REPORT FHWA/NY/SR-99/131

PB99-156762

A Simplified Design Procedure For End-Plates and Base-Plates Of Cantilevered Traffic Structures

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SPECIAL REPORT 131
TRANSPORTATION RESEACH AND DEVELOPMENT BUREAU
NEW YORK STATE DEPARTMENT OF TRANSPORTATION
George E. Pataki, Governor/Joseph H. Boardman, Commissioner

A SIMPLIFIED DESIGN PROCEDURE FOR END-PLATES AND BASE-PLATES OF CANTILEVERED TRAFFIC STRUCTURES

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Final Report on a Study Conducted in Cooperation
With The U.S. Department of Transportation, Federal Highway Administration

Special Report 131
June 1999

TRANSPORTATION RESEARCH AND DEVELOPMENT BUREAU
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ABSTRACT

Although end-plates and base-plates are routinely used in cantilevered structures supporting traffic signs, signals, and lights, no standard procedure has been established for the design of these plates. In this report, a simplified procedure is developed for design of end-plates and base-plates of these structures, and also for base-plates of span-wire-mounted traffic-signal structures. The proposed procedure is based on beam-and-plate bending and torsion theories, and is intended for routine application by practicing engineers when designing plates of square configurations. Plate thicknesses and stresses obtained using this procedure compared well with those estimated using finite-element analysis, and also supported earlier conclusions reached through physical testing. In all, 35 base-plates from five major manufacturers of traffic poles used in New York State were analyzed in the study. A spreadsheet program implementing the proposed procedure was also developed as an efficient tool for engineers designing these items. Using this tool will not only expedite the design process, but also facilitate optimizing plate sizes, because various design alternatives can be easily investigated and the optimal alternative rationally selected.

CONTENTS

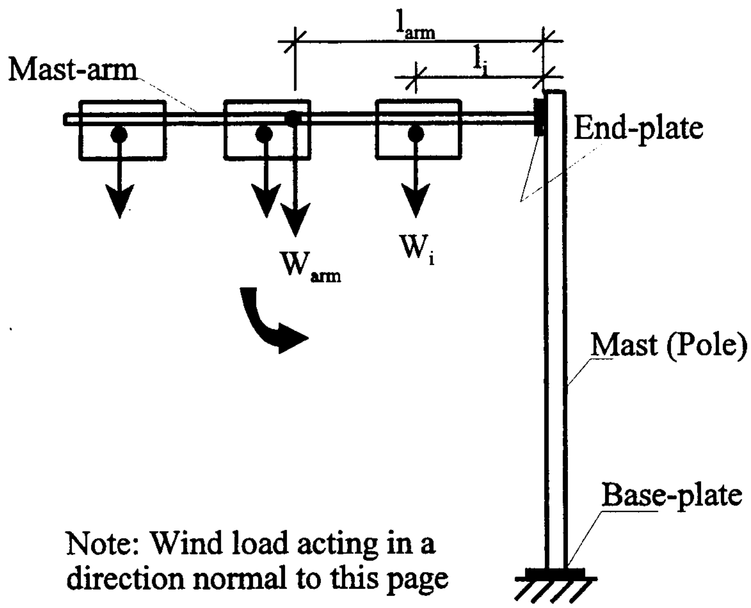
NOTATION

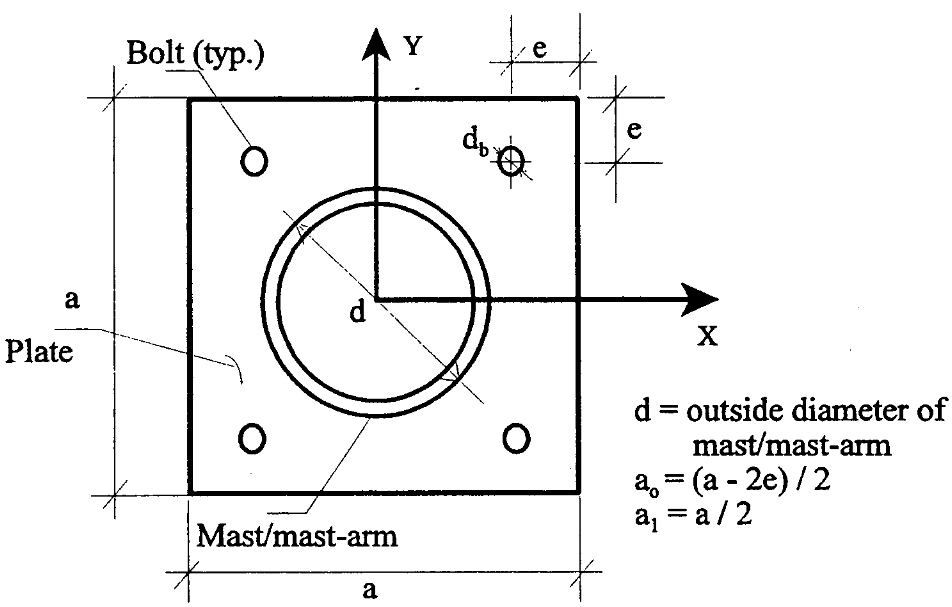
The following symbols are used in this report:

Γ	=	Section modulus for torsion
ϕ	=	Angle of incidence defined in Figure 2
a	=	Square plate side length
a_0	=	Distance defined in Figure 1
a_1	=	Distance defined in Figure 1
A_{vb}	=	Area for checking shear stress due to bearing stresses at the resultant pressure line
A_{vy}	=	Area for checking shear stress at the yield line
b'	=	Distance defined in Figure 4
b''	=	Yield line approximated in Figure 9
C_t	=	Torsion coefficient
d	=	Outer diameter of mast/mast-arm
d_b	=	Bolt diameter
e	=	Edge distance from center of bolt
f_{b_max}	=	Maximum bearing stress
F_b	=	Allowable bending stress
f_{b_fb}	=	Maximum bending stress for fixed-end beam approximation
f_{by}	=	Maximum bending stress at the yield line
F_P	=	Allowable bearing stress
F_v	=	Allowable shear stress
$f_{v\_torsion}$	=	Maximum shear stress for torsional beam approximation
f_{vb}	=	Maximum shear stress due to bearing stresses at the resultant pressure line
f_{bb}	=	Maximum bending stress due to bearing stresses at the resultant pressure line
f_{vy}	=	Maximum shear stress at the yield line
F_y	=	Yield stress of plate material
l_{arm}	=	Distance from center of gravity of mast-arm to end plate
l_i	=	Distance from sign/signal i to end plate
M_{by}	=	Maximum moment at yield line
M_{fb}	=	Maximum moment for fixed-end beam approximation
M_t	=	Total applied moment
$M_{torsion}$	=	Maximum torsional moment
M_x	=	Moment component about the X-axis due to dead load and ice load
M_y	=	Moment component about the Y-axis due to wind load
P_a	=	Total axial load on plate
P_b	=	Bearing force calculated from the assumed pressure distribution

P_{bb} = Effective forces due to bearing stresses at the resultant pressure line
P_l = Left point where the resultant pressure line intersects the plate side
P_r = Right point where the resultant pressure line intersects the plate side
r_{bb} = Arm for moment about resultant pressure line due to bearing stresses
r_{by} = Arm for moment about yield line due to maximum bolt tension
r_{c2} = Normal distance from corner C_2 to the resultant pressure line
r_{c3} = Normal distance from corner C_3 to the resultant pressure line
S_l = Distance defined in Figure 5
S_r = Distance defined in Figure 5
S_x = Section modulus for bending
t = Plate thickness
T_i = Force on Bolt i
T_{total} = Total force on all tension bolts
W_{arm} = Weight of mast arm
W_i = Weight of sign/signal i
x_i = Normal distance from Bolt i to the resultant pressure line

Figure 1. Typical cantilevered support structure and plate.





INTRODUCTION

Although end-plates and base-plates are routinely used in traffic signal, sign, and light support structures, a standard procedure has never been developed for the design of these plates. Manufacturers of these items normally maintain lists in their inventories including plate designs developed through a combination of experience, minimal analysis, and physical testing. Concern about adequacy of current New York State procedures for checking and designing these plates has led to investigation of new alternatives. Such an alternative is proposed here for designing end-plates and base-plates of the typical configurations shown in Figure 1 -- square plates supported by four bolts at their four corners, in turn supporting a cylindrical (or multi-sided) mast or mast-arm load-transferring member. Under service loads, these are essentially plate elements subjected to out-of-plane bending and shearing forces, and sometimes to in-plane torsional forces. Unlike base-plates and end-plates in building structures (*1,2,3,4,5,6,7,8*), the similarity between these two plates, in both geometry and loading conditions, allows for their design in the proposed unified approach.

Common AASHTO procedures for estimating design loads on these structures (*9*) have recently undergone intensive review by researchers (*10*). The intent of the study reported here was not to pursue similar efforts, but rather to focus on developing a simplified procedure for design of end-plates and base-plates of these structures, and also for base-plates of span-wire-mounted traffic-signal structures. Design loads to be used in the proposed procedure are limited to those due to dead load, ice load, and wind load, and the mast/mast-arm is assumed to be sufficiently strong to transfer these loads to the plate. End-plates and base-plates are further assumed to be sufficiently strong to transfer in-plane torsional forces, resulting from structural configuration and/or applied loads, to the bolts. Although tensile forces on the bolts and bolt size are estimated for the proposed design procedure, additional checks for shear and fatigue are required to confirm adequate bolt capacities to resist these forces (*1,2,10,11,12,13*).

Spans of the cantilevered arm of traffic support structures have increased significantly in recent years, due to requirements that masts to be set back greater distances from the roadway for safety reasons, and the need to span over more traffic lanes (*10*). Development of the simple, yet reliable proposed procedure should promote routine application by engineers in custom-designing these plates to meet varying structural demands, thus assuring safety and increasing the potential for savings by owner agencies.

A. BACKGROUND

According to current Department practice for design of base-plates, manufacturers may design these items based on a simple method that checks flexural stresses at only one location on the plate where stresses are assumed to be critical, or may use comparable methods of their own. Briefly, this method -- referred to here as "NYS method" -- assumes a yield line around the most stressed bolt as the most critical section for bending, and attempts to approximate the arm for this moment using an averaging procedure. For end-plates, depending on magnitude of design loads, current practice calls on manufacturers to use a fixed-thickness plate [1.25 or 1.5 in. (31.75 or 38.10 mm)] or other thicknesses that are designed based on other procedures. Absence of satisfactory methods for design of these plates has raised concerns about adequacy of current practice for their checking and design, and has created interest in investigation of new alternatives.

The first of these efforts was introduced in Research Report 159 (*14*) for design of base-plates for signal-pole structures. In response to a request from the Structures Design and Construction Division, the Transportation Research and Development Bureau conducted a study to evaluate the structural adequacy of span-wire traffic-signal poles, which included full-scale testing and finite-element analysis and resulted in a proposed procedure for designing base-plates of the supporting poles. Full-scale testing of four of these poles indicated structural inadequacy of base-plates and anchor bolts. Finite-element models were then developed to evaluate a representative sample of poles from the three manufacturers of poles then used in New York State. That evaluation, based on structural adequacy of base-plates and bolts of these poles, confirmed deficiency of manufacturers' methods in designing adequate base-plates and bolts to carry anticipated design loads. Accordingly, a procedure was proposed for their design (*14*), referred to here as the Research Report 159 or "RR 159" procedure. However, this procedure requires evaluation of semi-empirical relationships derived statistically for the plate parameters included in that study, thus limiting its applicability to plates having parameters falling within the ranges of those investigated.

B. REPORT OBJECTIVES AND APPROACH

The objectives of this study were 1) to develop a reliable, simple procedure for design of end-plates and base-plates of cantilevered traffic sign, signal, and light support structures, and for base-plates of span-wire-mounted traffic-signal structures, which can be routinely used by engineers designing these items, and 2) to develop a design tool for implementation of the proposed procedure. The approach to accomplish these objectives for an end-plate or a base-plate may be summarized in terms of the following six tasks:

1. Develop a procedure for estimating load distribution to anchor bolts, and also bearing stresses and forces,

2. Determine critical sections for design by investigating plate behavior under applied loads and the resulting stress states,

3. Propose approaches for simplifying design forces and estimating design stresses at critical sections,

4. Recommend a simplified design procedure,

5. Develop a spreadsheet design tool implementing the proposed procedure, and

6. Compare plate thicknesses obtained using the proposed procedure with those obtained using the RR 159 procedure (*14*), those supplied by manufacturers, and those based on the NYS method, and then compare the results with those obtained using finite-element analysis and physical testing.

C. REPORT ORGANIZATION

This report has three chapters. Procedures for determining load distribution to anchor bolts and estimating bearing forces and bearing stress distribution are introduced in Chapter 1. In Chapter 2, plate behavior under applied loads is investigated, critical stress states on the plate are identified, and a simplified procedure is proposed to estimate these stresses. In Chapter 3, spreadsheet implementation and application of the proposed procedure are discussed. Also in Chapter 3, plate designs obtained using the proposed procedure are compared with those obtained using the RR 159 procedure, those supplied by manufacturers, and those estimated using the NYS method, together with comparisons with finite-element analysis and physical testing results. Finally, conclusions are presented and an appendix gives examples illustrating application of the proposed procedure.

Figure 2. Applied moments on mast/mast-arm and bearing stresses on plate (positive = uplift, negative = positive bearing).

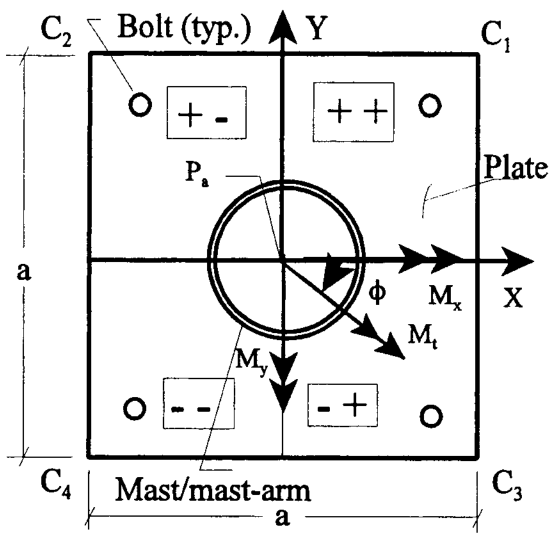
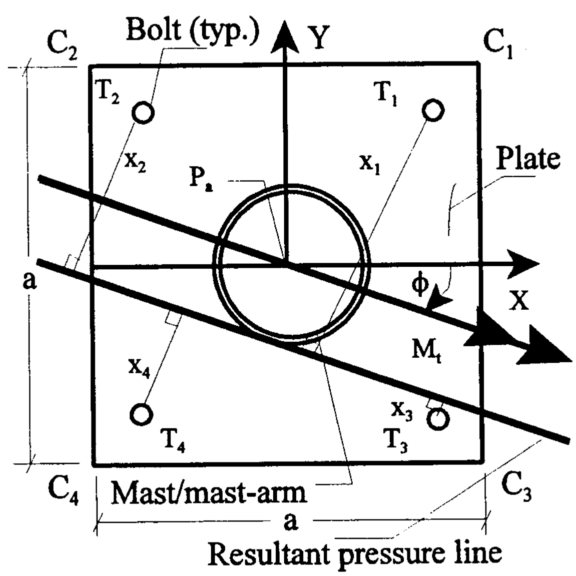


Figure 3. Bolt tension and normal distance to resultant pressure line.



1. FORCES ON BOLTS AND PLATES

Both end-plates and base-plates (of the special type discussed in this report) must resist moment and shear forces due to dead load, ice load, and wind load, and sometimes shear forces due to torsional loads. Base-plates also must resist axial forces due to gravity loads. As a result of this axial load and moment, compressive bearing stresses may exist fully or partially between the foundation and base-plate. When these stresses are partially maintained, some anchor bolts must carry tensile loads for static equilibrium of forces to be satisfied. Under service loads, axial loads on the traffic structures addressed here are normally of insufficient magnitude to maintain compressive stresses through the entire contact area between the foundation and plate. [Design is typically governed by Group II or III loadings in New York State (*9*)]. This allows for treatment of end-plates and base-plates in the proposed unified approach, which assumes that some bolts carry tensile forces under service loads. Accordingly, an end-plate would be a special case of a base-plate with no active axial load.

A. ESTIMATING TENSILE FORCES ON BOLTS

For calculated design moments due to gravity load M_x and wind load M_y, total moment on plate M_t and its angle of incidence ϕ (as defined in Figure 2) can be obtained from Eqs. 1 and 2, respectively, the angle ϕ in Eq. 2 being less than 90°:

$$M_t = \sqrt{M_x^2 + M_y^2} \tag{1}$$

$$\phi = \tan^{-1} (M_y/M_x) \tag{2}$$

Assuming flexibility of the plate, the resultant of compressive bearing forces may be assumed to lie at the toe of the mast/mast-arm at the location shown in Figure 3. From this figure, the normal distances x_1 to x_4 from the bolts to the resultant pressure line may be determined from the following equations:

$$x_1 = -\tfrac{1}{2} [2(\sin \phi + \cos \phi)\, a_0 + d] \tag{3A}$$

$$x_2 = \tfrac{1}{2} [2(\sin \phi - \cos \phi)\, a_0 - d] \tag{3B}$$

$$x_3 = -\tfrac{1}{2} [2(\sin \phi - \cos \phi)\, a_0 + d] \tag{3C}$$

$$x_4 = \tfrac{1}{2} [2(\sin \phi + \cos \phi)\, a_0 - d] \tag{3D}$$

where a_0 is as defined in Figure 1 and d is the mast/mast-arm outer diameter. A bolt is under tension only if the sign of its normal distance from these equations is negative.

Assuming that the tensile force T_i, i =1,...,4, on any bolt is proportional to its distance from the resultant pressure line, this force may be obtained in terms of the most stressed bolt force T_1 using the following equation:

$$T_i = T_1 \cdot x_i / x_1 \tag{4}$$

Taking moments about the resultant pressure line, one finds

$$\sum T_i \cdot x_i = M_t - (P_a \cdot d/2) \tag{5}$$

where P_a is the applied axial load. Substituting for T_i from Eq. 4 into Eq. 5,

$$\sum T_1 \, x_i^{\,2} / x_1 = M_t - P_a \cdot d/2 \tag{6}$$

Solving for T_1,

$$T_1 = (M_t - P_a \cdot d/2) / (\sum x_i^{\,2} / x_1) \tag{7}$$

Once T_1 is determined, remaining bolt forces can be determined from Eq. 4.

B. ESTIMATING BEARING STRESSES AND BEARING FORCES

To estimate these stresses and forces, coordinates must first be determined for the left and right points where the resultant pressure line intersects the plate sides. This can be achieved by calculating the normal distances r_{C2} and r_{C3} (as defined in Figure 4) from corners C_2 and C_3 to the resultant pressure line, using the following equations:

$$r_{C2} = \tfrac{1}{2} [\, 2 (\sin\phi - \cos\phi)\, a_1 - d] \tag{8A}$$

$$r_{C3} = - \tfrac{1}{2} [\, 2 (\sin\phi - \cos\phi)\, a_1 + d] \tag{8B}$$

where a_1 is as defined in Figure 1. If r_{C2} is negative, the pressure line intersects side C_2 - C_4 of the plate, and the coordinates of point P_l (x_l, y_l) may be obtained as

$$x_l = -a_1 \tag{9A}$$

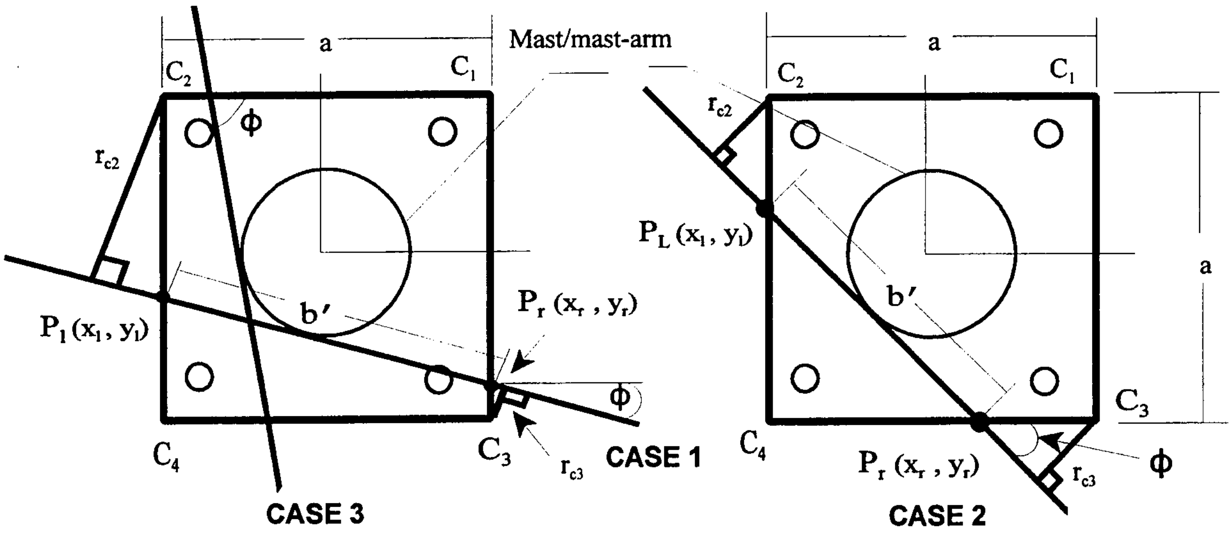
$$y_l = -(-2 \cdot \sin\phi \cdot a_1 + d)/(2 \cdot \cos\phi) \tag{9B}$$

Otherwise, it intersects side C_2 - C_1, and

$$x_l = -(2 \cdot \cos\phi \cdot a_1 + d)/(2 \cdot \sin\phi) \tag{10A}$$

$$y_l = a_1 \tag{10B}$$

Figure 4. Points of intersection of resultant pressure line with plate sides.



Similarly, if r_{C3} is negative the pressure line intersects side C_4 - C_3 of the plate, and the coordinates of point P_r (x_r , y_r) may be determined using

$$x_r = -(-2 \cdot \cos \phi \cdot a_1 + d)/(2 \cdot \sin \phi) \tag{11A}$$

$$y_r = -a_1 \tag{11B}$$

Otherwise, it intersects side C_1 - C_3 and

$$x_r = a_1 \tag{12A}$$

$$y_r = -(2 \cdot \sin \phi \cdot a_1 + d)/(2 \cdot \cos \phi) \tag{12B}$$

Assuming the linear distribution of bearing stresses shown in Figure 5, and based on the assumption that the resultant of the bearing force lies at the toe of the mast/mast-arm, the maximum bearing stress f_{b_max} in Figure 5 can be obtained from static equilibrium of forces as follows. For equilibrium of forces in the axial direction

$$P_b = T_{Total} + P_a \tag{13}$$

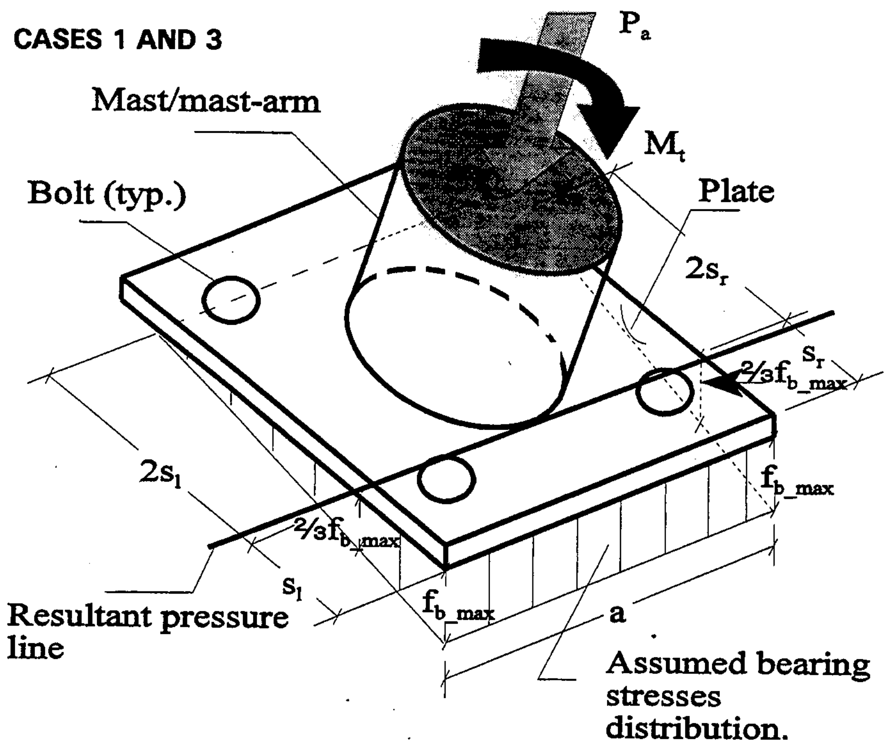
where P_b is the bearing force calculated from the assumed pressure distribution in Figure 5, and T_{Total} is the sum of tension bolt forces. From this distribution, P_b may be calculated as

$$P_b = (3/4)\, f_{b\_max} \cdot a\,(s_l + s_r) \text{ for Cases 1 and 3} \tag{14}$$

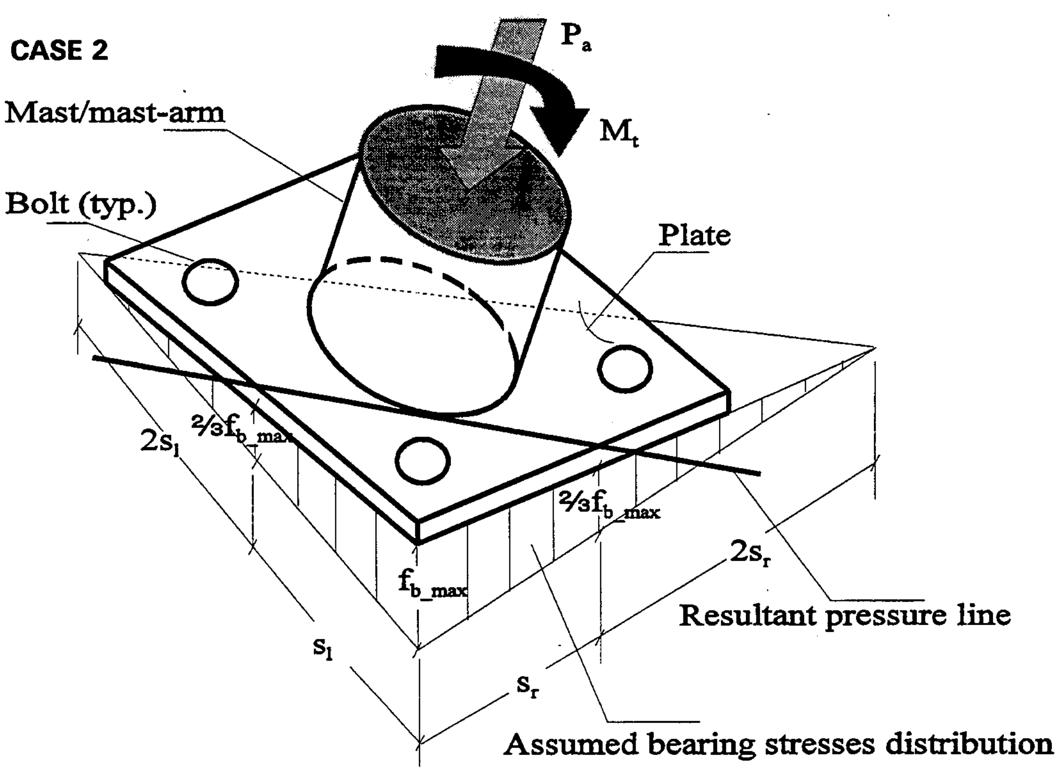
$$P_b = (3/2)\, f_{b\_max} \cdot s_l \cdot s_r \text{ for Case 2} \tag{15}$$

Figure 5. Assumed bearing stress distribution.

CASES 1 AND 3



CASE 2



where s_l and s_r are as defined in Figure 5, and Cases 1, 2, and 3 are as defined in Figure 4. Case 3 is a very rare instance for a very high a/d ratio existing when the resultant pressure line defining Case 1 is rotated counterclockwise to intersect sides C_1 - C_2 and C_3 - C_4 of the plate. Substituting for P_b from Eqs. 14 or 15 into Eq.13 and solving for f_{b_max}

$$f_{b\_max} = (4/3)\,(T_{Total} + P_a)/[a\,(s_l + s_r)] \text{ for Cases 1 and 3} \tag{16}$$

$$f_{b\_max} = (2/3)\,(T_{Total} + P_a)/(s_l \cdot s_r) \text{ for Case 2} \tag{17}$$

For a base-plate bearing directly on concrete, f_{b_max} should not exceed the allowable bearing stress for concrete ($0.70\,f'_c$), but if it bears directly on leveling nuts, f_{b_max} should not exceed the allowable bearing stress for steel ($0.90\,F_y$). For an end-plate, it should not exceed the allowable bearing stress for steel (*15*).

Figure 6. Critical stress regions on a base plate (*9*).

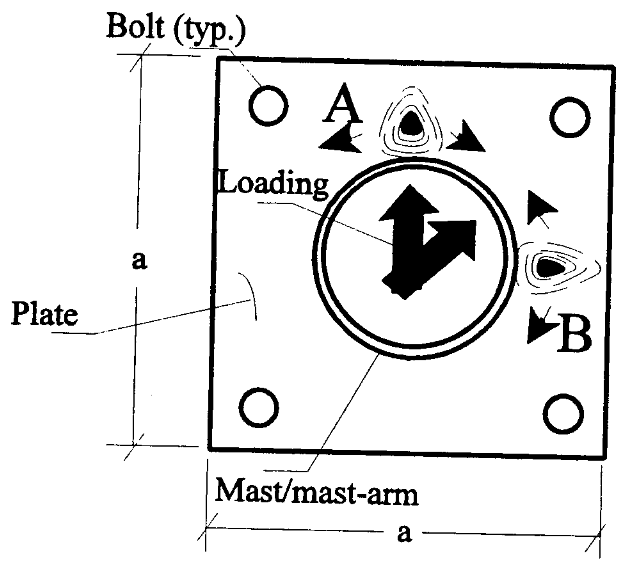
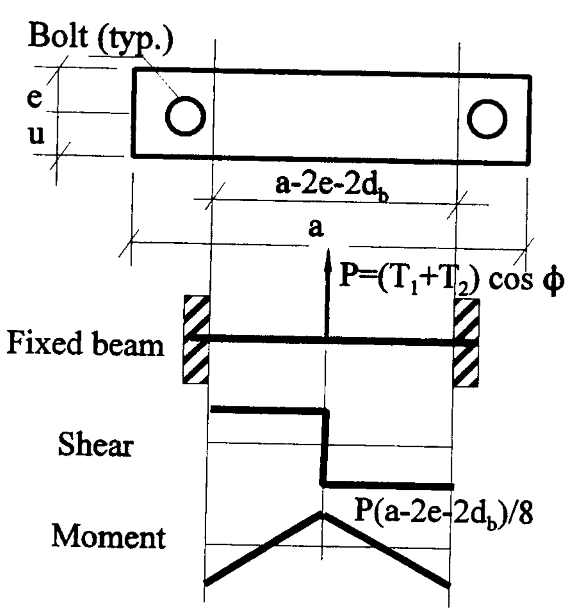


Figure 7. Fixed beam approximation.



2. PROPOSED DESIGN PROCEDURE

By investigating plate behavior under applied loads and expected failure modes, complex stresses on a plate can be approximated by those calculated for assumed plate elements due to estimated forces acting on those elements. Such stresses are now discussed for four locations where stresses have been identified as critical.

A. FLEXURAL STRESSES BETWEEN TENSION-SIDE BOLTS

Full-scale testing and finite-element analysis of base-plates have indicated that under applied moments, the stress field on the plate is characterized by presence of the two critical regions shown in Figure 6 (*14*). In Region A, bending stresses are the prominent stress state, indicating resistance to mainly flexural forces. For this case, a fixed rectangular beam element of the width and span length shown in Figure 7 is assumed to resist a bending moment due to effective reactions created by the bolt tension component $(T_1+T_2) \cdot \cos\phi$. Maximum moment on the fixed-end beam element M_{fb} may be calculated from

$$M_{fb} = (1/8)\,(a - 2e - 2d_b) \cdot (T_1+T_2) \cdot \cos\phi \qquad (18)$$

where d_b is the bolt diameter and e is the edge distance defined in Figure 1, and the length $(a - 2e - 2d_b)$ approximates the clear distance between bolts, defining the span of the fixed-end beam. Section modulus for bending for this beam may be obtained as

$$S_x = (e + u) \cdot t^2/6 \qquad (19)$$

where distance u is assumed to be equal to edge distance e. Maximum bending stress f_{b_fb} for the fixed beam may then be estimated using

$$f_{b\_fb} = M_{fb}/S_x \;\leq\; F_b = 1.40 \cdot 0.66 \cdot F_y \qquad (20)$$

A factor of 1.4 is used for the allowable stress F_b to reflect design stress limits for Group II or III loadings (*9*).

B. TORSIONAL STRESS AT THE SIDES OF THE MAST/MAST-ARM

In Region B of Figure 6, shear stresses are shown to dominate the stress state, indicating resistance to primarily torsional forces (*14*). To estimate these stresses, a rectangular beam element of the dimensions shown in Figure 8 may be assumed to resist a torsional moment due to the bolt reaction on the plate acting at the eccentricity shown in the figure. Design torsional moment $M_{torsion}$ may thus

Figure 8. Torsional beam approximation.

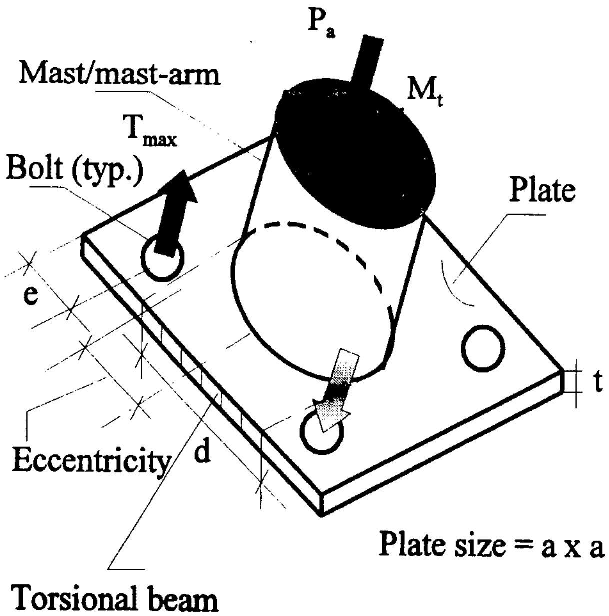
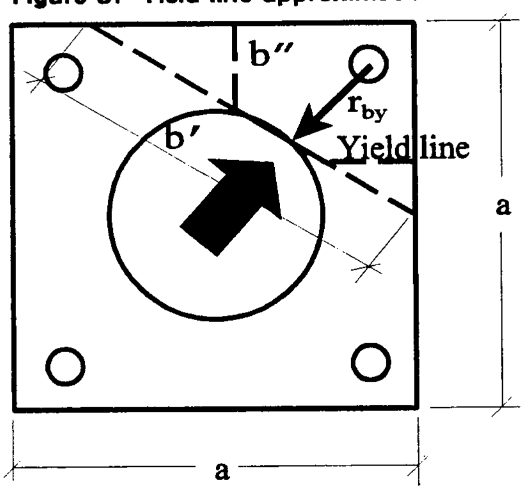


Table 1. Torsion coefficient C_t (*16*).

d/t	C_t	d/t	C_t
1	0.208	3	0.267
1.2	0.219	4	0.282
1.5	0.231	5	0.291
2	0.246	10	0.312
2.5	0.258	>10	0.333

Figure 9. Yield line approximation.



be obtained from

$$M_{torsion} = T_l \cdot (a - 2e - 2d_b)/2 \tag{21}$$

where $(a - 2e - 2d_b)/2$ is assumed to approximate bolt force eccentricity. Torsional section modulus Γ for the beam may be calculated using

$$\Gamma = d \cdot t^2 \cdot C_t \tag{22}$$

where C_t is the torsion coefficient given in Table 1 for various d/t ratios (*16*). Torsional shear stress $f_{v\_torsion}$ may then be obtained using

$$f_{v\_torsion} = M_{torsion}/\Gamma \leq F_v = 1.4 \cdot 0.4 \cdot F_y \tag{23}$$

Again, a factor of 1.4 is used for the allowable stress F_v to reflect design stress limits for Group II or III loadings (*9*).

C. FLEXURAL STRESSES AT THE YIELD LINE

The critical regions in Figure 6 are important not only for identifying the dominant stress components, but also for defining plate yielding behavior. As the load is monotonically increased, yielding eventually is expected to start inside the critical regions, and with continued increase in load the yielded area is expected to grow in size until failure is reached. At that stage, yielded areas will become contiguous and a yield line can be defined (*17*). This line may be approximated by line b″ in Figure 9. Possibility of this failure mode requires evaluation of shear and bending stresses at this section. For this evaluation, punching shear and bending stresses at this line must be checked, respectively, for a force equivalent to maximum bolt tension and a bending moment due to this force acting at the eccentricity r_{by} shown in Figure 9. Bending moment M_{by} at the yield line may be estimated as

$$M_{by} = T_l \cdot r_{by} \tag{24}$$

where $r_{by} = x_l - d - d_b/2$ approximates the moment arm. Length of the effective plate section for yield line bending b″ in Figure 9 may be approximated from the following equation

$$b'' = [b' - (0.25d)(a/d)]\cos\phi \tag{25}$$

in which b' is calculated from

$$b' = \sqrt{(y_l - y_r)^2 + (x_l - x_r)^2} \tag{26}$$

where y_l, y_r, x_l, and x_r were defined in Section B of Chapter 1. Section modulus S_x for bending, assuming a rectangular beam element of area $b'' \cdot t$, may be obtained as

$$S_x = b'' \cdot t^2/6 = \{[b' - (0.25d)(a/d)] \cdot \cos\phi\} \cdot t^2/6 \tag{27}$$

and maximum bending stress at the yield line f_{by} may be calculated using

$$f_{by} = M_{by}/S_x = 6T_1 \cdot r_{by}/\{[b' - (0.25\ d)\ (a/d)] \cdot \cos\phi \cdot t^2\} \le F_b = 1.4 \cdot 0.66 \cdot F_y \qquad (28)$$

Equation 28 can be solved for an estimate of required thickness t as follows:

$$t \ge \sqrt{6T_1 r_{by} / \{[b' - (0.25d)(a/d)]\cos\Phi \cdot F_b\}} \qquad (29)$$

Shear stress at the yield line may be estimated for an area A_{vy} given by

$$A_{vy} = b'' \cdot t \qquad (30)$$

and corresponding shear stress f_{vy} may thus be obtained using

$$f_{vy} = T_1/A_{vy} \le F_v = 1.4 \cdot 0.4 \cdot F_y \qquad (31)$$

D. FLEXURAL STRESSES AT THE RESULTANT PRESSURE LINE

The plate section defined by the pressure line at the mast/mast-arm toe is also critical for shear and bending due to bearing stresses. Shear and bending stresses at this section may be calculated based on the assumed bearing stress distribution in Figure 5 for the compression side of the pressure line. The effective force P_{bb}, due to bearing stresses, acting at this section for Cases 1 and 3, and Case 2, respectively, is given by

$$P_{bb} = (5/12)\ f_{b\text{-}max} \cdot a \cdot (s_l + s_r) \text{ for Cases 1 and 3} \qquad (32A)$$

and

$$P_{bb} = (7/18)\ f_{b\text{-}max} \cdot s_l \cdot s_r \text{ for Case 2} \qquad (32B)$$

This force may be used for checking shear stress on a section of area A_{vb}:

$$A_{vb} = b' \cdot t \qquad (33)$$

and thus the resulting shear stress f_{vb} may be obtained using

$$f_{vb} = P_{bb}/(b' \cdot t) \le 1.4 \cdot 0.4 \cdot F_y \qquad (34)$$

To calculate bending moment at this section, moment arm may be assumed as

$$r_{bb} = (s_l + s_r)/4 \quad \text{for Cases 1 and 3} \qquad (35A)$$

and

$$r_{bb} = (1/6)\ [2 \cdot (\sin\phi + \cos\phi)\ a_1 - d] \quad \text{for Case 2} \qquad (35B)$$

Section modulus S_x for bending at the resultant pressure line may be obtained using

$$S_x = b' \, t^2/6 \qquad (36)$$

and maximum bending stress f_{bb} due to bearing stresses may be calculated from

$$f_{bb} = P_{bb} \cdot r_{bb} / S_x \;\leq\; F_b = 1.4 \cdot 0.66 \cdot F_y \qquad (37)$$

E. SUMMARY OF THE PROPOSED PROCEDURE

The proposed procedure may be summarized in the following 14 steps:

Step 1: Determine design moments M_x and M_y and axial load P_a (if applicable) and determine the required mast/mast-arm diameter at the plate. For a base-plate-span-wire configuration, the following two cases should be checked when wire orientation relative to the plate is not specified: 1) M_x = calculated total moment and $M_y = 0$, and 2) $M_x = M_y$ = calculated total moment x cos 45°. For an end-plate, use calculated M_x and M_y directly.

Step 2: Assume a plate size (about 1.5 times the diameter of the mast or mast-arm) and thickness (Eq. 29 can be used for an initial estimate).

Step 3: Assume a bolt size and determine an edge distance as specified by the design codes (*15,18*).

Step 4: Determine total moment and angle of incidence ϕ from Eqs. 1 and 2, respectively. To account for plate symmetry, ϕ must be adjusted to (90°- ϕ) if it is greater than 45°.

Step 5: Calculate bolt distances x_1 to x_4 from the resultant pressure line using Eqs. 3, and bolt forces T_1 to T_4 using Eqs. 4 and 7. Readjust bolt size d_b based on maximum calculated bolt tension.

Step 6: Calculate normal distances to the resultant pressure line r_{c2} and r_{c3} from corners C_2 and C_3, using Eqs. 8. A negative sign of a normal distance indicates that the corresponding corner lies above the resultant pressure line. This determines the sides of the plate on which points $P_l(x_l, y_l)$ and $P_r(x_r, y_r)$ fall.

Step 7: Calculate the coordinates of points $P_l(x_l, y_l)$ and $P_r(x_r, y_r)$ from the appropriate equations, (Eqs. 9 and 10 or 11 and 12), and the lengths b′ and b″ from Eqs. 26 and 25, respectively.

Step 8: Calculate maximum bearing stress f_{b_max} from Eqs. 16 or 17, as appropriate, and ensure that it is within allowable bearing stress limits. If not, plate size must be increased or edge distances readjusted.

In Steps 9 through 12, for each step calculate required stress and ensure that it does not exceed allowable stress limits:

Step 9: Bending stress between tension side bolts f_{fb}, from fixed beam approximation, using Eq. 20.

Step 10: Shear stress at the sides of the mast/mast-arm $f_{v\_torsion}$ from torsional beam approximation, using Eq. 23.

Step 11: Bending and shear stresses at the yield line, from maximum bolt tension and assumed eccentricity r_{by} , using Eqs. 28 and 31, respectively.

Step 12: Bending and shear stresses at the pressure line, from assumed bearing pressure distribution, using Eqs. 37 and 34, respectively.

Step 13: Readjust plate size or thickness, as appropriate, until all conditions are met.

Step 14: Check adequacy of bolts for all applicable stresses, including tension, shear, and fatigue.

3. SPREADSHEET IMPLEMENTATION AND APPLICATIONS

A. SPREADSHEET IMPLEMENTATION

A spreadsheet implementing the proposed procedure has been developed as an efficient tool for engineers designing the types of base-plates and end-plates discussed in this report. The user first has to specify the type of plate to be analyzed and the desired system of units, and enter the following information:

a	Square plate side length
t	Trial plate thickness
d_b	Bolt diameter
e	Edge distance from the center of bolt hole
d	Mast/mast-arm outside diameter
F_y	Plate material yield stress
M_x	Gravity load moment (or total moment in case of a span wire)
M_y	Wind load moment
P_a	Axial load

Once this information is entered, the critical stress cases discussed in the previous chapter are calculated and checked against allowable stresses, and a message will be displayed indicating the checked result for each case. Allowable stresses for bending, shear, and bearing (F_b, F_v, and F_p, respectively) are calculated and factored assuming Group II or III design loads from the AASHTO specifications (*9*). Also displayed on the spreadsheet are bolt forces, load effects, and section modulus for each case, as well as other parameters used in calculating these stresses. Trial plate thickness can be fine-tuned by resetting and incrementing t to displayed initial thickness, which is estimated based on yield line bending, until all stress checks are met. Plate dimensions or edge distances should also be modified in case maximum calculated bearing stress exceeds allowable bearing stress F_p. When span-wire orientation relative to a base-plate is not specified, plate thickness should be governed by the larger of the two thicknesses estimated for the following loading cases: 1) M_x = calculated total moment and $M_y = 0$, and 2) $M_x = M_y$ = calculated total moment x cos 45°. For an end-plate, calculated M_x and M_y should be used directly.

B. APPLICATIONS

The developed spreadsheet was used to design two sets of base-plates from five major suppliers of traffic poles in this state. The first set consisted of 23 plates manufactured by three of those suppliers and was part of the RR 159 study, which included full-scale testing and finite-element analysis (*14*). The second set included 12 plates, randomly selected from recent plate designs

Table 2. Plate parameters, design moments, and comparison of first-set plate thicknesses obtained by three methods.

Plate Parameters and Moments						Plate Thickness (in.)			
						Manufacturers		RR 159 Procedure	Proposed Procedure
Plate ID*	a (in.)	e (in.)	d (in.)	d_b (in.)	Moment (lb-in.)	Used	NYS Method		
C326	17	2.5	10.75	1.25	882000	1.5	1.82	1.96	1.87
C328	18	2.65	10.75	1.25	954000	1.5	1.86	1.99	2
C430	18	2.65	12.75	1.5	1368000	1.75	2.12	2.27	2.11
C530	23	3.35	12.75	1.5	1710000	1.75	2.24	2.32	2.53
C530'	18	2.65	12.75	1.75	1710000	2	2.37	2.52	2.3
C732	25	3.65	16	1.5	2562000	2.5	2.55	2.77	2.67
C832	22	3.2	16	2	2928000	2.25	2.78	3.17	2.75
S324	21	3.1	10	1.5	810000	1.5	1.39	1.55	1.72
S328	21	3.1	11	1.5	954000	1.5	1.49	1.57	1.73
S334	27	4.65	13	1.5	1170000	1.5	1.46	1.6	1.8
S434	27	4.65	14	1.75	1560000	1.75	1.67	1.79	1.93
S530	27	4.65	14.5	1.75	1710000	1.75	1.73	1.87	1.96
S632	27	3.25	16.5	2	2196000	2	1.99	2.27	2.23
S832	32	4.7	19	2	2928000	2.25	2.08	2.35	2.25
S934	33	4.65	21	2.25	3510000	2.25	2.22	2.55	2.32
S1036	35	5.66	22.5	2.25	4140000	2.25	2.28	3.27	2.34
U226	14.1	2.1	10.5	1.25	588000	1.5	1.53	1.82	1.52
U530	20.5	3.2	14	1.75	1710000	2	2.23	2.29	2.15
U636	26	4.7	16.72	1.75	2484000	2	2.36	2.51	2.4
U832	24.5	3.95	16.5	2	2928000	2.5	2.66	2.75	2.6
U840	26	4.15	18.5	2.25	3696000	2.5	2.84	3	2.71
U1040	27.5	4.03	21	2.25	4620000	2.75	3.05	4.3	3.09
U1044	29	4.6	21.5	2.25	5100000	2.75	3.1	3.55	3.05

*Yield stresses for plates starting with initials C, S, and U are, respectively, 36, 50, and 36 ksi.

Table 3. Plate parameters, design moments, and comparison of second-set plate thicknesses obtained by three methods.

Plate Parameters and Moments						Plate Thickness (in.)			
						Manufacturers		RR 159 Procedure	Proposed Procedure
Plate ID*	a (in.)	e (in.)	d (in.)	d_b (in.)	Moment (lb-in.)	Used	NYS Method		
VA728	19.5	3	14	1.75	2352000	2	2.03	2.17	1.97
VA538	22	2.87	15.5	2	2280000	2	1.97	2.2	2.1
VA1038	27.5	3.64	18	2	4560000	2.5	2.54	2.82	2.67
UN442	22	3.375	13	1.25	1510656	2.25	2.11	2.27	2.32
UN451	25	3.82	16.5	1.5	2529000	2.5	2.49	2.66	2.56
UN470	29	4.625	21	2.37	5091720	3.25	3.14	3.39	3.03
SU430	19	2.78	13	1.5	1537632	2	1.88	2	1.89
SU633	22	3.25	15.5	1.75	2549232	2.25	2.23	2.31	2.23
SU836	24	3.5	17	2	3722688	2.75	2.57	2.73	2.57
VS226	15	1.85	12.24	1	667585	1.73	1.35	2.1	1.53
VS629	20.98	2.36	18.31	1.5	2268276	2.52	2.06	3.31	2.45
VS1139	27	3.25	23	2	5108004	3.27	2.72	3.75	3.2

*Yield stresses for plates starting with initials VA, UN, SU, and VS are, respectively, 60, 36, 50, and 50 ksi.

submitted to the Department by five manufacturers, including two who participated in the RR 159 study. All plates in the two sets were also designed according to the NYS method and the RR 159 procedure (*14*).

Estimated plate thicknesses using the spreadsheet program are compared with those obtained using the RR 159 procedure and those supplied by manufacturers. These are given in Tables 2 and 3, and also shown in Figures 10A and 10B, respectively, for the first and second sets. (Plate dimensions, bolt sizes, and design moments for each plate are also included in Tables 2 and 3). Estimated plate thicknesses using the proposed procedure in Table 2 and Figure 10A show clear agreement with the RR 159 conclusion regarding deficiency of the plates in this set, which was evident by the nonlinear behavior of stresses and displacements exhibited during the testing program of that study (*14*). This deficiency was observed to result primarily because of the need for thicker plates in some cases, when moment components were applied at a 45° angle from the horizontal axis and a plate was diagonally stressed.

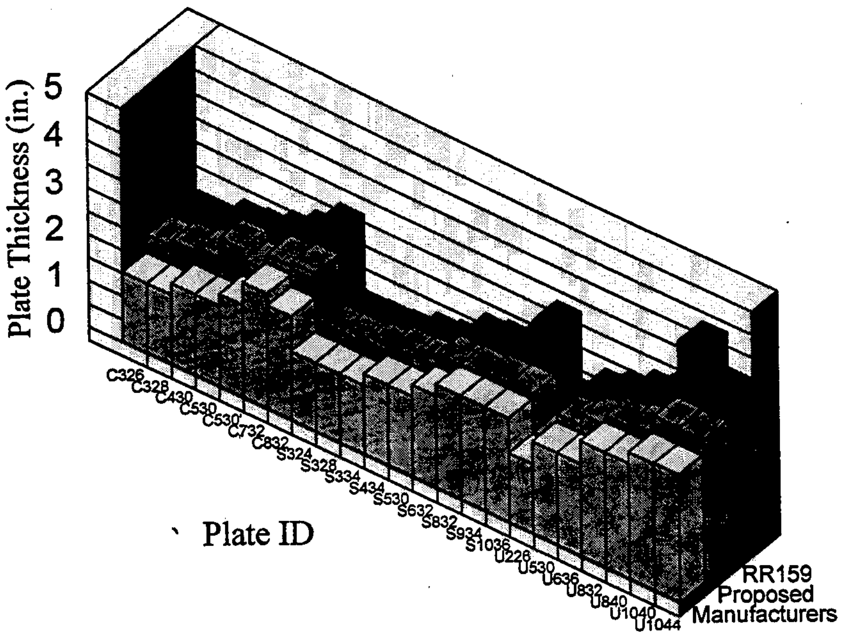
The spikes in Figure 10 represent plate thicknesses estimated using the RR 159 procedure for plates having dimensions falling outside the applicable ranges for validity of that procedure. Table 3 and Figure 10B indicate a much improved agreement between thicknesses obtained using the proposed procedure and those supplied by manufacturers. This suggests that some manufacturers may have revised their designs in response to the deficiency of these plates noted in Research Report 159. For both sets, the proposed procedure generally resulted in thicknesses much closer to those supplied by manufacturers than those based on RR 159.

Estimated stresses on base-plates in the first set obtained using the proposed procedure indicated that, on average, these stresses are within 11 to 20 percent (with a standard deviation of 7 to12 percent) of those obtained using finite-element analysis in RR 159 (*14*).

Estimated plate thicknesses using the spreadsheet program were also compared with those obtained using the NYS method and those supplied by manufacturers. This is shown in Tables 2 and 3, and also graphically in Figures 11A and 11B, respectively, for the first and second sets. Based on these results, the NYS method in some cases resulted in thicknesses comparable to those obtained using the proposed procedure. This was expected because the NYS method is based on checking stresses at one of the four sections considered critical in the proposed procedure (Sections A through D of Chapter 2). Smaller plate thicknesses obtained using the NYS method may also be explained by the fact that plate design is not necessarily governed by the single criterion checked using that method. Slightly larger thicknesses estimated using the NYS method (than those based on the proposed procedure) are mainly due to the different approaches used in the two methods for estimating moments and stresses for the same checking criterion. The results also show that methods used by manufacturers, other than the NYS method, sometimes may result in thicknesses smaller than those required by the NYS method.

Figure 10. Comparison of plate thicknesses obtained using the proposed procedure against those supplied by manufacturers and those estimated using the RR 159 procedure (*14*).

A. FIRST-SET PLATE THICKNESSES



B. SECOND-SET PLATE THICKNESSES

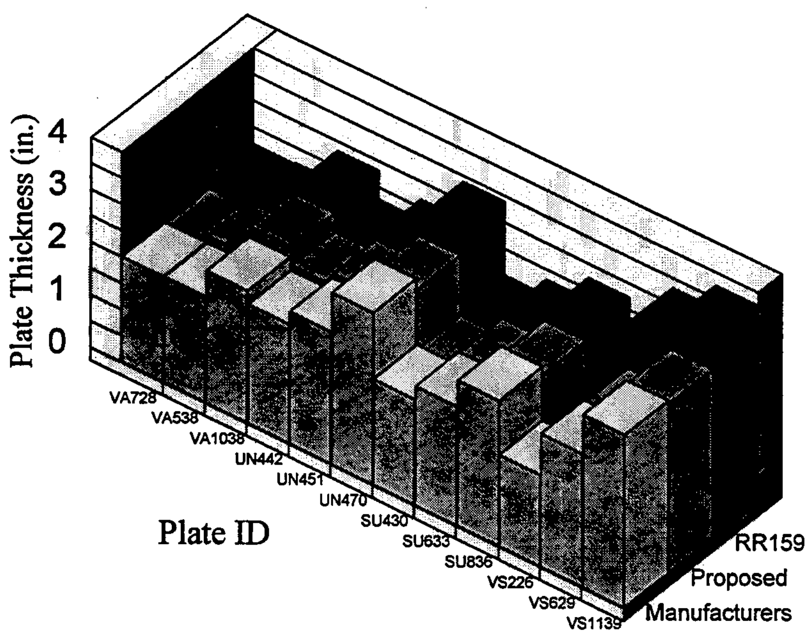
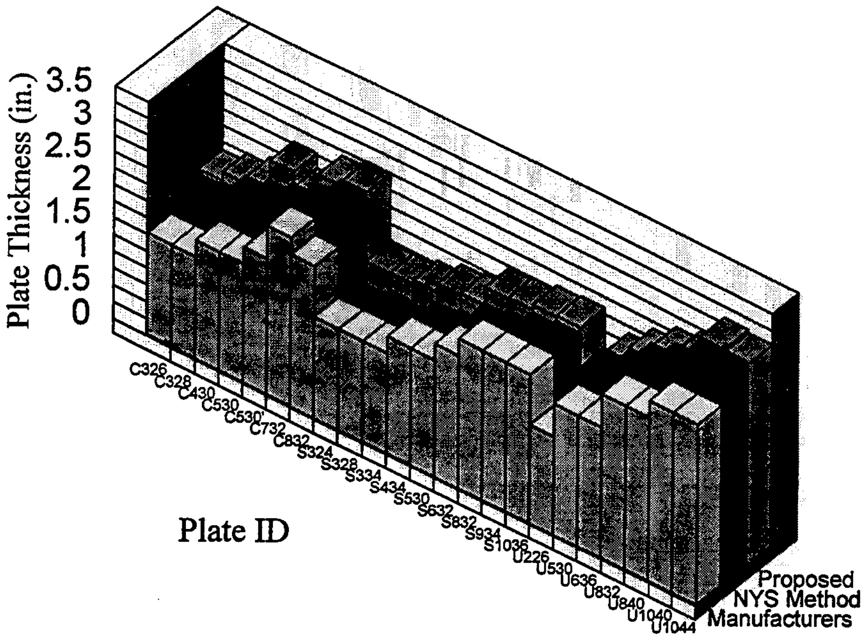
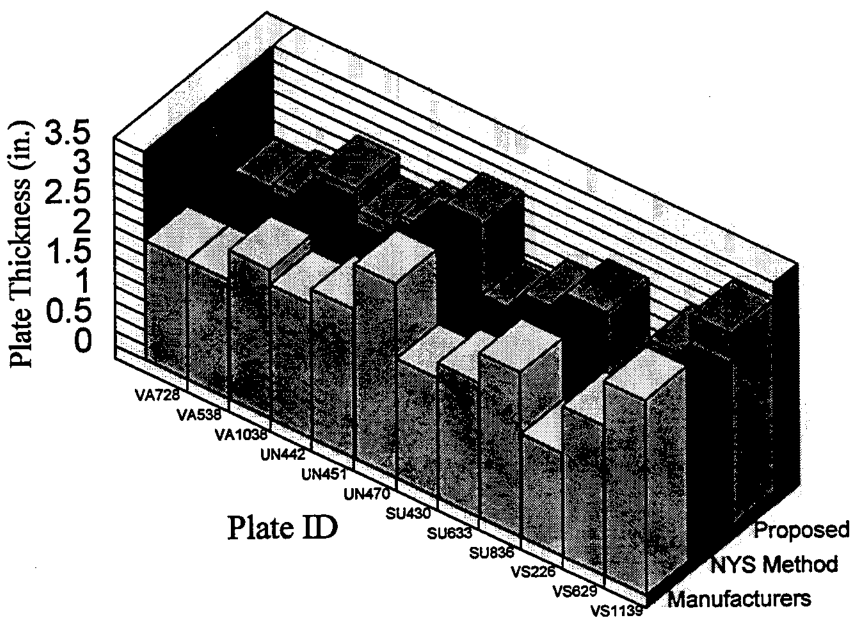


Figure 11. Comparison of plate thicknesses obtained using the proposed procedure against those supplied by manufacturers and those estimated using the NYS method.

A. FIRST-SET PLATE THICKNESSES



B. SECOND-SET PLATE THICKNESSES



CONCLUSIONS

A simplified procedure has been proposed here for design of end-plates and base-plates of cantilevered traffic sign, signal, and light structures, and for base-plates of span-wire-mounted traffic-signal structures. The procedure is based on identifying critical stress locations on the plate, and estimating the forces producing these stresses and the effective plate sections they act upon. For now, applicability of the procedure is limited only to square plates, but with additional physical testing and finite-element analysis its applicability could also be extended to include rectangular plates.

Plate designs obtained using the proposed procedure showed good agreement with those estimated using finite-element analysis, and also confirmed conclusions reached in Research Report 159 through physical testing (*14*). The developed spreadsheet implementing the proposed procedure is an efficient tool that should allow plate designers to optimize their designs, or check those designed by others, reliably and with very little effort.

The proposed procedure has five advantages:

1. It gives the Department a general, rational procedure for design of both end- and base-plates for the types of traffic support structures discussed here.

2. It is based on simple engineering mechanics concepts that verify critical stresses at more than one location on the plate, unlike the current New York method for base-plates which assumes stresses to be critical in only one section.

3. It has demonstrated a capability to provide reliable and consistent results.

4. It is more appropriate for cases when span-wire orientation relative to a base-plate is specified or field-verified, and

5. Its user-friendly spreadsheet makes it appealing for implementation as a ready-to-use tool for designing these items.

ACKNOWLEDGMENTS

The authors recognize the contribution of Dr. Ruijia Mu, formerly with the Transportation Research and Development Bureau, to the early stage of this study. Thanks are due to Jonathan Kunin of the Transportation Research and Development Bureau for his assistance in preparing the figures in the report. Thanks are also extended to Larry Johanson of the Structures Division, and to Larry Brown, Richard Stempel, and Richard Marriott of the Design Quality Assurance Bureau, for reviewing this report and for their invaluable comments. Thanks are also offered to the certified traffic structures manufacturers for the State of New York for their cooperation in providing various information related to the design of end-plates and base-plates.

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APPENDIX
Examples Illustrating Application of the Proposed Procedure

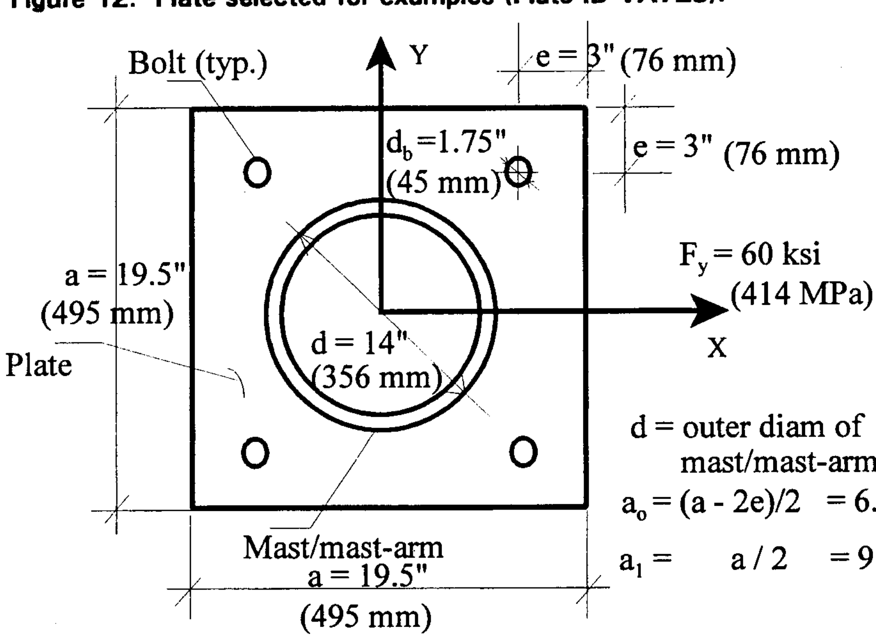
Plate VA728, shown in Figure 12, was selected to illustrate application of the proposed procedure, using the spreadsheet tool, in both U.S. customary and SI (metric) units. In the first example, plate thickness is determined assuming it to be a base-plate bearing on steel, to be designed for a total moment of 2,352,000 lb-in. (2.7×10^8 N-mm). In the second example, thickness is estimated assuming it to be an end-plate resisting applied moments $M_x = 2,352,000 \times \cos 30° = 2,036,892$ lb-in. (2.3×10^8 N-mm) and $M_y = 2,352,000 \times \sin 30° = 1,176,000$ lb- in. (1.3×10^8 N-mm). For both cases, axial load on the plates is neglected and spreadsheet results are compared to those obtained using Mathcad programs.

Example 1: Base-Plate Design

Spreadsheet design for this case is shown on pages 31 and 41, respectively, for two cases:

1. Total moment applied at $\phi = 0°$ and
2. Total moment applied at $\phi = 45°$.

Figure 12. Plate selected for examples (Plate ID VA728).



d = outer diam of mast/mast-arm

$a_0 = (a - 2e)/2 = 6.75"$ (171mm)

$a_1 = a / 2 = 9.75"$ (248mm)

From Cases 1 and 2, design thickness is selected as 1.97 in. (50 mm), the larger of the two thicknesses, 1.97 and 1.89 in. (50 and 48 mm), that are required to satisfy all stress-checking requirements. Thicknesses obtained for these two cases were also used in the following Mathcad programs of the proposed procedure, for comparison with the spreadsheet results. Similarity between these results confirms correctness of the spreadsheet implementation of the proposed procedure.

Example 2: End-Plate Design

The spreadsheet solution for this case is shown on page 51 for the design moments M_x and M_y applied simultaneously, and the plate is assumed to be bearing on a steel plate of similar strength. The resulting design thickness for this case is 1.87 in. (48 mm). Again, this thicknesses was employed to verify spreadsheet results for this case using the following Mathcad program. Agreement between the spreadsheet results and Mathcad program is also noted for this example.

EXAMPLE 1: ϕ = O° (U.S. Customary Units)

DESIGN OF END- AND BASE-PLATES				PLATE TYPE:	ENDPLATE / BASEPLATE	UNIT TYPE:	METRIC / US CUST	PLATE TYPE:	BASEPLATE
								UNIT TYPE:	US CUST

PLATE INPUT PARAMETERS

a (in.)	e (in.)	d (in.)	t (in.)	Pa (lb)	Fy (psi)	DESIGN ADEQUACY
19.5	3.0	14.0	1.970	0.0	60000.0	OK

ALLOWABLE STRESSES (psi)

BENDING Fb	SHEAR Fv	STEEL Fp
55440.0	33600.0	54000.0

BOLT DATA

db (in.)
1.8

DESIGN MOMENTS (lb-in.)

DL (Mx)	Wind (My	Total (Mt)
2.35E+06	0.00E+00	2.35E+06

ANGLE ϕ	
0.0	SHW HELP / CLR HELP

Initial Trial t
1.0

PLATE BEARING

STRESS fp (psi)	STEEL CHECK
2163.0	OK

YIELD LINE BENDING STRESS

r	Moment	b"	Sx	fb (psi)	CHECK
CASE	DOES	NOT	APPLY		OK

YIELD LINE SHEAR STRESS

Force	Area			fv (psi)	CHECK
85440.4	21.1			4048.7	OK

FIXED BEAM BENDING STRESS

Force	Moment	Sx		fb (psi)	CHECK
170880.8	2.1E+05	3.9		55039.1	OK

TORSIONAL BEAM SHEAR STRESS

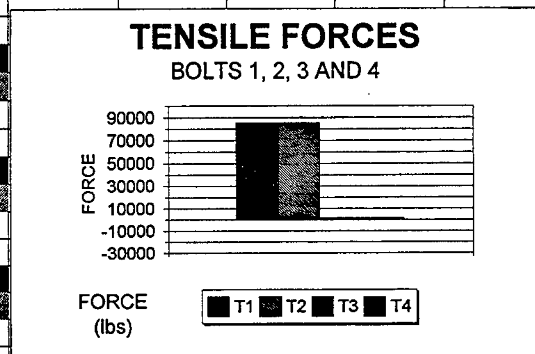
Force	Moment	r		fv (psi)	CHECK
85440.42	4.3E+05	16.7		25528.3	OK

PRESSURE LINE BENDING STRESS

Force	Moment	Sx		fb (psi)	CHECK
96659.9	1.3E+05	12.6		10537.4	OK

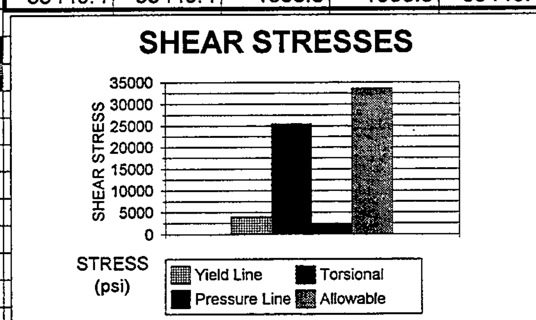
PRESSURE LINE SHEAR STRESS

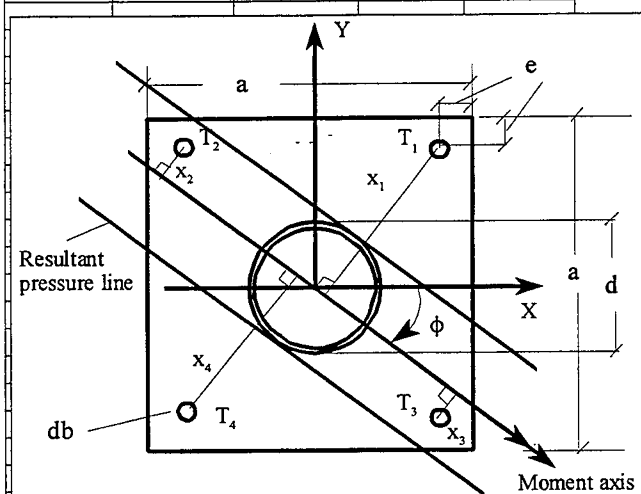
Force	Area			fv (psi)	CHECK
96659.9	38.4			2516.2	OK

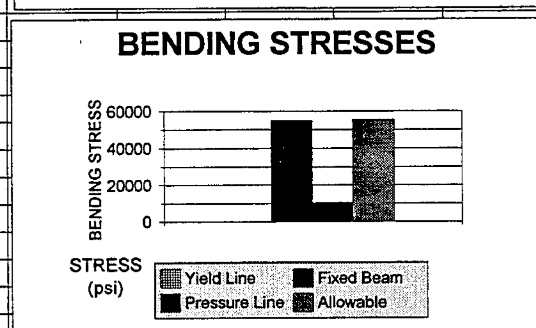


BOLT FORCES (lb)

T1	T2	T3	T4	Tmax
85440.4	85440.4	1553.5	1553.5	85440.4







EXAMPLE 1: Base Plate:

Plate ID: VA728

Case: $\phi = 0$

Units: US Customary

$t := 1.97 \cdot in$ Assumed thickness

INPUT DATA:

$M_x := 2352000 \cdot lb \cdot in$ $M_y := 0 \cdot lb \cdot in$ $P_a := 0 \cdot lb$ $F_y := 60 \cdot 10^3 \cdot \frac{lb}{in^2}$

$a := 19.5 \cdot in$ $e := 3 \cdot in$ $d := 14 \cdot in$ $d_b := 1.75 \cdot in$

$a_0 := \frac{a - 2 \cdot e}{2}$ $a_1 := \frac{a}{2}$ $\frac{d}{t} = 7.107$ $C_t := 0.30$ $u := e$

CALCULATIONS:

Calculate total moment and angle of incidence:

$M_t := \sqrt{M_x^2 + M_y^2}$ $\phi := \operatorname{atan}\left(\frac{M_y}{M_x}\right)$ $\phi := \begin{vmatrix} \phi \text{ if } \phi < 45 \\ (90 \cdot deg - \phi) \text{ otherwise} \end{vmatrix}$ $M_t = 2.352 \cdot 10^6 \cdot lb \cdot in$

$\phi = 0 \cdot deg$

Calculate bolt distances to the resultant pressure line:

$x_1 := -\left(\frac{1}{2}\right) \cdot \left[2 \cdot (\sin(\phi) + \cos(\phi)) \cdot a_0 + d\right]$ $x_2 := \frac{1}{2} \cdot \left[2 \cdot (\sin(\phi) - \cos(\phi)) \cdot a_0 - d\right]$

$x_3 := -\left(\frac{1}{2}\right) \cdot \left[2 \cdot (\sin(\phi) - \cos(\phi)) \cdot a_0 + d\right]$ $x_4 := \frac{1}{2} \cdot \left[2 \cdot (\sin(\phi) + \cos(\phi)) \cdot a_0 - d\right]$

Calculate bolt distances from the resultant pressure line:

$x_1 = -13.75 \cdot in$ $x_2 = -13.75 \cdot in$ $x_3 = -0.25 \cdot in$ $x_4 = -0.25 \cdot in$

Calculate bolt distances from the center of the mast:

$x_{11} := -x_1 - \frac{d}{2}$ $x_{12} := -x_2 - \frac{d}{2}$ $x_{13} := -x_3 - \frac{d}{2}$ $x_{14} := -x_4 - \frac{d}{2}$

$x_{11} = 6.75 \cdot in$ $x_{12} = 6.75 \cdot in$ $x_{13} = -6.75 \cdot in$ $x_{14} = -6.75 \cdot in$

Calculate bolt forces:

$$T_1 := \frac{M_t - P_a \cdot \frac{d}{2}}{\left(\frac{x_1^2 + x_2^2 + x_3^2 + x_4^2}{x_1}\right)}$$

$T_1 = -8.55 \cdot 10^4 \cdot lb$ (Tension)

$T_2 := T_1 \cdot \frac{x_2}{x_1}$ $T_2 = -8.55 \cdot 10^4 \cdot lb$ (Tension)

$$T_3 := T_1 \cdot \frac{x_3}{x_1}$$ $T_3 = -1.555 \cdot 10^3 \cdot lb$ (Tension)

$$T_4 := T_1 \cdot \frac{x_4}{x_1}$$ $T_4 = -1.555 \cdot 10^3 \cdot lb$ (Tension)

Calculate coordinates of the points where the resultant pressure line intersects plate sides:

First determine which sides of the plate the resultant pressure line intersects:

$$r_{c2} := \frac{1}{2} \cdot \left[2 \cdot (\sin(\phi) - \cos(\phi)) \cdot a_1 - d \right]$$ $r_{c2} = -16.75 \cdot in$ -ve implies Side C2-C4

$$r_{c3} := -\left(\frac{1}{2}\right) \cdot \left[2 \cdot (\sin(\phi) - \cos(\phi)) \cdot a_1 + d \right]$$ $r_{c3} = 2.75 \cdot in$ +ve implies Side C1-C3

Left and right points coordinates:

$$x_l := \begin{cases} -a_1 & \text{if } r_{c2} < 0 \\ -\frac{1}{2 \cdot \sin(\phi)} \cdot (2 \cdot \cos(\phi) \cdot a_1 + d) & \text{otherwise} \end{cases}$$

$$y_l := \begin{cases} -\frac{1}{2 \cdot \cos(\phi)} \cdot (-2 \cdot \sin(\phi) \cdot a_1 + d) & \text{if } r_{c2} < 0 \\ a_1 & \text{otherwise} \end{cases}$$

$x_l = -9.75 \cdot in$ $y_l = -7 \cdot in$

$$x_r := \begin{cases} -\frac{1}{2 \cdot \sin(\phi)} \cdot (-2 \cdot \cos(\phi) \cdot a_1 + d) & \text{if } r_{c3} < 0 \\ a_1 & \text{otherwise} \end{cases}$$

$$y_r := \begin{cases} -a_1 & \text{if } r_{c3} < 0 \\ -\frac{1}{2 \cdot \cos(\phi)} \cdot (2 \cdot \sin(\phi) \cdot a_1 + d) & \text{otherwise} \end{cases}$$

$x_r = 9.75 \cdot in$ $y_r = -7 \cdot in$

Calculate left and right distances used in estimating bearing stresses distribution:

$$s_l := \begin{cases} \sqrt{(-a_1 - x_l)^2 + (-a_1 - y_l)^2} & \text{if } r_{c2} < 0 \\ \sqrt{(-a_1 - x_l)^2 + (a_1 - y_l)^2} & \text{otherwise} \end{cases}$$ $s_l = 2.75 \cdot in$

$$s_r := \begin{cases} \sqrt{(-a_1 - x_r)^2 + (-a_1 - y_r)^2} & \text{if } r_{c3} < 0 \\ \sqrt{(a_1 - x_r)^2 + (-a_1 - y_r)^2} & \text{otherwise} \end{cases}$$ $s_r = 2.75 \cdot in$

Calculate maximum bearing stress:

$$f_{b\_max} := \begin{cases} \frac{2}{3}\cdot\frac{-(T_1+T_2+T_3)+P_a}{s_l\cdot s_r} & \text{if } r_{c3}<0 \\ \frac{4}{3}\cdot\frac{-(T_1+T_2+T_3)+P_a}{a\cdot(s_l+s_r)} & \text{otherwise} \end{cases} \qquad f_{b\_max} = 2.145\cdot 10^3 \cdot \frac{lb}{in^2}$$

$$F_p := 0.9\cdot F_y \qquad F_p = 5.4\cdot 10^4 \cdot \frac{lb}{in^2}$$

Calculate maximum bending stress between the bolts on the tension side:

$$M_{fb} := (a - 2\cdot e - 2\cdot d_b)\cdot\frac{T_1+T_2}{8}\cdot\cos(\phi) \qquad M_{fb} = -2.137\cdot 10^5 \cdot lb\cdot in$$

$$S_x := \frac{(e+u)\cdot t^2}{6} \qquad S_x = 3.881\cdot in^3$$

$$f_{b\_fb} := \frac{M_{fb}}{S_x} \qquad f_{b\_fb} = -5.508\cdot 10^4 \cdot \frac{lb}{in^2}$$

$$F_b := 1.4\cdot 0.66\cdot F_y \qquad F_b = 5.544\cdot 10^4 \cdot \frac{lb}{in^2}$$

Calculate maximum torsional shear stress at the sides of the mast:

$$M_{torsion} := T_1\cdot\frac{a - 2\cdot e - 2\cdot d_b}{2} \qquad M_{torsion} = -4.275\cdot 10^5 \cdot lb\cdot in$$

$$\Gamma := d\cdot t^2\cdot C_t \qquad \Gamma = 16.3\cdot in^3$$

$$f_{vt} := \frac{M_{torsion}}{\Gamma} \qquad f_{vt} = -2.623\cdot 10^4 \cdot \frac{lb}{in^2}$$

$$F_v := 1.4\cdot 0.4\cdot F_y \qquad F_v = 3.36\cdot 10^4 \cdot \frac{lb}{in^2}$$

Calculate maximum bending and shear stresses at the yield line:

$$r_{by} := -(x_1) - d - \frac{d_b}{2} \qquad r_{by} = -1.125\cdot in$$

Since r_by is -ve, case does not apply.

$$b_{prime} := \sqrt{(y_l - y_r)^2 + (x_l - x_r)^2} \qquad b_{prime} = 19.5\cdot in$$

$$b_{dprime} := \left(b_{prime} - 0.25\cdot\frac{a}{d}\cdot d\right)\cdot\cos(\phi) \qquad b_{dprime} = 14.625\cdot in$$

Calculate maximum bending and shear stresses at the resultant pressure line:

$f_{b\_max} = 2.145 \cdot 10^3 \cdot \frac{lb}{in^2}$ $\qquad s_l = 2.75 \cdot in$ $\qquad s_r = 2.75 \cdot in$

$$P_{bb} := \begin{vmatrix} \frac{7}{18} \cdot f_{b\_max} \cdot s_l \cdot s_r \text{ if } r_{c3} < 0 \\ \frac{5}{12} \cdot f_{b\_max} \cdot (s_l + s_r) \cdot a \text{ otherwise} \end{vmatrix}$$

$P_{bb} = 9.586 \cdot 10^4 \cdot lb$

$$f_{vb} := \frac{P_{bb}}{b_{prime} \cdot t}$$

$f_{vb} = 2.495 \cdot 10^3 \cdot \frac{lb}{in^2}$

$F_v := 1.4 \cdot 0.4 \cdot F_y$ $\qquad F_v = 3.36 \cdot 10^4 \cdot \frac{lb}{in^2}$

$$r_{bb} := \begin{vmatrix} \frac{1}{6} \cdot \left[2 \cdot (\sin(\phi) + \cos(\phi)) \cdot a_1 - d \right] \text{ if } r_{c3} < 0 \\ \frac{s_l + s_r}{4} \text{ otherwise} \end{vmatrix}$$

$r_{bb} = 1.375 \cdot in$

$$S_x := \frac{b_{prime} \cdot t^2}{.6}$$

$S_x = 12.613 \cdot in^3$

$M_{bb} := P_{bb} \cdot r_{bb}$ $\qquad M_{bb} = 1.318 \cdot 10^5 \cdot lb \cdot in$

$$f_{bb} := \frac{M_{bb}}{S_x}$$

$f_{bb} = 1.045 \cdot 10^4 \cdot \frac{lb}{in^2}$

$F_b := 1.4 \cdot 0.66 \cdot F_y$ $\qquad F_b = 5.544 \cdot 10^4 \cdot \frac{lb}{in^2}$

End of Example 1 Case ϕ = 0 deg.

EXAMPLE 1: ϕ = O° (Metric Units)

DESIGN OF END- AND BASE-PLATES				PLATE TYPE:	ENDPLATE / BASEPLATE	UNIT TYPE:	METRIC / US CUST	PLATE TYPE:	BASEPLATE
								UNIT TYPE:	METRIC

PLATE INPUT PARAMETERS								ALLOWABLE STRESSES (MPa		
a (mm)	e (mm)	d (mm)	t (mm)	Pa (N)	Fy (MPa)	DESIGN ADEQUACY		BENDING Fb	SHEAR Fv	STEEL Fp
495.3	76.2	355.6	50.000	0.0	413.7		OK	382.3	231.7	372.3

BOLT DATA		DESIGN MOMENTS (N-mm)				ANGLE	
db (mm)		DL (Mx)	Wind (My	Total (Mt)		ϕ	SHW HELP
44.5		2.66E+08	0.00E+00	2.66E+08		0.0	CLR HELP

PLATE BEARING	
STRESS fp (MPa)	STEEL CHECK
14.9	OK

YIELD LINE BENDING STRESS					CHECK	Initial
r	Moment	b"	Sx	fb (MPa)		Trial t
CASE	DOES	NOT	APPLY		OK	25.4

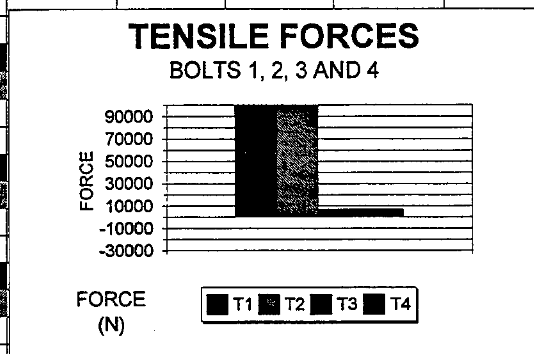
YIELD LINE SHEAR STRESS					CHECK
Force	Area			fv (MPa)	
380753.1	13604.7			28.0	OK

FIXED BEAM BENDING STRESS					CHECK
Force	Moment	Sx		fb (MPa)	
761506.2	2.4E+07	63500.0		380.6	OK

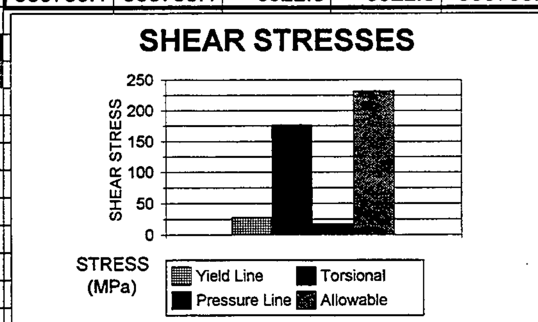
TORSIONAL BEAM SHEAR STRESS					CHECK
Force	Moment	r		fv (MPa)	
380753.1	4.8E+07	273812.0		176.5	OK

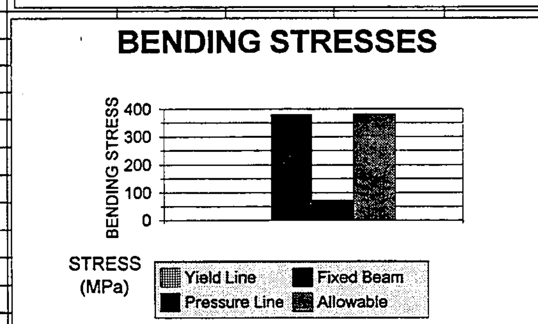
PRESSURE LINE BENDING STRESS					CHECK
Force	Moment	Sx		fb (MPa)	
430751.0	1.5E+07	206375.0		72.9	OK

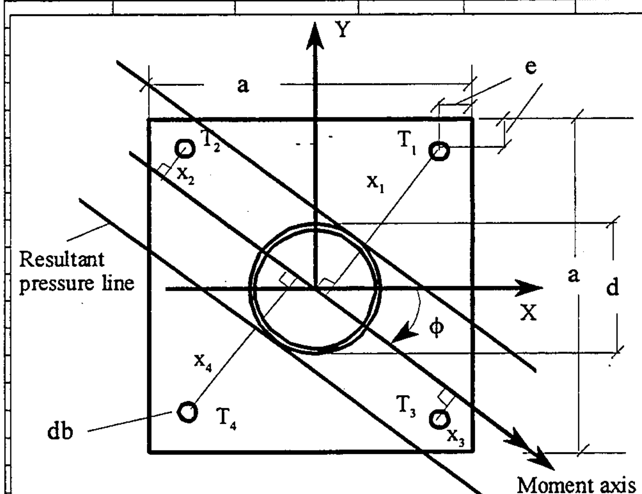
PRESSURE LINE SHEAR STRESS					CHECK
Force	Area			fv (MPa)	
430751.0	24765.0			17.4	OK



BOLT FORCES (N)				
T1	T2	T3	T4	Tmax
380753.1	380753.1	6922.8	6922.8	380753.1







EXAMPLE 1: Base Plate:

Plate ID: VA728

Case: ϕ =0

Units: SI (Metric) $N := kg \cdot \frac{m}{sec^2}$

$t := 50.0 \cdot mm$ Assumed thickness

$MPa := Pa \cdot 10^6$

INPUT DATA:

$M_x := 2.66 \cdot 10^8 \cdot N \cdot mm$ $\quad M_y := 0 \cdot N \cdot mm$ $\quad P_a := 0 \cdot N$ $\quad F_y := 413.7 \cdot MPa$

$a := 495.3 \cdot mm$ $\quad e := 76.2 \cdot mm$ $\quad d := 355.6 \cdot mm$ $\quad d_b := 44.5 \cdot mm$

$a_0 := \frac{a - 2 \cdot e}{2}$ $\quad a_1 := \frac{a}{2}$ $\quad \frac{d}{t} = 7.112$ $\quad C_t := 0.30$ $\quad u := e$

CALCULATIONS:

Calculate total moment and angle of incidence:

$$M_t := \sqrt{M_x^2 + M_y^2} \qquad \phi := atan\left(\frac{M_y}{M_x}\right) \qquad \phi := \begin{cases} \phi & \text{if } \phi < 45 \\ (90 \cdot deg - \phi) & \text{otherwise} \end{cases}$$

$M_t = 2.66 \cdot 10^8 \cdot N \cdot mm$

$\phi = 0 \cdot deg$

Calculate bolt distances to the resultant pressure line:

$$x_1 := -\left(\frac{1}{2}\right) \cdot \left[2 \cdot (\sin(\phi) + \cos(\phi)) \cdot a_0 + d\right] \qquad x_2 := \frac{1}{2} \cdot \left[2 \cdot (\sin(\phi) - \cos(\phi)) \cdot a_0 - d\right]$$

$$x_3 := -\left(\frac{1}{2}\right) \cdot \left[2 \cdot (\sin(\phi) - \cos(\phi)) \cdot a_0 + d\right] \qquad x_4 := \frac{1}{2} \cdot \left[2 \cdot (\sin(\phi) + \cos(\phi)) \cdot a_0 - d\right]$$

Calculate bolt distances from the resultant pressure line:

$x_1 = -349.25 \cdot mm$ $\quad x_2 = -349.25 \cdot mm$ $\quad x_3 = -6.35 \cdot mm$ $\quad x_4 = -6.35 \cdot mm$

Calculate bolt distances from the center of the mast:

$x_{11} := -x_1 - \frac{d}{2}$ $\quad x_{12} := -x_2 - \frac{d}{2}$ $\quad x_{13} := -x_3 - \frac{d}{2}$ $\quad x_{14} := -x_4 - \frac{d}{2}$

$x_{11} = 171.45 \cdot mm$ $\quad x_{12} = 171.45 \cdot mm$ $\quad x_{13} = -171.45 \cdot mm$ $\quad x_{14} = -171.45 \cdot mm$

Calculate bolt forces:

$$T_1 := \frac{M_t - P_a \cdot \frac{d}{2}}{\left(\frac{x_1^2 + x_2^2 + x_3^2 + x_4^2}{x_1}\right)}$$

$T_1 = -3.807 \cdot 10^5 \cdot N$ (Tension)

$$T_2 := T_1 \cdot \frac{x_2}{x_1}$$

$T_2 = -3.807 \cdot 10^5 \cdot N$ (Tension)

$$T_3 := T_1 \cdot \frac{x_3}{x_1}$$ $T_3 = -6.922 \cdot 10^3 \cdot N$ (Tension)

$$T_4 := T_1 \cdot \frac{x_4}{x_1}$$ $T_4 = -6.922 \cdot 10^3 \cdot N$ (Tension)

Calculate coordinates of the points where the resultant pressure line intersects plate sides:

First determine which sides of the plate the resultant pressure line intersects:

$$r_{c2} := \frac{1}{2} \cdot \left[2 \cdot (\sin(\phi) - \cos(\phi)) \cdot a_1 - d \right]$$ $r_{c2} = -425.45 \cdot mm$ -ve implies Side C2-C4

$$r_{c3} := -\left(\frac{1}{2}\right) \cdot \left[2 \cdot (\sin(\phi) - \cos(\phi)) \cdot a_1 + d \right]$$ $r_{c3} = 69.85 \cdot mm$ +ve implies Side C1-C3

Left and right points coordinates:

$$x_l := \begin{cases} -a_1 & \text{if } r_{c2} < 0 \\ -\frac{1}{2 \cdot \sin(\phi)} \cdot (2 \cdot \cos(\phi) \cdot a_1 + d) & \text{otherwise} \end{cases}$$

$$y_l := \begin{cases} -\frac{1}{2 \cdot \cos(\phi)} \cdot (-2 \cdot \sin(\phi) \cdot a_1 + d) & \text{if } r_{c2} < 0 \\ a_1 & \text{otherwise} \end{cases}$$

$x_l = -247.65 \cdot mm$ $y_l = -177.8 \cdot mm$

$$x_r := \begin{cases} -\frac{1}{2 \cdot \sin(\phi)} \cdot (-2 \cdot \cos(\phi) \cdot a_1 + d) & \text{if } r_{c3} < 0 \\ a_1 & \text{otherwise} \end{cases}$$

$$y_r := \begin{cases} -a_1 & \text{if } r_{c3} < 0 \\ -\frac{1}{2 \cdot \cos(\phi)} \cdot (2 \cdot \sin(\phi) \cdot a_1 + d) & \text{otherwise} \end{cases}$$

$x_r = 247.65 \cdot mm$ $y_r = -177.8 \cdot mm$

Calculate left and right distances used in estimating bearing stresses distribution:

$$s_l := \begin{cases} \sqrt{(-a_1 - x_l)^2 + (-a_1 - y_l)^2} & \text{if } r_{c2} < 0 \\ \sqrt{(-a_1 - x_l)^2 + (a_1 - y_l)^2} & \text{otherwise} \end{cases}$$

$s_l = 69.85 \cdot mm$

$$s_r := \begin{cases} \sqrt{(-a_1 - x_r)^2 + (-a_1 - y_r)^2} & \text{if } r_{c3} < 0 \\ \sqrt{(a_1 - x_r)^2 + (-a_1 - y_r)^2} & \text{otherwise} \end{cases}$$

$s_r = 69.85 \cdot mm$

Calculate maximum bearing stress:

$$f_{b\_max} := \begin{cases} \frac{2}{3}\cdot\frac{-(T_1+T_2+T_3)+P_a}{s_l\cdot s_r} & \text{if } r_{c3}<0 \\ \frac{4}{3}\cdot\frac{-(T_1+T_2+T_3)+P_a}{a\cdot(s_l+s_r)} & \text{otherwise} \end{cases} \qquad f_{b\_max} = 14.805\cdot\text{MPa}$$

$$F_p := 0.9\cdot F_y \qquad F_p = 372.33\cdot\text{MPa}$$

Calculate maximum bending stress between the bolts on the tension side:

$$M_{fb} := (a - 2\cdot e - 2\cdot d_b)\cdot\frac{T_1+T_2}{8}\cdot\cos(\phi) \qquad M_{fb} = -2.416\cdot10^7\cdot\text{N}\cdot\text{mm}$$

$$S_x := \frac{(e+u)\cdot t^2}{6} \qquad S_x = 6.35\cdot10^4\cdot\text{mm}^3$$

$$f_{b\_fb} := \frac{M_{fb}}{S_x} \qquad f_{b\_fb} = -380.54\cdot\text{MPa}$$

$$F_b := 1.4\cdot0.66\cdot F_y \qquad F_b = 382.259\cdot\text{MPa}$$

Calculate maximum torsional shear stress at the sides of the mast:

$$M_{torsion} := T_1\cdot\frac{a - 2\cdot e - 2\cdot d_b}{2} \qquad M_{torsion} = -4.833\cdot10^7\cdot\text{N}\cdot\text{mm}$$

$$\Gamma := d\cdot t^2\cdot C_t \qquad \Gamma = 2.667\cdot10^5\cdot\text{mm}^3$$

$$f_{vt} := \frac{M_{torsion}}{\Gamma} \qquad f_{vt} = -181.21\cdot\text{MPa}$$

$$F_v := 1.4\cdot0.4\cdot F_y \qquad F_v = 231.672\cdot\text{MPa}$$

Calculate maximum bending and shear stresses at the yield line:

$$r_{by} := -(x_l) - d - \frac{d_b}{2} \qquad r_{by} = -28.6\cdot\text{mm}$$

Since r_by is -ve, case does not apply.

$$b_{prime} := \sqrt{(y_l - y_r)^2 + (x_l - x_r)^2} \qquad b_{prime} = 495.3\cdot\text{mm}$$

$$b_{dprime} := \left(b_{prime} - 0.25\cdot\frac{a}{d}\cdot d\right)\cdot\cos(\phi) \qquad b_{dprime} = 371.475\cdot\text{mm}$$

Calculate maximum bending and shear stresses at the resultant pressure line:

$f_{b\_max} = 14.805 \cdot MPa$ $s_l = 69.85 \cdot mm$ $s_r = 69.85 \cdot mm$

$$P_{bb} := \begin{cases} \frac{7}{18} \cdot f_{b\_max} \cdot s_l \cdot s_r & \text{if } r_{c3} < 0 \\ \frac{5}{12} \cdot f_{b\_max} \cdot (s_l + s_r) \cdot a & \text{otherwise} \end{cases}$$ $P_{bb} = 4.268 \cdot 10^5 \cdot N$

$$f_{vb} := \frac{P_{bb}}{b_{prime} \cdot t}$$ $f_{vb} = 17.235 \cdot MPa$

$$F_v := 1.4 \cdot 0.4 \cdot F_y$$ $F_v = 231.672 \cdot MPa$

$$r_{bb} := \begin{cases} \frac{1}{6} \cdot \left[2 \cdot (\sin(\phi) + \cos(\phi)) \cdot a_1 - d\right] & \text{if } r_{c3} < 0 \\ \frac{s_l + s_r}{4} & \text{otherwise} \end{cases}$$ $r_{bb} = 34.925 \cdot mm$

$$S_x := \frac{b_{prime} \cdot t^2}{6}$$ $S_x = 2.064 \cdot 10^5 \cdot mm^3$

$$M_{bb} := P_{bb} \cdot r_{bb}$$ $M_{bb} = 1.491 \cdot 10^7 \cdot N \cdot mm$

$$f_{bb} := \frac{M_{bb}}{S_x}$$ $f_{bb} = 72.234 \cdot MPa$

$$F_b := 1.4 \cdot 0.66 \cdot F_y$$ $F_b = 382.259 \cdot MPa$

End of Example 1 Case $\phi = 0$ deg.

EXAMPLE 1: ϕ = 45° (U.S. Customary Units)

DESIGN OF END- AND BASE-PLATES	PLATE TYPE:	UNIT TYPE:	PLATE TYPE:	UNIT TYPE:
	ENDPLATE / BASEPLATE	METRIC / US CUST	BASEPLATE	US CUST

PLATE INPUT PARAMETERS

a (in.)	e (in.)	d (in.)	t (in.)	Pa (lb)	Fy (psi)	DESIGN ADEQUACY
19.5	3.0	14.0	1.890	0.0	60000.0	OK

ALLOWABLE STRESSES (psi)

BENDING Fb	SHEAR Fv	STEEL Fp
55440.0	33600.0	54000.0

BOLT DATA db (in.)	DESIGN MOMENTS (lb-in.) DL (Mx)	Wind (My)	Total (Mt)	ANGLE ϕ
1.8	1.66E+06	1.66E+06	2.35E+06	45.0

SHW HELP | CLR HELP

PLATE BEARING

STRESS fp (psi)	STEEL CHECK
1395.2	OK

YIELD LINE BENDING STRESS

r	Moment	b"	Sx	fb (psi)	CHECK
1.7	1.7E+05	6.1533595	3.7	47656.2	OK

Initial Trial t
1.8

YIELD LINE SHEAR STRESS

Force	Area	fv (psi)	CHECK
104482.2	20.2	5160.5	OK

FIXED BEAM BENDING STRESS

Force	Moment	Sx	fb (psi)	CHECK
105136.1	1.3E+05	3.6	36790.7	OK

TORSIONAL BEAM SHEAR STRESS

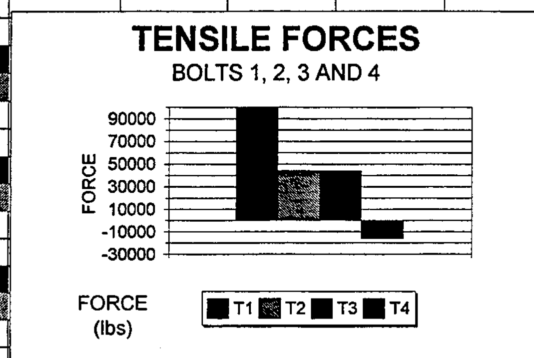
Force	Moment	r	fv (psi)	CHECK
104482.2	5.2E+05	15.6	33481.6	OK

PRESSURE LINE BENDING STRESS

Force	Moment	Sx	fb (psi)	CHECK
50007.9	1.1E+05	8.1	13999.6	OK

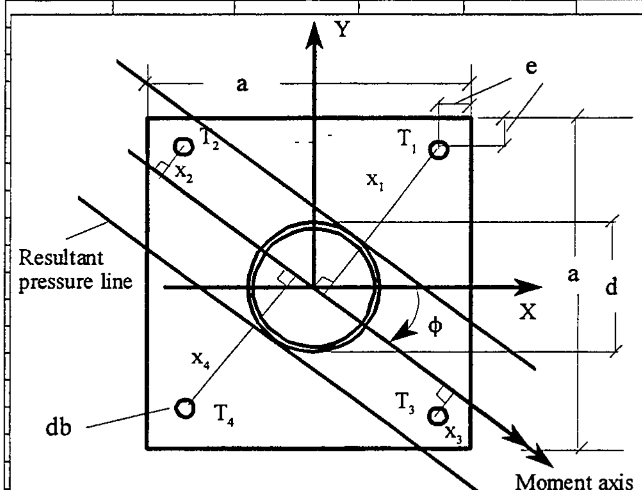
PRESSURE LINE SHEAR STRESS

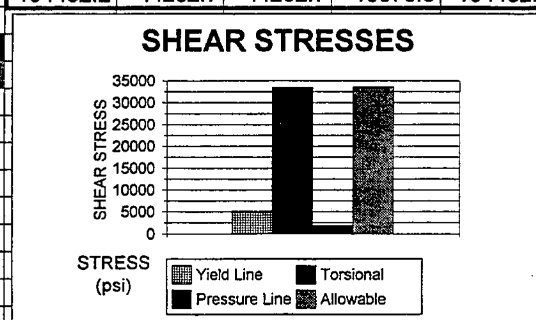
Force	Area	fv (psi)	CHECK
50007.9	25.7	1948.8	OK

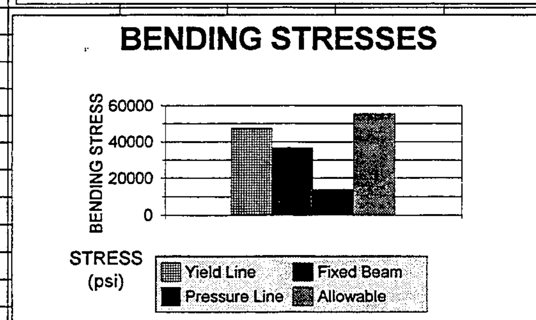


BOLT FORCES (lb)

T1	T2	T3	T4	Tmax
104482.2	44202.7	44202.7	-16076.8	104482.2







EXAMPLE 1: Base Plate:

Plate ID: VA728

Case: ϕ =45 deg

Units: US Customary

$t := 1.89 \cdot in$ Assumed thickness

INPUT DATA:

$M_x := 1663115 \cdot lb \cdot in$ $M_y := 1663115 \cdot lb \cdot in$ $P_a := 0 \cdot lb$ $F_y := 60 \cdot 10^3 \cdot \frac{lb}{in^2}$

$a := 19.5 \cdot in$ $e := 3 \cdot in$ $d := 14 \cdot in$ $d_b := 1.75 \cdot in$

$a_0 := \frac{a - 2 \cdot e}{2}$ $a_1 := \frac{a}{2}$ $\frac{d}{t} = 7.407$ $C_t := 0.30$ $u := e$

CALCULATIONS:

Calculate total moment and angle of incidence:

$M_t := \sqrt{M_x^2 + M_y^2}$ $\phi := atan\left(\frac{M_y}{M_x}\right)$ $\phi := \begin{cases} \phi & \text{if } \phi < 45 \\ (90 \cdot deg - \phi) & \text{otherwise} \end{cases}$ $M_t = 2.352 \cdot 10^6 \cdot lb \cdot in$

$\phi = 45 \cdot deg$

Calculate bolt distances to the resultant pressure line:

$x_1 := -\left(\frac{1}{2}\right) \cdot \left[2 \cdot (\sin(\phi) + \cos(\phi)) \cdot a_0 + d\right]$ $x_2 := \frac{1}{2} \cdot \left[2 \cdot (\sin(\phi) - \cos(\phi)) \cdot a_0 - d\right]$

$x_3 := -\left(\frac{1}{2}\right) \cdot \left[2 \cdot (\sin(\phi) - \cos(\phi)) \cdot a_0 + d\right]$ $x_4 := \frac{1}{2} \cdot \left[2 \cdot (\sin(\phi) + \cos(\phi)) \cdot a_0 - d\right]$

Calculate bolt distances from the resultant pressure line:

$x_1 = -16.546 \cdot in$ $x_2 = -7 \cdot in$ $x_3 = -7 \cdot in$ $x_4 = 2.546 \cdot in$

Calculate bolt distances from the center of the mast:

$x_{11} := -x_1 - \frac{d}{2}$ $x_{12} := -x_2 - \frac{d}{2}$ $x_{13} := -x_3 - \frac{d}{2}$ $x_{14} := -x_4 - \frac{d}{2}$

$x_{11} = 9.546 \cdot in$ $x_{12} = 1.093 \cdot 10^{-15} \cdot in$ $x_{13} = -1.093 \cdot 10^{-15} \cdot in$ $x_{14} = -9.546 \cdot in$

Calculate bolt forces:

$$T_1 := \frac{M_t - P_a \cdot \frac{d}{2}}{\left(\frac{x_1^2 + x_2^2 + x_3^2 + x_4^2}{x_1}\right)}$$ $T_1 = -1.029 \cdot 10^5 \cdot lb$ (Tension)

$T_2 := T_1 \cdot \frac{x_2}{x_1}$ $T_2 = -4.353 \cdot 10^4 \cdot lb$ (Tension)

$T_3 := T_1 \cdot \frac{x_3}{x_1}$ $\qquad T_3 = -4.353 \cdot 10^4 \cdot lb$ (Tension)

$T_4 := T_1 \cdot \frac{x_4}{x_1}$ $\qquad T_4 = 1.583 \cdot 10^4 \cdot lb$ (Compression)

<u>Calculate coordinates of the points where the resultant pressure line intersects plate sides:</u>

<u>First determine which sides of the plate the resultant pressure line intersects:</u>

$r_{c2} := \frac{1}{2} \cdot \left[2 \cdot (\sin(\phi) - \cos(\phi)) \cdot a_1 - d\right]$ $\qquad r_{c2} = -7 \cdot in$ -ve implies Side C2-C4

$r_{c3} := -\left(\frac{1}{2}\right) \cdot \left[2 \cdot (\sin(\phi) - \cos(\phi)) \cdot a_1 + d\right]$ $\qquad r_{c3} = -7 \cdot in$ -ve implies Side C3-C4

<u>Left and right points coordinates:</u>

$$x_l := \begin{cases} -a_1 & \text{if } r_{c2}<0 \\ -\frac{1}{2 \cdot \sin(\phi)} \cdot (2 \cdot \cos(\phi) \cdot a_1 + d) & \text{otherwise} \end{cases} \qquad y_l := \begin{cases} -\frac{1}{2 \cdot \cos(\phi)} \cdot (-2 \cdot \sin(\phi) \cdot a_1 + d) & \text{if } r_{c2}<0 \\ a_1 & \text{otherwise} \end{cases}$$

$x_l = -9.75 \cdot in$ $\qquad y_l = -0.149 \cdot in$

$$x_r := \begin{cases} -\frac{1}{2 \cdot \sin(\phi)} \cdot (-2 \cdot \cos(\phi) \cdot a_1 + d) & \text{if } r_{c3}<0 \\ a_1 & \text{otherwise} \end{cases} \qquad y_r := \begin{cases} -a_1 & \text{if } r_{c3}<0 \\ -\frac{1}{2 \cdot \cos(\phi)} \cdot (2 \cdot \sin(\phi) \cdot a_1 + d) & \text{otherwise} \end{cases}$$

$x_r = -0.149 \cdot in$ $\qquad y_r = -9.75 \cdot in$

<u>Calculate left and right distances used in estimating bearing stresses distribution:</u>

$$s_l := \begin{cases} \sqrt{(-a_1 - x_l)^2 + (-a_1 - y_l)^2} & \text{if } r_{c2}<0 \\ \sqrt{(-a_1 - x_l)^2 + (a_1 - y_l)^2} & \text{otherwise} \end{cases}$$

$s_l = 9.601 \cdot in$

$$s_r := \begin{cases} \sqrt{(-a_1 - x_r)^2 + (-a_1 - y_r)^2} & \text{if } r_{c3}<0 \\ \sqrt{(a_1 - x_r)^2 + (-a_1 - y_r)^2} & \text{otherwise} \end{cases}$$

$s_r = 9.601 \cdot in$

Calculate maximum bearing stress:

$$f_{b\_max} := \begin{cases} \frac{2}{3}\cdot\frac{-(T_1+T_2+T_3)+P_a}{s_l\cdot s_r} & \text{if } r_{c3}<0 \\ \frac{4}{3}\cdot\frac{-(T_1+T_2+T_3)+P_a}{a\cdot(s_l+s_r)} & \text{otherwise} \end{cases}$$

$$f_{b\_max} = 1.374\cdot 10^3 \cdot\frac{lb}{in^2}$$

$$F_p := 0.9\cdot F_y \qquad F_p = 5.4\cdot 10^4 \cdot\frac{lb}{in^2}$$

Calculate maximum bending stress between the bolts on the tension side:

$$M_{fb} := (a - 2\cdot e - 2\cdot d_b)\cdot\frac{T_1+T_2}{8}\cdot\cos(\phi) \qquad M_{fb} = -1.294\cdot 10^5 \cdot lb\cdot in$$

$$S_x := \frac{(e+u)\cdot t^2}{6} \qquad S_x = 3.572\cdot in^3$$

$$f_{b\_fb} := \frac{M_{fb}}{S_x} \qquad f_{b\_fb} = -3.623\cdot 10^4 \cdot\frac{lb}{in^2}$$

$$F_b := 1.4\cdot 0.66\cdot F_y \qquad F_b = 5.544\cdot 10^4 \cdot\frac{lb}{in^2}$$

Calculate maximum torsional shear stress at the sides of the mast:

$$M_{torsion} := T_1\cdot\frac{a - 2\cdot e - 2\cdot d_b}{2} \qquad M_{torsion} = -5.144\cdot 10^5 \cdot lb\cdot in$$

$$\Gamma := d\cdot t^2\cdot C_t \qquad \Gamma = 15.003\cdot in^3$$

$$f_{vt} := \frac{M_{torsion}}{\Gamma} \qquad f_{vt} = -3.429\cdot 10^4 \cdot\frac{lb}{in^2}$$

$$F_v := 1.4\cdot 0.4\cdot F_y \qquad F_v = 3.36\cdot 10^4 \cdot\frac{lb}{in^2}$$

Calculate maximum bending and shear stresses at the yield line:

$$r_{by} := -(x_1) - d - \frac{d_b}{2} \qquad r_{by} = 1.671\cdot in$$

$$M_{by} := T_1\cdot r_{by} \qquad M_{by} = -1.719\cdot 10^5 \cdot lb\cdot in$$

$$b_{prime} := \sqrt{(y_l - y_r)^2 + (x_l - x_r)^2} \qquad b_{prime} = 13.577\cdot in$$

$$b_{dprime} := \left(b_{prime} - 0.25\cdot\frac{a}{d}\cdot d\right)\cdot\cos(\phi) \qquad b_{dprime} = 6.153\cdot in$$

$S_x := \frac{b_{dprime} \cdot t^2}{6}$ $S_x = 3.663 \cdot in^3$

$f_{by} := \frac{M_{by}}{S_x}$ $f_{by} = -4.693 \cdot 10^4 \cdot \frac{lb}{in^2}$

$F_b := 1.4 \cdot 0.66 \cdot F_y$ $F_b = 5.544 \cdot 10^4 \cdot \frac{lb}{in^2}$

Calculate maximum bending and shear stresses at the resultant pressure line:

$f_{b\_max} = 1.374 \cdot 10^3 \cdot \frac{lb}{in^2}$ $s_l = 9.601 \cdot in$ $s_r = 9.601 \cdot in$

$$P_{bb} := \begin{cases} \frac{7}{18} \cdot f_{b\_max} \cdot s_l \cdot s_r & \text{if } r_{c3} < 0 \\ \frac{5}{12} \cdot f_{b\_max} \cdot (s_l + s_r) \cdot a & \text{otherwise} \end{cases}$$

$P_{bb} = 4.924 \cdot 10^4 \cdot lb$

$f_{vb} := \frac{P_{bb}}{b_{prime} \cdot t}$ $f_{vb} = 1.919 \cdot 10^3 \cdot \frac{lb}{in^2}$

$F_v := 1.4 \cdot 0.4 \cdot F_y$ $F_v = 3.36 \cdot 10^4 \cdot \frac{lb}{in^2}$

$$r_{bb} := \begin{cases} \frac{1}{6} \cdot \left[2 \cdot (\sin(\phi) + \cos(\phi)) \cdot a_l - d \right] & \text{if } r_{c3} < 0 \\ \frac{s_l + s_r}{4} & \text{otherwise} \end{cases}$$

$r_{bb} = 2.263 \cdot in$

$S_x := \frac{b_{prime} \cdot t^2}{6}$ $S_x = 8.083 \cdot in^3$

$M_{bb} := P_{bb} \cdot r_{bb}$ $M_{bb} = 1.114 \cdot 10^5 \cdot lb \cdot in$

$f_{bb} := \frac{M_{bb}}{S_x}$ $f_{bb} = 1.379 \cdot 10^4 \cdot \frac{lb}{in^2}$

$F_b := 1.4 \cdot 0.66 \cdot F_y$ $F_b = 5.544 \cdot 10^4 \cdot \frac{lb}{in^2}$

End of Example 1 Case ϕ = 45 deg.

EXAMPLE 1: ϕ = 45° (Metric Units)

DESIGN OF END- AND BASE-PLATES	PLATE TYPE:		UNIT TYPE:		PLATE TYPE:	BASEPLATE
		ENDPLATE		METRIC	UNIT TYPE:	METRIC
		BASEPLATE		US CUST		

PLATE INPUT PARAMETERS							
a (mm)	e (mm)	d (mm)	t (mm)	Pa (N)	Fy (MPa)	DESIGN ADEQUACY	
495.3	76.2	355.6	48.000	0.0	413.7		OK

ALLOWABLE STRESSES (MPa		
BENDING	SHEAR	STEEL
Fb	Fv	Fp
382.3	231.7	372.3

BOLT DATA	DESIGN MOMENTS (N-mm)			ANGLE
db (mm)	DL (Mx)	Wind (My	Total (Mt)	ϕ
44.5	1.88E+08	1.88E+08	2.66E+08	45.0

SHW HELP

CLR HELP

PLATE BEARING	
STRESS	STEEL
fp (MPa)	CHECK
9.6	OK

Initial
Trial t
44.5

YIELD LINE BENDING STRESS					CHECK
r	Moment	b"	Sx	fb (MPa)	
42.4	2.0E+07	156.29533	60017.4	329.2	OK

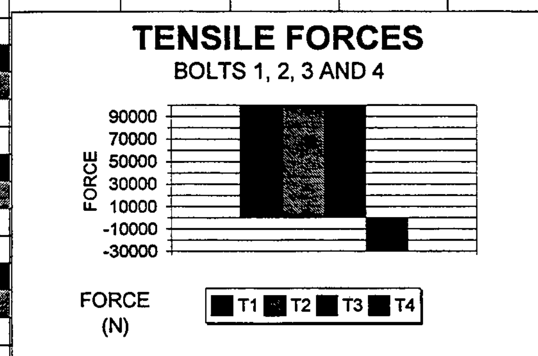
YIELD LINE SHEAR STRESS					CHECK
Force	Area			fv (MPa)	
465863.2	13060.5			35.7	OK

FIXED BEAM BENDING STRESS					CHECK
Force	Moment	Sx		fb (MPa)	
468778.8	1.5E+07	58521.6		254.2	OK

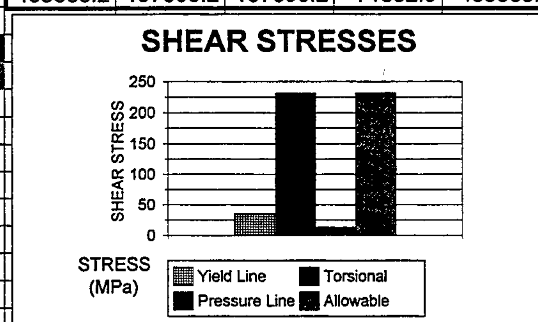
TORSIONAL BEAM SHEAR STRESS					CHECK
Force	Moment	r		fv (MPa)	
465863.2	5.9E+07	255622.3		231.4	OK

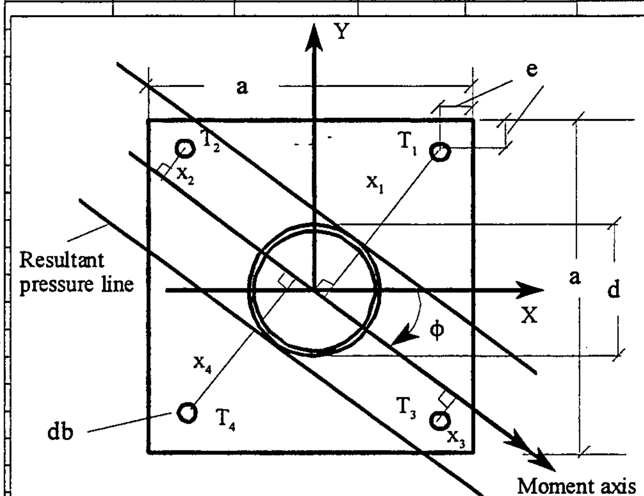
PRESSURE LINE BENDING STRESS					CHECK
Force	Moment	Sx		fb (MPa)	
222974.2	1.3E+07	132426.2		96.8	OK

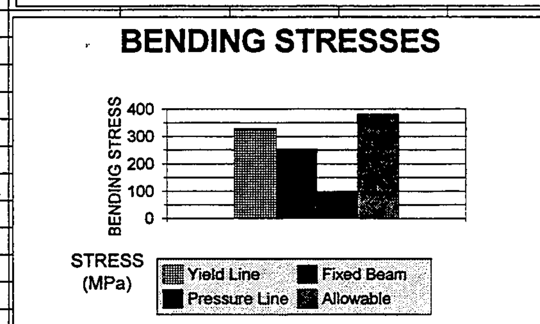
PRESSURE LINE SHEAR STRESS					CHECK
Force	Area			fv (MPa)	
222974.2	16553.3			13.5	OK



BOLT FORCES (N)				
T1	T2	T3	T4	Tmax
465863.2	197090.2	197090.2	-71682.9	465863.2







EXAMPLE 1: Base Plate:

Plate ID: VA728

Case: ϕ =45 deg

Units: SI (Metric)

$t := 48 \cdot mm$ Assumed thickness

$N := kg \cdot \frac{m}{sec^2}$ $MPa := Pa \cdot 10^6$

INPUT DATA:

$M_x := 1.88 \cdot 10^8 \cdot N \cdot mm$ $M_y := 1.88 \cdot 10^8 \cdot N \cdot mm$ $P_a := 0 \cdot N$ $F_y := 413.7 \cdot MPa$

$a := 495.3 \cdot mm$ $e := 76.2 \cdot mm$ $d := 355.6 \cdot mm$ $d_b := 44.5 \cdot mm$

$a_0 := \frac{a - 2 \cdot e}{2}$ $a_1 := \frac{a}{2}$ $\frac{d}{t} = 7.408$ $C_t := 0.30$ $u := e$

CALCULATIONS:

Calculate total moment and angle of incidence:

$M_t := \sqrt{M_x^2 + M_y^2}$ $\phi := atan\left(\frac{M_y}{M_x}\right)$ $\phi := \begin{vmatrix} \phi \text{ if } \phi<45 \\ (90 \cdot deg - \phi) \text{ otherwise} \end{vmatrix}$ $M_t = 2.659 \cdot 10^8 \cdot N \cdot mm$

$\phi = 45 \cdot deg$

Calculate bolt distances to the resultant pressure line:

$x_1 := -\left(\frac{1}{2}\right) \cdot \left[2 \cdot (\sin(\phi) + \cos(\phi)) \cdot a_0 + d\right]$ $x_2 := \frac{1}{2} \cdot \left[2 \cdot (\sin(\phi) - \cos(\phi)) \cdot a_0 - d\right]$

$x_3 := -\left(\frac{1}{2}\right) \cdot \left[2 \cdot (\sin(\phi) - \cos(\phi)) \cdot a_0 + d\right]$ $x_4 := \frac{1}{2} \cdot \left[2 \cdot (\sin(\phi) + \cos(\phi)) \cdot a_0 - d\right]$

Calculate bolt distances from the resultant pressure line:

$x_1 = -420.267 \cdot mm$ $x_2 = -177.8 \cdot mm$ $x_3 = -177.8 \cdot mm$ $x_4 = 64.667 \cdot mm$

Calculate bolt distances from the center of the mast:

$x_{11} := -x_1 - \frac{d}{2}$ $x_{12} := -x_2 - \frac{d}{2}$ $x_{13} := -x_3 - \frac{d}{2}$ $x_{14} := -x_4 - \frac{d}{2}$

$x_{11} = 242.467 \cdot mm$ $x_{12} = 2.776 \cdot 10^{-14} \cdot mm$ $x_{13} = -2.776 \cdot 10^{-14} \cdot mm$ $x_{14} = -242.467 \cdot mm$

Calculate bolt forces:

$$T_1 := \frac{M_t - P_a \cdot \frac{d}{2}}{\left(\frac{x_1^2 + x_2^2 + x_3^2 + x_4^2}{x_1}\right)}$$

$T_1 = -4.579 \cdot 10^5 \cdot N$ (Tension)

$T_2 := T_1 \cdot \frac{x_2}{x_1}$ $T_2 = -1.937 \cdot 10^5 \cdot N$ (Tension)

$$T_3 := T_1 \cdot \frac{x_3}{x_1} \qquad T_3 = -1.937 \cdot 10^5 \cdot N \qquad \text{(Tension)}$$

$$T_4 := T_1 \cdot \frac{x_4}{x_1} \qquad T_4 = 7.045 \cdot 10^4 \cdot N \qquad \text{(Compression)}$$

<u>Calculate coordinates of the points where the resultant pressure line intersects plate sides:</u>

<u>First determine which sides of the plate the resultant pressure line intersects:</u>

$$r_{c2} := \frac{1}{2} \cdot \left[2 \cdot (\sin(\phi) - \cos(\phi)) \cdot a_1 - d \right] \qquad r_{c2} = -177.8 \cdot mm$$ -ve implies Side C2-C4

$$r_{c3} := -\left(\frac{1}{2}\right) \cdot \left[2 \cdot (\sin(\phi) - \cos(\phi)) \cdot a_1 + d \right] \qquad r_{c3} = -177.8 \cdot mm$$ -ve implies Side C3-C4

<u>Left and right points coordinates:</u>

$$x_l := \begin{cases} -a_1 & \text{if } r_{c2} < 0 \\ -\frac{1}{2 \cdot \sin(\phi)} \cdot (2 \cdot \cos(\phi) \cdot a_1 + d) & \text{otherwise} \end{cases} \qquad y_l := \begin{cases} -\frac{1}{2 \cdot \cos(\phi)} \cdot (-2 \cdot \sin(\phi) \cdot a_1 + d) & \text{if } r_{c2} < 0 \\ a_1 & \text{otherwise} \end{cases}$$

$$x_l = -247.65 \cdot mm \qquad y_l = -3.797 \cdot mm$$

$$x_r := \begin{cases} -\frac{1}{2 \cdot \sin(\phi)} \cdot (-2 \cdot \cos(\phi) \cdot a_1 + d) & \text{if } r_{c3} < 0 \\ a_1 & \text{otherwise} \end{cases} \qquad y_r := \begin{cases} -a_1 & \text{if } r_{c3} < 0 \\ -\frac{1}{2 \cdot \cos(\phi)} \cdot (2 \cdot \sin(\phi) \cdot a_1 + d) & \text{otherwise} \end{cases}$$

$$x_r = -3.797 \cdot mm \qquad y_r = -247.65 \cdot mm$$

<u>Calculate left and right distances used in estimating bearing stresses distribution:</u>

$$s_l := \begin{cases} \sqrt{(-a_1 - x_l)^2 + (-a_1 - y_l)^2} & \text{if } r_{c2} < 0 \\ \sqrt{(-a_1 - x_l)^2 + (a_1 - y_l)^2} & \text{otherwise} \end{cases} \qquad s_l = 243.853 \cdot mm$$

$$s_r := \begin{cases} \sqrt{(-a_1 - x_r)^2 + (-a_1 - y_r)^2} & \text{if } r_{c3} < 0 \\ \sqrt{(a_1 - x_r)^2 + (-a_1 - y_r)^2} & \text{otherwise} \end{cases} \qquad s_r = 243.853 \cdot mm$$

Calculate maximum bearing stress:

$$f_{b\_max} := \begin{cases} \frac{2}{3} \cdot \frac{-(T_1 + T_2 + T_3) + P_a}{s_l \cdot s_r} & \text{if } r_{c3} < 0 \\ \frac{4}{3} \cdot \frac{-(T_1 + T_2 + T_3) + P_a}{a \cdot (s_l + s_r)} & \text{otherwise} \end{cases}$$

$f_{b\_max} = 9.477 \cdot MPa$

$F_p := 0.9 \cdot F_y$ $\qquad$ $F_p = 372.33 \cdot MPa$

Calculate maximum bending stress between the bolts on the tension side:

$M_{fb} := (a - 2 \cdot e - 2 \cdot d_b) \cdot \frac{T_1 + T_2}{8} \cdot \cos(\phi)$ $\qquad$ $M_{fb} = -1.462 \cdot 10^7 \cdot N \cdot mm$

$S_x := \frac{(e + u) \cdot t^2}{6}$ $\qquad$ $S_x = 5.852 \cdot 10^4 \cdot mm^3$

$f_{b\_fb} := \frac{M_{fb}}{S_x}$ $\qquad$ $f_{b\_fb} = -249.872 \cdot MPa$

$F_b := 1.4 \cdot 0.66 \cdot F_y$ $\qquad$ $F_b = 382.259 \cdot MPa$

Calculate maximum torsional shear stress at the sides of the mast:

$M_{torsion} := T_1 \cdot \frac{a - 2 \cdot e - 2 \cdot d_b}{2}$ $\qquad$ $M_{torsion} = -5.813 \cdot 10^7 \cdot N \cdot mm$

$\Gamma := d \cdot t^2 \cdot C_t$ $\qquad$ $\Gamma = 2.458 \cdot 10^5 \cdot mm^3$

$f_{vt} := \frac{M_{torsion}}{\Gamma}$ $\qquad$ $f_{vt} = -236.493 \cdot MPa$

$F_v := 1.4 \cdot 0.4 \cdot F_y$ $\qquad$ $F_v = 231.672 \cdot MPa$

Calculate maximum bending and shear stresses at the yield line:

$r_{by} := -(x_l) - d - \frac{d_b}{2}$ $\qquad$ $r_{by} = 42.417 \cdot mm$

$M_{by} := T_1 \cdot r_{by}$ $\qquad$ $M_{by} = -1.942 \cdot 10^7 \cdot N \cdot mm$

$b_{prime} := \sqrt{(y_l - y_r)^2 + (x_l - x_r)^2}$ $\qquad$ $b_{prime} = 344.86 \cdot mm$

$b_{dprime} := \left(b_{prime} - 0.25 \cdot \frac{a}{d} \cdot d\right) \cdot \cos(\phi)$ $\qquad$ $b_{dprime} = 156.295 \cdot mm$

$S_x := \frac{b_{dprime} \cdot t^2}{6}$ $S_x = 6.002 \cdot 10^4 \cdot mm^3$

$f_{by} := \frac{M_{by}}{S_x}$ $f_{by} = -323.604 \cdot MPa$

$F_b := 1.4 \cdot 0.66 \cdot F_y$ $F_b = 382.259 \cdot MPa$

Calculate maximum bending and shear stresses at the resultant pressure line:

$f_{b\_max} = 9.477 \cdot MPa$ $s_l = 243.853 \cdot mm$ $s_r = 243.853 \cdot mm$

$$P_{bb} := \begin{cases} \frac{7}{18} \cdot f_{b\_max} \cdot s_l \cdot s_r & \text{if } r_{c3} < 0 \\ \frac{5}{12} \cdot f_{b\_max} \cdot (s_l + s_r) \cdot a & \text{otherwise} \end{cases}$$

$P_{bb} = 2.192 \cdot 10^5 \cdot N$

$f_{vb} := \frac{P_{bb}}{b_{prime} \cdot t}$ $f_{vb} = 13.239 \cdot MPa$

$F_v := 1.4 \cdot 0.4 \cdot F_y$ $F_v = 231.672 \cdot MPa$

$$r_{bb} := \begin{cases} \frac{1}{6} \cdot \left[2 \cdot (\sin(\phi) + \cos(\phi)) \cdot a_1 - d \right] & \text{if } r_{c3} < 0 \\ \frac{s_l + s_r}{4} & \text{otherwise} \end{cases}$$

$r_{bb} = 57.477 \cdot mm$

$S_x := \frac{b_{prime} \cdot t^2}{6}$ $S_x = 1.324 \cdot 10^5 \cdot mm^3$

$M_{bb} := P_{bb} \cdot r_{bb}$ $M_{bb} = 1.26 \cdot 10^7 \cdot N \cdot mm$

$f_{bb} := \frac{M_{bb}}{S_x}$ $f_{bb} = 95.119 \cdot MPa$

$F_b := 1.4 \cdot 0.66 \cdot F_y$ $F_b = 382.259 \cdot MPa$

End of Example 1 Case ϕ = 45 deg.

EXAMPLE 2: ϕ = 30° (U.S. Customary Units)

DESIGN OF END- AND BASE-PLATES	PLATE TYPE:	ENDPLATE / BASEPLATE	UNIT TYPE:	METRIC / US CUST	PLATE TYPE:	ENDPLATE
					UNIT TYPE:	US CUST

PLATE INPUT PARAMETERS

a (in.)	e (in.)	d (in.)	t (in.)	Pa (lb)	Fy (psi)	DESIGN ADEQUACY
19.5	3.0	14.0	1.870	0.0	60000.0	OK

ALLOWABLE STRESSES (psi)

BENDING Fb	SHEAR Fv	STEEL Fp
55440.0	33600.0	54000.0

BOLT DATA

db (in.)
1.8

DESIGN MOMENTS (lb-in.)

DL (Mx)	Wind (My)	Total (Mt)
2.04E+05	1.18E+06	2.36E+06

ANGLE ϕ
30.0

SHW HELP

CLR HELP

PLATE BEARING

STRESS fp (psi)	STEEL CHECK
1379.5	OK

YIELD LINE BENDING STRESS

r	Moment	b"	Sx	fb (psi)	CHECK
1.3	1.4E+05	8.4056554	4.9	28173.2	OK

Initial Trial t
1.3

YIELD LINE SHEAR STRESS

Force	Area	fv (psi)	CHECK
102413.2	20.0	5112.4	OK

FIXED BEAM BENDING STRESS

Force	Moment	Sx	fb (psi)	CHECK
140363.8	1.8E+05	3.5	50174.4	OK

TORSIONAL BEAM SHEAR STRESS

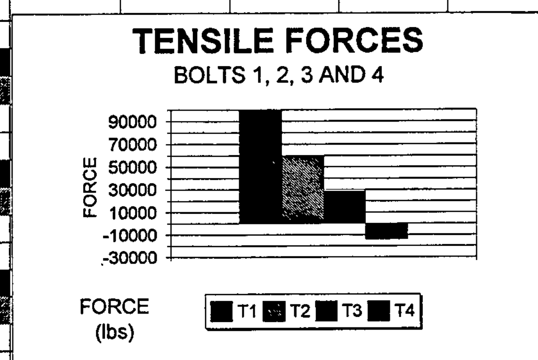
Force	Moment	r	fv (psi)	CHECK
102413.2	5.1E+05	15.3	33524.3	OK

PRESSURE LINE BENDING STRESS

Force	Moment	Sx	fb (psi)	CHECK
49465.3	1.0E+05	8.5	12261.8	OK

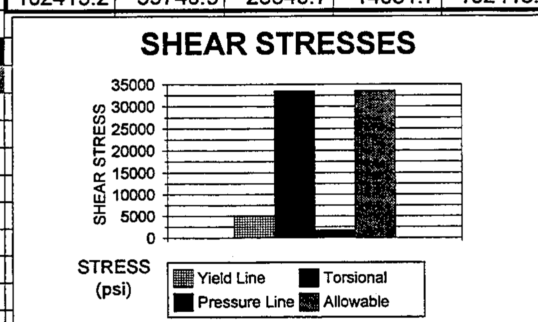
PRESSURE LINE SHEAR STRESS

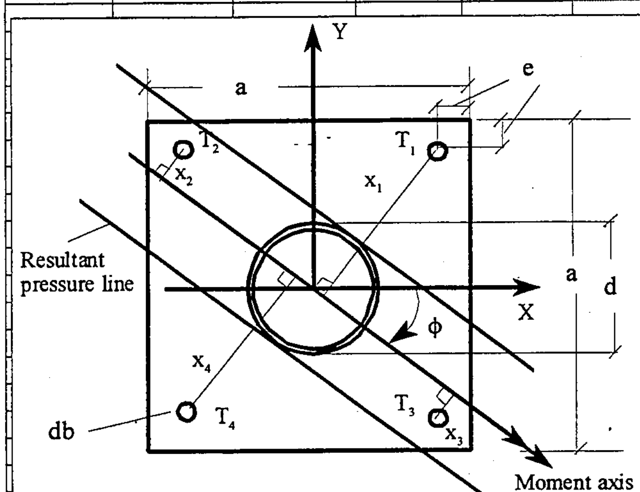
Force	Area	fv (psi)	CHECK
49465.3	27.3	1813.6	OK

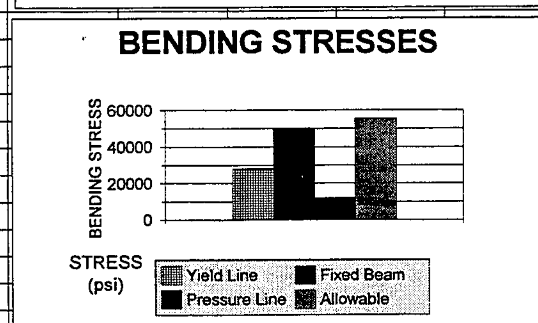


BOLT FORCES (lb)

T1	T2	T3	T4	Tmax
102413.2	59740.9	28640.7	-14031.7	102413.2







EXAMPLE 2: End Plate:

Plate ID: VA728

Case: ϕ =30 deg $t := 1.87\cdot in$ Assumed thickness

Units: US Customary

INPUT DATA:

$M_x := 2036892\cdot lb\cdot in$ $M_y := 1176000\cdot lb\cdot in$ $P_a := 0\cdot lb$ $F_y := 60\cdot 10^3\cdot\frac{lb}{in^2}$

$a := 19.5\cdot in$ $e := 3\cdot in$ $d := 14\cdot in$ $d_b := 1.75\cdot in$

$a_0 := \frac{a - 2\cdot e}{2}$ $a_1 := \frac{a}{2}$ $\frac{d}{t} = 7.487$ $C_t := 0.302$ $u := e$

CALCULATIONS:

Calculate total moment and angle of incidence:

$M_t := \sqrt{M_x^2 + M_y^2}$ $\phi := \operatorname{atan}\left(\frac{M_y}{M_x}\right)$ $\phi := \begin{cases} \phi & \text{if } \phi<45 \\ (90\cdot deg - \phi) & \text{otherwise}\end{cases}$ $M_t = 2.352\cdot 10^6\cdot lb\cdot in$

$\phi = 30\cdot deg$

Calculate bolt distances to the resultant pressure line:

$x_1 := -\left(\frac{1}{2}\right)\cdot\left[2\cdot(\sin(\phi) + \cos(\phi))\cdot a_0 + d\right]$ $x_2 := \frac{1}{2}\cdot\left[2\cdot(\sin(\phi) - \cos(\phi))\cdot a_0 - d\right]$

$x_3 := -\left(\frac{1}{2}\right)\cdot\left[2\cdot(\sin(\phi) - \cos(\phi))\cdot a_0 + d\right]$ $x_4 := \frac{1}{2}\cdot\left[2\cdot(\sin(\phi) + \cos(\phi))\cdot a_0 - d\right]$

Calculate bolt distances from the resultant pressure line:

$x_1 = -16.221\cdot in$ $x_2 = -9.471\cdot in$ $x_3 = -4.529\cdot in$ $x_4 = 2.221\cdot in$

Calculate bolt distances from the center of the mast:

$x_{11} := -x_1 - \frac{d}{2}$ $x_{12} := -x_2 - \frac{d}{2}$ $x_{13} := -x_3 - \frac{d}{2}$ $x_{14} := -x_4 - \frac{d}{2}$

$x_{11} = 9.221\cdot in$ $x_{12} = 2.471\cdot in$ $x_{13} = -2.471\cdot in$ $x_{14} = -9.221\cdot in$

Calculate bolt forces:

$$T_1 := \frac{M_t - P_a\cdot\frac{d}{2}}{\left(\frac{x_1^2 + x_2^2 + x_3^2 + x_4^2}{x_1}\right)}$$ $T_1 = -1.009\cdot 10^5\cdot lb$ (Tension)

$T_2 := T_1\cdot\frac{x_2}{x_1}$ $T_2 = -5.889\cdot 10^4\cdot lb$ (Tension)

$$T_3 := T_1 \cdot \frac{x_3}{x_1}$$ $$T_3 = -2.816 \cdot 10^4 \cdot lb$$ (Tension)

$$T_4 := T_1 \cdot \frac{x_4}{x_1}$$ $$T_4 = 1.381 \cdot 10^4 \cdot lb$$ (Compression)

Determine coordinates of the points where the resultant pressure line intersects plate sides:

First determine which sides of the plate the resultant pressure line intersects:

$$r_{c2} := \frac{1}{2} \cdot \left[2 \cdot (\sin(\phi) - \cos(\phi)) \cdot a_1 - d \right]$$ $$r_{c2} = -10.569 \cdot in$$ -ve implies Side C2-C4

$$r_{c3} := -\left(\frac{1}{2}\right) \cdot \left[2 \cdot (\sin(\phi) - \cos(\phi)) \cdot a_1 + d \right]$$ $$r_{c3} = -3.431 \cdot in$$ -ve implies Side C3-C4

Left and right points coordinates:

$$x_l := \begin{cases} -a_1 & \text{if } r_{c2} < 0 \\ -\frac{1}{2 \cdot \sin(\phi)} \cdot (2 \cdot \cos(\phi) \cdot a_1 + d) & \text{otherwise} \end{cases}$$

$$y_l := \begin{cases} -\frac{1}{2 \cdot \cos(\phi)} \cdot (-2 \cdot \sin(\phi) \cdot a_1 + d) & \text{if } r_{c2} < 0 \\ a_1 & \text{otherwise} \end{cases}$$

$$x_l = -9.75 \cdot in \qquad y_l = -2.454 \cdot in$$

$$x_r := \begin{cases} -\frac{1}{2 \cdot \sin(\phi)} \cdot (-2 \cdot \cos(\phi) \cdot a_1 + d) & \text{if } r_{c3} < 0 \\ a_1 & \text{otherwise} \end{cases}$$

$$y_r := \begin{cases} -a_1 & \text{if } r_{c3} < 0 \\ -\frac{1}{2 \cdot \cos(\phi)} \cdot (2 \cdot \sin(\phi) \cdot a_1 + d) & \text{otherwise} \end{cases}$$

$$x_r = 2.887 \cdot in \qquad y_r = -9.75 \cdot in$$

Calculate left and right distances used in estimating bearing pressure distribution:

$$s_l := \begin{cases} \sqrt{(-a_1 - x_l)^2 + (-a_1 - y_l)^2} & \text{if } r_{c2} < 0 \\ \sqrt{(-a_1 - x_l)^2 + (a_1 - y_l)^2} & \text{otherwise} \end{cases}$$ $$s_l = 7.296 \cdot in$$

$$s_r := \begin{cases} \sqrt{(-a_1 - x_r)^2 + (-a_1 - y_r)^2} & \text{if } r_{c3} < 0 \\ \sqrt{(a_1 - x_r)^2 + (-a_1 - y_r)^2} & \text{otherwise} \end{cases}$$ $$s_r = 12.637 \cdot in$$

Calculate maximum bearing stress:

$$f_{b\_max} := \begin{vmatrix} \frac{2}{3} \cdot \frac{-(T_1 + T_2 + T_3) + P_a}{s_l \cdot s_r} & \text{if } r_{c3} < 0 \\ \frac{4}{3} \cdot \frac{-(T_1 + T_2 + T_3) + P_a}{a \cdot (s_l + s_r)} & \text{otherwise} \end{vmatrix}$$

$$f_{b\_max} = 1.359 \cdot 10^3 \cdot \frac{lb}{in^2}$$

$$F_p := 0.9 \cdot F_y$$

$$F_p = 5.4 \cdot 10^4 \cdot \frac{lb}{in^2}$$

Calculate maximum bending stress between the bolts on the tension side:

$$M_{fb} := (a - 2 \cdot e - 2 \cdot d_b) \cdot \frac{T_1 + T_2}{8} \cdot \cos(\phi) \qquad M_{fb} = -1.729 \cdot 10^5 \cdot lb \cdot in$$

$$S_x := \frac{(e + u) \cdot t^2}{6} \qquad S_x = 3.497 \cdot in^3$$

$$f_{b\_fb} := \frac{M_{fb}}{S_x} \qquad f_{b\_fb} = -4.945 \cdot 10^4 \cdot \frac{lb}{in^2}$$

$$F_b := 1.4 \cdot 0.66 \cdot F_y \qquad F_b = 5.544 \cdot 10^4 \cdot \frac{lb}{in^2}$$

Calculate maximum torsional shear stress at the sides of the pole:

$$M_{torsion} := T_1 \cdot \frac{a - 2 \cdot e - 2 \cdot d_b}{2} \qquad M_{torsion} = -5.043 \cdot 10^5 \cdot lb \cdot in$$

$$\Gamma := d \cdot t^2 \cdot C_t \qquad \Gamma = 14.785 \cdot in^3$$

$$f_{vt} := \frac{M_{torsion}}{\Gamma} \qquad f_{vt} = -3.411 \cdot 10^4 \cdot \frac{lb}{in^2}$$

$$F_v := 1.4 \cdot 0.4 \cdot F_y \qquad F_v = 3.36 \cdot 10^4 \cdot \frac{lb}{in^2}$$

Calculate maximum bending and shear stresses at the yield line:

$$r_{by} := -(x_1) - d - \frac{d_b}{2} \qquad r_{by} = 1.346 \cdot in$$

$$M_{by} := T_1 \cdot r_{by} \qquad M_{by} = -1.357 \cdot 10^5 \cdot lb \cdot in$$

$$b_{prime} := \sqrt{(y_l - y_r)^2 + (x_l - x_r)^2} \qquad b_{prime} = 14.593 \cdot in$$

$$b_{dprime} := \left(b_{prime} - 0.25 \cdot \frac{a}{d} \cdot d\right) \cdot \cos(\phi) \qquad b_{dprime} = 8.416 \cdot in$$

$$S_x := \frac{b_{dprime} \cdot t^2}{6} \qquad S_x = 4.905 \cdot in^3$$

$$f_{by} := \frac{M_{by}}{S_x} \qquad f_{by} = -2.767 \cdot 10^4 \cdot \frac{lb}{in^2}$$

$$F_b := 1.4 \cdot 0.66 \cdot F_y \qquad F_b = 5.544 \cdot 10^4 \cdot \frac{lb}{in^2}$$

<u>Calculate maximum bending and shear stresses at the resultant pressure line:</u>

$$f_{b\_max} = 1.359 \cdot 10^3 \cdot \frac{lb}{in^2} \qquad s_l = 7.296 \cdot in \qquad s_r = 12.637 \cdot in$$

$$P_{bb} := \begin{cases} \frac{7}{18} \cdot f_{b\_max} \cdot s_l \cdot s_r & \text{if } r_{c3} < 0 \\ \frac{5}{12} \cdot f_{b\_max} \cdot (s_l + s_r) \cdot a & \text{otherwise} \end{cases} \qquad P_{bb} = 4.872 \cdot 10^4 \cdot lb$$

$$f_{vb} := \frac{P_{bb}}{b_{prime} \cdot t} \qquad f_{vb} = 1.785 \cdot 10^3 \cdot \frac{lb}{in^2}$$

$$F_v := 1.4 \cdot 0.4 \cdot F_y \qquad F_v = 3.36 \cdot 10^4 \cdot \frac{lb}{in^2}$$

$$r_{bb} := \begin{cases} \frac{1}{6} \cdot \left[2 \cdot \left(\sin(\phi) + \cos(\phi)\right) \cdot a_1 - d\right] & \text{if } r_{c3} < 0 \\ \frac{s_l + s_r}{4} & \text{otherwise} \end{cases} \qquad r_{bb} = 2.106 \cdot in$$

$$S_x := \frac{b_{prime} \cdot t^2}{6} \qquad S_x = 8.505 \cdot in^3$$

$$M_{bb} := P_{bb} \cdot r_{bb} \qquad M_{bb} = 1.026 \cdot 10^5 \cdot lb \cdot in$$

$$f_{bb} := \frac{M_{bb}}{S_x} \qquad f_{bb} = 1.207 \cdot 10^4 \cdot \frac{lb}{in^2}$$

$$F_b := 1.4 \cdot 0.66 \cdot F_y \qquad F_b = 5.544 \cdot 10^4 \cdot \frac{lb}{in^2}$$

<u>End of Example 2.</u>

EXAMPLE 2: ϕ = 30° (Metric Units)

DESIGN OF END- AND BASE-PLATES	PLATE TYPE:	ENDPLATE	UNIT TYPE:	METRIC	PLATE TYPE:	ENDPLATE
		BASEPLATE		US CUST	UNIT TYPE:	METRIC

PLATE INPUT PARAMETERS

a (mm)	e (mm)	d (mm)	t (mm)	Pa (N)	Fy (MPa)	DESIGN ADEQUACY
495.3	76.2	355.6	48.000	0.0	413.7	OK

ALLOWABLE STRESSES (MPa

BENDING Fb	SHEAR Fv	STEEL Fp
382.3	231.7	372.3

BOLT DATA

db (mm)
44.5

DESIGN MOMENTS (N-mm)

DL (Mx)	Wind (My	Total (Mt)
2.30E+08	1.33E+08	2.66E+08

ANGLE ϕ
30.0

SHW HELP

CLR HELP

PLATE BEARING

STRESS fp (MPa)	STEEL CHECK
9.5	OK

YIELD LINE BENDING STRESS

r	Moment	b"	Sx	fb (MPa)	CHECK	Initial Trial t
34.2	1.6E+07	213.54351	82000.7	189.6	OK	33.8

YIELD LINE SHEAR STRESS

Force	Area	fv (MPa)	CHECK
454545.9	13060.5	34.8	OK

FIXED BEAM BENDING STRESS

Force	Moment	Sx	fb (MPa)	CHECK
623063.1	2.0E+07	58521.6	337.9	OK

TORSIONAL BEAM SHEAR STRESS

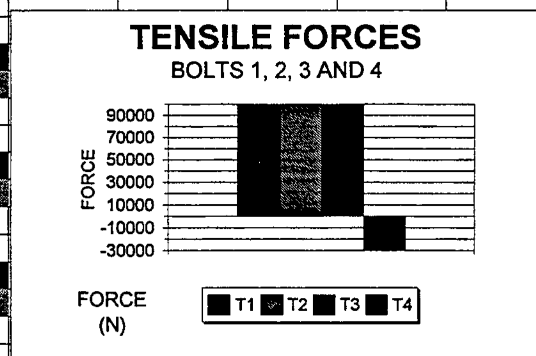
Force	Moment	r	fv (MPa)	CHECK
454545.9	5.8E+07	255622.3	225.7	OK

PRESSURE LINE BENDING STRESS

Force	Moment	Sx	fb (MPa)	CHECK
219546.5	1.2E+07	142272.4	82.6	OK

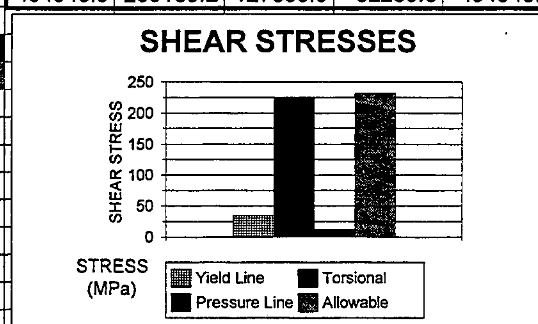
PRESSURE LINE SHEAR STRESS

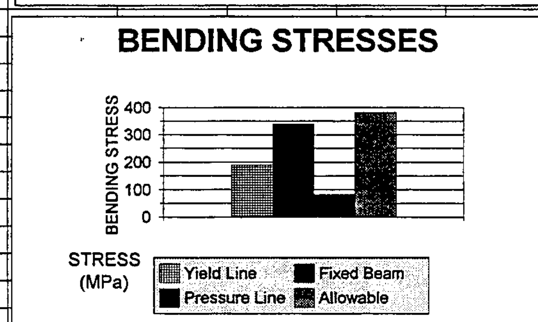
Force	Area	fv (MPa)	CHECK
219546.5	17784.1	12.3	OK

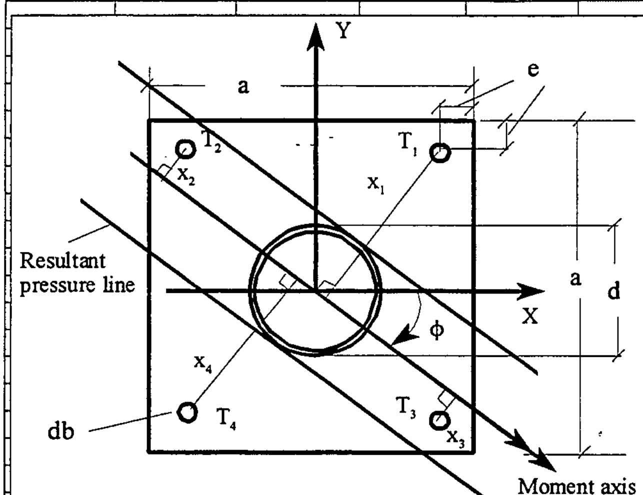


BOLT FORCES (N)

T1	T2	T3	T4	Tmax
454545.9	265189.2	127086.9	-62269.8	454545.9







EXAMPLE 2: End Plate:

Plate ID: VA728

Case: ϕ =30 deg $t := 48 \cdot mm$ Assumed thickness

Units: SI (Metric) $N := kg \cdot \frac{m}{sec^2}$ $MPa := Pa \cdot 10^6$

INPUT DATA:

$M_x := 2.3 \cdot 10^8 \cdot N \cdot mm$ $M_y := 1.33 \cdot 10^8 \cdot N \cdot mm$ $P_a := 0 \cdot N$ $F_y := 413.7 \cdot MPa$

$a := 495.3 \cdot mm$ $e := 76.2 \cdot mm$ $d := 355.6 \cdot mm$ $d_b := 44.5 \cdot mm$

$a_0 := \frac{a - 2 \cdot e}{2}$ $a_1 := \frac{a}{2}$ $\frac{d}{t} = 7.408$ $C_t := 0.302$ $u := e$

CALCULATIONS:

Calculate total moment and angle of incidence:

$M_t := \sqrt{M_x^2 + M_y^2}$ $\phi := atan\left(\frac{M_y}{M_x}\right)$ $\phi := \begin{vmatrix} \phi \text{ if } \phi<45 \\ (90 \cdot deg - \phi) \text{ otherwise} \end{vmatrix}$ $M_t = 2.657 \cdot 10^8 \cdot N \cdot mm$

$\phi = 30.039 \cdot deg$

Calculate bolt distances to the resultant pressure line:

$\phi = 30.039 \cdot deg$

$x_1 := -\left(\frac{1}{2}\right) \cdot \left[2 \cdot (\sin(\phi) + \cos(\phi)) \cdot a_0 + d\right]$ $x_2 := \frac{1}{2} \cdot \left[2 \cdot (\sin(\phi) - \cos(\phi)) \cdot a_0 - d\right]$

$x_3 := -\left(\frac{1}{2}\right) \cdot \left[2 \cdot (\sin(\phi) - \cos(\phi)) \cdot a_0 + d\right]$ $x_4 := \frac{1}{2} \cdot \left[2 \cdot (\sin(\phi) + \cos(\phi)) \cdot a_0 - d\right]$

Calculate bolt distances from the resultant pressure line:

$x_1 = -412.048 \cdot mm$ $x_2 = -240.395 \cdot mm$ $x_3 = -4.536 \cdot in$ $x_4 = 56.448 \cdot mm$

Calculate bolt distances from the center of the mast:

$x_{11} := -x_1 - \frac{d}{2}$ $x_{12} := -x_2 - \frac{d}{2}$ $x_{13} := -x_3 - \frac{d}{2}$ $x_{14} := -x_4 - \frac{d}{2}$

$x_{11} = 234.248 \cdot mm$ $x_{12} = 62.595 \cdot mm$ $x_{13} = -62.595 \cdot mm$ $x_{14} = -234.248 \cdot mm$

Calculate bolt forces:

$T_1 := \frac{M_t - P_a \cdot \frac{d}{2}}{\left(\frac{x_1^2 + x_2^2 + x_3^2 + x_4^2}{x_1}\right)}$ $T_1 = -4.486 \cdot 10^5 \cdot N$ (Tension)

$T_2 := T_1 \cdot \frac{x_2}{x_1}$ $T_2 = -2.617 \cdot 10^5 \cdot N$ (Tension)

$T_3 := T_1 \cdot \frac{x_3}{x_1}$ $\qquad T_3 = -1.254 \cdot 10^5 \cdot N$ (Tension)

$T_4 := T_1 \cdot \frac{x_4}{x_1}$ $\qquad T_4 = 6.146 \cdot 10^4 \cdot N$ (Compression)

Determine coordinates of the points where the resultant pressure line intersects plate sides:

First determine which sides of the plate the resultant pressure line intersects:

$r_{c2} := \frac{1}{2} \cdot \left[2 \cdot (\sin(\phi) - \cos(\phi)) \cdot a_1 - d\right]$ $\qquad r_{c2} = -268.215 \cdot$mr-ve implies Side C2-C4

$r_{c3} := -\left(\frac{1}{2}\right) \cdot \left[2 \cdot (\sin(\phi) - \cos(\phi)) \cdot a_1 + d\right]$ $\qquad r_{c3} = -87.385 \cdot$mm-ve implies Side C3-C4

Left and right points coordinates:

$$x_l := \begin{cases} -a_1 & \text{if } r_{c2} < 0 \\ -\frac{1}{2 \cdot \sin(\phi)} \cdot (2 \cdot \cos(\phi) \cdot a_1 + d) & \text{otherwise} \end{cases} \qquad y_l := \begin{cases} -\frac{1}{2 \cdot \cos(\phi)} \cdot (-2 \cdot \sin(\phi) \cdot a_1 + d) & \text{if } r_{c2} < 0 \\ a_1 & \text{otherwise} \end{cases}$$

$x_l = -247.65 \cdot mm \qquad y_l = -62.18 \cdot mm$

$$x_r := \begin{cases} -\frac{1}{2 \cdot \sin(\phi)} \cdot (-2 \cdot \cos(\phi) \cdot a_1 + d) & \text{if } r_{c3} < 0 \\ a_1 & \text{otherwise} \end{cases} \qquad y_r := \begin{cases} -a_1 & \text{if } r_{c3} < 0 \\ -\frac{1}{2 \cdot \cos(\phi)} \cdot (2 \cdot \sin(\phi) \cdot a_1 + d) & \text{otherwise} \end{cases}$$

$x_r = 73.087 \cdot mm \qquad y_r = -247.65 \cdot mm$

Calculate left and right distances used in estimating bearing pressure distribution:

$$s_l := \begin{cases} \sqrt{(-a_1 - x_l)^2 + (-a_1 - y_l)^2} & \text{if } r_{c2} < 0 \\ \sqrt{(-a_1 - x_l)^2 + (a_1 - y_l)^2} & \text{otherwise} \end{cases}$$

$s_l = 185.47 \cdot mm$

$$s_r := \begin{cases} \sqrt{(-a_1 - x_r)^2 + (-a_1 - y_r)^2} & \text{if } r_{c3} < 0 \\ \sqrt{(a_1 - x_r)^2 + (-a_1 - y_r)^2} & \text{otherwise} \end{cases}$$

$s_r = 320.737 \cdot mm$

Calculate maximum bearing stress:

$$f_{b\_max} := \begin{cases} \frac{2}{3}\cdot\frac{-(T_1+T_2+T_3)+P_a}{s_l\cdot s_r} & \text{if } r_{c3}<0 \\ \frac{4}{3}\cdot\frac{-(T_1+T_2+T_3)+P_a}{a\cdot(s_l+s_r)} & \text{otherwise} \end{cases}$$

$f_{b\_max} = 9.366 \cdot MPa$

$F_p := 0.9\cdot F_y$

$F_p = 372.33 \cdot MPa$

Calculate maximum bending stress between the bolts on the tension side:

$M_{fb} := (a - 2\cdot e - 2\cdot d_b)\cdot\frac{T_1+T_2}{8}\cdot\cos(\phi)$ $\qquad M_{fb} = -1.952\cdot 10^7 \cdot N\cdot mm$

$S_x := \frac{(e+u)\cdot t^2}{6}$ $\qquad S_x = 5.852\cdot 10^4 \cdot mm^3$

$f_{b\_fb} := \frac{M_{fb}}{S_x}$ $\qquad f_{b\_fb} = -333.488 \cdot MPa$

$F_b := 1.4\cdot 0.66\cdot F_y$ $\qquad F_b = 382.259 \cdot MPa$

Calculate maximum torsional shear stress at the sides of the pole:

$M_{torsion} := T_1\cdot\frac{a - 2\cdot e - 2\cdot d_b}{2}$ $\qquad M_{torsion} = -5.695\cdot 10^7 \cdot N\cdot mm$

$\Gamma := d\cdot t^2\cdot C_t$ $\qquad \Gamma = 2.474\cdot 10^5 \cdot mm^3$

$f_{vt} := \frac{M_{torsion}}{\Gamma}$ $\qquad f_{vt} = -230.171 \cdot MPa$

$F_v := 1.4\cdot 0.4\cdot F_y$ $\qquad F_v = 231.672 \cdot MPa$

Calculate maximum bending and shear stresses at the yield line:

$r_{by} := -(x_1) - d - \frac{d_b}{2}$ $\qquad r_{by} = 34.198 \cdot mm$

$M_{by} := T_1\cdot r_{by}$ $\qquad M_{by} = -1.534\cdot 10^7 \cdot N\cdot mm$

$b_{prime} := \sqrt{(y_l - y_r)^2 + (x_l - x_r)^2}$ $\qquad b_{prime} = 370.501 \cdot mm$

$b_{dprime} := \left(b_{prime} - 0.25\cdot\frac{a}{d}\cdot d\right)\cdot\cos(\phi)$ $\qquad b_{dprime} = 213.544 \cdot mm$

$S_x := \frac{b_{dprime} \cdot t^2}{6}$ $S_x = 8.2 \cdot 10^4 \cdot mm^3$

$f_{by} := \frac{M_{by}}{S_x}$ $f_{by} = -187.09 \cdot MPa$

$F_b := 1.4 \cdot 0.66 \cdot F_y$ $F_b = 382.259 \cdot MPa$

Calculate maximum bending and shear stresses at the resultant pressure line:

$f_{b\_max} = 9.366 \cdot MPa$ $s_l = 185.47 \cdot mm$ $s_r = 320.737 \cdot mm$

$$P_{bb} := \begin{cases} \frac{7}{18} \cdot f_{b\_max} \cdot s_l \cdot s_r & \text{if } r_{c3} < 0 \\ \frac{5}{12} \cdot f_{b\_max} \cdot (s_l + s_r) \cdot a & \text{otherwise} \end{cases}$$ $P_{bb} = 2.167 \cdot 10^5 \cdot N$

$f_{vb} := \frac{P_{bb}}{b_{prime} \cdot t}$ $f_{vb} = 12.184 \cdot MPa$

$F_v := 1.4 \cdot 0.4 \cdot F_y$ $F_v = 231.672 \cdot MPa$

$$r_{bb} := \begin{cases} \frac{1}{6} \cdot \left[2 \cdot (\sin(\phi) + \cos(\phi)) \cdot a_1 - d \right] & \text{if } r_{c3} < 0 \\ \frac{s_l + s_r}{4} & \text{otherwise} \end{cases}$$ $r_{bb} = 53.519 \cdot mm$

$S_x := \frac{b_{prime} \cdot t^2}{6}$ $S_x = 1.423 \cdot 10^5 \cdot mm^3$

$M_{bb} := P_{bb} \cdot r_{bb}$ $M_{bb} = 1.16 \cdot 10^7 \cdot N \cdot mm$

$f_{bb} := \frac{M_{bb}}{S_x}$ $f_{bb} = 81.51 \cdot MPa$

$F_b := 1.4 \cdot 0.66 \cdot F_y$ $F_b = 382.259 \cdot MPa$

End of Example 2.