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Estimates of the Initial Vortex Separation Distance, b_o, of Commercial Aircraft from Pulsed Lidar Data

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An aircraft in flight generates multiple wake vortices, the largest of which are a result of the lift on the wings. These vortices rapidly roll up into a counter-rotating vortex pair behind the aircraft. The initial separation between the centroids of vorticity behind the aircraft, or the cores of the resulting vortex pair, is commonly denoted as b_o. This separation distance is used to estimate the initial vortex strength (circulation) and to nondimensionalize both time and circulation behind the aircraft. Thus, b_o is an important parameter in nondimensionalizing aircraft wake measurements. For an elliptically loaded wing, $b_o = (\pi/4)s$, where s is the wingspan of the generating aircraft. The initial descent velocity of the vortex system behind the aircraft, V_o, is proportional to b_o^{-2}. Thus, from estimates of V_o from lidar measurements, we can estimate b_o. Estimates of V_o from lidar measurements of vortex altitude vs. time are presented in the Out of Ground Effect (OGE) region for aircraft arriving at Denver International Airport and San Francisco International Airport. Estimates of b_o for these aircraft are then obtained. These measurements suggest that, while some estimates of b_o from commercial aircraft are close to the value based on elliptical wing loading, some aircraft have distinctly nonelliptical values of b_o.

Nomenclature

b_o	=	initial separation between the vortex cores (or the centroids of vorticity for times before the vortices have completely rolled up)
DEN	=	Denver International Airport
DFW	=	Dallas/Fort Worth International Airport
EDR	=	Eddy Dissipation Rate (same as ε)
EDR*	=	Normalized Eddy Dissipation Rate, $(\varepsilon b_o)^{1/3}/V_o$
g	=	acceleration due to gravity
HW	=	headwind component
LMCT	=	Lockheed Martin Coherent Technologies
M	=	mass of the aircraft
MLM	=	Maximum Landing Mass of the aircraft
N	=	Brunt-Vaisala frequency, $((-g/\rho)d\rho/dz)^{1/2}$
N*	=	nondimensional stratification parameter, $(Nb_o)/V_o$

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s	=	wingspan
SFO	=	San Francisco International Airport
t	=	dimensional time
U	=	true airspeed of the aircraft
UTC	=	Coordinated Universal Time
V_o	=	initial vertical descent velocity of a vortex pair
V_{obs}	=	observed vertical descent velocity of a vortex pair
z	=	vertical coordinate
ε	=	eddy dissipation rate (same as EDR)
ρ	=	air density
Γ	=	vortex circulation
Γ_o	=	normalized vortex circulation, $2\pi V_o b_o$
θ	=	glide slope angle

I. Introduction

Lidar measurements of aircraft wake vortices have been obtained during the past 25 years and have proven useful in furthering our understanding of the interaction of the ambient environment on vortex evolution. In order to determine how vortices from one aircraft evolve relative to those from another aircraft, we nondimensionalize the vortex altitude, strength, and time. The vortex altitude is nondimensionalized using z/b_o, where z is the vortex altitude at a given time, t, and b_o is the initial separation between the vortex cores (or the centroids of vorticity for times before the vortices have completely rolled up). The vortex strength is nondimensionalized by Γ/Γ_o, where Γ is the circulation measured at the time t, and Γ_o is defined as the initial vortex circulation, $= 2\pi V_o b_o$, where V_o is the initial descent velocity of the centroid of vorticity of the vortex system. Time is nondimensionalized by $V_o t/b_o$. Note that in all three nondimensionalizations, b_o is used. Note that V_o is proportional to b_o^{-2}, and nondimensional time is proportional to b_o^{-3}. Thus, the quality of our nondimensional parameters depends on how well we can estimate b_o for each aircraft.

Because we typically do not know the value of b_o for any given aircraft, it is common to use the value of b_o associated with an elliptically loaded wing. For such an elliptical loading, $b_o = (\pi/4)s \approx 0.78s$, where s is the wingspan of the aircraft.

In this paper, we show how, using altitude vs. time data, we can estimate b_o for different aircraft. Our current findings suggest that, while some aircraft have values of b_o that are close to the elliptical values, other aircraft have values of b_o that are distinctly nonelliptical.

II. Method

The initial vertical descent velocity of a vortex pair, V_o, can be estimated from the equation

$$V_o = \frac{gM}{2\pi\rho U b_o^{2}} \tag{1}$$

where g is the acceleration due to gravity, M is the mass of the aircraft, ρ is the air density, and U is the true air speed of the aircraft.

For a Lockheed Martin Coherent Technologies (LMCT) WindTracer® lidar and the manufacturer supplied wake processing algorithm, a previous study showed that the mean bias in the range direction is about 4-7m whereas the cross range counterpart is about 1-3m. For a typical side scan geometry, these biases translate to 4-7m in lateral position and 1-3m in vertical position, respectively[1]. These low biases for vertical position suggest that vortex altitude is well measured by the pulsed lidar. We use the vortex altitude vs. time measurements to estimate the initial value of V_o.

Given a value of V_o for a landing, we use Eq. (1) to solve for b_o. For the aircraft mass, M, and air speed, U, we use estimates based on 85 percent of the Maximum Landing Mass (MLM) and the appropriate approach speed. The rationale for using 85 percent of the MLM was determined by examining the NASA AVOSS data for Memphis and DFW[2] where actual weights and speeds were measured. The standard atmosphere value of air density, ρ, was used for DEN and SFO, respectively. Changing air density has some effect on V_o but has little or no effect on b_o.

III. Aircraft Data

Data from two airports will be shown. First, data from the Denver International Airport were obtained with an LMCT pulsed lidar at a glide slope altitude of approximately 243m. This test was conducted by NASA in 2006. Second, data from the Denver International Airport were obtained with an LMCT pulsed lidar at a glide slope altitude of approximately 212m. This test was conducted by NASA in 2003. Finally, data from the San Francisco International Airport were obtained with an LMCT pulsed lidar at a glide slope altitude of approximately 162m. This test was conducted by the FAA in 2001.

IV. Estimates of V_o

To determine V_o, we fit the altitude vs. time data for each track with a linear fit, excluding the first data point, which we assume is associated with the rollup phase of the wake evolution. Figure 1 shows altitude vs. time measurements for one landing of an A320 from the Denver 2006 database. In this example, the linear fit is over the data in the first 33 seconds after passage of the aircraft. Note that the first data point is excluded from the fit, since including this data point would bias the result. The error bars on the measurements come from Ref. 1.

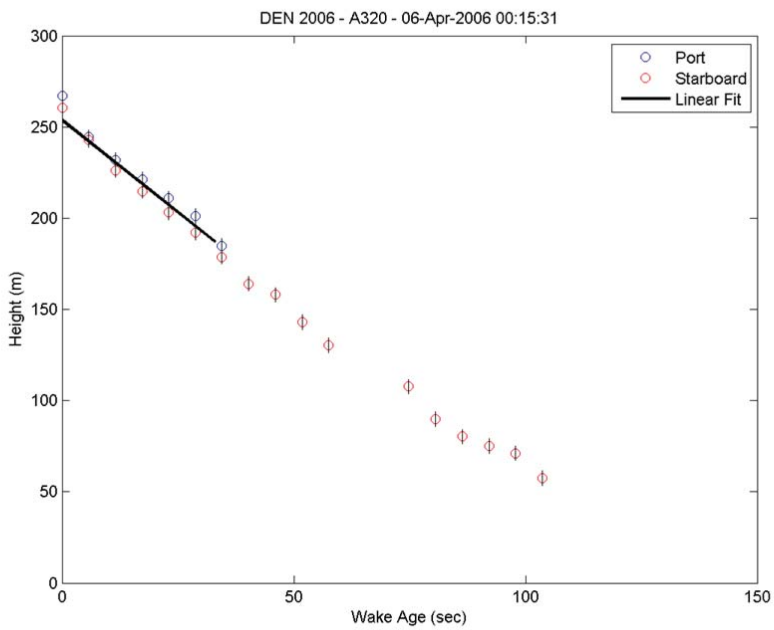


Figure 1. Altitude vs. wake age measurements for vortices from one landing of an A320 in the Denver 2006 database.

For each landing and each fit like that in Figure 1, we determine the variance in the fit, and rank the landings according to this variance, from the smallest variance to the largest variance. We then estimate the cumulative median V_o for the cumulative distribution, taking more and more data landings into account.

Figure 2 shows this process for the 653 A320 aircraft in the Denver 2006 database. In this figure, the V_o were determined from each run over the first 33 seconds, excluding the first data point. The blue line in Figure 2 shows the cumulative median V_o, and the green line shows the fraction of the total A320 landings that went into the cumulative median V_o. We then use the 50% fraction and take an average V_o value around that fraction. We use the 50% fraction to determine V_o since those measurements are the best fits (smallest variances) to the data.

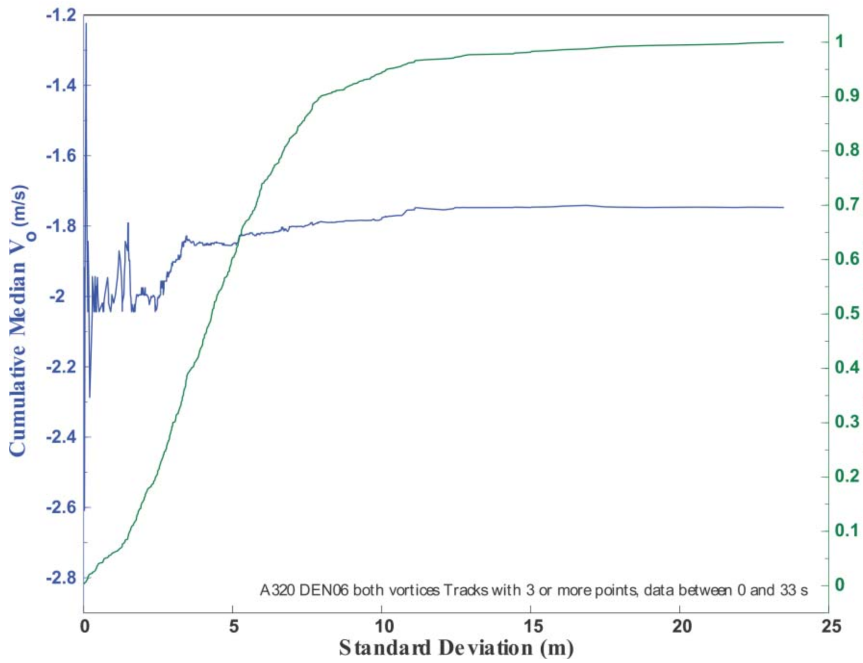


Figure 2. Estimates of the cumulative median V_o for Airbus A320 aircraft at Denver in 2006.

V. Atmospheric Effects

Three atmospheric effects were examined to determine their effect on V_o: stratification, turbulence, and headwind.

Stable stratification effects are generally observed during the night, during atmospheric cooling, and in the early morning hours, before solar heating. Figure 3 shows estimates of V_o during landings of Airbus A320 aircraft at Denver vs. Time of Day in 2006. (Although there are 653 A320 landings in the Denver 2006 database, there are only 423 landings that have estimates of stratification associated with them.) Here, the nondimensional stratification parameter is given by $N^* = (Nb_o)/V_o$, and is the inverse of the Froude number, where N is the Brunt-Vaisala frequency, defined by $N^2 = (-g/\rho)\, d\rho/dz$ and z is the vertical coordinate[3]. The value of N is determined for each landing from temperature measurements obtained with an MTP5 temperature profiler. For these estimates, the average temperature gradient was obtained over the altitudes of the vortex measurements for each landing. In Figure 3, V_o is estimated with a linear fit over the first 33 seconds of the data after the passage of the aircraft, after excluding the first data point. This figure shows that, when the stratification is large (larger values of N*), in the early morning around 1300 UTC, the range of V_o is reduced.

Figure 4 shows the V_o in Figure 3 vs. N*. The solid line in the figure is a linear least squares fit to the data and shows that there is a slightly slower average descent of the vortices when the atmosphere is stratified (larger values of N*).

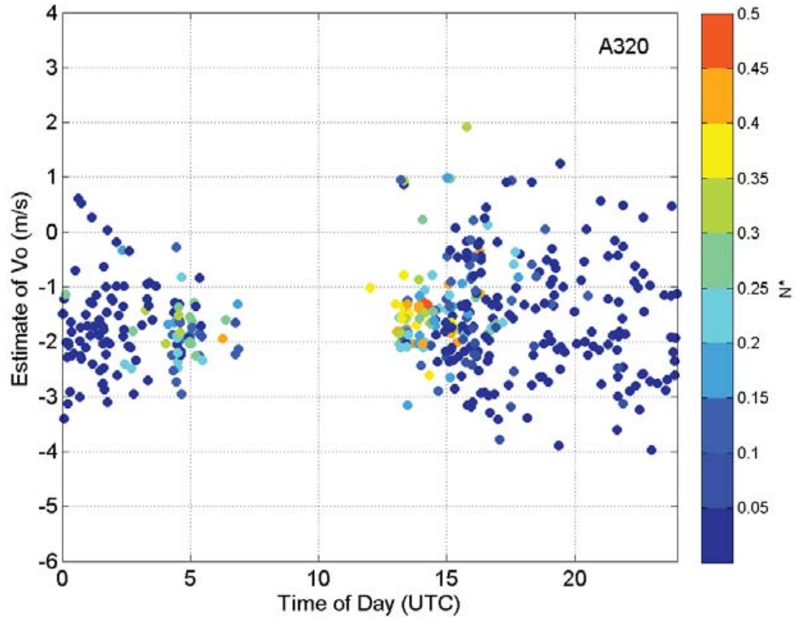


Figure 3. Estimates of V_o for the Airbus A320 aircraft landings in 2006 at Denver vs. Time of Day for different values of N* (stratification; see the color bar at the right in the figure). Local time during the data collection period is UTC – 6 hours.

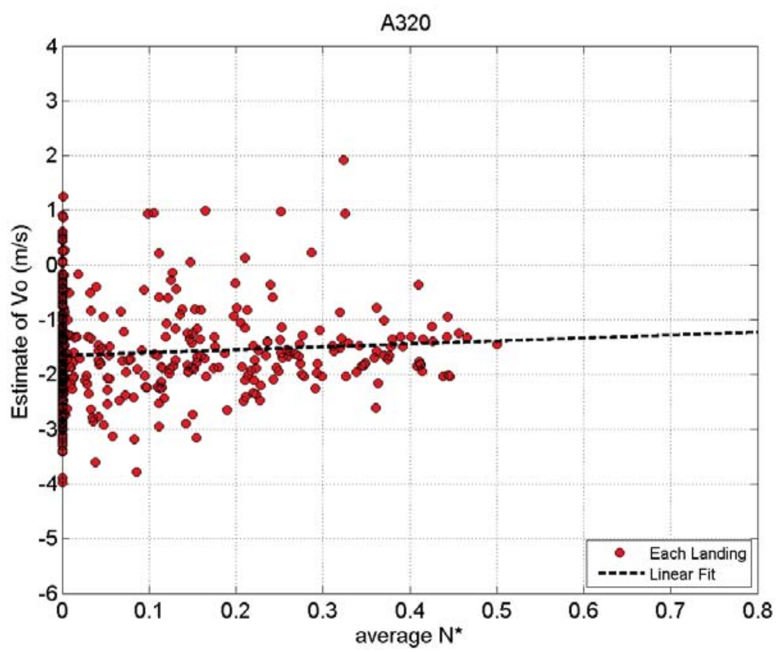


Figure 4. Estimates of V_o vs. N* for the Airbus A320 aircraft landings at Denver in 2006 and a linear least squares fit to the data (dashed line).

Turbulence effects were analyzed using estimates of the eddy dissipation rate (EDR). These EDR estimates were determined two ways. First, for the Denver 2006 database, EDR was estimated using velocity measurements from ultrasonic anemometers which were deployed at two heights on a tower. Estimates of EDR for each anemometer were obtained using a −5/3 power law which was fit to the portion of the spectrum that was contained within the inertial sub-range[4]. The EDR estimates at the two heights on the tower were extrapolated to the altitudes of the vortices using a similarity profile generator[5]. For the San Francisco 2001 and Denver 2003 databases, where there was no tower, the last data point was used as a proxy for EDR by using a functional relationship in Ref. 6 relating the linking time of vortices to the ambient EDR.

Figure 5 shows the same data as in Figures 3 and 4, but with descent speed plotted vs. average EDR*, where EDR* is the nondimensional EDR defined as $(\varepsilon b_o)^{1/3}/V_o$, where ε is the eddy dissipation rate. Individual landings are color coded with N*, shown on the color scale to the right of the figure. Figure 5 shows that there is an apparent minimum in the variations in V_o for EDR* around 0.1. For values around EDR* = 0, there is a larger variation of V_o, corresponding to small values of stratification. For values above approximately 0.15, the turbulent eddies modify V_o. The dashed black line in Figure 5 is a linear least squares fit to the data and shows there is essentially no variation in V_o with EDR*.

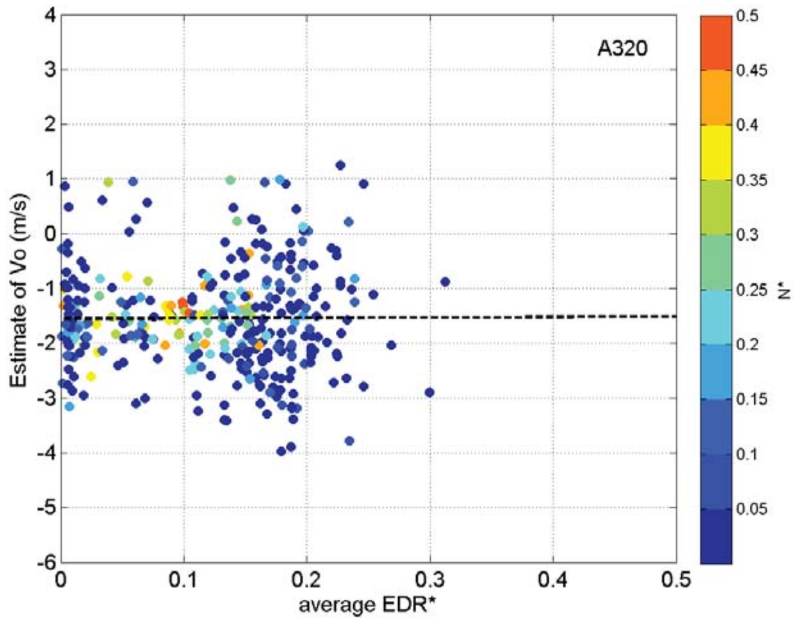


Figure 5. Estimates of V_o vs. EDR* for Airbus A320 aircraft at Denver in 2006.

The headwind component of the total wind (hereafter called "headwind") was also investigated for an effect on V_o. For these estimates, the headwind was obtained from the ASOS (Automated Surface Observing System) sensor on the airport, at a height of 10m. For a landing with headwind (tailwind), earlier (older) wakes are transported into the Range-Height-Indicator (RHI) lidar scan plane traditionally used in wake vortex measurements, thus contaminating the estimates of V_o. We do not know the exact correlation of the headwind at the ground to the headwind at altitude, so we will show results using all values of the headwind.

The equation for estimating the observed vortex vertical velocity, V_{obs}, to the true vortex vertical velocity, V_o, with a headwind, HW, and a glide slope, θ, is given by:

$$V_{obs} = -\left(V_o + HW\tan\theta\right)/\left(1 + \left(\frac{HW}{(U-HW)\cos\theta}\right)\right) \quad (2)$$

where U is the aircraft speed.

The numerator in Eq. (2) is often used as an approximation of the effect of headwind on V_o. However, the second term in the denominator can be an important one. Using an aircraft speed of 150 kt, a glide slope of 3 degrees, and no headwind, we will set V_o to -2.00 m/s. From Eq. (2), with a headwind of 10 kt and the same parameters, the observed V_o, V_{obs} = -2.12 m/s. This increase in V_o results in a 2.9% decrease in the estimate of b_o.

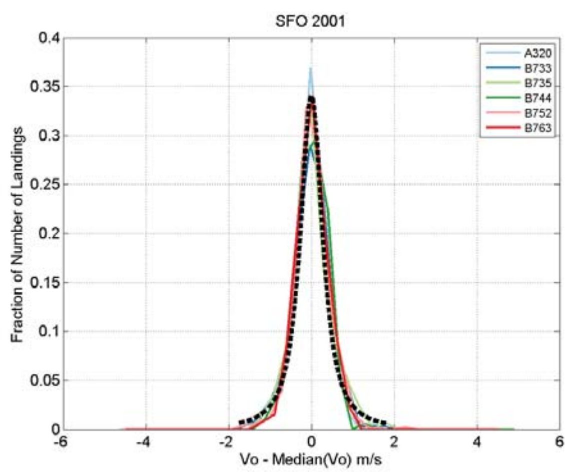
Had we used only the numerator in Eq. (2), the observed V_{obs} would be -2.27 m/s, resulting in a 6.5% decrease in the estimate of b_o.

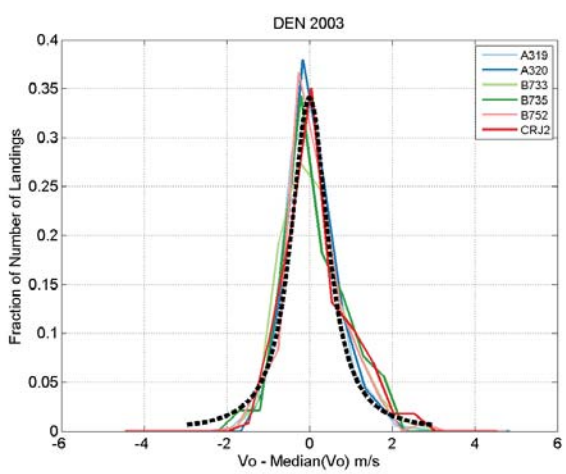
We do not know the value of the headwind at the altitudes of the aircraft vortex observations. However, at a 10m altitude, the median ASOS headwind for the Denver 2006 database is 5.0 kt. The above estimates of the error in estimating b_o from V_o used a headwind of 10 kt, or twice that commonly observed at the ground. Since there is typically a headwind of some magnitude for landing aircraft, the above analysis indicates that this method of estimating b_o from V_o will result in an estimated value of b_o that is smaller than the true value of b_o. We note from above that the result of using a headwind of 10 kt with a 3 degree glide slope results in an error in estimating b_o of less than 3%.

VI. Distributions of V_o

Using a calculated V_o for each landing using the methodology of Section IV, we compute the distribution of V_o for all the landings. We can also compute the residuals of V_o using the median V_o from the distribution. Figure 6 shows the residuals (V_o for each landing minus the median value of V_o) for all three databases with more than 100 landings for a given aircraft in each database. For each landing, V_o was determined using a 33-second linear fit of vortex altitude vs. time, ignoring the first data point, and no restrictions were placed on headwind. The black dashed line in each plot shows the Student's t-distribution.

The top plot in Figure 6 shows the residuals for six aircraft at SFO. In all plots in this figure, the distributions of the residuals are fairly consistent for all the aircraft shown, from the aircraft with the largest Maximum Take-Off Mass (MTM) down to the smallest MTM. Distributions of V_o residuals for the Denver data, shown in the middle and bottom plots, are also consistent for all the aircraft shown and are also consistent between the 2003 plot and the 2006 plot. The SFO data shows a smaller variance than observed at Denver.





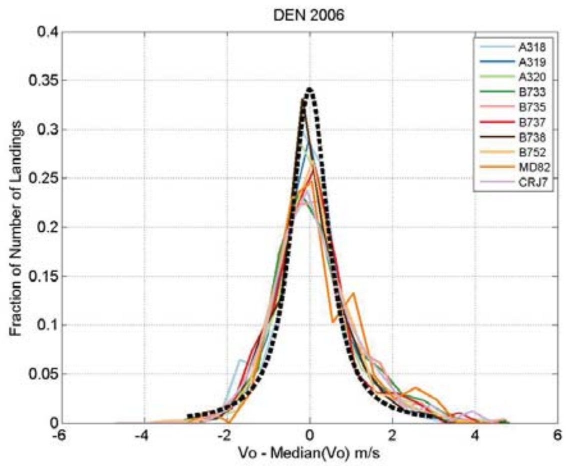


Figure 6. Residuals of V_o for SFO 2001 (top), DEN 2003 (middle), and DEN 2006 (bottom) data. Residuals are shown only for aircraft where there were more than 100 landings available for analysis in the database. This restriction allowed distributions for 6 aircraft in the SFO 2001 database, 6 aircraft in the DEN 2003 database, and 10 aircraft in the DEN 2006 database. The Student's t-distribution is shown as the black dashed line in each plot, which seems to provide a reasonable fit to the data.

VII. Estimates of b_o

Using the median V_o estimated from the altitude vs. time data, we can estimate b_o from Eq. (1). We note the importance of using the correct value of density in the estimates of b_o. Although the measured values of V_o differ between Denver and San Francisco, the values of b_o determined for a given aircraft from both locations are nearly the same when corrected for atmospheric density.

Estimates of b_o can also be obtained directly from the lidar measurements themselves by calculating the distance between the vortex cores. However, it is known that the LMCT lidar algorithm underestimates b_o[1].

Figure 7 shows estimates of b_o from both the indirect method, using V_o to infer b_o (blue squares), and the direct method of obtaining b_o from the lidar positions (black circles). In this plot, the direct measurements were the average distance between the vortex cores from 3 seconds to 45 seconds after the passage of the aircraft. In all cases, the direct lidar measurements of b_o are smaller than the indirect measurements from V_o, in agreement with the results of Ref. (1).

The important result in Figure 7 is that, using the method described in the paper, some aircraft (the B777-200 and the B747-400) appear to show a much smaller than elliptical value of b_o (elliptical is a value of 0.78 on the vertical axis in Figure 7) and some aircraft have a nearly elliptical value of b_o (the A319 and the A320).

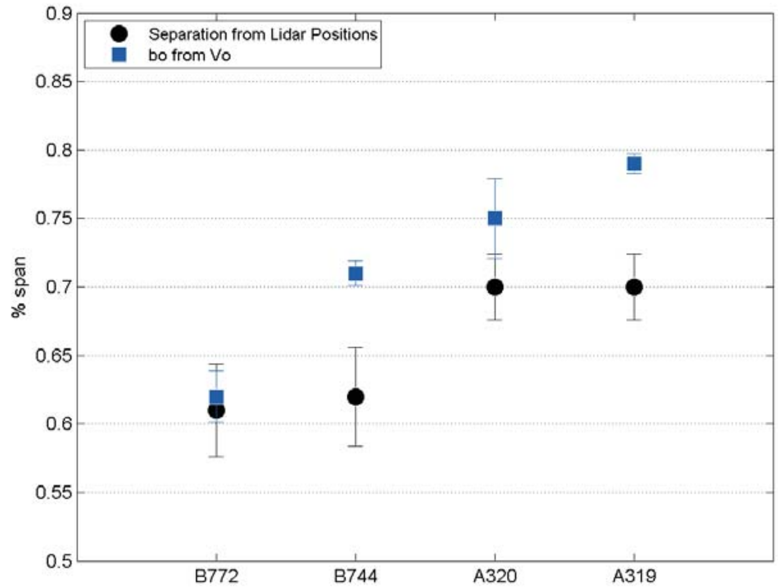


Figure 7. Estimates of mean b_o and standard deviation divided by wingspan for the A319, A320, B747-400, and B777-200 aircraft from the three databases. An elliptical value of b_o is a value of 0.78.

VIII. Conclusions

The parameter b_o, the initial separation of aircraft vortices, is used to nondimensionalize vortex altitude, vortex strength, and time. Thus, the quality of our nondimensional parameters depends on how well we can estimate b_o for each aircraft. Any differences in b_o from the commonly used elliptical values are important for correctly nondimensionalizing aircraft data so we can examine vortex evolution from different aircraft to determine how the vortex evolution is similar or different.

A methodology has been presented in this paper to estimate b_o from the lidar measurements of altitude vs. time. This method suggests that some aircraft have nearly elliptical values of b_o, while other aircraft have values of b_o that are nonelliptical. We are continuing our study of b_o to include other aircraft makes and models.

Acknowledgments

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