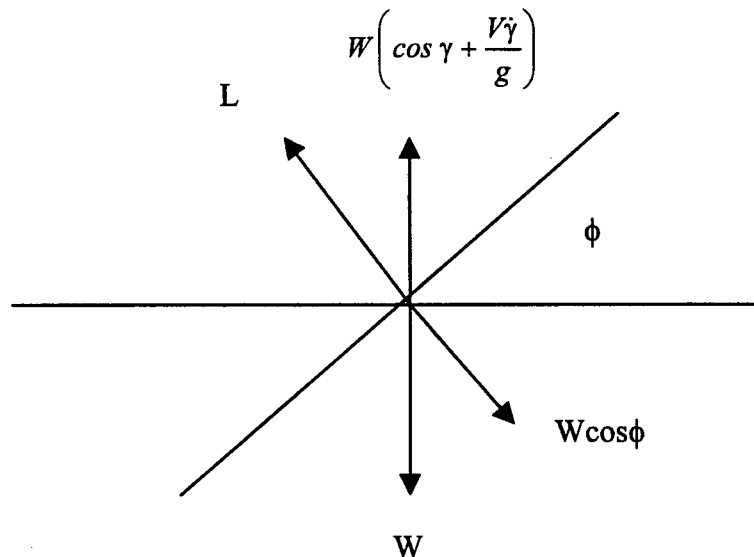


FIGURE A-5. FORCES OF A PITCHING MANEUVER IN A BANK

Considering that  $\dot{\gamma} = \dot{\theta} - \dot{\alpha}$  and that for steady maneuvers  $\dot{\alpha} = 0$  and for slow maneuvers  $\dot{\alpha} \cong 0$  and assuming  $\alpha \cong 0$ :

$$L \cos \phi = W \left( \cos \theta + \frac{V \dot{\theta}}{g} \right)$$

$$L = \frac{W}{\cos \phi} \left( \cos \theta + \frac{V \dot{\theta}}{g} \right)$$

$$n = \frac{L}{W} = \frac{\left( \cos \theta + \frac{V \dot{\theta}}{g} \right)}{\cos \phi}$$

