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**MULTISPAN ELEVATED GUIDEWAY DESIGN FOR PASSENGER
TRANSPORT VEHICLES
VOLUME II. APPENDIXES**

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

**PREPARED FOR
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16. Abstract Analysis techniques, a design procedure and design data are described for passenger vehicle, simply supported, single span and multiple span elevated guideway structures. Analyses and computer programs are developed to determine guideway deflections, moments and stresses and vehicle accelerations resulting from a two-dimensional vehicle with finite pad length front and rear suspensions traversing a multispans elevated guideway. A preliminary design procedure is described to estimate guideway beam structural requirements so that a vehicle-guideway system will meet specified levels of passenger comfort. Design data for 150 mph and 300 mph intercity 40,000, 80,000 and 120,000 lb. air cushion vehicle single and multiple span (span lengths of 50 to 150 ft.) guideways is summarized. For both urban and intercity operating regimes, the data indicates that improvements in the vehicle suspension and the use of multiple span structures rather than single span structures may result in reduced guideway material requirements. The nomenclature is contained in Appendix H, Volume II.		
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PREFACE

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APPENDIX A
DERIVATION OF VEHICLE-GUIDEWAY INTERACTION EQUATIONS

A.1 The Guideway Model

A.1.1 General Formulation

The guideway analysis is developed from the partial differential equation describing the motion of a single beam which may span several supports. In the derivation of the equation of motion the following assumptions are used:

- (1) Consideration is restricted to the transverse motion of a homogeneous, isotropic beam with a linear constitutive law relating stress and strain.
- (2) The beam rests upon rigid supports.
- (3) An individual beam is considered to be a Bernoulli-Euler beam. The neglect of rotary inertia and transverse shear which are present in a Timoshenko beam model is justified for spans in which length-to-height ratios are much greater than one and in which span forcing functions are generated by vehicles traveling less than 10% of the velocity of sound in a span. These assumptions are valid for typical vehicle-guideway systems in which vehicle velocities are less than 400 mph.
- (4) Damping is assumed to be linear, viscous damping and is assumed to be small, i.e. less than 10% of critical damping.
- (5) A beam is assumed to be flat with no surface irregularities under its own weight.*

Utilizing these assumptions, the partial differential equation of motion for a beam resting upon multiple supports and excited by an arbitrary forcing function may be derived as:

*The effects of camber and surface irregularity may be included in the analysis with little additional complexity and do not alter the basic modal shape functions or eigenvalues derived.

$$EI \frac{\partial^4 y}{\partial x^4} + \rho a \frac{\partial^2 y}{\partial t^2} + b \frac{\partial y}{\partial t} = f(x,t) \quad (A.1)$$

where:

EI = beam bending rigidity
 ρa = beam mass per unit length
 b = beam damping per unit length
 f(x,t) = time and spatially varying force per unit length
 y = guideway transverse displacement
 x = spatial horizontal coordinate
 t = time

To complete the description of the multi-span beam, the boundary conditions at the supports and the initial state of the beam at $t = 0$ must be specified. The following boundary conditions may be derived using the nomenclature illustrated in Fig. A.1.

(1) At all supports: $p = 0$ to k

The vertical displacement y is zero since the supports are rigid.

(2) At all internal supports: $p = 1$ to $k - 1$

(a) The span slope is continuous from span s to $s + 1$

(b) The span moment is continuous from span s to $s + 1$

(3) At the external supports: $p = 0$ and $p = k$ the span is considered to rest freely upon the support, i.e., a pinned end condition, with the moments at supports $p = 0$ and $p = k$ zero



Fig. A-1. A k Span Simply Supported Beam

The explicit use of these boundary conditions and the specification of span initial conditions are described in a following section.

In the modal analysis technique it is assumed that the transverse motion $y(x,t)$ of the beam which satisfies the partial differential equation (A.1) and the boundary conditions may be written as an infinite sum* of the products of time varying modal coefficients $A_m(t)$ which are functions only of time and modal shape functions $\phi_m(x)$:

$$y(x,t) = \sum_{m=1}^{\infty} A_m(t) \phi_m(x) \quad (A.2)$$

where ϕ_m are functions only of x and are orthogonal over the interval $0 < x < L$:

$$\int_0^L \phi_n(x) \phi_m(x) dx = 0 \text{ for } m \neq n \quad (A.3)$$

where: L = beam length = $\sum_{s=1}^k l_s$ for a k span beam

l_s = length of span

ϕ_m = mode m shape function

ϕ_n = mode n shape function

A_m = time varying modal coefficient

*In practice [1] only the first few terms of the sum and in some cases only the first term are required in many cases of practical interest to achieve results of acceptable accuracy for engineering purposes.

The modal shape functions $\phi_m(x)$ are determined from the natural unforced vibration of the span and represent the natural modes of vibration of the span while the time-varying coefficients $A_m(t)$ depend upon the forcing function $f(x,t)$ and must be determined from the complete forced equation.

The quantities A_m and ϕ_m may be determined by substituting (A.2) into (A.1) with the result:

$$\sum_{m=1}^{\infty} A_m EI \phi_m'''' + \rho a c_{12} \dot{A}_m + b \dot{A}_m \phi_m' = f(x,t) \quad (A.4)$$

where:

$$(\quad)' = \frac{d}{dx}(\quad)$$

$$(\dot{\quad}) = \frac{d}{dt}(\quad)$$

The natural modal shape functions must satisfy the homogeneous part of (A.4). Setting $f(x,t) = 0$ introducing the constant of separation ω_m and noting that ϕ_m are orthogonal functions, an equation for each mode m is derived as:

$$\frac{EI \phi_m''''}{\rho a \phi_m} = - \frac{\omega_m^2 + b \dot{A}_m / \rho a}{A_m} = \omega_m^2 \quad (A.5)$$

where:

ω_m = constant of separation

b_m = damping associated with mode m

Equation (A.5) may be written as two separate equations:

$$\phi_m'''' - \frac{\rho a \omega_m^2}{EI} \phi_m = 0 \quad (A.6)$$

which is expressed solely in terms of $\phi_m(x)$ and its derivatives with respect to x and:

$$\ddot{A}_m + \frac{b_m \dot{A}_m}{\rho a} + \omega_m^2 A_m = 0 \quad (A.7)$$

which is expressed solely in terms of $A_m(t)$ and its derivatives with respect to t . Directly from (A.7) it may be seen that ω_m represents the undamped natural frequency of the m^{th} mode of beam vibration.

A set of orthogonal functions $\phi_m(x)$ and values of ω_m are required which satisfy (A.6) and the support boundary conditions.

The derivation of appropriate functions $\phi_m(x)$ and values of ω_m for the discrete and semi-continuous span systems considered in this study are described in Section A.1.2. For the single span system $\phi_m(x)$ and ω_m are relatively simple and may be expressed as:

$$\phi_m = \sin\left(\frac{m\pi x}{L}\right) \quad (A.8)$$

$$\omega_m = \frac{\pi^2 m^2}{L^2} \sqrt{\frac{EI}{\rho a}} \quad (A.9)$$

while for other span systems they are more complex as shown in Section A.1.2.

The time varying modal coefficients depend upon the force distribution acting on the beam $f(x,t)$. When (A.6) which the $\phi_{11}(x)$ must satisfy, is combined with the forced equation of motion (A.4) the following complete equation of motion may be derived:

$$\sum_{m=1}^{\infty} \left[\ddot{A}_m + \frac{b_m}{\rho a} \dot{A}_m + \omega_m^2 A_m \right] \phi_m = \frac{1}{\rho a} f(x,t) \quad (A.10)$$

When each side of (A.10) is multiplied by the modal shape function $\phi_m(x)$ for a given mode m and both sides are integrated from 0 to L , an equation is obtained for each individual modal coefficient A_m in terms only of the m^{th} mode shape ϕ_m , natural frequency ω_m and damping ratio ξ_m since ϕ_m are orthogonal as indicated in (A.3). If in addition the mode shape functions used are required to be normalized so that:

$$\frac{1}{L} \int_0^L \bar{\phi}_m^2(x) dx = 1 \quad (A.11)$$

where:

$\bar{\phi}_m(x)$ = a normalized mode shape function satisfying (A.11)

the resulting equation derived from (A.10) is:

$$\ddot{A}_m + 2\xi_m \omega_m \dot{A}_m + \omega_m^2 A_m = \frac{1}{\rho a L} \int_0^L f(x,t) \bar{\phi}_m(x) dx \quad (A.12)$$

where

$$\xi_m = \frac{b_m}{2\rho a \omega_m}$$

For many cases of interest, the integral on the right side of (A.12) may be evaluated prior to solving (A.12) completely. The force distribution on the span in this study is assumed to result from the passage of a vehicle suspension air cushion or magnetic suspension pad or wheel. The force spatial distribution along the pad is considered to be uniform and varying in magnitude as a function of time; thus $f(x,t)$ may be written as a sum of the products of a time varying function and a uniform spatial distribution function:

$$f(x,t) = \sum_{i=1}^q F_i(t) p_i(x) \quad (A.13)$$

where:

$p_i(x)$ = uniform spatial distribution for i^{th} suspension pad

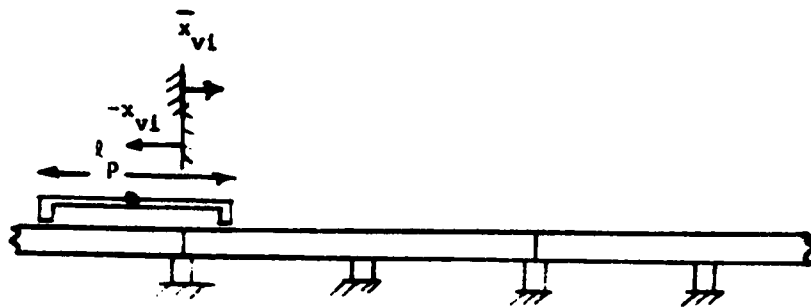
$F_i(t)$ = net force generated by the i^{th} suspension pad

q = number of pads

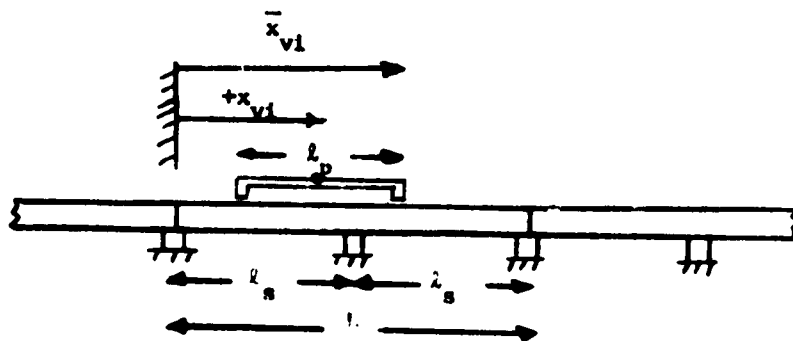
A guideway excited by a single suspension pad of length l_p is illustrated in Fig. A.2 for the three types of loading situations which must be considered. Using the nomenclature introduced in the Figure, $p_i(x)$ may be written as:

$$p_i(x) = \begin{cases} 0 & \text{for } x < x_{vi} - l_p/2 \\ 1/l_p & \text{for } x_{vi} - l_p/2 < x < x_{vi} + l_p/2 \\ 0 & \text{for } x_{vi} + l_p/2 < x \end{cases} \quad (A.14)$$

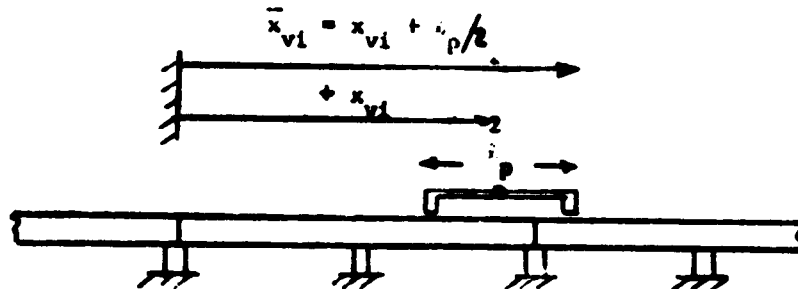
where l_p has been assumed to be identical for all pads under consideration at any given time and x_{vi} is the midpad position on the beam.



State 1: Pad Approaching Beam



State 2: Pad on Beam



State 3: Pad Leaving Beam

Fig. A-2. Suspension Pad States on a Semi-Continuous Span Guideway

The right-hand side of (A.12) may be written using (A.14)

as,

$$\sum_{i=1}^q \frac{F_i(t)}{\rho a L} \int_0^L p_i(x) \bar{\phi}_m(x) dx = \sum_{i=1}^q \frac{F_i(t)}{\rho a L l_p} \int_{x_{v1} - l_p/2}^{x_{v1} + l_p/2} \bar{\phi}_m(x) dx = \sum_{i=1}^q \frac{F_i(t)}{\rho a L} \psi_m(x_{v1}) \quad (\text{A.15})$$

where

$$\psi_m(x_{v1}) = \frac{1}{l_p} \int_{x_{v1} - l_p/2}^{x_{v1} + l_p/2} \bar{\phi}_m(x) dx \quad (\text{A.16})$$

The function $\psi_m(x_{v1})$ depends only upon the length of the pad l_p and the pad location and can be computed for a given pad and modal shape function $\bar{\phi}_m(x)$. The computation and tabulation of these functions are described in Section A.1.2. When the pad length approaches zero $\psi_m(x_{v1})$ reduces to simply $\bar{\phi}_m(x_{v1})$.

When (A.12) through (A.16) are combined, a single equation is obtained which yields the modal coefficients resulting from the passage of multiple finite pad length suspensions:

$$\ddot{A}_m + 2 \xi_m \omega_m \dot{A}_m + \omega_m^2 A_m = \sum_{i=1}^q \frac{F_i(t)}{\rho a L} \psi_m(x_{v1}) \quad (\text{A.17})$$

For each mode of beam vibration (A.17) may be used to determine $A_m(t)$ for $0 < t$ when at $t = 0$ the initial conditions $A_m(0)$ and $\dot{A}_m(0)$ are specified and when the forcing functions $F_i(t)$ and $\psi_m(x_{v1})$ are specified for $0 < t$.

With the solution of (A.17) the guideway deflection profile $y(x,t)$ may be determined directly since $\bar{\phi}_m(x)$ are determined from (A.6) and (A.11):

$$y(x,t) = \sum_{m=1}^{\infty} \Lambda_m(t) \bar{\phi}_m(x) \quad (A.18)$$

and, in turn, the bending moment and stress in the span may be determined as:

$$M_t = EI \frac{\partial^2 y}{\partial x^2} = EI \sum_{m=1}^{\infty} \Lambda_m(t) \frac{\partial^2 \bar{\phi}_m}{\partial x^2} \quad (A.19)$$

$$\sigma_t = \frac{M_t c^*}{I} = Ec^* \sum_{m=1}^{\infty} \Lambda_m(t) \frac{\partial^2 \bar{\phi}_m}{\partial x^2} \quad (A.20)$$

where:

M_t = bending moment

σ_t = stress due to bending moment

c^* = distance from beam centroid axis to stress surface

The primary feature of the modal analysis technique is that for many typical semi-continuous guideway spans where vehicle traverse velocities are less than 300 mph, the number of modes of vibration required in an analysis to obtain an estimate of the vehicle-span interaction with sufficient accuracy for engineering purposes is approximately equal to k where k equals the number of spans.

A.1.2 Beam Eigenvalues and Modal Shape Functions

The beam eigenvalues and modal shape functions are determined by seeking the solution to Equation (A.6) which satisfies the internal and external boundary conditions represented at each support. Equation (A.6) is of fourth order and has a solution of the general form:

$$\phi_m(x) = a'_m \sin \lambda_m x + b'_m \cos \lambda_m x + c'_m \sinh \lambda_m x + d'_m \cosh \lambda_m x \quad (\text{A.21})$$

where the parameter λ_m has been introduced with:

$$\lambda_m^4 = \frac{\rho A}{EI} \omega_m^2 \quad (\text{A.22})$$

and where a'_m , b'_m , c'_m , d'_m are arbitrary coefficients to be determined by boundary conditions.

It is noted that in order to satisfy the internal boundary conditions as well as the external boundary conditions, the values of a'_m , b'_m , c'_m and d'_m are, in general, different in each span of a beam; thus, it is convenient to express $\phi_m(x)$ as:

$$\phi_m(x) = \sum_{s=1}^k \phi_{ms}(x_s) \quad (\text{A.23})$$

where:

s = span number 1, 2, 3, from left to right

k = total number of spans in a beam

ϕ_{ms} = individual shape function for mode m of span s ,
defined as zero outside of span s

x_s = horizontal coordinate for span s extending over
the interval $0 < x_s < l_s$

l_s = length of span s : $\sum_{s=1}^k l_s = L$

Each ϕ_{ms} may be written explicitly in terms of the coefficients
for each span as:

$$\phi_{ms} = a_{ms} \sin \lambda_m x_s + b_{ms} \cos \lambda_m x_s + c_{ms} \sinh \lambda_m x_s + d_{ms} \cosh \lambda_m x_s \quad (A.24)$$

where:

a_{ms} , b_{ms} , c_{ms} , d_{ms} are the individual coefficients for
each span.

It is noted that each ϕ_{ms} is defined only over the part of
 x represented by span s ; i.e., ϕ_{ms} is defined for $0 < x_s < l_s$ or
for $l_1 + l_2 < x < \sum_{s=1}^k l_s$ and is zero elsewhere.

The general form of the modal shape functions are given by
(A.23) and (A.24) where the specific external and internal boundary
conditions are used to evaluate the coefficients in the general
functions. It is noted that the formulation allows individual spans
of various lengths l_s .

At each support the condition that the beam displacement is
zero requires for each mode m that:

$$\phi_{ms}(0) = \phi_{ms}(l_s) = 0 \text{ for } s = 1 \text{ to } k \quad (A.25)$$

The boundary condition (A.25) may be used to simplify the form of the modal shape functions. For (A.24) to satisfy (A.25) requires:

$$d_{ms} = -b_{ms}$$

and results directly in the simplification of (A.24) to:

$$\begin{aligned} \phi_{ms} &= a_{ms} \sin \lambda_{ms} x_s + b_{ms} (\cos \lambda_{ms} x_s + \cosh \lambda_{ms} z_s) + \\ & c_{ms} \sinh \lambda_{ms} x_s \end{aligned} \quad (A.26)$$

which will be used in further work.

The boundary conditions at the internal supports $p = 1$ to $k - 1$ require that for each mode m :

(a) The beam slope is continuous

$$\phi'_{ms}(\lambda_s) = \phi'_{m(s+1)}(0) \text{ for } s = 1 \text{ to } k - 1 \quad (A.27)$$

(b) The beam moment is continuous

$$\phi''_{ms}(\lambda_s) = \phi''_{m(s+1)}(0) \text{ for } s = 1 \text{ to } k - 1 \quad (A.28)$$

The boundary conditions at the external supports $p = 0$ and $p = k$ require for each mode m that:

The beam is pinned with zero moments

$$\phi''_{m1}(0) = \phi''_{mk}(\lambda_k) = 0 \quad (A.29)$$

The eigenvalues and modal shape functions may be determined directly by utilizing the boundary conditions of (A.27) and (A.28) and

(A.29) and the shape function of (A.26). First the eigenvalues are determined.

The eigenvalues of a semi-continuous beam may be conveniently determined by deriving a set of homogeneous equations to describe the unforced beam in terms of the moments at each support. Since the moments may be directly expressed in terms of the second derivative of the modal shape functions of (A.26), such a set of equations involves terms in λ_m . The condition for the occurrence of a natural vibration corresponds to the requirement that a nontrivial solution to the homogeneous equation set in terms of the support moments exists; thus, the determinant of the moment coefficient matrix must be zero. Setting the coefficient matrix determinant to zero yields a characteristic equation in terms of λ_m which may be solved to determine the eigenvalues of the beam. The details of the procedure are described below.

Let M_{ms} denote the moment at the right-hand end of a span s resting on support p due to the deflection of the m^{th} natural mode of vibration. The moment for any mode m at every interior support $p = 1$ to $k - 1$ may be expressed in terms of ϕ_{ms} as:

$$M_{ms} = -EI \phi''_{ms}(L_s) = -EI \phi''_{m(s+1)}(0) = 2EI\lambda_m^2 b_{m(s+1)} \quad (\text{A.30})$$

The equations relating the moment at a given internal support to the moments at other internal supports may be derived using the expression for ϕ_{ms} given in (A.26), the two conditions of moment and slope continuity at each internal support given in (A.27) and (A.28) and finally using (A.30) to relate M_{ms} to the mode shapes. When the internal support moment and slope conditions of (A.27) and (A.28) are applied to (A.26) the coefficients a_{ms} and c_{ms} may be expressed in terms of b_{ms} and $b_{m(s+1)}$:

$$a_{ms} = \frac{-b_{ms} \cos \lambda_m l_s + b_{m(s+1)}}{\sin \lambda_m l_s} \quad (\text{A.31})$$

$$c_{ms} = \frac{b_{ms} \cosh \lambda_m l_s - b_{m(s+1)}}{\sinh \lambda_m l_s} \quad (\text{A.32})$$

By using (A.31) and (A.32) with (A.26), utilizing (A.25) ($\phi_{ms}(l_s) = 0$) and finally substituting for b_{ms} with $M_{m(s-1)}$ for $b_{m(s+1)}$ with M_{ms} and for $b_{m(s+2)}$ with $M_{m(s+1)}$ as given by (A.30), the following recursion relationship may be derived for each internal support point:

$$M_{ms} M_{m(s-1)} - (G_{ms} + G_{m(s+1)}) M_{ms} + H_{m(s+1)} M_{m(s+1)} = 0 \quad (\text{A.33})$$

where the two quantities G_{ms} and H_{ms} have been introduced as:

$$G_{ms} = \coth \lambda_m l_s - \cot \lambda_m l_s \quad (\text{A.34})$$

$$H_{ms} = \operatorname{csch} \lambda_m l_s - \csc \lambda_m l_s \quad (\text{A.35})$$

Equation (A.33) may be used directly to develop the moment equations for each of the internal support points on the beam, i.e. support $p = 1$ to $k - 1$ corresponding to spans $s = 1$ to $k - 1$. It is noted that in deriving (A.33) a division by $\sin \lambda_m l_s$ has been performed and systems for which $\lambda_m l_s = m\pi$ are eigenvalues represent singularities to the equation set. Conditions for which these singularities exist* will be discussed subsequently.

To complete the formulation the conditions at the exterior right and left supports must be coupled with the equation set generated using (A.33). For the case of interest, the external ends of the beam are pinned, which requires that:

At support $p = 0$

$$M_{10} = 0$$

at support $p = k$

$$M_{nk} = 0$$

(A.36)

Thus, the equation set describing the free vibration of a multi-span beam may be developed by successively applying (A.33) at each internal support and then using (A.36) to eliminate M_{10} and M_{nk} .

In the general case, the equations describing the natural vibration of a beam consisting of equal length spans may be summarized

*The singularity exists for a system with identical length spans which has pinned ends and exists, in general, for any span for which $\sin m\pi x/l_s$ is a mode shape.

in matrix form as shown in Fig. A.3. Because the moments are zero at each end of the beam, the coefficient matrix is a $(k - 1)$ by $(k - 1)$ matrix.

Since the set of equations in Fig. A.3 is homogeneous, the condition for a nontrivial solution to exist, i.e., a natural mode of vibration, requires the determinant of the coefficient matrix to be zero. Values of λ_m^* for which the determinant equals zero are the beam eigenvalues and define modes of free vibration.

For calculation purposes, the expression for the determinant of the coefficient matrix can be somewhat simplified for beams with spans of equivalent length. By expanding the general determinant of Fig. A.3 by cofactors it may be noted that the determinant of a k span beam D_k may be written in terms of the determinants of $k - 1$ and $k - 2$ span beams:

$$D_k = -2 G_m D_{k-1} - H_m^2 D_{k-2} \quad (A.37)$$

Noting also that

$$D_2 = -2G_m \quad (A.38a)$$

and

$$D_3 = 4G_m^2 - H_m^2 \quad (A.38b)$$

*Note the ω_m , the beam natural frequencies, are related to the λ_m by (A.22).

$$\begin{bmatrix}
 -G_{m1} - G_{m2} & H_{m2} & 0 & 0 & 0 \\
 H_{m2} & -G_{m2} - G_{m3} & H_{m3} & 0 & 0 \\
 0 & H_{m3} & -G_{m3} - G_{m4} & H_{m4} & 0 \\
 \vdots & \vdots & \vdots & \vdots & \vdots \\
 H_{m(k-3)} & -G_{m(k-3)} - G_{m(k-2)} & H_{m(k-2)} & 0 & 0 \\
 0 & H_{m(k-2)} & -G_{m(k-2)} - G_{m(k-1)} & H_{m(k-1)} & 0 \\
 0 & 0 & H_{m(k-1)} & -G_{mk} & 0
 \end{bmatrix}
 \begin{bmatrix}
 V_{m1} \\
 V_{m2} \\
 V_{m3} \\
 \vdots \\
 V_{m(k-3)} \\
 V_{m(k-2)} \\
 V_{m(k-1)}
 \end{bmatrix}
 = 0$$

Fig. A-3. General Form of the Support Moment Equations for a k Span Simply Supported Beam

higher order determinants can be generated from (A.37) using lower order determinants beginning with (A.38).

To determine the eigenvalues of a k span beam, the characteristic equation

$$D_k = 0 \quad (A.39)$$

must be solved for the roots λ_m . Because the equation is transcendental, a two step procedure is useful in finding the solution. First, the equation is plotted using a digital computer to find the approximate location of the roots. A Newton-Raphson iteration is then used to converge to more accurate values. Also as noted previously, the eigenvalues for a single span pinned end beam,

$$\lambda_m l_s = m\pi \quad m = 1, 2, 3, \dots$$

constitute a singularity in the above defined equations. These are also eigenvalues of all multi-span pinned end beams.

Eigenvalues $\tilde{\lambda}_m = \lambda_m l_s$ for 1 through 9 span beams are tabulated in Table A.1. Note that they occur in clusters or bands defined by $\pi < \lambda_m l_s < 4.730$, $2\pi < \lambda_m l_s < 7.853$, etc.

The lower limits of each band are eigenvalues of a single span pinned end beam as previously noted, while the upper bounds are eigenvalues of a single span fixed end beam. The number of eigenvalues in each band corresponds to the number of spans in the beam. This 'bandpass' property is characteristic of all periodic structures.

TABLE A.1 EIGENVALUES $\bar{\lambda}_m = \lambda_m L$ IN FIRST TWO BANDS FOR ONE THROUGH

NINE PINNED END SPAN BEAMS

Number of Spans								
1	2	3	4	5	6	7	8	9
π	π	π	π	π	π	π	π	π
	3.927	3.556	3.393	3.309	3.261	3.230	3.210	3.196
		4.298	3.927	3.700	3.556	3.460	3.393	3.345
			4.463	4.153	3.927	3.764	3.645	3.556
				4.550	4.293	4.089	3.927	3.800
					4.601	4.315	4.208	4.053
						4.634	4.463	4.293
							4.655	4.513
								4.571
2π	2π	2π	2π	2π	2π	2π	2π	2π
	7.069	6.708	6.545	6.460	6.410	6.375	6.357	6.342
		7.430	7.069	6.849	6.708	6.612	6.545	6.497
			7.592	7.289	7.069	6.911	6.795	6.708
				7.677	7.430	7.226	7.069	6.946
					7.727	7.525	7.342	7.192
						7.759	7.592	7.430
							7.780	7.640
								7.795

The modal shape functions $\phi_m(x)$ are found by determining an expression $\phi_{ms}(x_s)$ for each span; the coefficients a_{ms} , b_{ms} and c_{ms} in (A.26) must be found which satisfy the beam boundary conditions. To define each mode shape $\phi_m(x)$ over all k spans thus requires the determination of $3k$ coefficients (a_{ms} , b_{ms} , and c_{ms} for $s = 1, 2, \dots, k$). Note that d_{ms} in (A.24) was previously eliminated from (A.26) using the boundary conditions stated in (A.25). Since a given mode shape has associated with it an arbitrary amplitude (i.e., if $\phi_m(x)$ is a mode shape, then any constant times $\phi_m(x)$ is also a mode shape), one of the $3k$ coefficients to be determined is arbitrary*, leaving $3k - 1$ independent coefficients. For convenience, a_{m1} is selected as the arbitrary constant for each mode in the work that follows. Subsequently, the value of a_{m1} will be selected to normalize the mode shape.

The basic determination of the $3k - 1$ independent coefficients may be performed in two steps:

- 1) Determine the coefficients of the first span, i.e., b_{m1} and c_{m1} in terms of a_{m1}
- 2) Determine a recursion relation for all spans $s = 2$ to k which relates $a_{m(s+1)}$, $b_{m(s+1)}$, and $c_{m(s+1)}$ to a_{ms} , b_{ms} , and c_{ms} .

The relations for the first span are considered first.

For a pinned end the boundary condition of (A.29) which requires zero moment yields:

*Note similarity between this and other eigenvalue problems where the magnitude of the eigenvectors is arbitrary.

$$\phi_{m1}''(0) = \lambda_m^2 b_{m1} = 0$$

$$b_{m1} = 0 \quad (A.40)$$

Noting (A.40) an expression for c_{m1} may be derived directly in terms of a_{m2} using the zero displacement condition $\phi_{m1}(l_1) = 0$:

$$c_{m1} = -a_{m1} \frac{\sin \lambda_m l_1}{\sinh \lambda_m l_1} \quad (A.41)$$

The derivation of the recursion relations for successive spans follows directly by using the boundary condition (A.25), (A.27) and (A.28) in manner similar to that used to derive the recursion relations for determination of beam eigenvalues. Using (A.31) and (A.32) derived with determination of the eigenvalues and noting $\phi_{ms}(l_s) = 0$ the following recursion relations may be obtained:

$$b_{m(s+1)} = a_{ms} \sin \lambda_m l_s + b_{ms} \cos \lambda_m l_s \quad (A.42)$$

$$a_{m(s+1)} = \left[\frac{\sinh \lambda_m l_{(s+1)}}{[\sinh \lambda_m l_{(s+1)} - \sin \lambda_m l_{(s+1)}]} \right] \cdot$$

$$\left[a_{ms} \cos \lambda_m l_s - b_{ms} (\sinh \lambda_m l_s + \sin \lambda_m l_s) + c_{ms} \cosh \lambda_m l_s \right] - b_{m(s+1)} \frac{\cosh \lambda_m l_{(s+1)} - \cos \lambda_m l_{(s+1)}}{\sinh \lambda_m l_{(s+1)} - \sin \lambda_m l_{(s+1)}} \quad (A.43)$$

$$\begin{aligned}
c_{m(s+1)} = & \left[\frac{\sin \lambda_m l (s+1)}{\sinh \lambda_m l (s+1) - \sin \lambda_m l (s+1)} \right] e \\
& \left[-a_{ms} \cos \lambda_m l_s + b_{ms} (\sinh \lambda_m l_s + \sin \lambda_m l_s) \right. \\
& \left. - c_{ms} \cosh \lambda_m l_s \right] + b_{m(s+1)} \frac{\cosh \lambda_m l (s+1) - \cos \lambda_m l (s+1)}{\sinh \lambda_m l (s+1) - \sin \lambda_m l (s+1)}
\end{aligned}
\tag{A.44}$$

For a given k-span beam the coefficients in ϕ_{ms} for $s = 1$ to k may be determined for each mode m in terms of a single constant a_{m1} using (A.40) and (A.41) for the first span and equations (A.42)---(A.44) for all spans $s = 2$ to k . The modal shape function $\phi_m(x)$ may be formed then by summing ϕ_{ms} over $s = 1$ to k (recall $\phi_{ms} = 0$ outside of span s) where for each mode m , $\phi_m(s)$ is determined in terms of the single arbitrary constant a_{m1} .

In numerical work it is convenient to use normalized modal shape functions. To obtain a normalized function, the value of the arbitrary constant a_{m1} is selected to yield:

$$\frac{1}{L} \int_0^L \bar{\phi}_m^2(x) dx = 1 = \sum_{s=1}^k \frac{1}{L_s} \int_0^{L_s} \bar{\phi}_{ms}^2(x_s) dx_s
\tag{A.45}$$

where $\bar{\phi}_m$ = normalized modal shape function
 $\bar{\phi}_{ms}$ = normalized span shape functions

While it is difficult to derive an explicit value for a_{m1} which will yield a normalized function, numerically the task may be accomplished by computing $\phi_m(x)$ with $a_{m1} = 1$ and then computing:

$$I_m^2 = \int_0^L [\phi_m^2(x)]_{a_{m1} = 1} dx/L \quad (A.46)$$

and finally forming:

$$\bar{\phi}_m(x) = \frac{1}{I_m} [\phi_m(x)]_{a_{m1} = 1} = \frac{k}{s_{m1}^2} \phi_{ms}/I_m = \frac{k}{s_{m1}^2} \bar{\phi}_{ms} \quad (A.47)$$

Normalized mode shapes may be determined directly with the use of (A.46) and (A.47).

A.1.3 Beam Forcing Functions

The forcing function for the m^{th} mode of vibration defined by Equation (A.15) is

$$\frac{q}{s_{m1}^2} \frac{F_1(t)}{\rho a L} \int_0^L p_1(x) \bar{\phi}_m(x) dx = \frac{q}{s_{m1}^2} \frac{F_1(t)}{\rho a L} \psi_m(x_{v1})$$

For a suspension with point force contact $p_1(x)$ becomes the dirac delta function $\delta(x - x_{v1})$ and the modal forcing function becomes

$$\frac{q}{l_{M1}} \frac{F_1(t)}{\rho a L} \int_0^L \delta(x - x_{v1}) \bar{\phi}_m(x) dx = \frac{q}{l_{M1}} \frac{F_1(t)}{\rho a L} \bar{\phi}_m(x_{v1}). \quad (A.48)$$

In this case the function $\psi_m(x_{v1})$ simply reduces to the mode shape $\bar{\phi}_m(x_{v1})$.

The function $\psi_m(x_{v1})$ for a finite length pad crossing a beam can be written:

$$\psi_m(x_{v1}) = \int_0^L p_1(x) \bar{\phi}_m(x) dx \quad (A.49)$$

where $p_1(x) = \frac{1}{l_p}$ within the region where the pad is on the beam and zero elsewhere as defined in (A.14). For convenience during the derivation that follows, (A.49) is written in terms of the variable $\bar{x}_{v1} = x_{v1} + l_p/2$ which denotes the position of the front of the pad as illustrated in Figure A.2 and the limits used to define $p_1(x)$ will reflect this change in variable.

$$\psi_m(\bar{x}_{v1}) = \int_0^L p_1(x) \bar{\phi}_m(x) dx \quad (A.50)$$

As a pressure pad crosses a guideway consisting of a series of semi-continuous beams, three states are possible with respect to each beam as shown in Fig. A.2. State 1 occurs while the pad is entering the beam, state 2 while the pad is fully on the beam, and state 3 as the pad is leaving the beam. Note that state 3 with respect to one beam appears as state 1 to the succeeding

beam and vice-versa. During states 1 and 3, the pad is influencing (forcing) two beams, while during state 2 only one beam interacts with the pad. Each of the above 3 states will now be examined in detail. It will be assumed in each case that the pad l_p is less than the length of each span of the beam.

State 1

During state 1

$$p_1(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{l_p} & 0 < x < \bar{x}_{v1} \\ 0 & \bar{x}_{v1} < x \end{cases} \quad (A.51)$$

Substituting (A.51) into (A.50)

$$\psi_m(\bar{x}_{v1}) = \frac{1}{l_p} \int_0^{\bar{x}_{v1}} \bar{\phi}_m(x) dx \quad (A.52)$$

Since the functions $\psi_m(\bar{x}_{v1})$ must be evaluated from the mode shapes $\bar{\phi}_m(x)$ which are written in terms of individual span mode shapes $\bar{\phi}_{ms}(x_s)$, it is convenient to write separate expressions corresponding to each span for $\psi_m(\bar{x}_{v1})$ similarly as $\psi_{ms}(\bar{x}_{s1})$. The subscript s denotes that ψ_{ms} is an expression for ψ_m which is valid within the span s and $\bar{x}_{s1}$ is the position of the front of the pad on span s . Using this nomenclature, (A.52) can be written

$$\psi_{ms}(\bar{x}_{s1}) = \frac{1}{l_p} \int_0^{\bar{x}_{s1}} \bar{\phi}_{ms}(x) dx$$

Substituting the expression for $\bar{\phi}_{ms}(x)$ defined by (A.26) and integrating,

$$\begin{aligned} \psi_{ms}(\bar{x}_{s1}) = & \frac{l_1}{l_p \lambda_m} b_{m1} \sin \lambda_m \bar{x}_{s1} - \frac{l_1}{l_p \lambda_m} a_{m1} \cos \lambda_m \bar{x}_{s1} \\ & + \frac{l_1}{l_p \lambda_m} b_{m1} \sinh \lambda_m \bar{x}_{s1} + \frac{l_1}{l_p \lambda_m} \cosh \lambda_m \bar{x}_{s1} \\ & + \frac{l_1}{l_p \lambda_m} (a_{m1} - c_{m1}) \end{aligned} \quad (A.53)$$

Note! (A.53) is valid for the region $0 < \bar{x}_{s1} < l_p$ for $s = 1$ only.

State 2

During state 2

$$p_1(x) = \begin{cases} 0 & x < \bar{x}_{v1} - l_p \\ \frac{1}{l_p} & \bar{x}_{v1} - l_p < x < \bar{x}_{v1} \\ 0 & \bar{x}_{v1} < x \end{cases} \quad (A.54)$$

Substituting (A.54) into (A.50)

$$\psi_m(\bar{x}_{v1}) = \frac{1}{l_p} \int_{\bar{x}_{v1} - l_p}^{\bar{x}_{v1}} \bar{\phi}_m(x) dx \quad (A.55)$$

For this state, two cases which require different expressions for $\bar{\phi}_m(x)$ must be considered depending upon the position of the pad on the span. For the first case, consider the pad to be supported completely on only one span s . Substituting the span no. 2 and the span position $\bar{x}_{s1}$ for the beam position x_{v1} as before, (A.55) can be written

$$\psi_{ms}(\bar{x}_{s1}) = \frac{1}{l_p} \int_{\bar{x}_{s1} - l_p}^{\bar{x}_{s1}} \phi_{ms}(x) dx$$

Substituting (A.26), integrating, and rearranging using the proper trigonometric identities

$$\begin{aligned} \psi_{ms}(\bar{x}_{s1}) = & \frac{l_s}{l_p \lambda_m} [a_{ms} \sin \lambda_m l_p + b_{ms} (1 - \cos \lambda_m l_p)] \sin \lambda_m \bar{x}_{s1} \\ & + \frac{l_s}{l_p \lambda_m} [a_{ms} (\cos \lambda_m l_p - 1) + b_{ms} \sin \lambda_m l_p] \cos \lambda_m \bar{x}_{s1} \\ & + \frac{l_s}{l_p \lambda_m} [b_{ms} (\cosh \lambda_m l_p - 1) + c_{ms} \sinh \lambda_m l_p] \sinh \lambda_m \bar{x}_{s1} \\ & + \frac{l_s}{l_p \lambda_m} [-b_{ms} \sinh \lambda_m l_p + c_{ms} (1 - \cosh \lambda_m l_p)] \cosh \lambda_m \bar{x}_{s1} \end{aligned}$$

(A.56)

(A.56) is valid for $l_p < \bar{x}_{s1} < l_s$, $s = 1, 2, \dots, k$

For the second case, the pad is crossing a support and is

supported by two spans, requiring that separate expressions be used for $\bar{\phi}_m(x)$ over each span. If the front of the pad is located on span s and the rear on span $s - 1$, (A.55) can be written

$$\psi_{ms}(\bar{x}_{s1}) = \frac{1}{l_p} \int_{l(s-1)-l_p+\bar{x}_{s1}}^{l(s-1)} \bar{\phi}_{m(s-1)}(x) dx + \frac{1}{l_p} \int_0^{\bar{x}_{s1}} \bar{\phi}_{ms}(x) dx$$

Substituting (A.26), integrating, and rearranging using the proper trigonometric identities

$$\psi_{ms}(\bar{x}_{s1}) =$$

$$\begin{aligned} & \left[\frac{l(s-1)}{l_p \lambda_m} \left[-a_{m(s-1)} \sin \lambda_m (l(s-1) - l_p) - b_{m(s-1)} \cos \lambda_m (l(s-1) - l_p) \right] \right. \\ & + \left. \frac{l_p}{l_p \lambda_m} b_{ms} \right] \sin \lambda_m \bar{x}_{s1} + \left[\frac{l(s-1)}{l_p \lambda_m} \left[a_{m(s-1)} \cos \lambda_m (l(s-1) - l_p) \right. \right. \\ & - \left. \left. b_{m(s-1)} \sin \lambda_m (l(s-1) - l_p) \right] - \frac{l_p}{l_p \lambda_m} a_{ms} \right] \cos \lambda_m \bar{x}_{s1} \\ & + \frac{l(s-1)}{l_p \lambda_m} \left[b_{m(s-1)} \cosh \lambda_m (l(s-1) - l_p) - c_{m(s-1)} \sinh \lambda_m (l(s-1) - l_p) \right] \\ & - \frac{l_p}{l_p \lambda_m} b_{ms} \left] \sinh \lambda_m \bar{x}_{s1} + \left[\frac{l(s-1)}{l_p \lambda_m} \left[b_{m(s-1)} \sinh \lambda_m (l(s-1) - l_p) \right. \right. \\ & - \left. \left. c_{m(s-1)} \cosh \lambda_m (l(s-1) - l_p) \right] + \frac{l_p}{l_p \lambda_m} c_{ms} \right] \cosh \lambda_m \bar{x}_{s1} \\ & + \left[\frac{l_p}{l_p \lambda_m} \left[-a_{m(s-1)} \cos \lambda_m l(s-1) + b_{m(s-1)} (\sin \lambda_m l(s-1) - \sinh \lambda_m l(s-1)) \right. \right. \\ & + \left. \left. c_{m(s-1)} \cosh \lambda_m l(s-1) \right] + \frac{l_p}{l_p \lambda_m} (a_{ms} - c_{ms}) \right] \end{aligned}$$

(A.57)

(A.57) is valid for $0 < \bar{x}_{s1} < l_p$, $s = 2, 3, \dots, k$

State 3

During state 3

$$p_i(x) = \begin{cases} 0 & x < \bar{x}_{v1} - l_p \\ \frac{1}{l_p} & \bar{x}_{v1} - l_p < x < L \\ 0 & L < x \end{cases} \quad (A.58)$$

where $L = \sum_{s=1}^k l_s$ is the length of the beam. Substituting (A.58) into (A.5):

$$\psi_n(\bar{x}_{v1}) = \frac{1}{l_p} \int_{\bar{x}_{v1} - l_p}^L \bar{\phi}_n(x) dx \quad (A.59)$$

Substituting the span number s and the span position $\bar{x}_{s1}$ as before, noting that for this case the front of the pack is on the first span of the next beam,

$$\psi_{ns}(\bar{x}_{s1}) = \frac{1}{l_p} \int_{l(n-1) - l_p + \bar{x}_{s1}}^{l(n-1)} \bar{\phi}_{n(n-1)}(x) dx$$

Using (A.26), integrating, and rearranging using the proper trigonometric identities,

$$\psi_{ms}(\bar{x}_{s1}) =$$

$$\begin{aligned} & \frac{\ell(s-1)}{\ell_p \lambda_m} [-a_{m(s-1)} \sin \lambda_m (\ell(s-1) - \ell_p) - b_{m(s-1)} \cos \lambda_m (\ell(s-1) - \ell_p)] \\ & \sin \lambda_m \bar{x}_{s1} + \frac{\ell(s-1)}{\ell_p \lambda_m} [\cos \lambda_m (\ell(s-1) - \ell_p) - b_{m(s-1)} \sin \lambda_m (\ell(s-1) - \ell_p)] \\ & \cos \lambda_m \bar{x}_{s1} + \frac{\ell(s-1)}{\ell_p \lambda_m} [b_{m(s-1)} \cosh \lambda_m (\ell(s-1) - \ell_p) - \\ & c_{m(s-1)} \sinh \lambda_m (\ell(s-1) - \ell_p)] \sinh \lambda_m \bar{x}_{s1} + \frac{\ell(s-1)}{\ell_p \lambda_m} \\ & [b_{m(s-1)} \sinh \lambda_m (\ell(s-1) - \ell_p) - c_{m(s-1)} \cosh \lambda_m (\ell(s-1) - \ell_p)] \\ & \cosh \lambda_m \bar{x}_{s1} + \frac{\ell(s-1)}{\ell_p \lambda_m} [-a_{m(s-1)} \cos \lambda_m \ell(s-1) + b_{m(s-1)} \sin \lambda_m \ell(s-1) \\ & - \sinh \lambda_m (s-1) + c_{m(s-1)}] \cosh \lambda_m \ell(s-1) \end{aligned}$$

(A.60)

Equation (A.60) is valid for $0 < \bar{x}_{s1} < \ell_p$ for $s = k+1$ only.

Equations (A.53), (A.56), (A.57) and (A.60) form the necessary expressions to determine $\psi_m(\bar{x}_{vi})$ for all $0 < \bar{x}_{vi} < L + \ell_p$, or for any position of the pad on the beam. Use of these equations can be simplified by noting that as the front of the pad crosses each individual span of the beam, two expressions for $\psi_{res}(\bar{x}_{s1})$ are required. One expression is valid for $0 < \bar{x}_{s1} < \ell_p$ and the other is valid for $\ell_p < \bar{x}_{s1} < \ell_s$. Also, since a portion of the pad remains on the beam

after the front of the pad has left the beam (State 3), $\psi_{ms}(\bar{x}_{s1})$ is defined for a region of the first span of the next beam which is designed as span $s = k + 1$ defined by $\tau < \bar{x}_{s1} < l_p$ for the beam in consideration. It therefore becomes convenient to divide each span into 2 segments, one of length l_p and the other of length $l_s - l_p$. In addition, a segment is defined of length l_p to the right of the beam as described above which corresponds to the first span of the next beam. The segments are numbered from left to right, the first segment of the next beam designated segment $2k + 1$ for the calculation for the beam of interest.

Each of the equations (A.53), (A.56), (A.57), and (A.60) are of the form

$$\begin{aligned} \psi_{ms}(\bar{x}_{s1}) = & H_1 \sin \lambda_m \bar{x}_{s1} + H_2 \cos \lambda_m \bar{x}_{s1} + H_3 \sinh \lambda_m \bar{x}_{s1} \\ & + H_4 \cosh \lambda_m \bar{x}_{s1} + H_5 \end{aligned} \quad (A.61)$$

where the coefficients, H_1 , H_2 , H_3 , H_4 , and H_5 are different for each of the segments.

For a k span beam with equal length spans of length l_s , the coefficients are summarized below. For convenience, the dimensionless parameters $\xi_s = \frac{\bar{x}_s}{l_s}$ and $\bar{l}_p = \frac{l_p}{l_s}$, and the eigenvalue parameter $\bar{\lambda}_m = \lambda_m l_s$ are used.

Range: $0 < \xi_s < \bar{\xi}_p$ [odd numbered segments]

$s = 1$ (from Equation (A.53))

$$H_1 = \frac{1}{\bar{\xi}_p \bar{\lambda}_m} b_{ms}$$

$$H_2 = \frac{-1}{\bar{\xi}_p \bar{\lambda}_m} a_{ms}$$

$$H_3 = \frac{1}{\bar{\xi}_p \bar{\lambda}_m} b_{ms} = H_1$$

$$H_4 = \frac{1}{\bar{\xi}_p \bar{\lambda}_m} c_{ms}$$

$$H_5 = \frac{1}{\bar{\xi}_p \bar{\lambda}_m} (a_{ms} - c_{ms})$$

$2 < s < k$ (from Equation (A.57))

$$H_1 = \frac{1}{\bar{\xi}_p \bar{\lambda}_m} [-a_m(s-1) \sin \bar{\lambda}_m (1 - \bar{\xi}_p) - b_m(s-1) \cos \bar{\lambda}_m (1 - \bar{\xi}_p) + b_{ms}]$$

$$H_2 = \frac{1}{\bar{\xi}_p \bar{\lambda}_m} [a_m(s-1) \cos \bar{\lambda}_m (1 - \bar{\xi}_p) - b_m(s-1) \sin \bar{\lambda}_m (1 - \bar{\xi}_p) - a_{ms}]$$

$$H_3 = \frac{1}{\bar{\xi}_p \bar{\lambda}_m} [b_m(s-1) \cosh \bar{\lambda}_m (1 - \bar{\xi}_p) - c_m(s-1) \sinh \bar{\lambda}_m (1 - \bar{\xi}_p) - b_{ms}]$$

$$H_4 = \frac{1}{\bar{\xi}_p \bar{\lambda}_m} [b_m(s-1) \sinh \bar{\lambda}_m (1 - \bar{\xi}_p) - c_m(s-1) \cosh \bar{\lambda}_m (1 - \bar{\xi}_p) + c_{ms}]$$

$$H_5 = \frac{1}{\bar{\xi}_p \bar{\lambda}_m} [-a_m(s-1) \cos \bar{\lambda}_m + b_m(s-1) (\sin \bar{\lambda}_m - \sinh \bar{\lambda}_m) + c_m(s-1) \cosh \bar{\lambda}_m + a_{ms} - c_{ms}]$$

$s = k + 1$ (From Equation (A.60))

$$H_1 = \frac{1}{\bar{\xi}_p \bar{\lambda}_m} [-a_{mk} \sin \bar{\lambda}_m (1 - \bar{\xi}_p) - b_{mk} \cos \bar{\lambda}_m (1 - \bar{\xi}_p)]$$

$$H_2 = \frac{1}{\bar{\xi}_p \bar{\lambda}_m} [a_{mk} \cos \bar{\lambda}_m (1 - \bar{\xi}_p) - b_{mk} \sin \bar{\lambda}_m (1 - \bar{\xi}_p)]$$

$$H_3 = \frac{1}{\bar{\xi}_p \bar{\lambda}_m} [b_{mk} \cosh \bar{\lambda}_m (1 - \bar{\xi}_p) - c_{mk} \sinh \bar{\lambda}_m (1 - \bar{\xi}_p)]$$

$$H_4 = \frac{1}{\bar{\xi}_p \bar{\lambda}_m} [b_{mk} \sinh \bar{\lambda}_m (1 - \bar{\xi}_p) - c_{mk} \cosh \bar{\lambda}_m (1 - \bar{\xi}_p)]$$

$$H_5 = \frac{1}{\bar{\xi}_p \bar{\lambda}_m} [-a_{mk} \cos \lambda_m + b_{mk} (\sin \lambda_m - \sinh \lambda_m) + c_{mk} \cosh \lambda_m]$$

Range: $\bar{\xi}_p < \xi_s < 1$ [even numbered segments]

$0 < s < k$

$$H_1 = \frac{1}{\bar{\xi}_p \bar{\lambda}_m} [a_{ms} \sin \bar{\lambda}_m \bar{\xi}_p + b_{ms} (1 - \cos \bar{\lambda}_m \bar{\xi}_p)]$$

$$H_2 = \frac{1}{\bar{\xi}_p \bar{\lambda}_m} [a_{ms} (\cos \bar{\lambda}_m \bar{\xi}_p - 1) + b_{ms} \sin \bar{\lambda}_m \bar{\xi}_p]$$

$$H_3 = \frac{1}{\bar{\xi}_p \bar{\lambda}_m} [b_{ms} (\cosh \bar{\lambda}_m \bar{\xi}_p - 1) + c_{ms} \sinh \bar{\lambda}_m \bar{\xi}_p]$$

$$H_4 = \frac{1}{\bar{\xi}_p \bar{\lambda}_m} [-b_{ms} \sinh \bar{\lambda}_m \bar{\xi}_p + c_{ms} (1 - \cosh \bar{\lambda}_m \bar{\xi}_p)]$$

$$H_5 = 0$$

A.2 THE VEHICLE MODEL

Directly from Fig. 2.1, the equations of motion for the two dimensional vehicle may be derived as:

$$0.5 m_v (\ddot{y}_{2f} + \ddot{y}_{2r} + 2g) + b_b (\dot{y}_{2f} + \dot{y}_{2r}) + k_b (y_{2f} + y_{2r}) = b_b (\dot{y}_{1f} + \dot{y}_{1r}) + k_b (y_{1f} + y_{1r}) \quad (A.62)$$

$$\frac{2I_v}{L_a^2} (\ddot{y}_{2f} - \ddot{y}_{2r}) + b_b (\dot{y}_{2f} - \dot{y}_{2r}) + k_b (y_{2f} - y_{2r}) = b_b (\dot{y}_{1f} - \dot{y}_{1r}) + k_b (y_{1f} - y_{1r}) \quad (A.63)$$

$$0.5 m_u (\ddot{y}_{1f} + g) + b_b \dot{y}_{1f} + (k_b + k_{sr}) y_{1f} = b_b \dot{y}_{2f} + k_b y_{2f} + k_{sr} y_{of} \quad (A.64)$$

$$0.5 m_u (\ddot{y}_{1r} + g) + b_b \dot{y}_{1r} + (k_b + k_{sr}) y_{1r} = b_b \dot{y}_{2r} + k_b y_{2r} + k_{sr} y_{or} \quad (A.65)$$

where:

m_v = vehicle sprung mass

m_u = vehicle total unsprung mass

I_v = vehicle moment of inertia with respect to geometric center

$y_{of}(y_{or})$ = guideway displacement directly beneath front (rear) pad midpoint

$y_{1f}(y_{1r})$ = front (rear) suspension pad midpoint displacement

$y_{2f}(y_{2r})$ = vehicle displacement at front (rear) suspension attachment point

g = acceleration due to gravity

The net force on the guideway generated by the front and rear suspension pads is:

$$F_{sf} = k_{sr} (y_{1f} - y_{of}) \quad (A.66)$$

$$F_{sr} = k_{sr} (y_{1r} - y_{or}) \quad (A.67)$$

where:

$F_{sf} (F_{sr})$ = net force acting on guideway due to front (rear) suspension pad

The model is valid only when F_{sf} and F_{sr} are negative since if they are positive the vehicle suspension has left the guideway, i.e. a wheel-hop condition in a wheeled suspension. For vehicles designed to meet desired levels of passenger safety and comfort this condition should not occur.

To complete the description of the vehicle the initial values of y_{2f} , $\dot{y}_{2f}$, y_{2r} , $\dot{y}_{2r}$, y_{1f} , $\dot{y}_{1f}$, y_{1r} and $\dot{y}_{1r}$ must be specified at $t = 0$.

A.3 Summary of Vehicle-Guideway Nondimensional Interaction Equations

The complete set of equations describing the two dimensional vehicle model and the guideway may be derived in nondimensional form using the fundamental parameters defined in Table 2.1. For a given guideway beam of interest the dynamic deflection profile is:

$$Y(X, \tau) = \sum_{n=1}^{\infty} a_n(\tau) \bar{\phi}_n(X) \quad (A.68)$$

where each a_n is determined from the equation:

$$\frac{d^2 a_n}{d\tau^2} + 2\bar{\xi}_n \bar{\omega}_n \frac{da_n}{d\tau} + \bar{\omega}_n^2 a_n = \frac{1}{2k_1 l_1} \bar{F}_1 \bar{\psi}_n \bar{K}_{v1, L_p} \quad (A.69)$$

where:

$$Y = y/y^*$$

$$X = x/l_g$$

$$\tau = 2\pi f^* t$$

$$a_n = A_n/y^*$$

$$\bar{\omega}_n = \omega_n/(2\pi f^*)$$

The normalizing deflection y^* is the deflection* of a single pinned end span of length l_s loaded at midspan by a single concentrated force equal to $(m_u + m_v)g$:

$$y^* = \frac{2(m_u + m_v)g l_s^3}{\pi^4 EI} \quad (A.70)$$

The vehicle suspension position X_{v1} may be related to time for a vehicle traveling at constant speed v :

$$X_{v1} = vt/l_s - l_1/l_s = v\tau/2\pi - \bar{l}_1 \quad (A.71)$$

where

$$\bar{l}_1 = l_1/l_s = \text{nondimensional suspension position at time } t = 0 \text{ as measured from } x = 0$$

The initial conditions for each beam represented by the values of $\frac{dy}{d\tau}$ and y_m must also be specified at $\tau = 0$.

For each vehicle considered, the equations which describe the two dimensional vehicle are:

$$\begin{aligned} M \left(\frac{d^2 y_{2f}}{d\tau^2} + \frac{d^2 y_{2r}}{d\tau^2} \right) + 1 + \frac{2E_M}{\Omega} \left(\frac{dy_{2f}}{d\tau} + \frac{dy_{2r}}{d\tau} \right) + \frac{M}{\Omega^2} (y_{2f} + y_{2r}) = \\ \frac{2E_M}{\Omega} \left(\frac{dy_{1f}}{d\tau} + \frac{dy_{1r}}{d\tau} \right) + \frac{M}{\Omega^2} (y_{1f} + y_{1r}) \end{aligned} \quad (A.72)$$

*The deflection is computed using the first mode deflection $2(m_u + m_v)gl_s^3/\pi^4 EI$, while the exact deflection from beam theory is $(m_u + m_v)gl_s^3/48EI$. The ratio $48/(\pi^4/2)$ is 0.985.

$$\begin{aligned}
& \frac{\Omega^2 \bar{I}_v}{3} \left(\frac{d^2 Y_{2f}}{d\tau^2} - \frac{d^2 Y_{2r}}{d\tau^2} \right) + 2 \xi_v \Omega \left(\frac{dY_{2f}}{d\tau} - \frac{dY_{2r}}{d\tau} \right) + (Y_{2f} - Y_{2r}) \\
& = 2 \xi_v \Omega \left(\frac{dY_{1f}}{d\tau} - \frac{dY_{1r}}{d\tau} \right) + (Y_{1f} - Y_{1r})
\end{aligned} \tag{A.73}$$

$$\begin{aligned}
& M M_u \frac{d^2 Y_{1f}}{d\tau^2} + 1 + \frac{2 \xi_v M}{\Omega} \frac{dY_{1f}}{d\tau} + \frac{(1+K)M}{\Omega^2} Y_{1f} = \frac{2 \xi_v M}{\Omega} \frac{dY_{2f}}{d\tau} \\
& + \frac{M}{\Omega^2} K Y_{of}
\end{aligned} \tag{A.74}$$

$$\begin{aligned}
& M M_u \frac{d^2 Y_{1r}}{d\tau^2} + 1 + \frac{2 \xi_v M}{\Omega} \frac{dY_{1r}}{d\tau} + \frac{(1+K)M}{\Omega^2} Y_{1r} = \frac{2 \xi_v M}{\Omega} \frac{dY_{2r}}{d\tau} + \\
& + \frac{M}{\Omega^2} Y_{2r} + \frac{MK}{\Omega^2} Y_{or}
\end{aligned} \tag{A.75}$$

The force generated by a given suspension may be written:

$$\bar{F}_i = \frac{M}{\Omega^2} \frac{1}{1 + M_u} K (Y_i - Y_o)_i \tag{A.76}$$

where for the front (rear) suspension of a single two dimensional vehicle $i = f$ ($i = r$) and where Y_{oi} is simply the displacement $Y_{oi} = Y(X_{vi}, \tau)$ computed directly from (A.68). The initial conditions at time $\tau = 0$ for the vehicle include the specification of $\frac{dY_{1f}}{d\tau}$, Y_{1f} , $\frac{dY_{2f}}{d\tau}$, Y_{2f} , $\frac{dY_{1r}}{d\tau}$, Y_{1r} , $\frac{dY_{2r}}{d\tau}$ and Y_{2r} .

Equations (A.68) through (A.76) summarize the behavior of a two dimensional vehicle with finite pad length suspensions traversing a semi-continuous guideway system. In addition to the

variables described by these equations the nondimensional moment and stress are also of interest:

$$\bar{M}_t = M_t / M_t^* = \frac{EI y^*}{l_s^2 M_t^*} \frac{\partial^2 y}{\partial x^2} \quad (A.77)$$

$$\bar{\sigma}_t = \sigma_t / \sigma_t^* = \bar{M}_t \quad (A.78)$$

where:

M_t^* = first mode midspan moment in a single discrete span due to a force equal to the vehicle weight concentrated at the midspan.

σ_t^* = first mode midspan stress in a single discrete span due to a force equal to the vehicle weight concentrated at the midspan.

and where:

$$M_t^* = \frac{EI \pi^2 y^*}{l_s^2} \quad (A.79)$$

$$\sigma_t^* = \frac{M_t^* c^*}{I} \quad (A.80)$$

When the normalizing factors are selected as given in (A.79) and (A.80) it is noted that the nondimensional stress is equal to the nondimensional moment.

A.4 SUMMARY OF VEHICLE TRANSFER FUNCTIONS

For the constant force analysis it is convenient to represent the vehicle equations in transfer function form. For the vehicle model described in Chapter 2 and represented by the equations summarized in A.2 the pitch and heave transfer functions may be derived as described below.

The pitch and heave transfer functions may be directly related to the transfer functions which yield the front and rear vehicle body suspension attachment point accelerations, due to the front and rear suspension motion:

$$\ddot{Y}_{2f} = T_s \ddot{Y}_{of} + T_c \ddot{Y}_{or} \quad (A.81)$$

$$\ddot{Y}_{2r} = T_c \ddot{Y}_{of} + T_s \ddot{Y}_{or} \quad (A.82)$$

where: T_s = front (back) acceleration nondimensional transfer function due to front (back) suspension input displacement

T_c = front (back) acceleration nondimensional transfer function due to back (front) suspension input displacement

When it is noted the vehicle heave and pitch accelerations are related to $\ddot{Y}_{2r}$ by:

$$\ddot{Y}_c = 0.5 (\ddot{Y}_{2f} + \ddot{Y}_{2r}) \quad (A.83)$$

$$\delta = \left(\frac{y^*}{0.54a} \right) \frac{\ddot{Y}_{2f} - \ddot{Y}_{2r}}{2} \quad (A.84)$$

With accelerations normalized by the vehicle suspension natural frequency ω_v and the midspan guideway displacement due to a constant force y^* , the nondimensional transfer functions T_g and T_c can be derived from the vehicle equations summarized in Appendix A.2 in terms of nondimensional frequency $\hat{\omega} = \omega/\omega_v$ as:

$$T_g(j\hat{\omega}_1) = \frac{c_{s7}(j\hat{\omega}_1)^7 + c_{s6}(j\hat{\omega}_1)^6 + c_{s5}(j\hat{\omega}_1)^5 + c_{s4}(j\hat{\omega}_1)^4 + c_{s3}(j\hat{\omega}_1)^3 + c_{s2}(j\hat{\omega}_1)^2}{DNM(j\hat{\omega}_1)} \quad (A.85)$$

$$T_c(j\hat{\omega}_1) = \frac{c_{c7}(j\hat{\omega}_1)^7 + c_{c6}(j\hat{\omega}_1)^6 + c_{c5}(j\hat{\omega}_1)^5 + c_{c4}(j\hat{\omega}_1)^4}{DNM(j\hat{\omega}_1)} \quad (A.86)$$

where the system characteristic equation is:

$$DNM(j\hat{\omega}_1) = d_8(j\hat{\omega}_1)^8 + d_7(j\hat{\omega}_1)^7 + d_6(j\hat{\omega}_1)^6 + d_5(j\hat{\omega}_1)^5 + d_4(j\hat{\omega}_1)^4 + d_3(j\hat{\omega}_1)^3 + d_2(j\hat{\omega}_1)^2 + d_1(j\hat{\omega}_1)^1 + d_0 \quad (A.87)$$

The coefficients in (A.85) to (A.87) are tabulated in Table A.2 in terms of the nondimensional parameters identified for the vehicle described in Chapter 2: M_u the unsprung mass ratio, I_v the inertia ratio, ξ_v the suspension damping ratio and K the suspension stiffness ratio.

The heave and pitch transfer functions may be determined from T_g and T_c using (A.81) and (A.82) directly.

TABLE A.2

VEHICLE TRANSFER FUNCTION COEFFICIENTS

$$c_{s7} = K\xi_v M_u \left[1 + \frac{\bar{I}_v}{3}\right]$$

$$c_{s6} = K M_u \left[\frac{1}{2} + \frac{\bar{I}_v}{6}\right] + 4\xi_v^2 K \left[M_u + \frac{1}{2} + \frac{\bar{I}_v}{6}\right]$$

$$c_{s5} = K\xi_v \left[4M_u + (2 + K) \left(1 + \frac{\bar{I}_v}{3}\right)\right]$$

$$c_{s4} = K \left(M_u + (1 + K) \left(\frac{1}{2} + \frac{\bar{I}_v}{6}\right)\right) + 4\xi_v^2 K^2$$

$$c_{s3} = 4K^2 \xi_v$$

$$c_{s2} = K^2$$

$$c_{c7} = -K\xi_v M_u \left[1 - \frac{\bar{I}_v}{3}\right]$$

$$c_{c6} = -K \left(M_u + 4\xi_v^2\right) \left(\frac{1}{2} - \frac{\bar{I}_v}{6}\right)$$

$$c_{c5} = -K\xi_v (2 + K) \left(1 - \frac{\bar{I}_v}{3}\right)$$

$$c_{c4} = -K(1 + K) \left(\frac{1}{2} - \frac{\bar{I}_v}{6}\right)$$

$$d_8 = M_u^2 \left[\left(\frac{1}{2} + \frac{\bar{I}_v}{6}\right)^2 - \left(\frac{1}{2} - \frac{\bar{I}_v}{6}\right)^2\right]$$

$$d_7 = 4\xi_v M_u \left[\left(\frac{1}{2} + \frac{\bar{I}_v}{6}\right) \left(M_u + \frac{1}{2} + \frac{\bar{I}_v}{6}\right) - \left(\frac{1}{2} - \frac{\bar{I}_v}{6}\right)^2\right]$$

$$d_6 = \xi_v^2 \left[2M_u + 1 + \frac{\bar{I}_v}{3}\right]^2 - (4\xi_v^2 + 2M_u(1 + K)) \left(\frac{1}{2} - \frac{\bar{I}_v}{6}\right)^2 \\ + M_u \left[1 + \frac{\bar{I}_v}{3}\right] \left[(1 + K) \left(\frac{1}{2} + \frac{\bar{I}_v}{6}\right) + M_u\right]$$

$$\begin{aligned}
d_5 &= 2\xi_v K M_u \left[1 + \frac{\bar{I}_v}{3}\right] - \xi_v \left(1 - \frac{\bar{I}_v}{3}\right)^2 (1 + K) + 2\xi \left[2M_u + 1 + \frac{\bar{I}_v}{3}\right] \\
&\quad + \xi_v \left[2M_u + 1 + \frac{\bar{I}_v}{3}\right] \left[(1 + K) \left(1 + \frac{\bar{I}_v}{3}\right) + 2M_u\right] \\
d_4 &= K M_u \left[1 + \frac{\bar{I}_v}{3}\right] + 4K\xi_v^2 \left[2M_u + 1 + \frac{\bar{I}_v}{3}\right] - (1 + K)^2 \left(\frac{1}{2} - \frac{\bar{I}_v}{6}\right)^2 \\
&\quad + \left[(1 + K) \left(\frac{1}{2} + \frac{\bar{I}_v}{6}\right) + M_u\right]^2 \\
d_3 &= 2\xi_v K \left[(2 + K) \left(1 + \frac{\bar{I}_v}{3}\right) + 4M_u\right] \\
d_2 &= 4\xi_v^2 K^2 + K \left[(1 + K) \left(1 + \frac{\bar{I}_v}{3}\right) + 2M_u\right] \\
d_1 &= 4\xi_v K^2 \\
d_0 &= K^2
\end{aligned}$$

APPENDIX B
EVALUATION OF PIER SUPPORT DYNAMICS

B.1 INTRODUCTION

The general guideway model described in Chapter 2 is based upon the assumption that the pier and foundation support structure are "rigid", i.e. their dynamic motions may be neglected in comparison to the span dynamic motions. In the paragraphs below a dynamic model is formulated for the pier-foundation-soil system. Methods for determining dynamic motion of the pier resulting from a vehicle passage are derived.

B.2 SUPPORT STRUCTURE MODEL

A sketch of the support structure model considered is shown in Figure B.1. The span, column and foundation are represented by the effective masses m_s , m_c and m_f , while the visco-elastic properties of the soil are represented by an effective stiffness k_f and damping b_f . The guideway mass m_s represents the span effective translational mass*. The column stiffness for the typical designs considered (column heights up to 26 ft.) is sufficiently greater than the soil effective stiffness, i.e. worst case column stiffness is at least three times the soil stiffness, so that the column is modeled as a mass**.

*The distributed mass and flexibility of the span are still present in the span model equations.

** If the column stiffness is reduced to the point where it is comparable to the soil stiffness, its stiffness should also be included in the model.

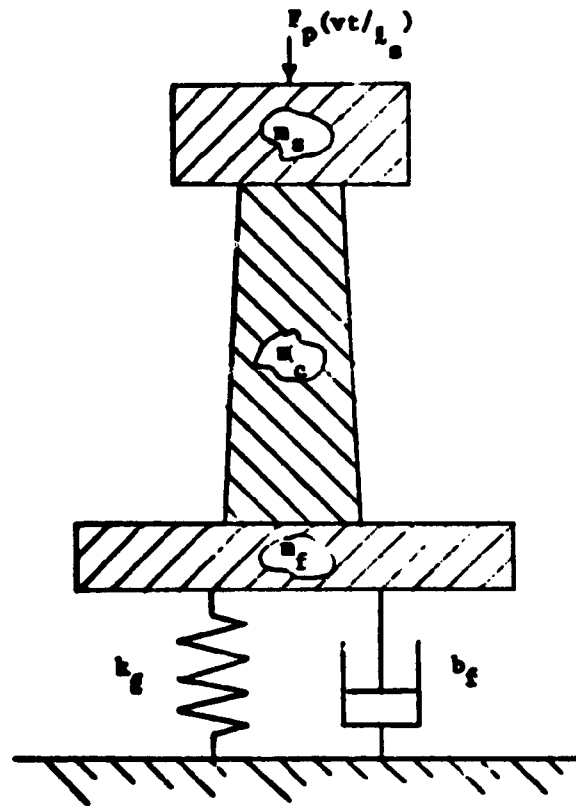


Fig. B-1. Idealized Lumped Element Span, Column, Footing and Soil Model

The foundation is modeled as a lumped mass since its stiffness is substantial, i.e. greater than 10 times the soil stiffness for all cases considered. The soil effective stiffness k_f and damping b_f depend upon the foundation area and the local soil properties. When the soil is considered as a semi-infinite region, the stiffness and damping properties may be derived using the Lysmer soil model [33] as:

$$k_f = \frac{4G_s r_o}{1 - \nu_s} \quad (B.1)$$

$$b_f = \frac{3.4 r_o^2 \sqrt{\rho_s G_s}}{(1 - \nu_s)} \quad (B.2)$$

where:

G_s = soil shear modulus

ρ_s = soil density

ν_s = poisson ratio of soil

r_o = equivalent footing radius^a

The principal damping in the model arises from radiation of energy through the soil. The damping is proportional to r_o^2 while stiffness is proportional to r_o .

The dynamic equation of motion for the model may be written in nondimensional form in terms of the pier motion, Y_p , and the force acting on the pier, $\bar{F}_p$:

^aFor a rectangular footing, $r_o = \sqrt{\frac{l_t l_b}{\pi}}$ where the footing length l_b and width l_t are defined in Fig. B.5.

$$\frac{d^2 Y_p}{d\tau^2} + 2\xi_p \Omega_p \frac{dY_p}{d\tau} + \Omega_p^2 Y_p = \frac{\bar{F}_p}{4 M_p} \quad (B.3)$$

where:

$$Y_p = \frac{y_p}{y^*}$$

$$\bar{F}_p = \frac{f_p}{0.5 (m_u + m_v) g}$$

$$\xi_p = \frac{0.5 b_f}{\sqrt{k_f (m_s + m_c + m_f)}}$$

$$\Omega_p = \frac{\omega_p}{\omega_1} = \frac{\sqrt{k_f / (m_s + m_c + m_f)}}{\omega_1} = \frac{\omega_p}{2\pi f^*}$$

$$M_p = \frac{m_s + m_c + m_f}{m_s}$$

and where it is noted the normalized factors are consistent with those used in the derivations of Appendix A.

With the specification of the force $\bar{F}_p$ the pier dynamic motion may be determined.

B.3. Computation of Pier Forces Due to a Vehicle Passage

An approximate method is used to compute the force acting on the pier. The pier force is computed as the sum of the shear forces generated in the spans resting on the pier. These shear forces are computed for the span resting upon "rigid" supports as it is traversed by a constant suspension force vehicle model. The computation of the pier forces using a rigid support assumption is adopted initially since this assumption will allow an estimate of the force magnitudes which can be used to check the

rigid support assumption. The pier force $\bar{F}_p$ may be written as:

$$\bar{F}_p = V_e + V_r \quad (B.4)$$

where: V_e is the left span shear force

V_r is the right span shear force

and where the shear force in a span may be written as:

$$V(X, \tau) = \frac{4}{\pi^4} \frac{\partial^3 Y(X, \tau)}{\partial X^3} = \frac{4}{\pi^4} \sum_{n=1}^{\infty} \frac{\partial^3 \phi_{n,s}}{\partial X^3} \alpha_n(\tau) \quad (B.5)$$

For interior continuous supports $V_e = V_r = \frac{1}{2} V$,

while for exterior supports (B.5) must be used to compute the individual left and right span shear forces.

The computation of $\bar{F}_p$ using (B.5) directly leads to numerical computation difficulties because the series convergence is slow, i.e. it converges as $1/n$, and the resultant computation involves differences of small numbers. To alleviate this convergence difficulty, the shear force may be expressed as:

$$V(X, \tau) = V_{st}(X) + V_{dn}(X, \tau) \quad (B.6)$$

where: V_{st} = shear due to a stationary load on the span

V_{dn} = shear due to traveling load effects on a beam.

The quantity V_{st} is computed from classical force-moment analysis of the loads on the span and represents the limit of a slowly moving load as the velocity approaches zero, while the shear due to the moving load V_{dn} is computed using the dynamic contributions of the first fifteen terms of (B.5). Terms above the fifteenth

term have span natural frequencies associated with them that are greater than fifteen times the crossing frequencies considered and thus only their static contributions are important. The static contributions of these higher order terms are included in V_{st} . The details of the computation are described in [34].

The force occurring at the pier of a system of single spans due to the passage of a single concentrated force at crossing velocity V_c has been computed using the method described and is plotted in Figure B.2. The case for $V_c = 0.0$ is the static case while the influence of dynamics are shown in the cases $V_c = 0.4$ and 0.8 . The case $V_c = 0.4$ essentially illustrates dynamics which only slightly modify the static results and results in the same maximum shear force, while the case $V_c = 0.8$ which excites span dynamics significantly results in a maximum shear force which is 50% greater than the static force. This higher speed case also illustrates that the spans on the pier are left in an oscillating mode* as indicated in the Figure.

Figure B.3 shows the pier force resulting from the passage of a pressure pad of length $l_p = 0.3l_g$. The same general effects shown in Figure B.2 are illustrated except that due to the finite pad length, the pier force magnitudes are reduced and the time histories are smoothed.

*The span damping is zero.

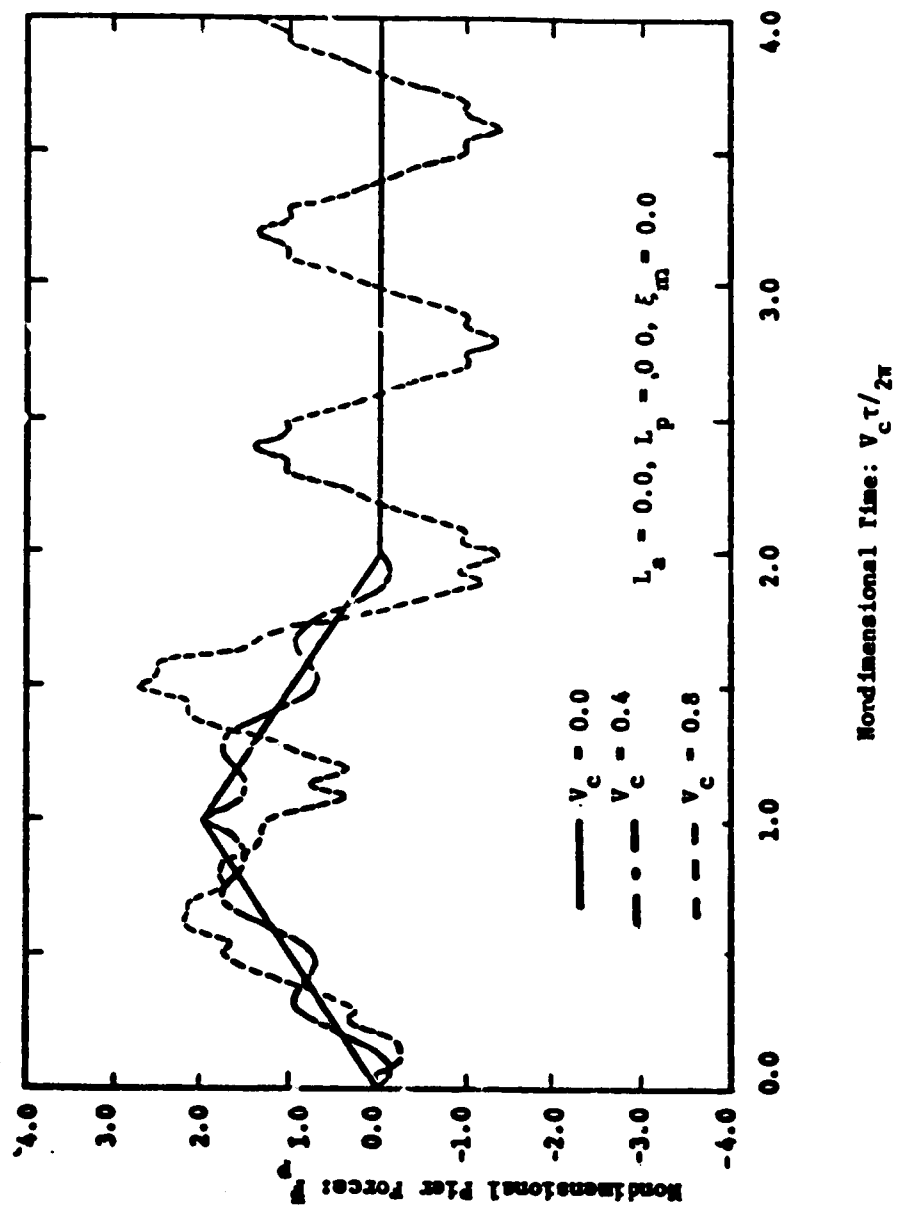


Fig. B-2. Single Span Nondimensional Pier Force Due to a Constant Force Passage

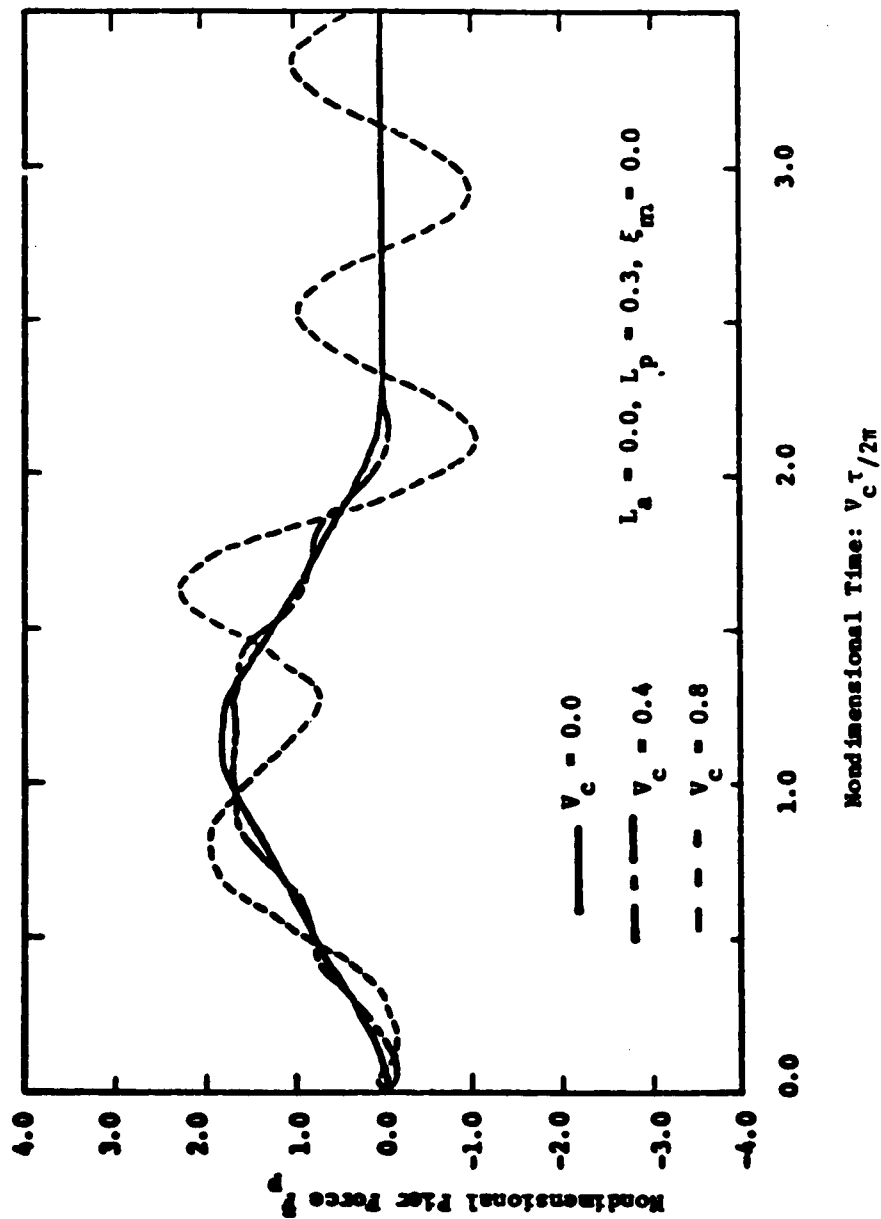


Fig. B-3. Single Span Nondimensional Pier Force Due to a Pressure Pad Traverse

Figure B.4 shows the pier force resulting from the passage of twin pressure pads of length $l_p = 0.5l_g$ and separated at length $l_s = 0.5l_g$. The further spreading of the vehicle forces across the twin pads results in a further decrease in pier force maximum magnitude and further smoothing of the time history in comparison to the single pressure pad case.*

Using the methods described, the pier forces due to an arbitrary vehicle suspension force distribution may be determined.

B.4 Evaluation of Pier Motions

To evaluate the motions of the pier-foundation system, the four design cases summarized in Table B.1 are considered. These cases consider the pier-foundation system sketched in Figure B.5. The designs represent 12 and 24 ft. high guideways for 150 and 300 mph vehicle systems. The dimensions of the footing and pier were determined using the design procedure outlined in [34] which is based upon AASHTO design codes and civil engineering practice. The procedure results in designs which are similar to those described for the urban air cushion vehicle guideway design described in [11]. The procedure is based upon determining the environmental and the dead loads on the pier. For the cases considered it was found that a principal design constraint is represented by a 100 mph cross wind on the structure

*The same total suspension load is used in Figures B-2, B-3 and B-4

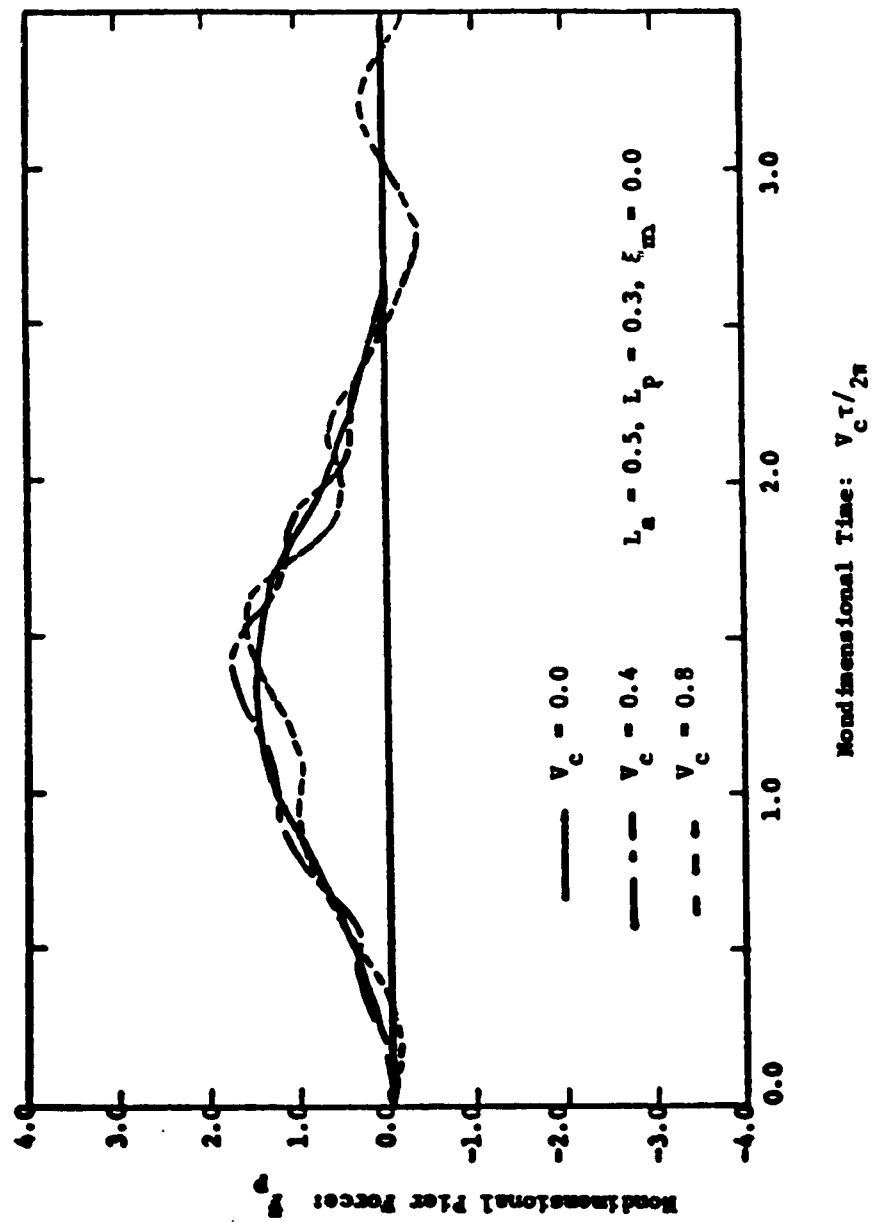


Fig. B-4. Single Span Nondimensional Pier Force Due to a Multiple Suspension Vehicle Passage

TABLE B.1

**PIER-FOUNDATION STRUCTURE PROTOTYPE DESIGNS
FOR 100 FOOT SINGLE SPAN GUIDEWAYS**

Design Case	1	2	3	4
Speed: mph	150	150	300	300
Column height: ft.	12	24	12	24
Span height: in.	69	69	84	84
Span weight: lb	233,700	233,700	268,400	268,400
Column weight: lb	6,000	17,150	6,500	18,600
Footing weight: lb	60,900	113,600	70,100	121,800
Soil stiffness, k_s : lb/ft	36×10^6	43×10^6	37×10^6	44×10^6
Soil damping, ξ_p	.44	.57	.46	.56
Frequency ω_p : rad/sec	61	62	58	58
Frequency Ratio, Ω_p	2.2	2.2	1.75	1.77
Span Stiffness: lb/ft	2.6×10^6	2.6×10^6	4.25×10^6	4.25×10^6
Footing Size: ft x ft	23 x 8.6	28 x 11	24 x 9	29 x 11
Footing Thickness: ft	2.0	2.4	2.1	2.5

Soil Parameters:

Shear modulus = 5000 lb/in^2
 Density = 110 lb/ft^3
 Poisson ratio = 0.333
 Bearing capacity = 3000 lb/ft^2

Vehicle Parameters:

Weight = 75,000 lbs
 $k_a/k_s = 0.5$
 $k_p/k_s = 0.3$

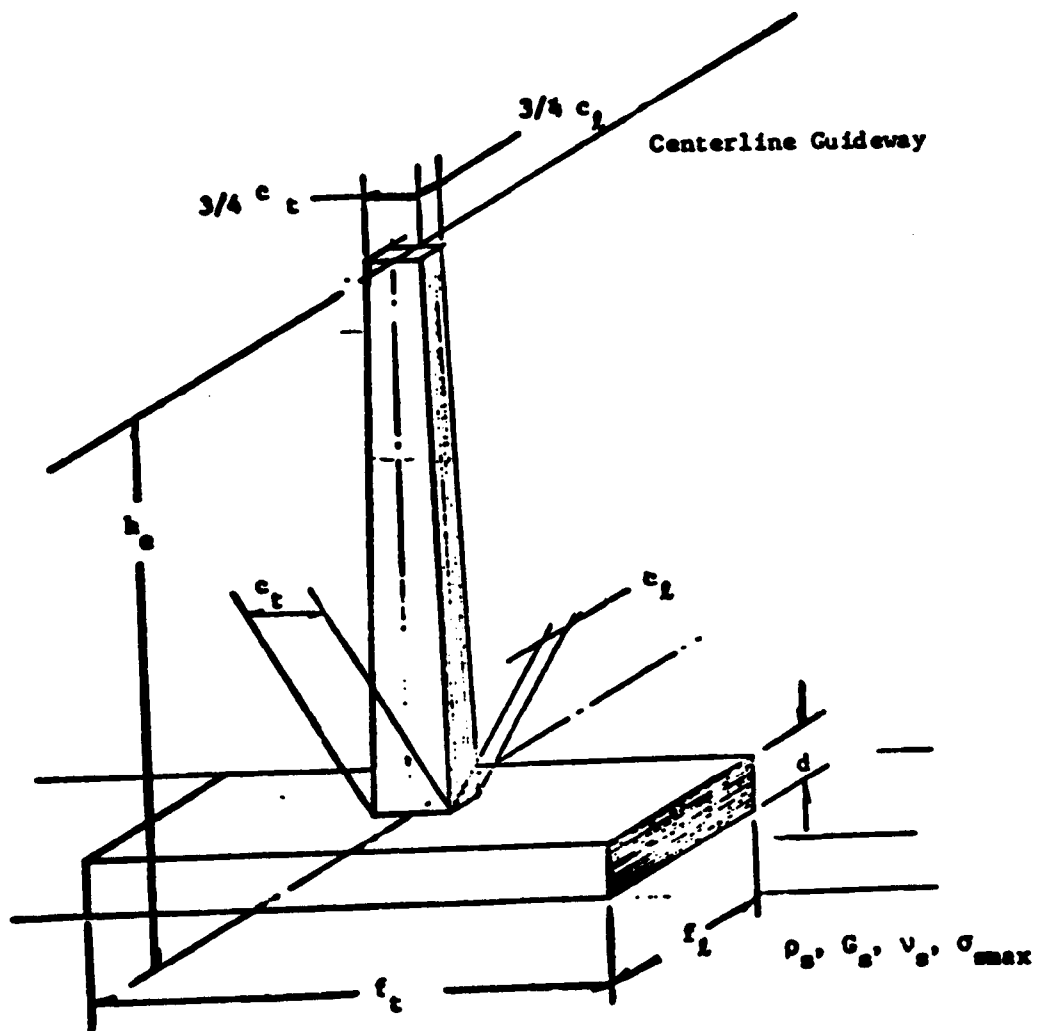


Fig. B-5. A Prototype Pier-Footing Design

with the vehicle present. The overall footing size is essentially set by balancing this moment against the soil bearing capacity at the outward edge of the footing.* Thus, as the soil bearing capacity decreases, the footing size increases. For the design examples cited, a bearing capacity of 3000 lb/ft.^2 was used.

Also tabulated are the values of the footing stiffness and damping ratio and the span-column-footing mass-soil stiffness natural frequency. For all the design cases the soil-footing stiffness is at least eight times the span stiffness, thus it is expected that pier motions will be small compared to span motions. Also it is noted that the pier system damping ratios are in the range of 0.5 and dynamic amplifications of motion will be small, particularly since the vehicle forcing frequencies are less than one half the pier system natural frequencies.

The detailed time histories of the pier motion due to the passage of a 75,000 lb. air cushion vehicle with pad lengths of $L_p = 0.3$ and suspension separation distance $L_s = 0.5$ across single 100 ft. spans are displayed in Figures B.6 to B.9 for the four pier designs summarized in Table B.1. These time responses were obtained by first determining the pier forces from the vehicle passage using equation (B.6) and then determining the pier motion by solving (B.3) with numerical integration. The maximum nondimensional pier deflections, dimensional deflections

*The pier-footing designs differ for 150 and 300 mph systems because span heights are different.

and ratios of pier to span maximum deflections are summarized in Table B.2. As noted in the table, the computed pier maximum deflections are small, i.e. all pier deflections are less than 0.015 in. and are similar. The ratios of the pier to span deflections are also small with the 150 mph system pier deflections less than 6% of the span deflections and the 300 mph system pier deflections less than 9% of the span deflection.

For the design cases considered, the pier deflections are small enough compared to the span deflections so that the assumption that vehicle-span interaction dynamics may be determined assuming rigid supports is justified. Also it is noted that as shown in the Figures B.6 to B.9, the pier motion is essentially independent of vehicle operating speed, i.e. the pier is responding in a quasi-static manner as is expected since the pier-footing system natural frequencies are in the 10 hertz range, while the maximum vehicle crossing frequency is less than 4 hertz, thus the pier is excited at a frequency less than its natural frequency and responds almost quasi-statically. For the cases considered, pier-footing dynamics do not significantly influence span dynamics.

The pier-designs considered are expected to be typical of those employed in tracked levitated vehicle systems and it is expected that the rigid support assumption will be valid for designs which are similar to those studied. For designs which are significantly different from those described, the methods outlined in this appendix may be used to evaluate the validity of the rigid pier support assumption.

TABLE B.2

**PIER MOTION DUE TO A 75,000 lb
AIR CUSHION VEHICLE PASSAGE**

Design Case	1	2	3	4
Speed: mph	150	150	300	300
Column height: ft	12	24	12	24
Maximum Pier Deflection: in	.015	.016	.014	.015
Maximum Span Deflection: in	.284	.284	.170	.170
Ratio of Maximum Pier to Span Deflections	.052	.056	.086	.087

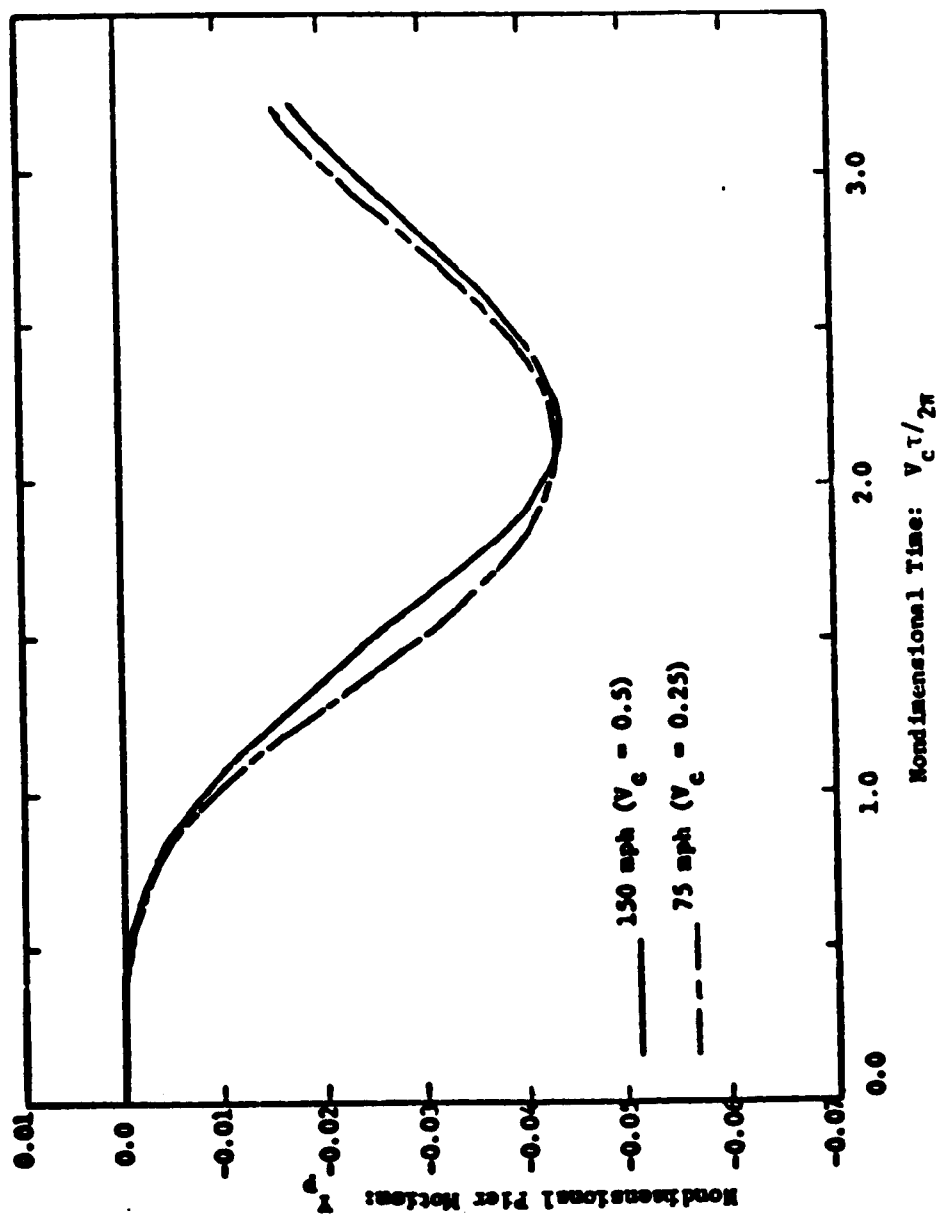


Fig. B-6. Nondimensional Pier Motion Due to a Vehicle Passage for Design Case 1

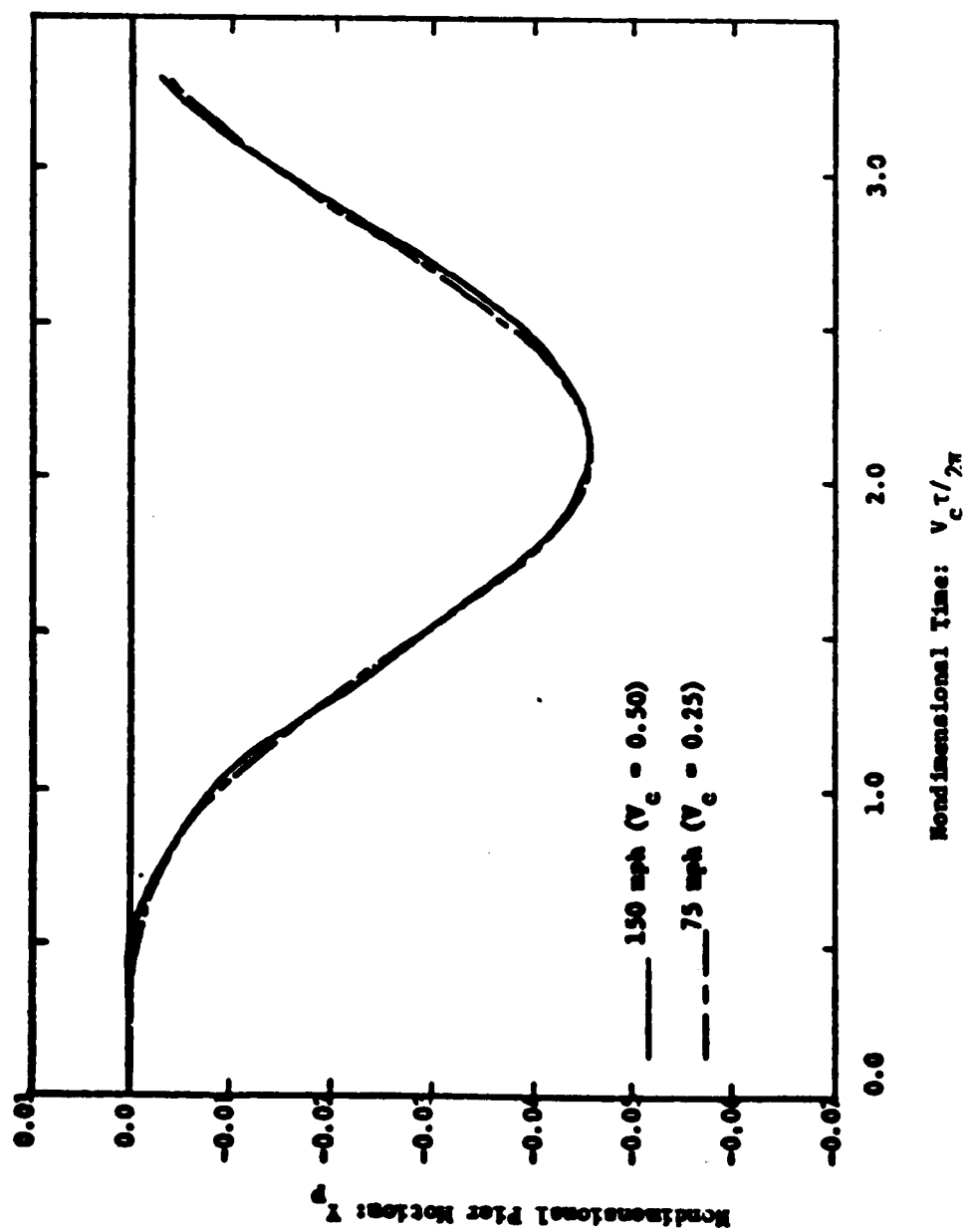


Fig. E-7. Nondimensional Pier Motion Due to a Vehicle Passage for Design Case 2

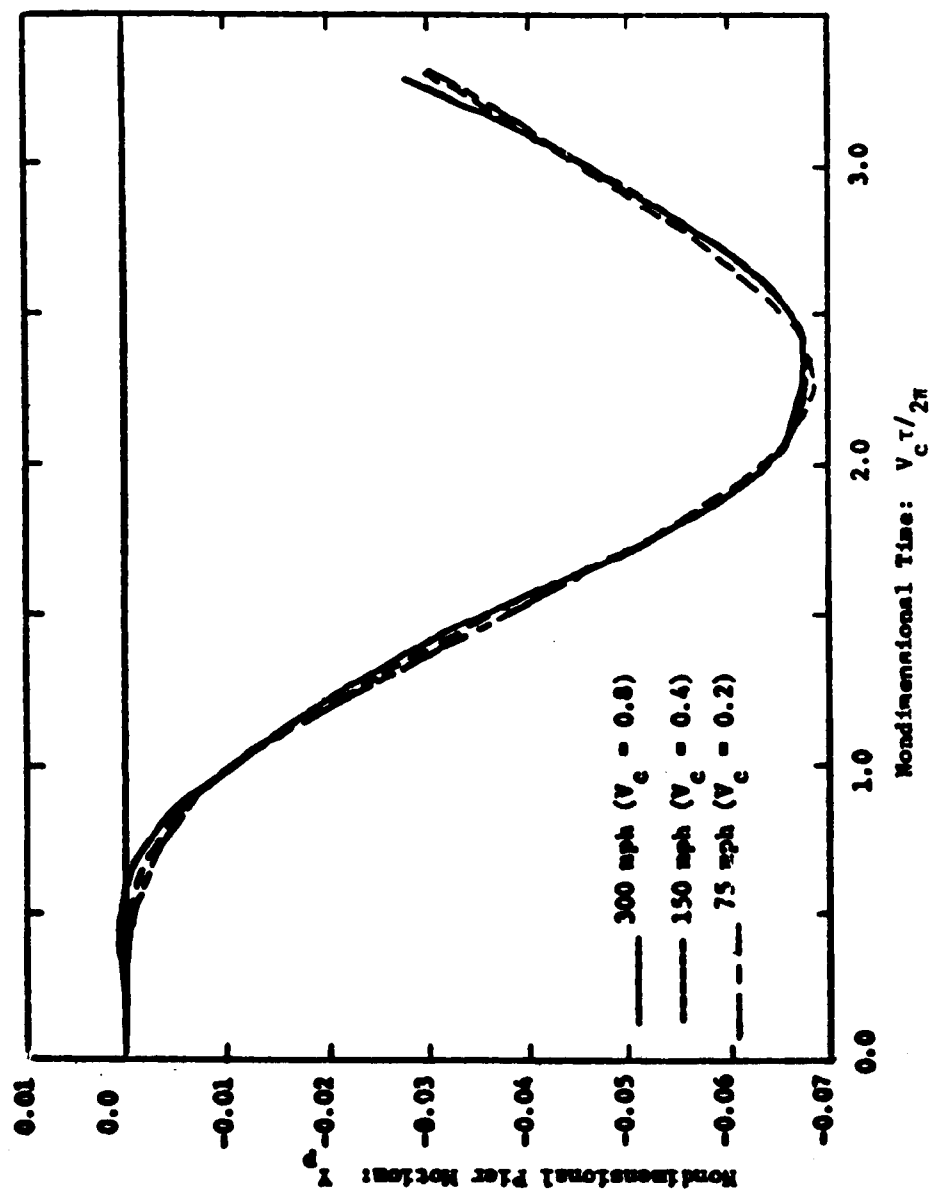


Fig. B-8. Nondimensional Pier Motion Due to a Vehicle Passage for Design Case 3

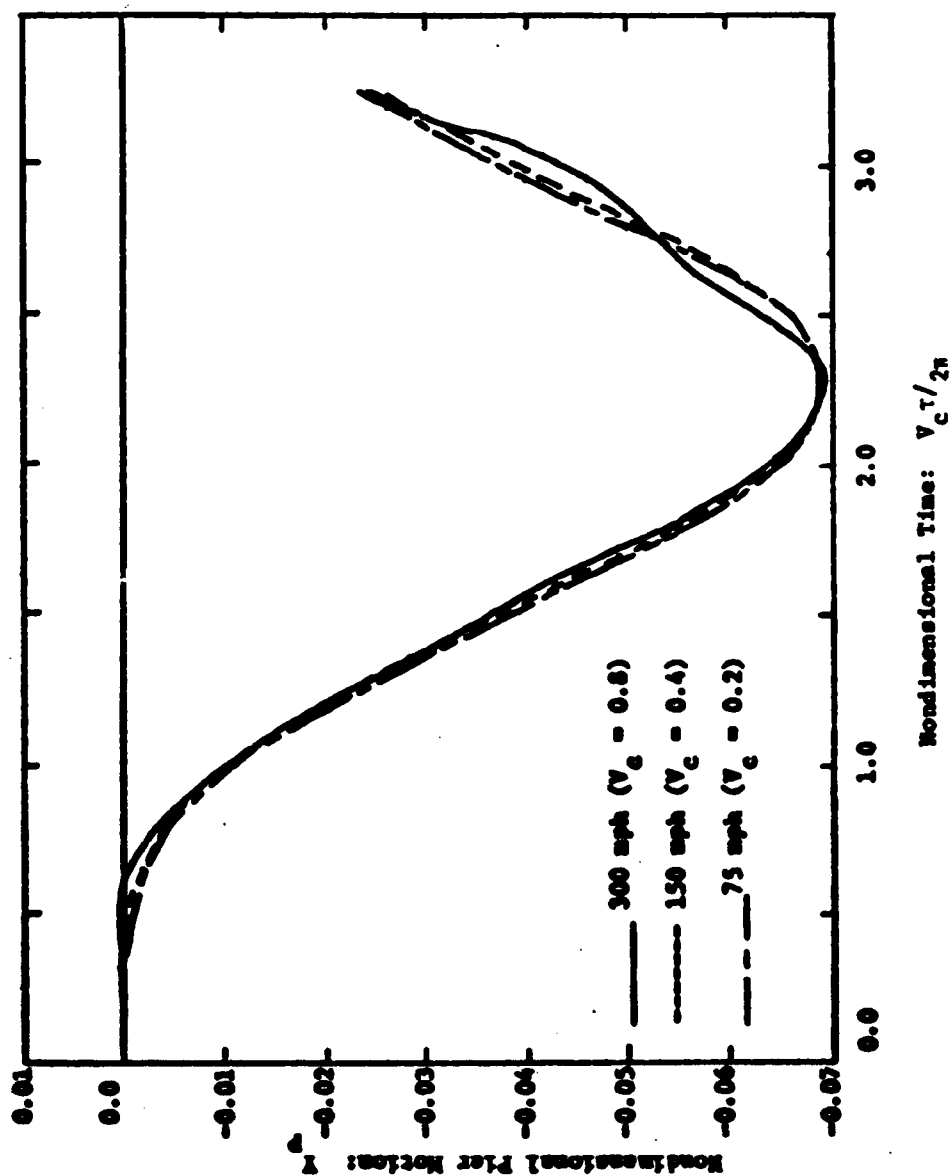


Fig. B-9. Nondimensional Pier Motion, Due to a Vehicle Passage for Design Case 4

APPENDIX C
COMPUTER SIMULATION PROGRAM OF TWO-DIMENSIONAL
VEHICLE OVER A MULTISPAN GUIDEWAY

This program simulates a two-dimensional vehicle traversing a semi-continuous guideway as described in Chapter 2. The program allows for either a sixth order vehicle model with no unsprung mass or an eighth order vehicle model including unsprung mass. The vehicle interacts with the guideway at each suspension via a uniform pressure pad with pad length to span length ratios in the range of $0.0 < \frac{l_p}{l_s} < 1.0$. The guideway deflection under the center of the pad provides the deflection input to each suspension. Vehicles with vehicle to span length ratios in the range $0 < \frac{l_v}{l_s} < 2k$, where k is the number of spans per beam, are permissible. Vehicles longer than this, which would interact with more than three beams simultaneously, are not considered. The guideway is assumed to be undisturbed as the vehicle approaches it. Either of two predetermined camber shapes, an arbitrary camber shape, or no camber are program options.

In the computer simulation the coupled vehicle-guideway system equations are integrated using a fourth order Runge-Kutta solution technique. This solution technique requires that some care be exercised in choosing the time step used to avoid numerical instabilities.

The inputs to the program which effectively govern the time step are the crossing frequency or velocity and the number of points at which the state variables are determined per span [CRFQ and INT respectively in the program]. The time step is inversely proportional to both of these parameters. Thus if one is decreased, the other should be increased proportionally to keep the time step constant. Experience has indicated that a crossing frequency--points per span product of about 40 provides good numerical stability for most systems with vehicles having no unsprung mass. Adding unsprung mass effectively adds a high frequency vibrational mode to the system and tends to require a decrease in the time step (or increase in the crossing frequency x points per span product).

Inputs to the program include the following:

- VC: Crossing frequency ratio or dimensionless velocity, V_c
- OM: Span to vehicle frequency ratio, Ω
- AK: Primary to secondary stiffness ratio, K
- ZV: Vehicle damping ratio, ξ_v
- ZN: Span damping ratio, ξ_m
- VL: Vehicle attachment length ratio. This is the ratio of the length between suspension attachments to the span length
- $$L_a = l_a / l_s = (l_v - l_p) / l_s.$$
- AM: Vehicle to span mass ratio, M .
- PLR: Pad to span length ratio, $L_p = l_p / l_s$
- AI: Vehicle inertia ratio, $\bar{I}_v$
- AMU: Unsprung mass ratio, M_u . If M_u is set equal to zero, the

order of the vehicle equations is automatically reduced from eight to six in the program.

- MODE:** The number of modes used to model each beam of the guideway. The program is dimensioned to accommodate up to six modes, but more can be added by changing only the appropriate dimension statements.
- NSPAN:** Parameter which designates the number of span crossings, or length of the simulation. (Note variation due to value of **MNT** below).
- IC:** Parameter which allows continuation of previous run. If **IC** is set equal to zero, initial conditions are chosen from static equations. If **IC** is set equal to one, initial conditions of state variables are read in as data.
- INT:** Number of time steps per span crossing. (See choice of time step above).
- NS:** Number of spans per beam, **k**. The program is dimensioned to accommodate up to 5, but more could be added by changing the appropriate dimension and format statements.
- MNT:** Parameter which varies output. If **MNT** is set equal to zero, midspan deflections are output. If **MNT** is set equal to 1, midspan moments are output. If **MNT** equals 2, midspan deflections are output for **NSPAN** span crossings and then midspan moments are output for **k** more span crossings (1 more beam).

CAMCD: Camber code 'A' for camber shape equal to $\bar{A}_c |\sin \pi X|$,
'C' for camber shape equal to $\bar{A}_c (1 - \cos 2\pi X)/2$, 'G' for
general camber shape equal to $\bar{A}_c (\gamma_c(X))$, where values
of $\gamma(X)$ are read from succeeding cards, or blank

AMCAM; Camber amplitude, $\bar{A}_c$.

AL(I): Eigenvalues of beam. Correspond to $\lambda_m l_s$ in text.
(See Appendix A for listing).

Program outputs are printed in columns with the following
headers from left to right.

F: The beam number the leading edge of the front suspension
is on. If IC is set equal to zero, the leading edge of
the front suspension begins on beam 1. (Also designated
beam A.) As time progresses, the front pad moves on to
beam 2 (also designated B), beam 3 (also designated C),
and then on to beam 1 again, and the sequence repeats.
As the vehicle approaches each new k-span beam, it is
initialized to static conditions and the vehicle "sees"
it as a new, undisturbed guideway beam.

R: The number of beam the leading edge of the rear suspension
is on.

YMA_n: These headings designate the midspan deflections of beams 1

YMB_n: (YMA_n), 2 (YMB_n), and 3 (YMC_n), where n represents the span
number. [See explanation for F above]. If NBT is read as
1 or 2, these headings are replaced at the appropriate time

with M_{AN} , M_{BN} , and M_{CN} , designating the corresponding midspan moments.

YACC: The acceleration in g's at the front attachment point of the vehicle.

ZACC: The acceleration in g's at the rear attachment point of the vehicle.

CGACC: The acceleration in g's at the center of the vehicle.

YF: The displacement (beam deflection) under the center of the front suspension pad.

YR: The displacement under the center of the rear suspension pad.

TWO-DIMENSIONAL VEHICLE - MULTISPAN GUIDEWAY, FULLY COUPLED MODEL

THIS PROGRAM CALCULATES THE TIME RESPONSE OF THE FRONT AND REAR
ACCELERATION FOR A TWO-DIMENSIONAL VEHICLE WITH LENGTH RATIO
GREATER OR LESS THAN NUMBER OF SPANS PER BEAM
WITH NO UNSPRUNG MASS (SIXTH ORDER MODEL) OR WITH
UNSPRUNG MASS (EIGHTH ORDER MODEL) (DEPENDENT UPON
VALUE OF MASS READ IN) AS VEHICLE CROSSES A
SERIES OF SEMICONTINUOUS GUIDEWAY BEAMS.

PROGRAM VARIABLE DEFINITIONS

TAU NON-DIMENSIONAL TIME (THE FUNDAMENTAL FREQUENCY OF A SIMPLE
ONE SPAN BEAM TIMES REAL TIME)

VC THE RATIO OF THE FREQUENCY OF SPAN CROSSINGS TO THE FUND-
AMENTAL FREQUENCY OF A SIMPLE ONE SPAN BEAM IN CPS
(ALSO DIMENSIONLESS VELOCITY)

W THE RATIO OF THE FUNDAMENTAL FREQUENCY OF A SIMPLE ONE SPAN
BEAM TO THE NATURAL FREQUENCY OF THE VEHICLE SECONDARY
SUSPENSION

K THE RATIO OF THE PRIMARY VEHICLE SUSPENSION STIFFNESS
TO THE SECONDARY SUSPENSION STIFFNESS (AK
INTERNALLY IN THE PROGRAM)

ZV THE VEHICLE DAMPING RATIO

ZR THE GUIDEWAY DAMPING RATIO OF THE NTH MODE

VL THE RATIO OF THE DISTANCE BETWEEN VEHICLE
SUPPORTS TO THE SPAN LENGTH

PLR THE RATIO OF THE ROAD LENGTH TO THE SPAN LENGTH

X THE RATIO OF THE MASS OF THE VEHICLE TO THE MASS OF THE
SPAN (AK INTERNALLY)

I THE RATIO OF THE VEHICLE MOMENT OF INERTIA
TO THE MOMENT OF INERTIA OF A THIN ROD OF LENGTH
VL AND SAME MASS AS THE VEHICLE SPRUNG MASS
(AK INTERNALLY)


```

C
C DATA CARD 5 * FINAL VALUE OF TAU FROM OPTIC RUN... (FLU,2)
C DATA CARDS 6,7,... * X(1),X(2),X(3),ETC. THE FINAL VALUES OF ALL
C STATE VARIABLES FROM PRIOR RUN... (8FLD,2)
C IF ON CARD 3 IC IS 0 FOR NEW RUN, CARDS 5,6,7... NOT REQUIRED
C
C -----
C
C PROGRAM OUTPUTS * MID-SPAN DEFLECTIONS, FRONT, REAR, AND MIDDLE
C ACCELERATIONS, AND DEFLECTIONS UNDER THE FRONT AND REAR
C SUSPENSIONS VS. TIME
C
C *****
C
C DIMENSION R1(98),B2(58),B3(98),B4(98)
C DIMENSION X0(98),XA(58),YMA(5),YMA(5),YMC(5)
C COMMON /SMODE/ AL(15),A(15,5),B(15,5),PHIM(15,5),CHIM(15,5)
C
C 1)
C COMMON /SPAD/ H1(15,11),H2(15,11),H3(15,11),H4(15,11),H5(15,11)
C COMMON /STATE/ X(98),IFC,NULD,NRF,NBR,YF,YR
C COMMON /PAPAM/ T,DM,AK,AM,ZN,ZV,VL,PLR,MUDE,NS,NN,NSYS,NF,NP,NV
C COMMON /PARAM/ PHIF(15),PHIR(15),PSIF(15),PSIR(15),PSIFI(15),PSIRI(
C 15),INT,ISEG,KRP,IM,IP
C COMMON /VEN/ C1,C2,C3,C4,C5,C6,C7,C8,C9,IMU
C COMMON /CAM/ ICAMS, CAPFR(51)
C DATA ALPHA, ALPHC, ALPHE / 'A', 'C', 'G' /
C DATA TEAD / 'PINN' /
C IR=0
C IM=6
C
C OUTPUT HEADER
C
C 9990 WRITE(16,1)
C 1 FORMAT(141,'//,2X,2HTWC-DIMENSIONAL VEHICLE,/')
C
C REAR SYSTEM PARAMETERS
C

```

*CAMBER
*CAMBER


```

PI=3.1415927
YACC=0.
ZACC=0.
CCACC=0.
NN=6+MODE
ISTEP=INT*NSPAN
LSEG=2*NS+1
H=2.*PI*F/FLCAT(INT)
C9=AK*AM/(1.+AMU)*OM**2)
IF(AMU-C.0001) 51,51,52
C1=(1.+AK)/(2.*ZV*OM)
C2=1./(2.*ZV*OM)
C3=AK/(2.*ZV*OM)
C4=AK*(1.+3./AI)/(2.*OM**2)
C5=AK*(1.-3./AI)/(2.*CM**2)
AMU=0
GC TO 53
C1=(1.+3./AI)/(2.*OM**2)
C2=(1.-3./AI)/(2.*OM**2)
C3=ZV*(1.+3./AI)/OM
C4=ZV*(1.-3./AI)/OM
C5=1./(AMU*OM**2)
C6=(1.+AK)/(AMU*OM**2)
C7=2.*ZV/(AMU*OM)
C8=AK/(AMU*OM**2)
IMU=1
NN=NN+2
CONTINUE
CALL EGN(MODE,NS,TEND)
CALL HCAL(MODE,NS,PLR)
NP=PLR*FLCAT(INT)*2.
NV=VL*FLCAT(INT)*2.
NB=2*INT*AS
NSYS=3*NB
VF=0.

```

51

52

53

C


```

C
C
      5 CONTINUE
      YAU=0.
      TAU2=0.0
      IFO=1
C
C      DETERMINE AT REST STATE VARIABLE INITIAL CONDITIONS
C
      DO 10 I=1,NN
      10 X(I)=0.
      NBF=1
      IIR=NSYS-NV+1
      NBR=(IIR-1)/NB+1
      IBR=IIR-(NBR-1)*NB
      CALL SHMCO(1,IBR)
      KRP=NBR-KRP
      KFP=1+2*KFP
      DO 11 I=1,MODE
      L1=7+6*(I-1)+2*(NBR-1)
      IF(IBR-NP) 48,48,28
      IF(NBR-3) 49,50,52
      48 WRITE(IW,100)
      49 FORMAT(1H0,1UX,'VEHICLE TOO LONG FOR SYSTEM',//)
      100 CALL EXIT
      L2=7+6*(I-1)+2*(NBR-2)
      X(L2)=-0.25*(PI/AL(I))*4*PSIR1(I)/FLOAT(NS)
      28 X(L1)=-0.25*(PI/AL(I))*4*PSIR(I)/FLOAT(NS)
      IF(1-NP) 26,26,11
      26 L1=11+6*(I-1)
      X(L1)=X(L1)-0.25*(PI/AL(I))*4*PSIF1(I)/FLOAT(NS)
      11 CCNTINUE
      DO 54 I=1,MODE
      L1=7+6*(I-1)+2*(KRP-1)
      L2=7+6*(I-1)+2*(KFP-1)
      VF=VF+X(L2)*PHIF(I)

```



```

      CALL EVAL(XD,XA,IFU)
      DO 16 I=1,NN
        P(I)=H*XC(I)
        16 X(I)=X(I)+(1./G.)*(R1(I)+C.W2(1)+2.*R3(1)+H*4(1))
      C
      C
      C      DETERMINE TAU AT NEXT TIME STEP
      TAU=TAU+H
      IFU=IFU+2
      DO 24 I=1,NN
        24 XA(I)=X(I)
      C
      C      CALL EVAL(XD,XA,IFU)
      C
      C      DETERMINE NORMALIZED TAU, ACCELERATIONS, AND MIDSPAN DEFLECTIONS
      YACC=XD(2)*2.*AM
      ZACC=XD(6)*2.*AM
      CGACC=.5*(YACC+ZACC)
      TAU2=TAU/(2.*PI**2)
      DO 55 I=1,5
        YMA(I)=C.O
        YMR(I)=C.O
        YMC(I)=C.O
      55 CONTINUE
      DO 35 I=1,NS
        CC 35 A=1,MJCT
        AMDA=7+C*(N-1)
        AMDB=9+C*(N-1)
        AMDC=11+C*(N-1)
        YMA(I)=YMA(I)+DHIM(N,I)*X(N,TA)
        YMR(I)=YMR(I)+DHIM(N,I)*X(N,RA)
        YMC(I)=YMC(I)+DHIM(N,I)*X(N,RC)
      35 CONTINUE
      C
      C      WRITE FRONT BEAM N.O., READ BEAM NO., AND ACCELERATIONS

```

```

36 WRITE(IH,17)NBF,NBR,TAU2,(YMA(I),I=1,5),(YMB(I),I=1,5),(YMC(I),I=1
1,5),YACC,ZACC,CGACC,YF,YR
17 FORMAT(1X,2I2,1X,21(F6.3))
C
C
C TEST FOR FINAL TIME STEP
C
C 10 CONTINUE
C
C
C TEST WHETHER TO RUN FOR ONE MORE BEAM WHILE PRINTING MOMENTS
C
C IF(MNT-2) 63,64,63
C MNT=1
C ISTEP=NS*INT
C WRITE(IH,300)
C FORMAT(IH1)
C GC TO 57
C
C 300
C
C WRITE TAU AND FINAL VALUES OF STATE VARIABLES
C
C 63 WRITE(IH,19)
C 19 FORMAT(IH1,/,2X,31HFINAL VALUES OF STATE VARIABLES,/)
C WRITE(IH,20)TAU
C 20 FORMAT(5X,5HTAU =F10.4,/)
C CO 21 I=1,NN
C 21 WRITE(IH,22)1,X(I)
C 22 FORMAT(5X,2HX(,I2,4H) = ,E15.5/)
C GC TO 9990
C END
C SUBROUTINE EVAL(XD,X,IIF)
C DIMENSION X(6),XD(6),F(3)
C COMMON /SMODE/ AL(15),A(15,5),B(15,5),C(15,5),PHIM(15,5),CHIM(15,5)
C 1)
C COMMON /STATE/ XP(98),IFO,NOLD,NRF,NBR,YF,YR
C COMMON /PARAM/ T,OM,AK,AM,ZN,ZV,VL,PIR,M,NS,NN,MSYS,NB,NP,NV
C COMMON /ARRA/ PHIF(15),PHIP(15),PSIF(15),PSIR(15),PSIF1(15),PSIR1(

```

```

115) NPTS, LSEG, KFP, KRP, LP, LY
COMMON /VEM/ C1,C2,C3,C4,C5,C6,C7,C8,C9, INU
COMMON /CAM/ ICAMS, CAMPR(61)
NRF=(IIF-1)/NR+1
IF(NHF-NCID) 2,2,1
1  IDF=IIF-IFF
  IF(NHF-3) 3,3,4
4  NRF=NRF-3
  IIF=IIF-NSYS
3  IF(IDF) 19,19,2
COMMON RESET HEAM VHF
19  IFU=IIF
  NCID=NHF
  WRITE(LP,10,1) VHF
  FORMAT(10) JFSET HEAM 1,12)
  DO 5 I=1,M
    L1=7+6*(I-1)+2*(NHF-1)
    X(L1)=0.
    X(L1+1)=0.
    XF(L1)=0.
    XP(L1+1)=0.
    IIP=IIF-NV
    IF(IIR) 5,6,16
    IIR=IIF+NSYS
    APP=(IIM-1)/NR+1
    IFF=IIF-(NHF-1)*NR
    I9=IIF-(NHF-1)*NR
    XF = FLCAT(IAP-1) / FLCAT(2*APTS)
    XR = FLCAT(IAP-1) / FLCAT(2*APTS)
    VF=0.
    VM=0.
    CALL SHARC(IFF,IIR)
    KFD=NHF-KFP
    KRD=N3M-KRP
    IF(KFP) 20,20,21
    KFD=KFD+2
20

```

CHANGED

CHANGED
CHANGED

```

21 IF(KRP) 22,22,23
22 KRP=KRP+3
23 DO 7 I=1,M
    L1=7+6*(I-1)+2*(KRP-1)
    L2=7+6*(I-1)+2*(KRP-1)
    YF=YF+X(L1)*PHIF(I)
    YA=YR+X(L2)*PHIR(I)
7
C
C --- UNIT TO IMPOSE INTERPOLATED CAMBER.
    IF (ICAMS.EQ. 1) GO TO 9020
C --- FIND POSITION ON BEAM AND INTERPOLATE.
C --- DELX,R RANGE FROM ZERO TO 1 BETWEEN EACH PAIR OF DATA POINTS.
C
    NCAMF=FIX(60.*XF/FLOAT(NS))*1
    NCAMR=FIX(60.*XR/FLOAT(NS))*1
    DELAF=60.*XF/FLOAT(NS)-FLOAT(NCAMF-1)
    UELX=60.*XR/FLOAT(NS)-FLOAT(NCAMR-1)
    YF = YF + CAMR(NCAMF)*(1.-DELXF) + CAMR(NCAMF+1)*DELXF
    YR = YR + CAMR(NCAMR)*(1.-DELYR) + CAMR(NCAMR+1)*DELYR
9020 CONTINUE
C --- END OF CAMBER ADJUSTMENTS. -----
    FF=((X(1)-YF)*C9-0.5)/(2.*FLOAT(NS))
    FR=((X(4)-YR)*C9-0.5)/(2.*FLOAT(NS))
    XD(2)=X(3)
    XC(5)=X(6)
    IF(I.NE. 24,24,25)
    XD(1)=-C1*X(1)+C2*X(2)+X(3)+C3=YF
    XD(3)=C4*(YF-X(1))+C5*(YR-X(4))
    XD(4)=-C1*X(4)+C2*X(5)+X(6)+C3=YR
    XC(6)=C5*(YF-X(1))+C4*(YR-X(4))
    GC TO 26
    XD(1)=X(NN-1)
    XD(3)=C1*(X(1)-X(2))-C3*(X(3)-X(NN-1))+C2*(X(4)-X(5))+C4*(X(NN)-
1 X(6))
    XD(4)=X(NN)
    XD(6)=C2*(X(1)-X(2))-C4*(X(3)-X(NN-1))+C1*(X(4)-X(5))+C3*(X(NN)-

```

```

*CAMBER
*CAMBER
*CAMBER
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*CAMBER
*CAMBER
*CAMBER

```

```

1 X(6))
XD(NN-1)=-C6*X(1)+C5*X(2)+C7*(X(3)-X(NN-1))+C8*YF
XC(NN)=-C6*X(4)+C5*X(5)+C7*(X(6)-X(NN))+C8*YR
CONTINUE
CO 0 I=1,M
NN=(AL(1)/3.1415927)*2
DO 17 J=1,3
F(J)=0.
F(NBF)=F(NBF)+FF*PSIF(I)
IF(1BF-NP) 9,10,10
NBF=NBF-1
IF(NBFR)11,11,12
NBFR=NBFR+3
1BF=NB+1BF
F(NBFR)=F(NBFR)+FF*PSIF(I)
F(NBR)=F(NBR)+FR*PSIR(I)
IF(1BR-NP) 13,14,14
NBR=NBR-1
IF(NBR) 15,15,16
NBR=NBR+3
1BR=NB+1BR
F(NBR)=F(NBR)+FR*PSIR(I)
DO 6 J=1,3
L1=7+6*(I-1)+2*(J-1)
XC(L1)=X(L1+1)
XD(L1+1)=-WN*2*X(L1)-2.*ZN*WN*X(L1+1)+F(J)
RETURN
END
SUBROUTINE EGN(M,NS,TEND)
REAL*8 SH,CH,G1,G2,G3,G4,G5,G6,SNM,SNCH,CSSH,CSCM,SMCH,CMCH,
ISA,SB,SC,SU,DALM,DEXP
COMMON /SMODE/ AL(15),A(15,5),B(15,5),C(15,5),PHIM(15,5),CHIM(15,5
1)
DATA PIN,'PINN',FIX,'FIXE'
DO 50 I=1,M
ALM=AL(I)

```



```

DALM=ALP
C----- COMPUTE TRIGONOMETRIC PROPERTIES ONCE FOR EACH MODE
      SA=SIN(ALM)
      SH=0.5*(DEXP(DALM)-DEXP(-DALM))
      CS=COS(ALM)
      CH=0.5*(DEXP(DALM)+DEXP(-DALM))
C----- COMPUTE GEOMETRIC PROPERTIES
      G1=SH/(1P-SN)
      G2=SH+SN
      G3=(CH-CS)/(SH-SN)
      G4=SN/(SH-SN)
C----- PREPARE TO NORMALIZE MODE BY COMPUTING THE MODE INTEGRAL FOR A(I,1)=1.0
C----- LS (SPAN SEGMENT LENGTH) = 1.0 (DEFINITION)
C----- INTEGRAL(PHIN**2)=NS
C----- QUADRATIC INTEGRALS FOR ANY SPAN
      SNSN=0.5*(ALM-SN*CS)
      CSCS=0.5*(ALP+SN*CS)
      SHSH=0.5*(SH*CH-ALM)
      CHCH=0.5*(SH*CH+ALM)
C----- 0.5 FOR MIXED TERMS CANCELS 2.0 FOR BINOMIAL EXPANSION
      SNCS=SN*SN
      SASH=CH*SN-SH*CS
      SMCH=SH*SN-CH*CS+1.0
      CSSN=SN*SN+CH*CS-1.0
      CSCM=SH*CS+CH*SN
      SMCH=SH*SH
      C=0.0
      DO 40 J=1,NS
      IF(J-1)10,10,20
C----- COMPUTE A,B,C FOR FIRST SPAN
10      SA=1.0
      IF(TEND.EQ.PIN) SB=0.
      IF(TEND.EQ.FIX) SB=-1./G3
      SC=-SN/SH+(CH-CS)*SB/SH
      GO TO 30
C----- USE RECURSION FOR REMAINING TERMS, WORKS EVEN IF SIN(ALM)=0.0

```

```

20  G5=SA*CS-G2*SH+SC*CH
    U6=SA*SN+S8*CS
C-----NOW UPDATE A,H, AND C
    SA=G1*G3-G3*G6
    S2=G6
    SC=G3*G6-U4*G5
    SD=-Sd
    A(I,J)=SA
    B(I,J)=SR
    C(I,J)=SC
C-----INTEGRATE ALONG TOTAL SPAN LENGTH BY ACCUMULATING INDIVIDUAL
C-----SPAN SEGMENT INTEGRALS
    C=O+SA*(SA*SN*SY+SB*SACS+SC*SN*SH+S7*SN*CH)
    1  +SB*(SA*CS*CS+SC*CS*SH+S7*CS*CH)
    2  +SC*(SC*SH*SH+SD*SH*CH)
    1  +SD*(SD*CH*CH)
40  CONTINUE
C-----NORMALIZE MODE
    C=SQRT(FLCAT(NS)*ALM/C)
    DO 50 J=1,NS
        A(I,J)=A(I,J)*Q
        H(I,J)=H(I,J)*Q
        C(I,J)=C(I,J)*Q
50  CONTINUE
    DO 5 I=1,M
        ALM=AL(I)*QJ.5
        CALP=ALP
        ALS=(AL(I)/3.1415927)*.2
        SA=SIN(ALM)
        CA=COS(ALM)
        SH=U.5*(DEXP(UALV)-DEXP(-UALV))
        CH=U.5*(DEXP(DALM)+DEXP(-DALM))
    DO 5 JK=1,NS
        PHM(I,JK)=A(I,JK)*SN+B(I,JK)*SC+CH*(CN+CH)+C(I,JK)*SH
        CHM(I,JK)=ALS*(-A(I,JK)*SN-B(I,JK)*SC+CH*(CN+CH)+C(I,JK)*SH)
5  CONTINUE

```

```

RETURN
END
SUBROUTINE MCAL(M,NS,PLF)
REAL*8 SINH,COSH,SHL,CHL1,SHM,CHM,SHR,CHR,DBPX,DALF,DALI,DR,DEXP
COMMON /SMODE/ AL(15),A(15,5),B(15,5),C(15,5),PHIM(15,5),CHIM(15,5
1)
COMMON /SPAD/ H1(15,11),H2(15,11),H3(15,11),H4(15,11),H5(15,11)
SINH(DBPX)=0.5*(DEXP(DBPX)-DEXP(-DBPX))
COSH(DBPX)=0.5*(DEXP(DBPX)+DEXP(-DBPX))
IF(PLR-0.0001) 41,41,40
DO 44 I=1,M
DO 42 J=1,NS
K=2*J-1
DO 42 KS=1,2
H1(I,K)=A(I,J)
H2(I,K)=B(I,J)
H3(I,K)=C(I,J)
H4(I,K)=-B(I,J)
H5(I,K)=0.0
K=K+1
H1(I,K)=0.0
H2(I,K)=0.0
H3(I,K)=0.0
H4(I,K)=0.0
H5(I,K)=0.0
RETURN
CCONTINUE
DO 2 I=1,M
CALI=AL(I)
ALI=1./(PLR*AL(I))
ALF=AL(I)*PLR
CALF=ALF
SL=SIN(ALF)
CL=COS(ALF)-1.0
SHL=SINH(CALF)
CHL1=COSH(DALF)-1.0

```

41

42

44

40

```

SM=SIGN(AL(I))
CM=COS(AL(I))
SHM=SINH(DALI)
CHM=CSH(DALI)
R=AL(I)*1.-PLR)
CH=R
SR=SIN(R)
CR=COS(R)
SHR=SINH(CR)
CHR=CSH(CR)
H1(I,1)=ALI+P(I,1)
H2(I,1)=-ALI+A(I,1)
H3(I,1)=P(I,1)
H4(I,1)=ALI+C(I,1)
H5(I,1)=ALI*(A(I,1)-C(I,1))
IF(NS-I) 3,3,4
DO 1 J=2,NS
IS=2+J-1
H1(I,IS)=ALI*(-A(I,J-1)*SR-B(I,J-1)*CR+R(I,J))
H2(I,IS)=ALI*( A(I,J-1)*CR-R(I,J-1)*SR-A(I,J))
H3(I,IS)=ALI*( H(I,J-1)*CR-C(I,J-1)*SR-B(I,J))
H4(I,IS)=ALI*( A(I,J-1)*SHR-C(I,J-1)*CH+C(I,J))
H5(I,IS)=ALI*(-A(I,J-1)*CR+3(I,J-1)*(SR-SHR)+C(I,J-1)*CH)
1 A(I,J)=C(I,J)
NS=2+NS+1
H1(I,NS)=ALI*(-A(I,NS)*SR-B(I,NS)*CR)
H2(I,NS)=ALI*( A(I,NS)*CR-R(I,NS)*SR)
H3(I,NS)=ALI*(B(I,NS)*CH-C(I,NS)*SH)
H4(I,NS)=ALI*(B(I,NS)*SHR-C(I,NS)*CHR)
H5(I,NS)=ALI*(-A(I,NS)*CR+P(I,NS)*(SR-SHR)+C(I,NS)*CH)
DC 2 J=1,NS
IS=2+J
H1(I,IS)=ALI*(A(I,J)*SL-R(I,J)*CL)
H2(I,IS)=ALI*(A(I,J)*CL+R(I,J)*SL)
H3(I,IS)=ALI*(A(I,J)*CHL+C(I,J)*SHL)
H4(I,IS)=ALI*(-R(I,J)*SHL-C(I,J)*CHL)

```

```

2      H5(1,15)=0.
      RETURN
      END

C
C
      SUBROUTINE SHMOD(IBF,IER)
      REAL*8 DXAF,EXF,SHF,CHF,DXAR,EXR,SHR,CHR,DXPFA,DXPRA,DEXP
      COMMON /SMODE/ AL(15),A(15,5),R(15,5),C(15,5),PHIM(15,5),CHIM(15,5)
1)    COMMON /SPAD/ H1(15,11),H2(15,11),H3(15,11),H4(15,11),H5(15,11)
      COMMON /PARAM/ T,OM,AK,AM,ZN,ZV,VL,PLR,M,NS,NN,NSYS,N8,NP,NV
      COMMON /ARRA/ PHIF(15),PHIR(15),PSIF(15),PSIR(15),PSIF1(15),PSIR1(
15),NPTS,LSEG,KFP,KRP,LP,LR
      KFP=0
      KRP=0
      XF=FLOAT(IBF-1)/FLOAT(2*NPTS)
      JSF=IFIX(XF)+1
      XSF=XF-FLOAT(JSF-1)
      IF(XSF-PLR) 18,19,19
18     JSEGF=2*JSF-1
      GO TO 20
19     JSEGF=2*JSF
20     XR=FLOAT(1BR-1)/FLOAT(2*NPTS)
      JSR=IFIX(XR)+1
      XSR=XR-FLOAT(JSR-1)
      IF(XSR-PLR) 21,22,22
21     JSEGR=2*JSR-1
      GO TO 23
22     JSEGR=2*JSR
23     CCNTINUE
      XFP=XF-.5*PLR
      IF(XFP) 28,29,29
28     XFP=XFP+FLOAT(NS)
      KFP=1
29     JSFP=IFIX(XFP)+1
      XSFP=XFP-FLOAT(JSFP-1)

```

```

30 XRP=XR-.5*PLR
   IF(XRP) 30,31,31
   XRP=XRP+FLOAT(NS)
   KRP=1
31 JSRP=JFIX(XRP)+1
   XSRP=XRP-FLOAT(JSRP-1)
   DO 1 I=1,M
     XAF=AL(I)*XSF
     DXAF=XAF
     SNF=SIN(XAF)
     CAF=COS(XAF)
     EXP=DEXP(DXAF)
     S+F=0.5*(EXP-1./EXP)
     CHF=0.5*(EXP+1./EXP)
     XAR=AL(I)*XSR
     DXAR=XAR
     SNR=SIN(XAR)
     CNR=COS(XAR)
     EXR=DEXP(CXAR)
     SHR=0.5*(EXR-1./EXR)
     C+R=0.5*(EXR+1./EXR)
     PSIF(I)=H1(I,JSEGF)*CHF+H3(I,JSEGF)
     PSIR(I)=H1(I,JSEGR)*CNR+H3(I,JSEGR)*SHR+
     1 H4(I,JSEGR)*CHR+H5(I,JSEGR)
     IF(IBF-NP) 24,24,25
     PSIF1(I)=H1(I,LSEG)*SNF+H2(I,LSEG)*CNF+H3(I,LSEG)*SHF+
     1 H4(I,LSEG)*CHF+H5(I,LSEG)
     GO TO 32
24 PSIF1(I)=0.
25 PSIF1(I)=0.
32 IF(IBR-NP) 26,26,27
26 PSIR1(I)=H1(I,LSEG)*SNR+H2(I,LSEG)*CNR+H3(I,LSEG)*SHR+
   1 H4(I,LSEG)*CHR+H5(I,LSEG)
   GO TO 33
27 PSIR1(I)=0.
33 CONTINUE

```

```

XPFA=AL(I)*XSFP
DXPFA=XPFA
SNF=SIN(XPFA)
CNF=COS(XPFA)
EXF=DEXP(DXPFA)
SF=0.5*(EXF-1./EXF)
CF=0.5*(EXF+1./EXF)
PHIF(I)=A(I,JSFP)*SNF+R(I,JSFP)*(CNF-CMF)+C(I,JSFP)*SHF
XPRA=AL(I)*XSRP
DXPRA=XPRA
SNR=SIN(XPRA)
CNR=COS(XPRA)
EXR=DEXP(DXPRA)
SRR=0.5*(EXR-1./EXR)
CRR=0.5*(EXR+1./EXR)
PHIR(I)=A(I,JSRP)*SNR+R(I,JSRP)*(CNR-CHR)+C(I,JSRP)*SHR
CONTINUE
RETURN
END

```

1 C

APPENDIX D

COMPUTER PROGRAM TO DETERMINE GUIDEWAY MIDSPAN DEFLECTIONS AND MOMENTS AND VEHICLE SUSPENSION DEFLECTIONS AND ACCELERATIONS BASED ON A CONSTANT FORCE MODEL

Program Description

A two suspension, constant force vehicle traversing a simply-supported multispan guideway is simulated based on the guideway vehicle model described in Chapter 3. The constant suspension forces are applied to the guideway through uniform pressure pads with pad to span length ratios of $0 \leq \ell_p/\ell_s < 1.0$. When the suspension forces are constant, the responses of each multispan beam in the guideway are identical, thus, only the response of a single beam is required.

The program determines the deflections under the mid-points of the front and rear suspensions as a function of time. Each suspension deflection profile which is periodic from beam-to-beam, is then fit with a Fourier series using the number of terms specified by the user. The resulting Fourier coefficients corresponding to multiples of the beam encounter frequency are then input to the vehicle transfer functions described in Appendix A to determine vehicle accelerations. The program also determines the guideway midspan deflections and moments as a function of time.

To provide an efficient method of computing the deflections and moments, the numerical solution to the guideway differential equations derived in Chapter 3 is obtained using a set of first order difference equations of the form:

$$[\underline{X}]_{n+1} = [e^{A\Delta t}] [\underline{X} - \underline{X}_p]_n + [\underline{X}_p]_{n+1}$$

where

$[\underline{X}]$ is the vector of state variables

$[e^{A\Delta t}]$ is the system transition matrix

$[\underline{X}_p]$ is a vector of particular solutions for the state variables and are calculated at each time step.

This solution procedure which is described in [35] avoids numerical instabilities and has the advantage that the difference equations represent exact solutions to the differential equations as long as the prescribed pad length and the prescribed vehicle length represent distances traveled in an integer number of time steps.

The effect of using guideway precamber can also be determined by the program. Normalized camber deflections of the guideway are calculated at each time step and are added to the modal amplitudes before the suspension deflection Fourier coefficients are computed. Three types of camber are included and are denoted by the first letter in the name of each type:

$$(1) \text{ Absolute sine: } y_{cam} = A_{cam} \left| \sin \pi \frac{x}{l_s} \right|$$

where y_{cam} is the camber deflection of the beam, x is distance along the beam, l_s is span length, and A_{cam} is the normalized camber amplitude. This camber type closely approximates thermal distortion ($A_{cam} > 0$) or sag ($A_{cam} < 0$) for a single-span simply-supported guideway beam.

$$(2) \text{ Cosine: } y_{\text{cam}} = A_{\text{cam}} \frac{1}{2} (1 - \cos 2\pi \frac{x}{l_s})$$

This camber type does not have discontinuities of slope as does type (1).

$$(3) \text{ General: } y_{\text{cam}} (i/n_{\text{cam}} x/l_b) = A_{\text{cam}} \hat{y}_i,$$

where l_b is the length of the entire beam, n_{cam} is the number of time steps in the simulation determining guideway deflections for one beam and $\hat{y}$ is an array of equally - spaced camber deflection points read from cards ($i=1, n_{\text{cam}}$). General camber allows complete flexibility in dealing with multi-span beam camber which need not be periodic in l_s .

Program Inputs:

The program inputs consist of the following parameters:

Card 1: (Format (8I10))

- M:** The number of modes used to simulate the beam. The program is dimensioned to accommodate up to 5 modes, but more can be added by changing the appropriate dimension statements.
- NS:** The number of spans of the beam. The program is dimensioned to accommodate up to 5 but more can be added by changing the appropriate dimension statement.
- NPTS:** The number of points per span at which the state variables are calculated as the vehicle traverses the beam. This parameter effectively controls the time

step at a given vehicle velocity.

NFC: The number of sine and cosine terms in the Fourier series representation of the suspension displacements.

IPOP: Specifies the print option
IPOP = 1 results in printing of all outputs
IPOP = 0 suppresses some outputs (see discussion of program outputs which follows)

Cards 2, 3, 4, 5: (FORMAT (20A4))

These cards are used to read in headings that are printed on each page of the output during the simulation. Cards 2 and 3 represent one line of printed output and 4 and 5 represent another. If headings are omitted from printout, blank cards must be read in. Typical headings used correspond to the columns of output of time, midspan deflections, midspan moments, and front and rear suspension deflections. (See description of output below to determine desired heading format).

Card 6: (Format (8F10.0))

AL: A vector of M (no. of modes) eigenvalues ($\lambda_m l_s$) as described and listed in Appendix A.

Card 7: (Format (3F10.0, A1, 9x, F10.0))

ZM: The span damping ratio, ξ_m in text.

PLR: The pad to span length ratio, l_p/l_s in text. The program accepts PLR within the range $0.0 < \text{PLR} < 1.0$.

VL: The vehicle attachment to span length ratio, $l_a/l_s = (l_v - l_p)/l_s$ in text.

CDCAM: Camber type where

'A' = Absolute sine

'C' = Cosine

'G' = general

'blank' = no camber

AMCAM: Normalized camber amplitude A_{cam} (see previous discussion on camber)

Card 8: (Format (8F10.0))

AI: Vehicle inertia ratio, $\bar{I}$, in text.

AM: Vehicle unsprung to sprung mass ratio, M_u in text.

AK: Vehicle stiffness ratio, K in text.

Z: Vehicle suspension damping ratio, ξ_v in text.

FV: Vehicle suspension natural frequency (Hz), $2\pi\omega_v$ in text.

SL: Span length (ft.), l_s in text.

VS: Vehicle speed (miles/hr.), V in text.

Cards 9 and 10: (Format (16F5.2))

VX: A vector of up to 32 crossing frequency ratios, V_c at which the guideway will be simulated using the damping ratio, vehicle length ratio, and camber specified on Card 7, as well as the vehicle parameters specified on Card 8. If less than 32 crossing frequency ratios are desired, the computer discontinues as soon as it encoun-

ters a zero (or blank field) on the card. Two cards must be included, however, even if only 16 or less crossing frequencies are desired [in this case card 10 would be blank].

Card 11: (Format (3F10.0, AI, 9x, F10.0))

If this card contains a 10.0 in the first field, the program terminates. If it encounters a 1.0, it reads a new set of cards for another run, beginning with Card 1. If the first field contains a number less than 1.0, the card is assumed to contain the same variables as Card 7 above and Cards 13 and 14 should contain a vector of crossing frequencies (similar to Cards 9 and 10) to be run at the damping, pad length, and vehicle length ratios specified in Card 11. Card 12 should contain vehicle parameters as on Card 8. This sequence of runs will continue until a value of 10.0 or greater is read in the first field of a card corresponding to Card 11.

Program Outputs

The program outputs, in addition to printing out headings specifying the values of system parameters that are read include the following system variables in columns from left to right.

(Format 5x,16F7.3)):

TIME: A dimensionless parameter in units of span crossings.
(Printed in column 1)

YM: A vector of NS (no. of spans) normalized midspan deflections, beginning in column 2 with YM(1).

BM: A vector of NS normalized midspan moments, beginning in column NS+2 with BM(1).

A: A vector of M normalized modal coefficients, α_m , beginning in column 2*NS+2 with A(1).

YF: The normalized deflection under the center of the front suspension pad (column 2NS+M+2).

YR: The normalized deflection under the center of the rear suspension pad (column 2NS+M+3).

The headings HEAD1 and HEAD2 defined above can be used to properly position headings above each of the output variables, since the number of output variables changes with the number of spans and number of modes. As the leading edge of the front pad of the vehicle enters a new span of the guideway, a new page of print is begun. On each page, at the bottom of each of the columns containing midspan deflections and moments, the maximum deflection or moment which occurs during the span crossing (labeled SMAX) and the maximum deflection or moment which has occurred up to that time in the simulation (labeled CMAX) is printed.

Optional Outputs

Item 1: At the end of the simulation, the suspension deflection profiles (guideway deflection under center of suspension pads) are printed out in the phase in which they would be seen by a vehicle traversing a guideway.

Item 2: A single line stating camber code and amplitude is printed if camber is included by reading an 'A', 'C', or 'G' on Card 7. If camber effects are included, the suspension deflection profiles front (YF) and rear (YR) will be printed out as a function of time step. In addition, the camber amplitude (CF) will be printed.

Item 3: The root mean square vehicle accelerations are printed out preceded by a heading listing the vehicle characteristics including speed and guideway span length. Included in this printed output from left to right is the non-dimensional frequency (beam encounter frequency/ vehicle natural frequency) labeled 'W/FV', the front r.m.s. vehicle acceleration at this frequency, labeled 'Y2F', the total r.m.s. front acceleration including all preceding frequencies ('SUMF'), the rear r.m.s. vehicle acceleration ('Y2R'), the total r.m.s. rear acceleration ('SUMR'), the vehicle center of gravity r.m.s acceleration at this frequency ('YCG') and the total center of gravity r.m.s. acceleration including all previous frequencies ('SUMC').

Item 4: The total r.m.s. acceleration for the front, rear, and center of gravity of the vehicle is printed out including the speed for which these values are calculated.

Item 5: The coefficients of the first NPF terms of a Fourier Series representation of the suspension displacement profiles are then printed. The coefficients of the cosine and sine terms of the front suspension profile are labeled 'AF' and 'BF' respectively, while the cosine and sine coefficients of the rear profile

are labeled 'AR' and 'BR' respectively. The frequency in hertz corresponding to each coefficient (a multiple of the beam encounter frequency) is also printed and labeled 'W'.

Item 6: A reconstruction of the front and rear vehicle acceleration using the Fourier series coefficients obtained by inputting the suspension deflection coefficients to the vehicle transfer function are printed for 51 points representing equal fractions of the time the vehicle takes to traverse a single guideway beam. The fraction of the traverse time is printed out (and labeled '1/TP') followed by the non-dimensional front and rear vehicle accelerations at each fractional period labeled as 'AY2F' and 'AY2R' respectively. Failure to specify IPOF=1 in data Card 1 will result in suppression of all optional outputs except items 2 and 4. A complete listing of the Fortran program follows.


```

C      CONSTANT FORCE TWO SUSPENSION VEHICLE TRAVERSING SEMI-
C      CONTINUOUS GUIDEWAY --- CALCULATES FOURIER COEFFICIENTS OF
C      GUIDEWAY DEFLECTIONS AND VEHICLE MOTIONS FOR NTH ORDER
C      VEHICLE MODEL
C
C      DIMENSION X(10),HEAD1(30),HEAD2(30),YMXS(5),
1YMXA(5),BPMXS(5),BMXA(5),VX(32),YMTV(2,32),BMTV(2,32),YMTA(2,32),
2 BPTA(2,32)
C      DIMENSION YVF(101),YVR(101)
C      COMMON AL(5),A(5,5),B(5,5),C(5,5),PHIM(5,5),CHIM(5,5),LR,LP
C      COMMON E1(5),E2(5),E3(5),E4(5)
C      COMMON C1(5,11),C2(5,11),C3(5,11),C4(5,11),C5(5,11),MF(5),IRFS(5)
C      COMMON H1(5,11),H2(5,11),H3(5,11),H4(5,11),H5(5,11)
C      COMMON /CCCAM/CCCAP,AMCAP,ICAPL
C      DATA TEND /'PIAN'./
C      LR=8
C      LP=5
99  CONTINUE
C      YMTA(1,1)=0.
C      YMTV(1,1)=0.
C      YMTA(1,1)=0.
C      BMTV(1,1)=0.
C      READ(ILR,100) M,NS,NPTS,MFC,IPOP
C      FORMAT(8I10)
C      READ(ILR,104) HEAD1
C      FORMAT(2CA4)
C      READ(ILR,104) HEAD2
C      M2=2*P
C      DTIME=1./FLCAT(NPTS)
C      READ(ILR,102)(AL(I),I=1,M)
C      FORMAT(8F10.0)
C      CALL EGN(P,NS,TEND)
3    READ (ILR,1025) ZM, PLR, VL, CDCAM, AMCAP
1025 FORMAT (3F10.0, A1, 9X, F10.0)
C      READ(ILR,102) AI,AP,AK,Z,FV,SL,VS

```

CAMBER

```

1013 WRITE(LP,1013)
      FORMAT(1H1)
      IF(ZN.GE.10.)CALL EXIT
      IF(ZN.GE.1.0)GO TO 99
      ISA=PLR*FLOAT(NPTS)*0.5
      CALL HCAL(M,NS,PLR)
      NSEG=2*NS+1
      IT=1
      READ(LR,103) VX
      FORMAT(16F5.2)
      VC=VX(IT)
      T=1./VC
      TAUD=6.2831854*T*VL
      TAUS=6.2831854*T
      TAUP=6.2831854*T*PLR
      ITMF=1
      ITHR=VL*FLOAT(NPTS)+1.5
      NPIT=NPTS*NS+1
150 IF(ITHR-LT.NPIT)GO TO 151
      ITPR=ITPR-(NPIT-1)
      GO TO 150
151 VYF(ITMF)=0.
      VYR(ITPR)=0.
      NC=1
      DTAL=2.*3.1415927*T/FLOAT(NPTS)
      CALL EXPRS(T,ZN,M,NS,CTAU)
      NSEG=2*NS+1
      NJ=1
      NCAS=FLOAT(NS)+VL*PLR+0.99
15 DO 5 I=1,M2
      X(I)=0.0
      DO 6 J=1,NS
      BMXA(J)=0.0
      YMXA(J)=C.0
      TIME=0.0
14 WRITE(LP,1007) PLR

```

```

1007  FORMAT(1M1,T16,'SEMICCONTINUOUS GUIDEWAY',T47,'PRESSURE PAD LENGTH
      IRATIO=',F4.2)
1012  WRITE(LP,1012) VC,NS,M,NJ,NPTS,ZN,VL
      FORMAT(1M,T16,'CRGSS. FREQ. RATIO=',F6.3,T47,'NUMBER OF SPANS=',
1 12,T72,'NUMBER OF MODES=',I2,/,T16,'VEHICLE IS ON SPAN ',I2,
2 T47,'POINTS PER SPAN=',I2,T72,'DAMPING RATIO=',F4.2,/,T16,
3 'VEHICLE LENGTH RATIO=',F4.2,/)
      DO 19 J=1,NS
        BMXS(J)=C.0
        YMXS(J)=0.0
        TAU=0.0
19      WRITE(LP,1008) HEAD1
1008  FORMAT(1M0,30A4)
      WRITE(LP,1009) HEAD2
1009  FORMAT(1M,30A4,/)
      CALL CPDIF(X,NPTS,NJ,M,NS,TAU,DTAL,TIME,CTIME,YMXS,BMXS,ISA,
1 TAU,TAUS,TAUP,PLR,VL,VYF,VYR,ITMF,ITMR,NPTT)
      DO 7 J=1,NS
        IF(YMXS(J)-YMXA(J))8,6,9
        YMXA(J)=YMXS(J)
        IF(BPMXS(J)-BMXA(J))7,7,10
        BMXA(J)=BPMXS(J)
        CONTINUE
        IF(IJ-NS) 31,31,22
        IF(YMXS(NJ)-YMTV(1,IT))24,24,4
        YMTV(1,IT)=YMXS(NJ)
        IF(BPMXS(NJ)-BMTV(1,IT)) 22,22,23
        BMTV(1,IT)=BPMXS(NJ)
        CONTINUE
        WRITE(LP,1010)(YMXS(J),J=1,NS),(BMXS(J),J=1,NS)
        FORMAT(7X,'SMAX=',I4F7.3)
        WRITE(LP,1011)(YMXA(J),J=1,NS),(BMXA(J),J=1,NS)
        FORMAT(7X,'CMAX=',I4F7.3)
        NJ=NJ+1
        IF(IJ-NCRS)14,14,13
13      DO 21 I=1,NS

```

```

25 IF (YMXA(1)-YMTA(1,IT)) 26,26,25
26 YMTA(1,IT)=YMXA(1)
27 IF (BMTA(1)-BMTA(1,IT)) 21,21,27
21 BMTA(1,IT)=BMTA(1)
CONTINUE
YVR(1)=YVR(NPTT)
CALL FCOEF (YVF,YVR,NS,NPTT,MFC,LP,LR,I,VL,PLR,ZN,AI,AP,AK,Z,
1FV,SL,VS,IPOP)
IT=IT+1
28 IF (IT.GT.32) GO TO 29
IF (VX(IT).EQ.0.0) GO TO 29
YMTA(1,IT)=0.0
YMTV(1,IT)=0.0
BMTA(1,IT)=0.0
BMTV(1,IT)=0.0
GO TO 12
35 YMTA(1,IT)=0.
BMTA(1,IT)=0.
YMTV(1,IT)=0.
BMTV(1,IT)=0.
DO 36 J=1,NS
YMXA(J)=0.
RMTA(J)=C.
GO TO 14
29 IT=IT-1
DO 37 I=1,IT
YMTA(2,I)=VX(I)
WRITE(LP,1007) PLR
WRITE(LP,1012) VC,NS,P,NJ,NPTS,ZN,VL
WRITE(LP,334) (VX(I),YMTA(1,I),BMTA(1,I),I=1,IT)
334 FORMAT('0',14X,'VC',7X,'MAX. DEFL.',/,(7X,3F12.3))
GO TO 3
C
C
C
ENC

```

```

C
SUBROUTINE EGN(P,NS,TEND)
COMMON AL(S),A(S,S),R(S,S),C(S,S),PHIM(S,S),CHIM(S,S),LR,LP
DATA PIA/'PIA',/FIX/'FIXE'/
DO 50 I=1,M
  ALP=AL(I)
  C-----CCPUTE TRIGHCMPETRIC PROPERTIES ONCE FOR EACH MODE
    SN=SIN(ALP)
    SH=0.5*(EXP(ALP)-EXP(-ALP))
    CS=COS(ALP)
    CH=0.5*(EXP(ALP)+EXP(-ALP))
  C-----COMPUTE GEOMETRIC PROPERTIES
    G1=SH/(SM-SN)
    G2=SH+SN
    G3=(CH-CS)/(SM-SN)
    G4=SN/(SM-SN)
  C-----PREPARE TO NORMALIZE MODE BY COMPUTING THE MODE INTEGRAL FOR A(I,1)=1.0
  C-----LS (SPAN SEGMENT LENGTH) = 1.0 (DEFINITION)
  C-----INTEGRAL(PHIN+2)=NS
  C-----QUADRATIC INTEGRALS FOR ANY SPAN
    SMSN=0.5*(ALM-SM+CS)
    CSCS=0.9*(ALM+SM+CS)
    SHSM=0.5*(SM+CH-ALM)
    CHCH=0.5*(SM+CH+ALP)
  C-----0.5 FOR FIXED TERMS CANCELS 2.0 FOR PINCMIAL EXPANSION
    SNCS=SM+SN
    SMSH=CH+SN-SM+CS
    SMCH=SM+SN-CH+CS+1.0
    CSSH=SM+SN+CH+CS-1.0
    CSCM=SM+CS+CH+SN
    SMCM=SM+SH
    Q=C.C
  DO 4C J=1,NS
    IF(J-1)10,10,20
  C-----CCPUTE A,B,C FOR FIRST SPAN
    10  SA=1.0

```

```

IF (TENC.EQ.PIN) SB=0.
IF (TEND.EC.FIX) SB=-1./G3
SC=-SN/SH+(CH-CS)*SB/SH
GO TO 30
C-----USE RECURSION FOR REMAINING TERMS, WORKS EVEN IF SIN(ALM)=G.O
2C G5=SA*CS-G2*SB+SC*CH
G6=SA*SN+SB*CS
C-----NCH UPDATE A,B, AND C
SA=G1*G5-G3*G6
SB=G6
SC=G3*G6-G4*G5
30 SD=-SB
A(I,J)=SA
B(I,J)=SB
C(I,J)=SC
C-----INTEGRATE ALONG TOTAL SPAN LENGTH BY ACCUMULATING INDIVIDUAL
C-----SPAN SEGMENT INTEGRALS
O=O+SA*(SA*SN+SB*SNCS+SC*SNSH+SD*SNCH)
1 +SB*(SB*CS+SC*CSSH+SD*SCH)
2 +SC*(SC*SN+SD*SNCH)
1 +SD*(SD*CH)
40 CONTINUE
C-----NORMALIZE MODE
O=SQRT(FLOAT(NS)*ALM/Q)
DO 50 J=1,NS
A(I,J)=A(I,J)*O
B(I,J)=B(I,J)*O
C(I,J)=C(I,J)*O
CONTINUE
50 DO 5 I=1,P
ALM=AL(I)*0.5
ALS=(AL(I)/3.1415927)*2
SN=SIN(ALM)
CN=COS(ALM)
SH=C.5*(EXP(ALM)-EXP(-ALM))
CH=0.5*(EXP(ALM)+EXP(-ALM))

```

```

DO 5 JK=1,NS
  PHIP(I,JK)=A(I,JK)*SN+B(I,JK)*(CN-CN)+C(I,JK)*SH
  CHIP(I,JK)=ALS*(-A(I,JK)*SN-B(I,JK)*(CN+CH)+C(I,JK)*SH)
CONTINUE
RETURN
END

SUBROUTINE MCALIP,NS,PLR)
COMMON AL(5),A(5,5),B(5,5),C(5,5),PHIP(5,5),CHIP(5,5),LR,LP
COMMON E1(5),E2(5),E3(5),E4(5)
COMMON C1(5,11),C2(5,11),C3(5,11),C4(5,11),C5(5,11),WF(5),IRES(5)
COMMON H1(5,11),H2(5,11),H3(5,11),H4(5,11),H5(5,11)
SINH(X)=0.5*(EXP(X)-EXP(-X))
COSH(X)=0.5*(EXP(X)+EXP(-X))
IF(PLR-G.COC1) 41,41,4C
DO 44 I=1,M
DO 42 J=1,NS
  K=2*J-1
DO 42 KS=1,2
  H1(I,K)=A(I,J)
  H2(I,K)=R(I,J)
  H3(I,K)=C(I,J)
  H4(I,K)=-B(I,J)
  H5(I,K)=0.0
  K=K+1
  H1(I,K)=C.0
  H2(I,K)=0.0
  H3(I,K)=0.0
  H4(I,K)=C.0
  H5(I,K)=C.0
RETURN
CONTINUE
DO 2 I=1,M
  ALI=1./(PLR*AL(I))
  ALF=AL(I)*PLR

```

```

SL=SIN(ALF)
CL1=COS(ALF)-1.
SHL=SIAM(ALF)
CHL1=CCSH(ALF)-1.
SM=SIN(AL(I))
CM=COS(AL(I))
SMP=SINM(AL(I))
CMP=COSM(AL(I))
R=AL(I)*I.-PLR)
SR=SIN(R)
CR=COS(R)
SMR=SIAM(R)
CMR=COSH(R)
H1(I,1)=ALI*O(I,1)
H2(I,1)=-ALI*O(I,1)
H3(I,1)=H1(I,1)
H4(I,1)=ALI*C(I,1)
H5(I,1)=ALI*(A(I,1)-C(I,1))
IF(NS-1) 3,3,4
DO 1 J=2,NS
IS=2*J-1
H1(I,IS)=ALI*(-A(I,J-1)*SR-B(I,J-1)*CR+O(I,J))
H2(I,IS)=ALI*( A(I,J-1)*CR-B(I,J-1)*SR-A(I,J))
H3(I,IS)=ALI*( B(I,J-1)*CHR-(I,J-1)*SHR-B(I,J))
H4(I,IS)=ALI*( B(I,J-1)*SHR-C(I,J-1)*CHR+C(I,J))
H5(I,IS)=ALI*(-A(I,J-1)*CP+B(I,J-1)*(SM-SM)*C(I,J-1)*CHM+
1 A(I,J)-C(I,J))
NHS=2*NS+1
H1(I,NHS)=ALI*(-A(I,NS)*SR-B(I,NS)*CR)
H2(I,NHS)=ALI*( A(I,NS)*CR-B(I,NS)*SR)
H3(I,NHS)=ALI*(B(I,NS)*CHR-C(I,NS)*SHR)
H4(I,NHS)=ALI*(B(I,NS)*SHR-C(I,NS)*CHR)
H5(I,NHS)=ALI*(-A(I,NS)*CP+B(I,NS)*(SM-SM)*C(I,NS)*CHM-
DO 2 J=1,NS
IS=2*J
H1(I,IS)=ALI*(A(I,J)*SL-B(I,J)*CL1)

```



```

H2(I,IS)=AL1*(A(I,J)*CL1+B(I,J)*SL)
H3(I,IS)=AL1*(B(I,J)*CHL1+C(I,J)*SML)
H4(I,IS)=AL1*(-B(I,J)*SHL-C(I,J)*CHL1)
H5(I,IS)=0.
RETURN
END

SUBROUTINE EXPRS(T,ZN,M,NS,DTAU)
COMMON AL(S),A(S,S),B(S,S),C(S,S),PHIM(S,S),CHIM(S,S),LR,LP
COMMON E1(S),E2(S),E3(S),E4(S)
COMMON C1(S,11),C2(S,11),C3(S,11),C4(S,11),C5(S,11),VFF(S,11),IRES(S)
COMMON H1(S,11),H2(S,11),H3(S,11),H4(S,11),H5(S,11)
NSEG=2*NS+1
DO 1 I=1,M
  WN=(AL(I)/3.1415927)**2
  WF=AL(I)/(2.*3.1415927*T)
  VFF(I)=WF
  P1=WN*WN-WF*WF
  P4=WN*WN+WF*WF
  P5=P1*P1
  P6=P4*P4
  AP1 = ABS(P1)
  IF((ZN.LT.0.00001).AND.(AP1.LT.0.00001))GO TO 3
  IRES(I)=0
  P2=ZN*WF*WN
  P3=4.*(ZN*WN*WF)**2
  RAD=SQRT(1.-ZN**2)
  WR=WN/RAD
  SN=SIN(WR*DTAU)
  CN=COS(WR*DTAU)
  ZR=ZN/RAD
  EX=EXP(-ZN*WN*DTAU)
  E1(I)=EX*(CN+ZR*SN)
  E2(I)=EX*SN/WR

```

2

C C C

```

E3(I)=-EX*SN*WN/RAD
E4(I)=EX*(CN-ZR*SN)
DO 2 NJ=1,NSEG
C1(I,NJ)=(-P1*H1(I,NJ)/2.-P2*H2(I,NJ))/((P5+P3)*FLOAT(NS))
C2(I,NJ)=(P2*H1(I,NJ)-P1*H2(I,NJ)/2.)/((P5+P3)*FLOAT(NS))
AC3=(-P4*H3(I,NJ)/2.+P2*H4(I,NJ))/((P6-P3)*FLOAT(NS))
AC4=(P2*H3(I,NJ)-P4*H4(I,NJ)/2.)/((P6-P3)*FLOAT(NS))
C3(I,NJ)=0.5*(AC3+AC4)
C4(I,NJ)=0.5*(AC4-AC3)
C5(I,NJ)=-H5(I,NJ)/(2.*WN*WN*FLOAT(NS))
GO TC 1
IRES(I)=1
CN=COS(WN*DTAU)
SN=SIN(WN*DTAU)
E1(I)=CN
E2(I)=SN*WN
E3(I)=-WN*SN
E4(I)=CN
DO 4 NJ=1,NSEG
C1(I,NJ) = -0.5*3.1415927*H2(I,NJ)/(FLOAT(NS)*AL(I))
C2(I,NJ) = +0.5*3.1415927*H1(I,NJ)/(FLOAT(NS)*AL(I))
AC3=-0.5*P4*H3(I,NJ)/P6/FLOAT(NS)
AC4=-0.5*P4*H4(I,NJ)/P6/FLOAT(NS)
C3(I,NJ)=0.5*(AC3+AC4)
C4(I,NJ)=0.5*(AC4-AC3)
C5(I,NJ)=-H5(I,NJ)/(2.*WN*WN*FLOAT(NS))
CONTINUE
RETURN
END

```

```

SUBROUTINE CPCIF(X,NPTS,ISPAN,M,NS,TAU,DTAU,TIME,DTIME,YMXS,BMXS,
1 ISA,TAUD,TAUS,TAUP,PLR,VL,VVF,VYR,ITMF,ITMR,NPTT)
DIMENSION X(1),YMXS(1),BMXS(1),YM(5),RM(5),XP(10),XPR(10)
DIMENSION VVF(1),VYR(1)
COMMON AL(5),A(5,5),B(5,5),C(5,5),PHIP(5,5),CHIP(5,5),LR,LP

```

```

COMMON E1(5),E2(5),E3(5),E4(5)
COMPCN C1(5,11),C2(5,11),C3(5,11),C4(5,11),C5(5,11),WF(5),IRES(5)
NS2=2*NS+1
NJ=2*ISPAN-1
KPTS=ISA
KS=2
IF(IISA)17,17,18
17 NJ=2*ISPAN
KPTS=NPTS
GO TC 15
18 IF(IISA-NPTS)14,15,16
15 KS=1
GO TC 14
16 WRITE(LP,200)
200 FORMAT('1 PAD LENGTH RATIO IS GREATER THAN 1',/)
RETURN
DO 13 KP=1,KS
NR0=-2C
DO 7 I=1,M
IDCT=M+I
XPR(I)=0.
XPR(IDOT)=0.
IF(INJ-AS2)8,8,4C
40 XPII)=C.
XP(IDOT)=C.
GO TC 7
8 XS=WF(I)*TAU
SN=SIN(XS)
CN=COS(XS)
EPX=EXP(XS)
IF(IRES(I).NE.1) GC TC 10
XPII)=(TAU*(C1(I,NJ)*SN+C2(I,NJ)*CN)+C3(I,NJ)*EPX+C4(I,NJ))/EPX+
1 C5(I,NJ))*0.5
XPR(IDOT)=((SN+WF(I)*TAU*CN)*C1(I,NJ)+(CN-WF(I)*TAU*SN)*C2(I,NJ)
1 +WF(I)*(C3(I,NJ)*EPX-C4(I,NJ)/EPX))*0.5
GO TO 7

```

```

10  XP(I)=(C1(I,NJ)*SN+C2(I,NJ)*CN+C3(I,NJ)*EPX+C4(I,NJ)/EPX+
    1  C5(I,NJ))*0.5
    XP(IDOT)=(WF(I)*(C1(I,NJ)*CN-C2(I,NJ)*SN+C3(I,NJ)*EPX-C4(I,NJ)/
    1  EPX))*0.5
    7  CONTINUE
    9  DO 1 K=1,KPTS
        NRCO=NR0
        TAU=TAU+CTAU
        XFP=(TAU/TAUS)-.5*PLR
        IF(XFP)46,46,45
            45  ISF=ISPAN
                GO TC 52
            46  ISF=ISPAN-1
                XFP=1.+XFP
            52  IF(1SF)57,57,58
            58  IF(1SF-NS) 59,59,57
            59  ITMF=ITMF+1
            57  TAUR=TAU-TAUD
                ISPNR=ISPAN
                IF(TAUR) 20,20,49
                TAUR=TAUS+TAUR
                ISPNR=ISPNR-1
                IF(TAUR) 20,20,49
            49  XRP=(TAUR/TAUS)-0.5*PLR
                IF(XRP)48,48,47
            48  XRP=XRP+1.0
                ISR=ISPNR-1
                GO TC 60
            47  ISR=ISPNR
                IF(ISR) 61,61,62
                IF(ISR-NS) 63,63,61
            63  ITMR=ITMR+1
                IF(ITMR.GT.NPT1)ITMR=2
            61  IF(TAUR-TAUP) 27,28,28
                27  NR=2+ISPNR-1
                    GO TC 34

```

```

28 NR=2*ISPNR
34 YF=0.
   YR=0.
   IF(NR-NS2)3,3,4
4   ON 5 I=1,M
   XI=X(I)
   IDOT=M+I
   X(I)=E1(I)*XI+E2(I)*X(IDOT)
   X(IDOT)=E3(I)*XI+E4(I)*X(IDOT)
5   GO TO 6
3   ON 4C0 I=1,P
   IDOT=M+I
   IF(NR-ARO) 37,38,37
37 XR=WF(I)*(TAUR-OTAU)
   XP(I)=XP(I)-XPR(I)*.5
   XP(IDOT)=XP(IDOT)-XPR(IDOT)*.5
   GO TO 35
38 XR=WF(I)*TAUR
35 IF(ISPNR-0)23,23,24
23 XPR(I)=0.
   XPR(IDOT)=0.
   GO TO 39
24 SNR=SIN(XR)
   CNR=COS(XR)
   EXR=EXP(XR)
29 IF(IRES(I)-1)3C,29,30
   XPR(I)=TAUR*(C1(I,NR)*SNR+C2(I,NR)*CNR)+C3(I,NR)*EXR+C4(I,NR)/EXR+
   IC5(I,NR)
   XPR(IDOT)=(SNR*WF(I)*TAUR+CNR)*C1(I,NR)+(CNR*WF(I)*TAUR*SNR)*C2(I,
   NR)*WF(I)*(C3(I,NR)*EXR-C4(I,NR)/EXR)
   GO TO 39
30 XPR(I)=C1(I,NR)*SNR+C2(I,NR)*CNR+C3(I,NR)*EXR+C4(I,NR)/EXR+
   IC5(I,NR)
   XPR(IDOT)=WF(I)*(C1(I,NR)*CNR-C2(I,NR)*SNR+C3(I,NR)*EXR-C4(I,NR)/
   EXR)
39 IF(NR-NRO) 36,31,36

```

```

36  XP(I)=XP(I)+XPR(I)*.5
    XP(IDOT)=XP(IDOT)+XPR(IDOT)*.5
    NRO=NR
    GO TC 30
31  XI=X(I)-XP(I)
    XD=X(IDOT)-XP(IDOT)
    IF (NJ-A:2) 42,42,43
43  XP(I)=C.
    XP(IDOT)=C.
    GO TC 12
42  XS=WF(I)*TAU
    SN=SIN(XS)
    CN=COS(XS)
    EPX=EXP(XS)
    IF (IRES(I).NE.1) GO TC 11
    XP(I)=TAU*(C1(I,NJ)*SN+C2(I,NJ)*CN)+C3(I,NJ)*EPX+C4(I,NJ)/EPX+
    1 C3(I,NJ)
    XP(IDOT)=(SN*WF(I)*TAU*CN)+C1(I,NJ)+(CN-WF(I)*TAL*SN)*C2(I,NJ)
    1 *WF(I)*(C3(I,NJ)*EPX-C4(I,NJ)/EPX)
    GO TO 12
11  XP(I)=C1(I,NJ)*SN+C2(I,NJ)*CN+C3(I,NJ)*EPX+C4(I,NJ)/EPX+C5(I,NJ)
    XP(IDOT)=WF(I)*(C1(I,NJ)*CN-C2(I,NJ)*SN+C3(I,NJ)*EPX-C4(I,NJ)/EPX)
    12 XP(I)=(XP(I)+XPR(I))**.5
    XP(IDOT)=(XP(IDOT)+XPR(IDOT))**.5
    X(I)=E1(I)*XI+E2(I)*XC+XP(I)
    X(IDOT)=E3(I)*XI+E4(I)*XD+XP(IDOT)
    IF (ISR) 50,50,51
51  IF (ISR-NS) 53,53,2
53  XLR=AL(I)*XRP
    SNRP=SIGN(XLR)
    CNRP=CCS(XLR)
    EXRP=EXP(XLR)
    PHIR=A(I,ISR)*SNRP+B(I,ISR)*(CNRP-.5*(EXRP+1./EXRP))+C(I,ISR)*.5*(
    1 EXRP-1./EXRP)
    YR=YR+PHIR*X(I)
50  IF (ISF) 2,2,55

```

```

55 IF (ISF-NS)56,56,2
56 XLP=AL(I)*XFP
   SNFP=SIN(XLF)
   CNFP=COS(XLF)
   EXFP=EXP(XLF)
   PHIF=A(I,ISF)*SNFP+B(I,ISF)*(CNFP-.5*(EXFP+1./EXFP))+C(I,ISF)*.5*(
1EXFP-1./EXFP)
   VF=YF+PHIF*X(I)
2 IF (I.LT.M)NRO=NROO
400 CONTINUE
6 DO 75 J=1,NS
   BJ=0.0
   YJ=0.0
   DO 9C I=1,M
     BJ=BJ+CHIM(I,J)*X(I)
     YJ=YJ+PHIM(I,J)*X(I)
90 IF (ABS(YJ)-YMXS(J))95,95,92
     YMXS(J)=ABS(YJ)
92 YH(J)=YJ
95 IF (ABS(BJ)-BMXS(J))75,75,72
     BMXS(J)=ABS(BJ)
72 BM(J)=BJ
75 TIME=TIME+DTIME
   TIM=TIME+C.COOS
   WRITE(LP,1CC) TIM , (YH(I),I=1,NS), (BM(I),I=1,NS), (X(I),I=1,M)
1 ,YF,YR
1CC FORMAT(5X,16F7.3)
   YVF(ITMF)=YF
   YVR(ITPR)=YR
1 CONTINUE
   KPTS=NPTS-ISA
   NJ=2+ISPAN
13 CONTINUE
   RETURN
   END
C

```

```
C SUBRCUTIME PCOEF(YVF,YVR,MS,NPTT,NFC,L,P,R,T,V,L,PLR,ZN,AI,AM,AK,Z,  
C IFV,SL,VS,IPOP)  
C  
C DETERMINE FOURIER COEFFICIENTS FOR SUSPENSION DEFLECTION PRO-  
C FILES  
C WRITE RESULTS  
C DETERMINE FRONT AND REAR RMS ACCELERATIONS AT NORMALIZED FRE-  
C QUENCIES FROM FOURIER COEFFICIENTS AND OTH ORDER VEHICLE TRANS-  
C FER FUNCTIONS  
C WRITE RESULTS  
C DIMENSION YVF(101),YVR(101),ARG(18)  
C DIMENSION X(101),S(101)  
C DIMENSION AF(18),BF(18),AR(18),BR(18)  
C DIMENSION WM(18)  
C DIMENSION CAMB(101)  
C COMMON /COCAM/ CDCAM, ANCAM, ICAML  
C DATA ALPHA, ALPHC, ALPHG / 'A', 'C', 'G' /  
C DATA PI, PIZ / 3.14159265 , 6.2831853 /  
C TNS=FLOAT(NS)  
C M=FLOAT(NS)/FLOAT(NPTT-1)  
C IF(IPOP.EQ.O)GC TC 994  
C WRITE(LP,201)  
C 201 FORMAT(1H1,3X,'SUSPENSION DISPLACEMENT PHASED AS SEEN',/,4X,'BY VE  
C HICLE TRAVERSING GUIDEMAY',///,4X,'TIME STEP',5X,'VF',14X,'VR',//)  
C WRITE(LP,200) ((YVF(I),YVR(I)),I=1,NPTT)  
C 200 FORMAT(7X,13,F10.3,5X,F10.3)  
C 994 CONTINUE  
C -----CAMBER UNIT-----  
C C ----CAMBER UNIT  
C  
C NPTX = NPTT - 1  
C ---- DEFAULT VALUE OF ANCAM IS UNITY.  
C IF (ANCAM .EQ. G.) ANCAM = 1.  
C ---- BRANCH ACCORDING TO CCCE.  
C IF (CDCAM .NE. ALPHA) GC TC 701
```

D-23


```

C --- HERE FOR ABSOLUTE VALUE OF SINE.
    DO 7011 I = 1, NPTT
      CARG = FLOAT((I-1)*NS) / FLOAT(NPTX)
      CAMBR(I) = ABS(SIN(PI*CARG)) * ANCAP
    7011 CONTINUE
    GO TO 750
  701 IF (CDCAP .NE. ALPHC) GO TO 702
C --- HERE FOR COSINE.
    DO 7021 I = 1, NPTT
      CARG = FLOAT((I-1)*NS) / FLOAT(NPTX)
      CAMBR(I) = 0.5 * (1.-COS(PI2*CARG)) * ANCAM
    7021 CONTINUE
    GO TO 750
  702 IF (CDCAM .NE. ALPHG) GO TO 799
C --- HERE FOR ARBITRARY CAMBER.
C --- NORMALIZED CAMBER IS READ IN FROM CARCS, ONE VALUE FOR EACH
C --- TIME STEP ALONG THE BEAM. ITEMS READ = NS*NPTS + 1.
    READ (LR,7030) (CAMBR(I), I = 1, NPTT)
  7030 FORMAT (6F10.0)
    DO 7031 I = 1, NPTT
      CAMBR(I) = CAMBR(I) * ANCAM
    7031 CONTINUE
    750 CONTINUE
C --- ADD CAMBER TO VVF, VYR.
    DO 7501 I = 1, NPTX
      VVF(I) = VVF(I) + CAMBR(I)
      ICAM1 = I - 1 + ICAML
      ICAP2 = MOD(ICAM1, NPTX) + 1
      VYR(ICAP2) = VYR(ICAP2) + CAMBR(I)
    7501 CONTINUE
    VVF(NPTT) = VVF(1)
    VYR(NPTT) = VYR(1)
C --- PRINT CAMBER ADJUSTMENTS.
    WRITE (LP,7040) CDCAP, ANCAM
  7040 FORMAT (11 CAMBER CODE '...', A1, '...', AMPLITUDE= ', F6.3 //

```

[illegible]

CAMBER
CAMBER
CAMBER
CAMBER
CAMBER

```
1 4X, 'TIME STEP', 5X, 'YF', 13X, 'YR', 13X, 'CF'//
WRITE (LP,7050) (I,VYF(I), YVR(I), CAMBR(I), I = 1, NPIT)
7050 FORMAT (7X, 13, F10.3, 5X, F10.3, 5X, F10.3 )
```

C
799 CONTINUE
C -----

```
DO 11 I=1,NPIT
11 X(I)=VYF(I)/TNS
CALL SINT(X,M,NPIT,S)
AF0=S(NPIT)
DO 13 I=1,NPIT
13 X(I)=VVR(I)/TNS
CALL SINT(X,M,NPIT,S)
ARO=S(NPIT)
DO 50 J=1,MFC
ARG(J)=FLOCAT(J)*2.03.1415927/TNS
DO 15 I=1,NPIT
15 X(I)=(2./TNS)*VYF(I)*COS(ARG(J)*FLOCAT(I-1)*PI)
CALL SINT(X,M,NPIT,S)
AF(J)=S(NPIT)
DO 17 I=1,NPIT
17 X(I)=(2./TNS)*VYF(I)*SIN(ARG(J)*FLOCAT(I-1)*PI)
CALL SINT(X,M,NPIT,S)
BF(J)=S(NPIT)
DO 19 I=1,NPIT
19 X(I)=(2./TNS)*VVR(I)*CCS(ARG(J)*FLOCAT(I-1)*PI)
CALL SINT(X,M,NPIT,S)
AR(J)=S(NPIT)
DO 21 I=1,NPIT
21 X(I)=(2./TNS)*VVR(I)*SIN(ARG(J)*FLOCAT(I-1)*PI)
CALL SINT(X,M,NPIT,S)
BR(J)=S(NPIT)
50 CONTINUE
VC=1./T
SUPF=0.
```

C


```

1+4.0Z0CUM30((1.0AK)0DUM2+AM)
D6--(4.0Z002+2.0AM0(1.0AK))0DUM1002
1+2.0AM0DUM20((1.0AK)0DUM2+AM)+4.0Z0020DUM3002
D7--4.0Z0AM0(DUM1002-CUM20CUM3)
D8=AP0020(DUM2002-DUM1002)
ARATM=C.
VLEN=SL0(PLR0VL)
IF(IIPCP.EC.0) GC TC 991
CFR=1.7
WRITE(LP,505) NS,SL,VS,PLR,VLEN,ZA
WRITE(LP,507) CFR,FV,VL
CONTINUE
991 IF(IPOP.EQ.0)GO TO 99C
WRITE(LP,111)
CONTINUE
990 CONTINUE
DO 69 I=1,NFC
WM(1)=FLOAT(1)+VS01.467/(TNS0SL)
h=WM(1)/FV
ANLSR=CS0-W0020CS2+h0040CS4-W0060CS6
ANUSI=h0CS1-W0030CS3+h0050CS5-W0070CS7
ANUCR=CC0-W0020CC2+h0040CC4-W0060CC6
ANLCI=h0CC1-W0030CC3+h0050CC5-W0070CC7
DNH=DO-W0020D2+h0040D4-W0060D6+h0080D8
DN1=h0C1-W0030C3+h0050C5-W0070D7
DUP4=SCRT(DNR002+DN1002)
APAGC=SQRT(ANUCI002+ANUCR002)/CLP4
AMAGS=SQRT(ANUSI002+ANUSR002)/DUM4
XS=ANUSR0DNR+ANLSI0DNI
YS=ANUSI0DNR-ANLSR0DNI
XC=ANUCR0DNR+ANLCI0DNI
YC=ANUCI0DNR-DNI0ANUCR
CALL PHASE(XS,YS,PH1)
PHAS=PH1
PHAS=PH10360./6.28310
CALL PHASE(XC,YC,PH2)
PHAC=PH2

```

```

FHAC=PH2*360./6.28318
SUMS=SIN(PHAS)*COS(PHAS)
DIFS=CCS(PHAS)-SIN(PHAS)
SUMC=SIN(PHAC)*CCS(PHAC)
DIFC=COS(PHAC)-SIN(PHAC)
Y2F1=AMAGS*(BF(I)*COS(PHAS)-AF(I)*SIN(PHAS))
1+APAGC*(BR(I)*CCS(PHAC)-AR(I)*SIN(PHAC))
Y2F2=AMAGS*(BF(I)*SIN(PHAS)+AF(I)*CCS(PHAS))
1+APAGC*(BR(I)*SIN(PHAC)+AR(I)*COS(PHAC))
Y2F=0.707*(Y2F1+Y2F2)*0.5
Y2R1=AMAGS*(BR(I)*CCS(PHAS)-AR(I)*SIN(PHAS))
1+AMAGC*(BF(I)*COS(PHAC)-AF(I)*SIN(PHAC))
Y2R2=AMAGS*(BR(I)*SIN(PHAS)+AR(I)*COS(PHAS))
1+AMAGC*(BF(I)*SIN(PHAC)+AF(I)*COS(PHAC))
Y2R=0.707*(Y2R1+Y2R2)*0.5
YCG=0.5*0.707*(Y2F1+Y2R1)*0.2+(Y2F2+Y2R2)*0.2*C.5
SUMF1=(Y2F1+Y2F2+Y2F2)*0.5
SUMR1=(Y2R1+Y2R2+Y2R2)*0.5
SUMG1=((Y2F1+Y2R1)*0.2+(Y2F2+Y2R2)*0.2)*0.125
SUMF=SUMF+SUMF1
SUMR=SUMR+SUMR1
SUMG=SUMG+SUMG1
SUMF=SQRT(SUMF)
SUMR=SQRT(SUMR)
SUMG=SQRT(SUMG)
IF(IPOP.FC.0)GC TO 69
WRITE(ILP,110)Y,Y2F,SUMF,Y2R,SUMR,YCG,SUMG
IF(1.EC.NFC)GO TO 69
SUMF=SUMF*SUMF
SUMR=SUMR*SUMR
SUMG=SUMG*SUMG
69 CONTINUE
WRITE(ILP,510)VS,SUMF,SUMR,SUMG
510 FORMAT(10X,10X,'THE TCTAL NON-DIV. ACC. AT ',F7.2,'MPH IS ',F7.2,'
1 FOR THE FRONT',5X,F7.2,' FOR THE REAR',5X,F7.2,' FOR THE CENTER')
IF(1POP.FC.0)GC TO 985

```

```

      WRITE(LP,112)
989  CONTINUE
      DO 75 K=1,NPC
      IF(1POP.EQ.0)GC TO 988
      WRITE(LP,110)NR(K),AF(K),BF(K),AR(K),BR(K)
988  CONTINUE
75  CONTINUE
C-----RECONSTRUCT VEHICLE ACCELERATIONS
      IF(1POP.EQ.C) GC TO 71
      WRITE(LP,202)
202  FORMAT(1H0,15X,'1/TP',11X,'AY2F',11X,'AY2R'//)
      DATA NPTS/50/
      NPTSP=NPTS+1
      DP=1./FLOAT(NPTS)
      P=0.0
      DO 71 KK=1,NPTSP
      AY2F=0.0
      AY2R=0.0
      DO 70 I=1,MFOUR
      AAG=NR(I)/NM(I)*P*P12
      AY2F=AY2F
      1+ (BF(I)*TSC+BR(I)*TCC-AF(I)*TSS-AR(I)*TCS)*SIN(AAG)
      2+COS(AAG)*(BF(I)*TSS+BR(I)*TCS+AF(I)*TSC+AR(I)*TCC)
      AY2R=AY2R
      1+ (BR(I)*TSC+BF(I)*TCC-AR(I)*TSS-AF(I)*TCS)*SIN(AAG)
      2+COS(AAG)*(BR(I)*TSS+BF(I)*TCS+AR(I)*TSC+AF(I)*TCC)
70  CONTINUE
      WRITE(LP,203) P,AY2F,AY2R
203  FORMAT(1H ,10X,F10.2,2F15.5)
      P=P+DP
71  CONTINUE
C-----END RECONSTRUCTION OF VEHICLE ACCELERATIONS
777 CONTINUE
505  FORMAT(1H ,10X,'NUMBER OF SPANS=',12/1H ,10X,'SPAN LENGTH (FT)
+ ',F7.2/1H ,10X,'VEHICLE SPEED (MPH)=' ,F7.2/1H ,10X,'PAC LENGTH
+ RATIO=' ,F5.2/1H ,10X,'VEHICLE LENGTH (FT) =' ,F7.2/1H ,10X,'BEAM D

```

```

507      +AMPING=' ,F7.2)
      FORMAT(1H ,10X,'CROSSING FREQUENCY RATIO=' ,F5.2/
+1H ,10X,'VEHICLE NATURAL FREQUENCY (HZ)=' ,F5.2/
+1H ,10X,'ATTACHMENT LENGTH RATIO=' ,F5.2/)
111      FORMAT(1H0,7X,'W/FV',7X,'Y2F ',7X,'SUMF',7X,'Y2R ',7X,'SUPR',7X,'Y
      12CG',7X,'SUMC'/)
112      FORMAT(1H0,9X,'W',8X,' AF ',7X,' RF ',7X,' AR ',7X,' BR '/)
110      FORMAT(1H ,F11.2,6F11.4)
      RETURN
      END
C
C
      SUBROUTINE PHASE(R,A1,P)
C
C      PHASE IS CALLED BY FCOEF IN THE SEQUENCE DETERMINING RMS ACCELE-
C      TIONS FROM THE FOURIER COEFFICIENTS AND THE VEHICLE TRANSFER FUNC-
C      TIONS
C
      IF(R.LT.0.) GO TO 101
      P=ATAN(A1/R)
      GO TO 100
101      IF(A1.LT.0.) GO TO 102
      P=ATAN(A1/R)+3.14159
      GO TO 100
102      P=ATAN(A1/R)-3.14159
100      CONTINUE
      RETURN
      END
C
C
      SUBROUTINE SINT(X,H,NPT,S)
C
C
C      SINT IS CALLED BY FCOEF AND IS A SIMPSON'S RULE NUMERICAL INTEGRA-
C      TION ROUTINE USED TO DETERMINE THE FOURIER COEFFICIENTS FROM THE
C      DEFLECTION PROFILES BENEATH THE FRONT AND REAR SUSPENSIONS

```

C

```
DIMENSION X(2),S(2)
S(1)=0.
S(2)=(5.*X(1)+8.*X(2))*H/12.+S(1)
H3=H/3.
DO 1 I=3,NPTT
1 S(1)=S(I-2)+H3*(X(I-2)+4.*X(I-1)+X(I))
RETURN
END
```


APPENDIX E
GUIDEWAY DESIGN PROGRAM BASED UPON A CONSTANT
FORCE VEHICLE MODEL

This computer program provides design data for the two suspension constant force vehicle models traversing the multispan configuration described in Chapter 4. The program computes span deflections and moments and vehicle accelerations and suspension deflections using the constant force vehicle model described in Chapter 4. The program permits beams having either of two predetermined camber shapes, an arbitrary camber shape, or uncambered beams. The program has two options. Option #1 implements directly the design procedure summarized in Table 4.2. In this option the minimum span stiffness and related cross-section properties required to provide a vehicle-guideway system that meets passenger comfort specifications at the vehicle operating speed are determined. As inputs to the program the vehicle characteristics, the span length and the number of spans in the semi-continuous guideway beam must be specified. As presently constituted the program is based upon the twin I beam prestressed concrete span configuration illustrated in Figures 4.2 and 4.3 and the passenger comfort specifications illustrated in Figure 4.1. Other span configurations and materials and alternate passenger comfort specifications may be implemented through modification of the program.

In option #2 span deflections and moments and vehicle accelerations and suspension deflections are determined for a completely specified vehicle-guideway system over a specified range of operating speeds. This option is convenient for determining

the system performance characteristics over the complete operating speed range after a preliminary design has been completed.

In addition to the two modes of operation, two levels of printing detail may be specified in the program. By specifying the detailed output mode the constant force simulation sequence for one beam will be printed in detail for a complete vehicle passage. Included in the constant force simulation printing are the normalized midspan deflections and moments, the modal coefficients, and the front and rear suspension displacements. At each time step during the passage the normalized front and rear suspension displacements are printed. From these phased displacements the suspension deflection Fourier coefficients are calculated and printed. The non-dimensional front and rear rms accelerations for each non-dimensional excitation frequency, the total non-dimensional acceleration for the speed being simulated and the maximum ratio of non-dimensional acceleration to the limiting specified acceleration are printed. If program option #1 is in effect, the guideway design parameters are also printed. Finally the maximum midspan deflections and moments at each crossing velocity are printed.

If detailed information is unnecessary, a limited printing mode may be specified in which only the accelerations and design data are printed for option #1, and only the total rms accelerations and the acceleration ratio for each speed are printed for option #2.

The input parameters required in the program and the specification of program options and printing modes are described below.

INPUTS

CARD #1	FORMAT 5F 10.0
FV(cps) (f_v)	vehicle suspension natural frequency
SL (ft) (l_s)	span length
WTOT (lbs) ($(m_v + m_u)g$)	total vehicle weight
VMAX (mph) (v_{max})	for option #1, the design velocity of interest for option #2, the maximum vehicle velocity of interest
VMIN (mph) (v_{min})	for option #1, blank for option #2, the minimum vehicle velocity of interest
VDEL (mph) (Δv)	for option #1, blank for option #2, the velocity increment (the program will simulate $v = v_{min} + (n-1) \Delta$ up to v_{max} $n = 1, 2, 3$)
<u>CARD #2</u>	FORMAT 4F 10.0
AI ($\bar{I}_v$)	vehicle moment of inertia ratio
AM (M_u)	ratio of total vehicle unsprung mass to vehicle sprung mass
AK (k)	vehicle stiffness ratio
$z(\xi_v)$	vehicle suspension damping ratio

CARD #3**FORMAT 5I10****M**
(subscript m)

specified number of modes used in simulation

NS
(k)

specified number of spans per beam in simulation

NPTS

specified number of time steps per span in simulation

NFC

number of Fourier coefficients to be determined in addition to the zeroth coefficients

IPOP

0 or blank to suppress bulk of printout any other integer for detailed printout

CARDS #4--#7**FORMAT 20A4**

These are header cards used to read in headings that are printed on each page of the output during the constant force simulation. Cards #4 and #5 represent one line of printed output. Cards #6 and #7 another line. If no headings are desired blank cards must be included. Typical headings used correspond to the columns of time, midspan deflections, midspan moments, and front and rear deflections. If limited printing option is in effect, cards #4--#7 must be included but may be blank

CARD #8**FORMAT 8F10.0****AL(I)**
($\lambda_1 I$)

The eigenvalues, one for each mode ($I = 1, N$)
(See Table A.1) If more than eight eigenvalues must be read, additional cards follow in sequence.

CARD #9**FORMAT 3F10.0, A1, F10.0****ZB**
(ξ_m)

beam damping ratio

PLR
(L_p)

pad length ratio

$VL(L_s)$ attachment length ratio
 CAMCD camber code--'A' for camber shape equal to $\bar{A}_c |\sin \pi X|$,
 'C' for camber shape equal to $\bar{A}_c (1 - \cos 2\pi X)/2$
 'G' for general camber shape equal to $\bar{A}_c (Y_c(X))$ where
 values of $Y(X)$ are read from succeeding cards, or
 blank for no camber.
 AMCAM camber amplitude
 $(\bar{A}_c)$

CARDS #9 a,b,c FORMAT 8F10.0

Optional cards used only when CAMCD above is 'G'.
 Values of $Y_c(X)$ for each time step from 1 to
 NS*NPTS are read. The program is set up for 60
 points.

3) If card #11 has values as specified for card #9,
 a card specifying VCMAX should follow (any number
 of pairs of cards #9 and #10 may follow after
 which the run may terminate or continue with new
 parameters read according to the other two options
 for Card #11.

OUTPUT

OPTION #1

ITEM #1 "Option 1 in effect"
 ITEM #2 the vehicle natural frequency, span length, design
 velocity, (echo check of card #1)
 ITEM #3 the constant force simulation sequence
 ITEM #4 suspension displacements front and rear
 ITEM #4a camber code and camber amplitude (camber option only)
 ITEM #4b suspension displacements including camber and camber
 displacements (camber option only)
 ITEM #5 the Fourier coefficients printed
 ITEM #6 complete list of simulation parameters

ITEM #7	non-dimensional front and rear RMS accelerations for each non-dimensional frequency
ITEM #8	total front and rear RMS accelerations for the vehicle design and velocity
ITEM #9	Fourier coefficients for each dimensional frequency
ITEM #10	maximum ratio of non-dimensional acceleration to the acceleration specification limit
ITEM #11	the guideway design sequence
ITEM #12	maximum non-dimensional midspan deflection and midspan moment for the design crossing velocity
OPTION #2	
ITEM #1	"Option #2 in effect"
ITEM #2	vehicle natural frequency, span length, maximum, minimum, increment velocities (echo check of card #1)
ITEM #3	constant force simulation sequence
ITEM #4	suspension displacements front and rear
ITEM #4a	camber code and camber amplitude (camber option only)
ITEM #4b	suspension displacements including camber and camber displacements (camber option only)
ITEM #5	the Fourier coefficients printed
ITEM #6	complete list of simulation parameters
ITEM #7	non-dimensional front and rear RMS accelerations for each non-dimensional frequency
ITEM #8	total front and rear RMS accelerations for the vehicle speed being simulated
ITEM #9	Fourier coefficients for each dimensional frequency

ITEM #10 maximum ratio of non-dimensional acceleration
 to the acceleration specification limit

ITEMS #3--#10 repeated for each velocity up
to v_{max}

ITEM #11 maximum non-dimensional mid-span deflections
 and mid-span moments for each crossing velocity

 If IPOF is 0 or blank only items #1, #2, #6, #8, #10, and
#11 are printed for option #2, only items #1, #2, #6, #10, #11 and
#12 are printed for option #1.

 The program listing is included on the following pages.

```

C C PARTIALLY COUPLED, CONSTANT FORCE MODEL FOR MULTISPAN GUIDEWAYS
C C WITH TWO-DIMENSIONAL VEHICLES. OPTION #1 DESIGNS GUIDEWAY.
C C OPTION #2 DETERMINES ACCELERATIONS AT VARIOUS OPERATING SPEEDS.
C C
C C
C DIMENSION X(30),HEAD1(30),HEAD2(30),YMXS(5),
C 1YMXA(5),8MXS(5),8MXA(5),VX(32),YMTV(2,32),BMTV(2,32),YMTA(2,32),
C 2 BMTA(2,32)
C DIMENSION YVF(201),YYR(201)
C COMMON AL(15),A(15,5),B(15,5),C(15,5),CHIM(15,5),LR,L,P
C COMMON E1(15),E2(15),E3(15),E4(15)
C COMMON C1(15,11),C2(15,11),C3(15,11),C4(15,11),C5(15,11),WF(15),IR
C 1ES(15)
C COMMON H1(15,11),H2(15,11),H3(15,11),H4(15,11),H5(15,11)
C DATA TEND /'PINN'/
C LR=5
C LP=6
C 99 CONTINUE
C *****
C READ DATA CARDS #1 THROUGH #8
C INITIALIZE PROGRAM VARIABLES
C DETERMINE OPERATING OPTION #1 OR #2
C READ(LP,98)FV,SL,WTOT,VMAX,VMIN,VDEL
C READ(LR,98)AI,AM,AK,Z
C FORMAT(8F10.0)
C IF(VMIN.LT.0.01)GOTO 94
C IOPT=2
C WRITE(LP,96)
C 96 FORMAT(1H1,'OPTION 2 IN EFFECT',/)
C WRITE(LP,97)FV,SL,WTOT,VMAX,VMIN,VDEL
C 97 FORMAT(1H0,T15,'VEH. FREQ. = ',F6.3,'CPS',T50,'SPAN LENGTH =
C 1 ',F6.2,'FT ',T80,'TOTAL VEH. WT. = ',F10.2,'LBS',/,T15,'MAX. VEL
C 2OCITY = ',F6.2,'MPH',T50,'MIN. VELOCITY = ',F6.2,'MPH',T80,'DEL. V
C 3VELOCITY = ',F6.2,'MPH',/)
C GO TO 85

```



```

94      IOPT=1
      WRITE(LP,93)
93      FORMAT(1H1,'OPTION 1 IN EFFECT',/)
      WRITE(LP,92)FV,SL,WTOT,VMAX
92      FORMAT(1H0,T15,'VEH. NATL. FREQ. = ',F6.3,'CPS',T50,'SPAN LENGTH =
1      ',F6.2,'FT',T80,'TOTAL VEH. WT. = ',F10.2,'LBS',/,T15,'DESIGN VEL
20CITY = ',F6.2,'MPH',/)
85      READ(LR,100)M,NS,NPTS,NFC,IPOP
100      FORMAT(8I10)
      READ(LR,104) HEAD1
104      FORMAT(20A4)
      READ(LR,104) HEAD2
      M2=2*M
      OTIME=1./FLOAT(NPTS)
      READ(LR,102)(AL(1),I=1,M)
102      FORMAT(MF10.0)
C*****
      CALL EGN(M,NS,TEND)
3      READ (LR,1025) ZN, PLR, VL, COCAM, AMCAM
1025      FORMAT (3F10.0, A1, 9X, F10.0)
      WRITE(LP,1013)
1013      FORMAT(1H1)
C
C
C      TERMINATE; CONTINUE WITH ZN,PLR,VL; READ NEW PARAMETERS?

      IF (ZN.GE.10.)CALL EXIT
      IF (ZN.GF.1.0)GO TO 99
      VMTA(1,1)=0.
      VMTV(1,1)=0.
      BMTA(1,1)=0.
      BMTV(1,1)=0.
      ISA=PLR*FLCAT(NPTS)+0.5
      CALL HCAL(M,IS,PLR)
      NSEG=2*NS+1
      READ(LR,98)VCMAX
      F1=1.467*VMAX/(SL*VCMAX)

```

CAMBER
CAMBER

```

IF(IOPT.EQ.1)GO TO 87
IT=0
91 IT=IT+1
VM=VMIN+FLOAT(IT-1)*VDEL
VMTST=1.001*VMAX
IF(VM.GT.VMTST)GO TO 88
VX(IT)=1.467*VM/(SL*F1)
GO TO 91
88 VX(IT)=0.
GO TO 86
87 VX(1)=VCMAX
VX(2)=0.
86 IT=1
12 VC=VX(IT)
T=1./VC
TAUD=6.2831854*PI*VL
TAUS=6.2831854*PI
TAUP=6.2831854*PI*PLR
ITMF=1
ITMR=VL*FLOAT(NPTS)+1.5
NPTT=NPTS*NS+1
150 IF(ITMR.LT.NPTT)GO TO 151
ITMR=ITMR-(NPTT-1)
GO TO 150
151 CONTINUE
ICAML = ITMR - ITMF
VVF(ITMF) = 0.
VVR(ITMR)=0.
NC=1
DTAU=2.*3.1415927*PI/FLOAT(NPTS)
CALL EXPRS(T,ZN,M,NS,DTAU)
NSEG=2*NS+1
NJ=1
NCRS=FLOAT(NS)+VL+PLR+0.99
DO 5 I=1,N2
5 X(I)=0.0

```

CAMBER
CAMBER
CAMBER

```

      DO 6 J=1,NS
      BMXA(J)=0.0
      YMXA(J)=0.0
      TIME=0.0
      14 IF(IPOP.EQ.0)GO TO 998
      C
      C      WRITE PAGE HEADING
      C
      WRITE(LP,1007) PLR
      1007 FORMAT(1H1,T16,'SEMICONTINUOUS GUIDEWAY',T47,'PRESSURE PAD LENGTH
      RATIO=',F4.2)
      WRITE(LP,1012) VC,NS,M,NJ,NPTS,Z,V,VL
      1012 FORMAT(1H ,T16,'CROSS. FREQ. RATIO=',F6.3,T47,'NUMBLR OF SPANS=',
      1 12,T72,'NUMBER OF MODES=',I2,/,T16,'VEHICLE IS ON SPAN ',I2,
      2 T47,'POINTS PER SPAN=',I2,T72,'DAMPING RATIO=',F4.2,/,T16,
      3'VEHICLE LENGTH RATIO=',F4.2,/)
      998 CONTINUE
      DO 19 J=1,NS
      BMXS(J)=0.0
      19  YMXS(J)=0.0
      TAU=0.0
      IF(IPCP.EQ.0)GO TO 997
      WRITE(LP,1008) HEAD1
      1008 FORMAT(1H0,30A4)
      WRITE(LP,1009) HEAD2
      1009 FORMAT(1H ,30A4,/)
      997 CONTINUE
      CALL CPUIF(X,NPTS,NJ,M,NS,TAU,DTAU,TIME,DTIME,YMXS,BMXS,ISA,
      1 TAU,TAUS,TAUP,PLR,VL,YVF,YVR,ITMF,ITMR,NPTT,IPOP)
      C
      C      NEW MIDSPAN DEFLECTIONS AND/OR MOMENTS?
      C
      DO 7 J=1,NS
      IF(YMXS(J)-YMXA(J))8,8,9
      YMXA(J)=YMXS(J)
      9  IF(BMXS(J)-BMXA(J))7,7,10
      10  BMXA(J)=BMXS(J)

```

```

7      CONTINUE
      IF(NJ-NS) 31,31,22
31     IF(YMXS(NJ)-YMTV(1,IT))24,24,4
4      YMTV(1,IT)=YMXS(NJ)
24     IF(BMXS(NJ)-BMTV(1,IT)) 22,22,23
23     BMTV(1,IT)=BMXS(NJ)
22     CCNTINUE
      IF(IPOP.EQ.C)GO TO 995
      WRITE(LP,1010)(YMXS(J),J=1,NS),(BMXS(J),J=1,NS)
1010   FORMAT(7X,'SMAX=',14F7.3)
      WRITE(LP,1011)(YMXA(J),J=1,NS),(BMXA(J),J=1,NS)
1011   FORMAT(7X,'CMAX=',14F7.3)
995    CONTINUE
      NJ=NJ+1
C      DESIRED NUMBER OF SPAN CROSSINGS COMPLETE?
      IF(NJ-NCRS)14,14,13
13     DO 21 I=1,NS
25     IF(YMXA(I)-YMTA(1,IT)) 26,26,25
26     YMTA(1,IT)=YMXA(I)
27     IF(BMXA(I)-BMTA(1,IT)) 21,21,27
21     BMTA(1,IT)=BMXA(I)
      CONTINUE
      YVR(1)=YVR(NPTT)
      CALL FCOEF(VYF,YVR,NS,NPTT,NFC,LP,LR,T,VL,PLR,ZN,AL,AM,AK,L,I,DPT,F
28     IV,F1,SL,IT,IPOP,VMIN,VMAX,WTOT)
      IT=IT+1
      IF(IT.GT.32) GO TO 29
      IF(VX(IT).EQ.0.0) GO TO 29
      YMTA(1,IT)=0.0
      YMTV(1,IT)=0.0
      BMTA(1,IT)=0.0
      BMTV(1,IT)=0.0
      GO TO 12
35     YMTA(1,IT)=0.
      BMTA(1,IT)=0.
      YMTV(1,IT)=0.

```



```

CH=0.5*(DEXP(DALM)+DEXP(-DALM))
C-----COMPUTE GEOMETRIC PROPERTIES
G1=SN/(SH-SN)
G2=SN*SN
G3=(CH-CS)/(SH-SN)
G4=SN/(SH-SN)
C-----PREPARE TO NORMALIZE MODE BY COMPUTING THE MODE INTEGRAL FUP A(I,1)=1.0
C-----LS (SPAN SEGMENT LENGTH) = 1.0 (DEFINITION)
C-----INTEGRAL(PHIN**2)=NS
C-----QUADRATIC INTEGRALS FOR ANY SPAN
SNSN=0.5*(ALM-SN*CS)
CSCS=0.5*(ALM*SN*CS)
SHSH=0.5*(SH*CH-ALM)
CHCH=0.5*(SH*CH+ALM)
C-----0.5 FOR MIXED TERMS CANCELS 2.0 FOR BINOMIAL EXPANSION
SNCS=SN*SN
SNCH=CH*SN-SH*CS
SNCH=SN*SN-CH*CS+1.0
CSSH=SH*SN+CH*CS-1.0
CSCM=SH*CS+CH*SN
SMCH=SH*SH
Q=0.0
DO 40 J=1,NS
IF(J-1)10,10,20
C-----COMPUTE A,H,C FOR FIRST SPAN
10 SA=1.0
IF(TEND.EQ.PIN) SR=0.
IF(TEND.EQ.FIX) SB=-1./G3
SC=-SN/SH+(CH-CS)*SR/SH
GO TO 30
C-----USE RECURSION FOR REMAINING TERMS, WORKS EVEN IF SIN(ALM)=J.0
20 G5=SA*CS-G2*SB+SC=CH
G6=SA*SN+SB*CS
C-----NOW UPDATE A,B, AND C
SA=G1+G5-G3*G6
SB=G6

```

```

30      SC=G3*G6-G4*G5
      SD=-SB
      A(I,J)=SA
      B(I,J)=SB
      C(I,J)=SC
C-----INTEGRATE ALONG TOTAL SPAN LENGTH BY ACCUMULATING INDIVIDUAL
C-----SPAN SEGMENT INTEGRALS
      Q=Q+SA*(SA*SN+SB*SNCS+SC*NSH+SD*SNCH)
      1  +SB*(S+CSCS+SC*SSH+SD*CSCH)
      2  +SC*(SC*SSH+SD*SHCH)
      1  +SD*(SD*CHCH)
40      CONTINUE
C-----NORMALIZE MODE
      Q=SQRT(FLOAT(NS)*ALM/Q)
      DO 50 J=1,NS
      A(I,J)=A(I,J)*Q
      B(I,J)=B(I,J)*Q
      C(I,J)=C(I,J)*Q
50      CONTINUE
      DO 5 I=1,M
      ALM=AL(I)*0.5
      DALM=ALM
      ALS=(AL(I)/3.1415927)**2
      SN=SIN(ALM)
      CN=COS(ALM)
      SH=0.5*(DEXP(DALM)-DEXP(-DALM))
      CH=0.5*(DEXP(DALM)+DEXP(-DALM))
      DO 5 JK=1,NS
      PHIM(I,JK)=A(I,JK)*SN+B(I,JK)*(CN-CH)+C(I,JK)*SH
      CHIM(I,JK)=ALS*(-A(I,JK)*SN-B(I,JK)* (CN+CH)+C(I,JK)*SH)
5      CONTINUE
      RETURN
      END
C
C      SUBROUTINE HCAL(M,NS,PLR)

```

```

C*****
C      CALCULATE COEFFICIENTS OF PRESSURE PAD FORCING FUNCTIONS
C*****
      REAL*8  SINH,COSH,SML,CML1,SMM,CHM,SHM,CHR,DBPX,DALF,DAL1,DR,DEXP
      COMMON  AL(15),A(15,5),B(15,5),C(15,5),PHIM(15,5),LK,LP
      COMMON  E1(15),E2(15),E3(15),E4(15)
      COMMON  C1(15,11),C2(15,11),C3(15,11),C4(15,11),C5(15,11),WF(15),IR
      IF5(15)
      COMMON  H1(15,11),H2(15,11),H3(15,11),H4(15,11),H5(15,11)
      SINH(DBPX)=0.5*(DEXP(DBPX)-DEXP(-DBPX))
      COSH(DBPX)=0.5*(DEXP(DBPX)+DEXP(-DBPX))
      IF(PLR-3.0001) 41,41,40
      DO 44 I=1,M
      DO 42 J=1,NS
      K=2*J-1
      DO 42 KS=1,2
      H1(I,K)=A(I,J)
      H2(I,K)=B(I,J)
      H3(I,K)=C(I,J)
      H4(I,K)=-B(I,J)
      H5(I,K)=0.0
      K=K+1
      H1(I,K)=0.0
      H2(I,K)=0.0
      H3(I,K)=0.0
      H4(I,K)=0.0
      H5(I,K)=3.0
      RETURN
      CONTINUE
      DO 2 I=1,M
      DALI=AL(I)
      ALI=1./((PLR*AL(I))
      ALF=AL(I)*PLR
      DALF=DALF
      SL=SIN(ALF)
      CL1=COS(ALF)-1.

```

41

42

44

40


```

SML=SINH(DALF)
CHL1=COSH(DALF)-1.
SM=SIN(AL(I))
CM=COS(AL(I))
SMH=SINH(DALI)
CHH=COSH(DALI)
R=AL(I)*(1.-PLR)
DR=R
SR=SIN(R)
CR=COS(R)
SHR=SINH(DR)
CHR=COSH(DR)
H1(I,1)=AL*B(I,1)
H2(I,1)=-AL*A(I,1)
H3(I,1)=H1(I,1)
H4(I,1)=AL*C(I,1)
H5(I,1)=AL*(A(I,1)-C(I,1))
IF(NS-1) 3,3,4
DO 1 J=2,NS
  IS=2*J-1
  H1(I,IS)=AL*(-A(I,J-1)*SR-B(I,J-1)*CR+B(I,J))
  H2(I,IS)=AL*( A(I,J-1)*CR-B(I,J-1)*SR-A(I,J))
  H3(I,IS)=AL*( B(I,J-1)*CHR-C(I,J-1)*SHR-B(I,J))
  H4(I,IS)=AL*( B(I,J-1)*SHR-C(I,J-1)*CHR+C(I,J))
  H5(I,IS)=AL*(-A(I,J-1)*CM+B(I,J-1)*(SM-SHM)+C(I,J-1)*CHM+
    1 A(I,J)-C(I,J))
  NMS=2*NS+1
  H1(I,NMS)=AL*(-A(I,NS)*SR-B(I,NS)*CR)
  H2(I,NMS)=AL*( A(I,NS)*CR-B(I,NS)*SR)
  H3(I,NMS)=AL*(B(I,NS)*CHR-C(I,NS)*SHR)
  H4(I,NMS)=AL*(B(I,NS)*SHR-C(I,NS)*CHR)
  H5(I,NMS)=AL*(-A(I,NS)*CM+B(I,NS)*(SM-SHM)+C(I,NS)*CHM+
    DO 2 J=1,NS
      IS=2*J
      H1(I,IS)=AL*(A(I,J)*SL-B(I,J)*CL1)
      H2(I,IS)=AL*(A(I,J)*CL1+B(I,J)*SL)

```

```

      M3(1,15)=AL1*(B(I,J)*CHL1+C(I,J)*SHL)
      M4(1,15)=AL1*(-B(I,J)*SHL-C(I,J)*CHL1)
      M5(1,15)=0.
      RETURN
      END

      C
      C
      C SUBROUTINE EXPRES(T,ZN,M,MS,DTAU)
      C *****
      C DETERMINE ELEMENTS IN EXPONENTIAL MATRIX AND COEFFICIENTS OF PAR-
      C TICULAR SOLUTIONS FOR EACH MODE
      C *****
      REAL*8 AC3,AC4
      COMMON AL(15),A(15,5),B(15,5),C(15,5),PHI4(15,5),CHIM(15,5),LR,LP
      COMMON E1(15),E2(15),E3(15),E4(15)
      COMMON C1(15,11),C2(15,11),C3(15,11),C4(15,11),C5(15,11),WFF(15),I
      RES(15)
      COMMON H1(15,11),H2(15,11),H3(15,11),H4(15,11),H5(15,11)
      NSEG=2*MS+1
      DO 1 I=1,M
      WN=(AL(I)/3.1415927)**2
      WF=AL(I)/(2.*3.1415927*T)
      WFF(I)=WF
      P1=WN*WN-WF*WF
      P4=WN*WN+WF*WF
      P5=P1*P1
      P6=P4*P4
      API = ABS(P1)
      IF (I*ZN.LT.0.00001).AND.(API.LT.0.00001)GO TO 3
      IRES(I)=0
      P2=ZN*WF*WN
      P3=4.*9*(ZN*WN*WF)**2
      RAO=SQRT(1.-ZN**2)
      WR=WN*RAO
      SN=SIN(WR*DTAU)
      CN=COS(WR*DTAU)

```

```

ZR=ZN/RAD
ER=EXP(-Z*WN*DTAU)
E1(I)=ER*(CN+ZR*SN)
E2(I)=ER*SN/WR
E3(I)=-ER*SN*WN/RAD
E4(I)=ER*(CN-ZR*SN)
DO 2 NJ=1,NSEG
  C1(I,NJ)=(-P1*H1(I,NJ)/2.-P2*H2(I,NJ))/((P5+P3)*FLOAT(NS))
  C2(I,NJ)=(P2*H1(I,NJ)-P1*H2(I,NJ)/2.)/((P5+P3)*FLOAT(NS))
  AC3=(-P4*H3(I,NJ)/2.+P2*H4(I,NJ))/((P6-P3)*FLOAT(NS))
  AC4=(P2*H3(I,NJ)-P4*H4(I,NJ)/2.)/((P6-P3)*FLOAT(NS))
  C3(I,NJ)=0.5*(AC3+AC4)
  C4(I,NJ)=0.5*(AC4-AC3)
  C5(I,NJ)=-H5(I,NJ)/(2.*WN*WN*FLOAT(NS))
GO TO 1
IRES(I)=1
CN=COS(WN*DTAU)
SN=SIN(WN*DTAU)
E1(I)=CN
E2(I)=SN/WR
E3(I)=-WN*SN
E4(I)=CN
DO 4 NJ=1,NSEG
  C1(I,NJ)=-0.5*3.1415927*T*H2(I,NJ)/FLOAT(NS)*AL(I)
  C2(I,NJ)=+0.5*3.1415927*T*H1(I,NJ)/FLOAT(NS)*AL(I)
  AC3=-0.5*P4*H3(I,NJ)/P6/FLOAT(NS)
  AC4=-0.5*P4*H4(I,NJ)/P6/FLOAT(NS)
  C3(I,NJ)=0.5*(AC3+AC4)
  C4(I,NJ)=0.5*(AC4-AC3)
  C5(I,NJ)=-H5(I,NJ)/(2.*WN*WN*FLOAT(NS))
CONTINUE
RETURN
END

```

SUBROUTINE CPDIF(X,NPTS,ISPAN,M,NS,TAU,DTAU,TIME,DTIME,YMXS,RMXS,

2

3

T19

4

1

C

C

```

1  ISA,TAUD,TAUS,TAUP,PLR,VL,VVF,VYR,ITMF,ITMR,NPTT,IPOP)
*****
C  CALCULATE NORMALIZED MIDSPAN DEFLECTIONS AND MOMENTS AND SUSPEN-
C  SION DEFLECTIONS AS A FUNCTION OF TIME IN CROSSINGS FROM CON-
C  STANT FORCE SIMULATION; ONE SPAN CROSSING EACH CALL
C  WRITE RESULTS
*****
      REAL*8 EPX,EXR,EXRP,EXFP,DPXS,DPXLR,DPXR,DEXP,DPXLF
      DIMENSION X(1),YMXS(1),RMXS(1),YM(5),RM(5),XP(30),XPR(30)
      DIMENSION YVF(1),YYR(1)
      COMMON AL(15),A(15,5),B(15,5),C(15,5),PHIM(15,5),CHIM(15,5),LR,LP
      COMMON EL(15),E2(15),E3(15),E4(15)
      COMMON C1(15,11),C2(15,11),C3(15,11),C4(15,11),C5(15,11),WF(15),IR
      IES(15)
      NS2=2*NS+1
      NJ=2*ISPAN-1
      KPTS=ISA
      KS=2
      IF(ISA)17,17,18
17  NJ=2*ISPAN
      KPTS=NPTS
      GO TO 15
18  IF((ISA-NPTS)14,15,16
15  KS=1
      GO TO 14
16  WRITE(LP,200)
200  FORMAT('1
      RETURN
14  DO 13 KP=1,KS
      MRJ=-20
      DO 7 I=1,M
      IDOT=M+I
      XPR(1)=0.
      XPR(IDOT)=0.
      IF(NJ-NS2)8,8,40
40  XP(1)=0.

```

```

1001 100000000.
1002 00 TO 7
1003 10=WP(I)*TAU
1004 SN=SIGN(S)
1005 CN=CON(NS)
1006 NP=NS
1007 EPX=ENP(DPLS)
1008 IF (INES(I),NE,1) 60 TO 10
1009 NP(I)=(TAU*(C1(I,NJ)*SN+C2(I,NJ)*CN)+C3(I,NJ)*EPX+C4(I,NJ)/EPX+
1010 1 C5(I,NJ))*S
1011 XP(IDOT)=(ISM*WF(I)*TAU*CN)=C1(I,NJ)+(CN*WF(I)*TAU*SN)*C2(I,NJ)
1012 1 *WF(I)*(C3(I,NJ)*EPX-C4(I,NJ)/EPX))*S
1013 60 TO 7
1014 XP(I)=(C1(I,NJ)*S+C2(I,NJ)*CN+C3(I,NJ)*EPX+C4(I,NJ)/EPX+
1015 1 C5(I,NJ))*S
1016 XP(IDOT)=(WF(I)*(C1(I,NJ)*CN-C2(I,NJ)*SN+C3(I,NJ)*EPX-C4(I,NJ)/
1017 1 EPX))*S
1018 7 CONTINUE
1019 00 1 K=1,KPTS
1020 NR00=NR0
1021 TAU=TAU*DTAU
1022 XFP=(TAU/TAUS)--.5*PLR
1023 IF (XFP)46,46,45
1024 45 ISF=ISPM
1025 60 TO 52
1026 ISF=ISPM-1
1027 XFP=1.+XFP
1028 52 IF (ISF)57,57,58
1029 IF (ISF-NS) 59,59,57
1030 ITWP=ITWP+1
1031 57 TAUR=TAU-TAUD
1032 ISPMR=ISPM
1033 IF (TAUR) 20,20,49
1034 TAUR=TAUS+TAUR
1035 ISPMR=ISPMR-1
1036 IF (TAUR) 20,20,49

```

```

49   XRP=(TAUR/TAUS)-G.5*PLR
    IF (XRP)48,48,47
48   XRP=XRP+1.0
    ISR=ISPNR-1
    GO TO 60
47   ISR=ISPNR
    IF (ISR) 61,61,62
60   IF (ISR-NS) 63,63,61
62   ITMR=ITMR+1
63   IF (ITMR.GT.NPTI) ITMR=2
    IF (TAUR-TAUP) 27,28,28
61   27 NR=2*ISPNR-1
    GO TO 34
28   NR=2*ISPNR
34   YF=0.
    YR=0.
    IF (NR-NS2)3,3,4
    4 DO 5 I=1,M
      XI=X(I)
      IDOT=M+1
      X(I)=E1(I)=XI+E2(I)*X(IDOT)
      X(IDOT)=E3(I)*XI+E4(I)*X(IDOT)
      GO TO 6
3 DO 400 I=1,M
  IDOT=M+1
  IF (NR-NRQ) 37,38,37
37  XR=WF(I)*(TAUR-DTAU)
  XP(I)=XP(I)-XPR(I)*.5
  XP(IDOT)=XP(IDOT)-XPR(IDOT)*.5
  GO TO 35
38  XR=WF(I)*TAUR
35  IF (ISPNR-0)23,23,24
23  XPR(I)=0.
  XPR(IDOT)=0.
  GO TO 39
24  SNR=SIN(XR)

```

```

CNR=COS(XR)
DPXA=XR
EXR=DEXP(DPXR)
IF (IRES(I)-1)30,29,30
29 XPR(I)=TAUR*(C1(I,NR)*SNR+C2(I,NR)*CNR)+C3(I,NR)*EXR+C4(I,NR)/EXR+
1 C5(I,NR)
XPR(IDOT)=(SNR+WF(I)*TAUR*CNR)=C1(I,NR)+(CNR-WF(I)*TAUR*SNR)=C2(I,
1NR)+WF(I)*(C3(I,NR)*EXR-C4(I,NR)/EXR)
GO TO 39
30 XPR(I)=C1(I,NR)*SNR+C2(I,NR)*CNR+C3(I,NR)*EXR+C4(I,NR)/EXR+
1 C5(I,NR)
XPR(IDOT)=WF(I)*(C1(I,NR)*CNR-C2(I,NR)*SNR+C3(I,NR)*EXR-C4(I,NR)/
1 EXR)
39 IF (NR-NR0) 35,31,36
36 XP(I)=XP(I)+XPR(I)*.5
XP(IDOT)=XP(IDOT)+XPR(IDOT)*.5
NR0=NR
GO TO 38
31 XI=X(I)-XP(I)
XD=X(IDOT)-XP(IDOT)
IF (NJ-NS2)42,42,43
43 XP(I)=0.
XP(IDOT)=0.
GO TO 12
42 XS=WF(I)*TAU
SN=SIN(XS)
CN=COS(XS)
DPXS=XS
EPX=DEXP(OPXS)
IF (IRES(I).NE.1) GO TO 11
XP(I)=TAU*(C1(I,NJ)*SN+C2(I,NJ)*CN)+C3(I,NJ)*EPX+C4(I,NJ)/EPX+
1 C5(I,NJ)
XP(IDOT)=(SN+WF(I)*TAU*CN)*C1(I,NJ)+(CN-WF(I)*TAU*SN)*C2(I,NJ)
1 +WF(I)*(C3(I,NJ)*EPX-C4(I,NJ)/EPX)
GO TO 12
11 XP(I)=C1(I,NJ)*SN+C2(I,NJ)*CN+C3(I,NJ)*EPX+C4(I,NJ)/EPX+C5(I,NJ)

```

```

XP(IDOT)=WF(1)=(C1(I,NJ)*CN-C2(I,NJ)*SN+C3(I,NJ)*EPX-C4(I,NJ)/EPX)
12 XP(I)=(XP(I)+XPR(I))*5
XP(IDOT)=(XP(IDOT)+XPR(IDOT))*5
X(I)=E1(I)*X1+E2(I)*XD+XP(I)
X(IDOT)=E3(I)*X1+E4(I)*XD+XP(IDOT)
IF(ISR)50,50,51
51 IF(ISR-NS)53,53,2
53 XLR=AL(I)*XRP
SNRP=SIN(XLR)
CNRP=COS(XLR)
DPXLR=XLR
EXRP=DEXP(DPXLR)
PHIR=A(I,ISR)*SNRP+B(I,ISR)*(CNRP-.5*(EXRP+1./EXRP))+C(I,ISR)*.5*(
1EXRP-1./EXRP)
VR=VR+PHIR*X(I)
50 IF(ISF)2,2,55
55 IF(ISF-NC)56,56,2
56 XLF=AL(I)*XFP
SNFP=SIN(XLF)
CNFP=COS(XLF)
DPXLF=XLF
EXFP=DEXP(DPXLF)
PHIF=A(I,ISF)*SNFP+B(I,ISF)*(CNFP-.5*(EXFP+1./EXFP))+C(I,ISF)*.5*(
1EXFP-1./EXFP)
VF=VF+PHIF*X(I)
2 IF(I.LT.M)NRO=NRO+1
400 CONTINUE
6 DO 75 J=1,NS
BJ=0.0
YJ=0.0
DO 90 I=1,M
BJ=BJ+CHIM(I,J)*X(I)
90 YJ=YJ+PHIM(I,J)*X(I)
IF(ABS(YJ)-YMXS(J))95,95,92
92 YMXS(J)=ABS(YJ)
95 YN(J)=YJ

```



```

72 IF(ABS(BJ)-BMAX(J))75,75,72
75 BMAX(J)=ABS(BJ)
   BM(J)=BJ
   TIME=TIME+DTIME
   TIM=TIME+0.0009
   IF(IPOP.EQ.0)GO TO 996
   WRITE(LP,100) TIM, (YM(I),I=1,NS), (BM(I),I=1,NS), (X(I),I=1,M)
1, YF,YR
100 FORMAT(5X,16F7.3)
996 CONTINUE
   VYF(ITMF)=YF
   YR(ITR)=YR
   CONTINUE
   KPTS=NPTS-ISA
   NJ=2*ISPAN
   CONTINUE
13 RETURN
   END

C
C
C SUBROUTINE FCOEF(VYF,YR,NS,NPTT,NFC,LP,LR,T,VL,PLR,ZV,AI,AM,AK,Z,
C LIOPT,FV,FI,SL,IT,IPOP,VMIN,VMAX,WTOT)
C
C DETERMINE FOURIER COEFFICIENTS FOR SUSPENSION DEFLECTION PRO-
C FILES
C WRITE RESULTS
C DETERMINE FRONT AND REAR RMS ACCELERATIONS AT NORMALIZED FRE-
C QUENCIES FROM FOURIER COEFFICIENTS AND 8TH ORDER VEHICLE TRANS-
C FER FUNCTIONS
C WRITE RESULTS
C DETERMINE DESIGN INFORMATION IF OPTION #1 IN EFFECT
C WRITE RESULTS
C DIMENSION VYF(101),YR(101),ARG(18)
C DIMENSION X(101),S(101)
C DIMENSION F(4,101),R(4,101)
C DIMENSION AF(18),BF(18),AR(18),BR(18)

```

```

DIMENSION WH(100)
DIMENSION CAMBR(101)
COMMON /COCAM/ COCAM, AMCAM, ICAML
DATA ALPHA, ALPHC, ALPHG / 'A', 'C', 'G' /
DATA PI, P12 / 3.14159265 , 6.2831853 /
TNS=FLOAT(NS)
H=FLOAT(NS)/FLOAT(NPTT-1)
IF(IPOP.EQ.0)GO TO 994
WRITE(LP,201)
201 FORMAT(1H1,3X,'SUSPENSION DISPLACEMENT PHASED AS SEEN',/,4X,'RY VE
1HICLE TRAVERSING GUIDEWAY',/,/,4X,'TIME STEP',5X,'YF',14X,'YR',/)
WRITE(LP,200) (1,VYF(I),VYR(I),I=1,NPTT)
200 FORMAT(7X,13,F10.3,5X,F10.3)
994 CONTINUE
C -----CAMBER UNIT-----
C
C
C NPTX = NPTT - 1
C --- DEFAULT VALUE OF AMCAM IS UNITY.
C IF (AMCAM.EQ. 0.) AMCAM = 1.
C --- BRANCH ACCORDING TO CODE.
C IF (CDCAM .NE. ALPHA) GO TO 701
C --- HERE FOR ABSOLUTE VALUE OF SINE.
DO 701 I = 1, NPTT
CARG = FLOAT((1-I)*NS) / FLOAT(NPTX)
CAMBR(I) = ABS(SIN(PI*CARG)) * AMCAM
701 CONTINUE
GO TO 750
701 IF (CDCAM .NE. ALPHC) GO TO 702
C --- HERE FOR COSINE.
DO 702 I = 1, NPTT
CARG = FLOAT((1-I)*NS) / FLOAT(NPTX)
CAMBR(I) = 0.5 * (1.-COS(PI2*CARG)) * AMCAM
702 CONTINUE
GO TO 750
702 IF (CDCAM .NE. ALPHG) GO TO 799

```

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C --- HERE FOR ARBITRARY CAMBER.
C --- NORMALIZED CAMBER IS READ IN FROM CARDS, ONE VALUE FOR EACH
C --- TIME STEP ALONG THE BEAM. ITEMS READ = NS*NPTS + 1.
      READ (LR,7030) (CAMBR(I), I = 1, NPTT)
7030 FORMAT (8F10.0)
      DO 7031 I = 1, NPTT
        CAMSR(I) = CAMBR(I) * AMCAN
7031 CONTINUE
750 CONTINUE
C --- ADD CAMBER TO YVF, YVR.
      DO 7501 I = 1, NPTX
        YVF(I) = YVF(I) + CAMBR(I)
        ICAM1 = I - 1 + ICAML
        ICAM2 = MOD(ICAM1, NPTX) + 1
        YVR(ICAM2) = YVR(ICAM2) + CAMBR(I)
7501 CONTINUE
        YVF(NPTT) = YVF(1)
        YVR(NPTT) = YVR(1)
C --- PRINT CAMBER ADJUSTMENTS.
      WRITE (LP,7040) CDCAM, AMCAN
7040 FORMAT ('1 CAMBER CODE ',A1, '...', AMPLITUDE=, F6.3 //
           1 4X, 'TIME STEP', 5X, 'YF', 13X, 'YR', 13X, 'CF'//)
      WRITE (LP,7050) (I,YVF(I), YVR(I), CAMBR(I), I = 1, NPTT)
7050 FORMAT (7X, 13, F10.3, 5X, F10.3, 5X, F10.3)
C
799 CONTINUE
C -----
      DO 11 I=1,NPTT
11 X(I)=YVF(I)/TMS
        CALL SINT(X,M,NPTT,S)
        AFO=S(NPTT)
      DO 13 I=1,NPTT
13 X(I)=YVR(I)/TMS
        CALL SINT(X,M,NPTT,S)
        ARO=S(NPTT)

```

```

DO 50 J=1,NFC
  ARG(J)=FLOAT(J)*2.*3.1415927/TNS
DO 15 I=1,NPTT
  15 X(I)=(2./TNS)*YYF(I)*COS(ARG(J)*FLOAT(I-1)*H)
  CALL SINT(X,H,NPTT,S)
  AF(J)=S(NPTT)
DO 17 I=1,NPTT
  17 X(I)=(2./TNS)*YYF(I)*SIN(ARG(J)*FLOAT(I-1)*H)
  CALL SINT(X,H,NPTT,S)
  BF(J)=S(NPTT)
DO 19 I=1,NPTT
  19 X(I)=(2./TNS)*YYR(I)*COS(ARG(J)*FLOAT(I-1)*H)
  CALL SINT(X,H,NPTT,S)
  AR(J)=S(NPTT)
DO 21 I=1,NPTT
  21 X(I)=(2./TNS)*YYR(I)*SIN(ARG(J)*FLOAT(I-1)*H)
  CALL SINT(X,H,NPTT,S)
  BR(J)=S(NPTT)
50 CONTINUE
VC=1./T

C
C
C
PUNCH FOURIER COEFFICIENTS ON DATA CARDS

IF(IPOP.EQ.0)GO TO 993
DO 501 J=1,NFC
  AJ=J
501 WRITE(7,500) AJ,AF(J),BF(J),AR(J),BR(J),VC,VL,PLR,ZN,TNS
500 FORMAT(5F10.3,5X,5F5.3)
C
WRITE(LP,100)
100 FORMAT(1H1,16X,'AF',10X,'BF',10X,'AR',10X,'BR')
WRITE(LP,101)AFO,ARO
101 FORMAT(6X,'0',5X,F7.3,17X,F7.3)
103 FORMAT(6X,12,5X,F7.3,5X,F7.3,5X,F7.3,5X,F7.3)
DO 102 J=1,NFC
  WRITE(LP,103)J,AF(J),BF(J),AR(J),BR(J)

```

```

102 CONTINUE
993 CONTINUE
C=====ROUTINE GOES BETWEEN THIS AND THE NEXT CARD
C=====
SUMF=0.
SUMR=0.0
SUMG=0.0
IF(IPOP.EQ.0)GO TO 666
GO TO 555
666 IF(IT.GT.1)GO TO 992
555 WRITE(LP,502)
502 FORMAT(1M,10X,'ROOT MEAN SQUARE VEHICLE ACCELERATION',/
+1M,10X,'VEHICLE CHARACTERISTICS')
WRITE(LP,503)AM,AK,AI,Z
503 FORMAT(1M,10X,'M=',F5.2,6X,'K=',F5.2,6X,'I=',F5.2,6X,
+Z=',F5.2/')
992 CONTINUE
CC0=0.0
CC1=0.0
CC2=0.0
CC3=0.0
DUM1=.5-AI/6.
CC4=-AK*(1.+AK)*DUM1
CC5=-AK*DUM1*2.+2*(AK+2.)
CC6=-AK*(AM+.5*Z*2)*DUM1
CC7=-AK*DUM1*2.+2*AM
CS0=0.0
CS1=0.0
CS2=AK**2
CS3=.4.*Z*AK**2
DUM2=.5+AI/6.
DUM3=AM+DUM2
CS4=AK*(1.+AK)*DUM2+AM*AK+.4.*Z**2*AK**2
CS5=2.*Z*AK*(2.*AM+(2.+AK)*DUM2)
CS6=AK*AM*DUM2+.4.*Z**2*AK*DUM3
CS7=2.*Z*AK*AM*DUM2

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```

D0=AK**2
D1=4.*Z**AK**2
D2=4.*Z**2*AK**2+2.*AK*((1.+AK)*DUM2+AM)
D3=4.*Z**AK*((2.+AK)*DUM2+2.*AM)
D4=-1.*(1.+AK)*2*DUM1**2+((1.+AK)*DUM2+AM)**2
1+2.*AM*AK*DUM2+8.*Z**2*AK*DUM3
D5=-4.*Z*(1.+AK)*DUM1**2+4.*Z*AK*AM*DUM2
1+4.*Z*DUM3*((1.+AK)*DUM2+AM)
D6=-(4.*Z**2+2.*AM*(1.+AK))*DUM1**2
1+2.*AM*DUM2*((1.+AK)*DUM2+AM)+4.*Z**2*DUM3**2
D7=-4.*Z*AM*(DUM1**2-DUM2*DUM3)
D8=AM**2*(DUM2**2-DUM1**2)
ARATN=0.
VS=VC*SL*F1/1.467
VLEN=SL*(PLR+VL)
IF(IPOP.EQ.0)GO TO 444
GO TO 333
444 IF(I7.GT.1)GO TO 991
IF(IOPT.EQ.1)GO TO 333
WRITE(ILP,900)NS,SL,VMIN,VMAX,PLR,VLEN,ZN,WTOT
900 FORMAT(1H ,10X,'NUMBER OF SPANS=',I2/1H ,10X,'SPAN LENGTH (FT)=' ,F
17.2/1H ,10X,'VEHICLE SPEED RANGE (MPH)=' ,F7.2,' - ',F7.2/1H ,10X,'
2PAS LENGTH RATIO=' ,F5.2/1H ,10X,'VEHICLE LENGTH (FT)=' ,F7.2/1H ,10
3X,'BEAR DAMPING=' ,F7.2/1H ,10X,'VEHICLE WEIGHT (LBS)=' ,F10.2)
VCNX=VC*(VMAX/VMIN)
WRITE(ILP,901)VC,VCNX,FV,VL
901 FORMAT(1H ,10X,'CROSSING FREQ. RATIO RANGE=' ,F5.2,' - ',F5.2/1H ,1
10X,'VEHICLE NATURAL FREQUENCY (HZ)=' ,F5.2/1H ,10X,'ATTACHMENT LENG
2TH RATIO=' ,F5.2/)
GO TO 991
333 WRITE(ILP,505)NS,SL,VS,PLR,VLEN,ZN,WTOT
WRITE(ILP,507)VC,FV,VL
991 CONTINUE
IF(IPOP.EQ.0)GO TO 990
WRITE(ILP,111)
990 CONTINUE

```

```

DO 69 I=1,NFC
WM(I)=FLOAT(I)*VS*1.467/(TNS*SL)
W=WM(I)/FV
ANUSR=CSO-W*2*CS2+W*4*CS4-W*6*CS6
ANUSI=W*CS1-W*3*CS3+W*5*CS5-W*7*CS7
ANUCR=CCO-W*2*CC2+W*4*CC4-W*6*CC6
ANUCI=W*CC1-W*3*CC3+W*5*CC5-W*7*CC7
DNR=DO-W*2*DD2+W*4*DD4-W*6*DD6+W*8*DD8
DNI=W*DI1-W*3*DI3+W*5*DI5-W*7*DI7
DUM4=SQRT(DNR**2+DNI**2)
AMAGC=SQRT(ANUCI**2+ANUCR**2)/DUM4
AMAGS=SQRT(ANUSI**2+ANUSR**2)/DUM4
XS=ANUSR/DNR+ANUSI/DNI
YS=ANUSI/DNR-ANUSR/DNI
XC=ANUCR/DNR+ANUCI/DNI
YC=ANUCI/DNR-DNI+ANUCR
CALL PHASE(XS,YS,PH1)
PHAS=PH1
PHAS=PH1*360./6.28318
CALL PHASE(XC,YC,PH2)
PHAC=PH2
PHAC=PH2*360./6.28318
SUMS=SIN(PHAS)+COS(PHAS)
DIFS=COS(PHAS)-SIN(PHAS)
SUNC=SIN(PHAC)+COS(PHAC)
DIFC=COS(PHAC)-SIN(PHAC)
Y2F1=AMAGS*(BF(I)*COS(PHAS)-AF(I)*SIN(PHAS))
1+AMAGC*(BR(I)*COS(PHAC)-AR(I)*SIN(PHAC))
Y2F2=AMAGS*(BF(I)*SIN(PHAS)+AF(I)*COS(PHAS))
1+AMAGC*(BR(I)*SIN(PHAC)+AR(I)*COS(PHAC))
Y2F=0.707*(Y2F1+Y2F2)*0.5
Y2R1=AMAGS*(BR(I)*COS(PHAS)-AR(I)*SIN(PHAS))
1+AMAGC*(BF(I)*COS(PHAC)-AF(I)*SIN(PHAC))
Y2R2=AMAGS*(BR(I)*SIN(PHAS)+AR(I)*COS(PHAS))
1+AMAGC*(BF(I)*SIN(PHAC)+AF(I)*COS(PHAC))
Y2R=0.707*(Y2R1+Y2R2)*0.5

```

YCG=0.5*0.707*((Y2F1+Y2R1)**2+(Y2F2+Y2R2)**2)**0.5

SUMF1=(Y2F1+Y2F1+Y2F2+Y2F2)*0.5

SUMR1=(Y2R1+Y2R1+Y2R2+Y2R2)*0.5

SUMG1=((Y2F1+Y2R1)**2+(Y2F2+Y2R2)**2)*0.125

SUMF=SUMF+SUMF1

SUMR=SUMR+SUMR1

SUMG=SUMG+SUMG1

SUMF=SQRT(SUMF)

SUMR=SQRT(SUMR)

SUMG=SQRT(SUMG)

PI=3.1415927

THE FOLLOWING PARAMETERS CAN BE CHOSEN BY THE DESIGNER:

ROC--DENSITY OF SPAN MATERIAL (LB/FT**3)

WB--WIDTH OF I-BEAM FLANGE (FT)

C--WIDTH OF I-BEAM WEB (FT)

WTH--THICKNESS OF I-BEAM FLANGE (FT)

ES--SPAN MODULUS OF ELASTICITY (PSI)

ROC=150.

WB=5.0

C=.833

WTH=.833

ES=5000000.

RMS ACCELERATION VS. FREQUENCY SPEC. DEFINED IN NEXT 10 STATEMENTS

IF(WH(1)-.45)1,1,2

1 Y200G=.10

GO TO 400

2 IF(WH(1)-3.13,3,4

3 Y200G=.1/(WH(1)/.45)**.483

GO TO 400

4 IF(WH(1)-12.15,5,6

5 Y200G=.04

GO TO 400

6 Y200G=.01*WH(1)/3.


```

C*****
400 ARATF=Y2F/Y2D0G
   ARATR=Y2R/Y2D0G
   IF(ARATF-ARATR)212,212,211
211 ARATI=ARATF
   GO TO 213
212 ARATI=ARATR
213 IF(ARATI.LT.ARATH)GO TO 79
   ARATH=ARATI
   IMX=1
   MINX=M(1)
   AGMAX=Y2D0G
79 CONTINUE
   IF(IPOP.EQ.0)GO TO 69
   WRITE(LP,110)M,Y2F,SUMF,Y2R,SUMR,YCG,SUMG
   IF(1.EQ.MFC)GO TO 69
   SUMF=SUMF+SUMF
   SUMR=SUMR+SUMR
   SUMG=SUMG+SUMG
69 CONTINUE
   WRITE(LP,510)VS,SUMF,SUMR,SUMG
510 FORMAT(1H0,10X,'THE TOTAL NON-DIM. ACC. AT ',F7.2,'MPH IS ',F7.2,'
1 FOR THE FRONT',5X,F7.2,' FOR THE REAR',5X,F7.2,' FOR THE CENTER')
   IF(IPOP.EQ.0)GO TO 989
   WRITE(LP,112)
989 CONTINUE
   DO 75 K=1,MFC
   IF(IPOP.EQ.0)GO TO 988
   WRITE(LP,110)M(K),AF(K),BF(K),AR(K),OR(K)
988 CONTINUE
75 CONTINUE
   WRITE(LP,888)ARATH
888 FORMAT(1H0,/,7X,'MAX. NON-DIM. ACC./SPEC. ACC. LIMIT = ',E15.8,'UN
117/CEE')
   IF(10PT.EQ.2)GO TO 777
   Y2DIX=ARATH*AGMAX

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```

Y2DDG=AGMAX
Y2DDF=32.2*Y2DDG
YSTAR=32.2/(ARATH*(2.*P[1*FV]**2)
WT=WTOT
EI=(288.*WT*SL**3)/(YSTAR*P[1**4]
SI=EI/ES
SIFT=SI/12.**4
C*****
C ITERATIVE ROUTINE TO FIND SPAN HEIGHT H FROM SPAN MOMENT OF IN-
C ERTIA SI
C H=1.0
DEL=0.1
22 H=(1.+DEL)*H
SITST=(WB-H**3-(WB-C)*(H-WTH)**3)/6.
IF(SIFT.GT.SITST)GO TO 22
H=H/(1.+DEL)
DEL=0.1*DEL
IF(DEL.LT.0.001)GO TO 2199
GO TO 22
C*****
2199 CONTINUE
SA=2.*(WB-H-(WB-C)*(H-WTH))*144.
CUBS=1.358*SA
F1CHK=(.5*PI/SL**2)*((32.2*EI/(ROC*SA))**.5)
VCCHK=VS*1.467/(SL*F1CHK)
WRITE(LP,300)Y2DMX,Y2DDG,Y2DDF,YSTAR,WT,EI,SI,SA,H,CUBS,F1CHK,VCCHK
1K
300 FORMAT(1M1,T17,'MAX. NGN-DIM. ACC.-----=',E12.4,/,/,
2T19,'SPEC. ACC. LIMIT-----=',E12.4,' GEES',/,/,
3T19,'SPEC. ACC. LIMIT-----=',E12.4,' FT/SEC**2',/,/,
4T12,'NORMALIZING DEFLECTION (Y*)=',E12.4,' FT',/,/,
CT20,'VEHICLE WEIGHT (WT)=',E12.4,' LB',/,/,
5T12,'SPAN STRENGTH MODULUS (E*)=',E12.4,' LB-IN**2',/,/,
6T13,'SPAN MOMENT OF INERTIA (I)=',E12.4,' IN**4',/,/,
7T12,'SPAN CROSS-SECTION AREA (A)=',E12.4,' IN**2',/,/,
8T24,'SPAN HEIGHT (H)=',E12.4,' FT',/,/,

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```

9120,'VOLUME CONCRETE-----',E12.4,' YD=3/MI',/,/,
AT13,'CHECK BEAM FREQ. (F1)=',E12.4,' CPS',/,/,
BT11,'CHECK CROSSING VELOCITY (VC)=',E12.4)
777 CONTINUE
505 FORMAT(1H ,10X,'NUMBER OF SPANS=',12/1H ,10X,'SPAN LENGTH (FT)
+ ',F7.2/1H ,10X,'VEHICLE SPEED (MPH)=',F7.2/1H ,10X,'PAD LENGTH
+ RATIO=',F5.2/1H ,10X,'VEHICLE LENGTH (FT) =',F7.2/1H ,10X,'BEAM D
+ AMPING=',F7.2/1H ,10X,'VEHICLE WEIGHT (LBS)=',F10.2)
506 FORMAT(2F10.0)
507 FORMAT(1H ,10X,'CROSSING FREQUENCY RATIO=',F5.2/
+1H ,10X,'VEHICLE NATURAL FREQUENCY (HZ)=',F5.2/
+1H ,10X,'ATTACHMENT LENGTH RATIO=',F5.2/)
301 FORMAT(5F10.0,5X,5F5.3)
610 FORMAT(2F10.0)
111 FORMAT(1H0,7X,'W/FV',7X,'YZF ',7X,'SUMF',7X,'Y2R ',7X,'SUMR',7X,'Y
12CG',7X,'SUMC',/)
112 FORMAT(1H0,9X,'W',8X,' AF ',7X,' BF ',7X,' AR ',7X,' BR ',/)
110 FORMAT(1H ,F11.2,6F11.4)
203 FORMAT(1H ,F11.2,8F11.4)
RETURN
END

C
C
SUBROUTINE PHASE(R,A1,P)
C
C
C PHASE IS CALLED BY FCODE IN THE SEQUENCE DETERMINING RMS ACCELERA-
C TIONS FROM THE FOURIER COEFFICIENTS AND THE VEHICLE TRANSFER FUNC-
C TIONS
C
IF(R.LT.0.) GO TO 101
P=ATAN(A1/R)
GO TO 100
101 IF(A1.LT.0.) GO TO 102
P=ATAN(A1/R)+3.14159
GO TO 100
102 P=ATAN(A1/R)-3.14159

```

```

100      CONTINUE
        RETURN
        END
C
C
C      SUBROUTINE SINT(X,M,NPTT,S)
C
C      SINT IS CALLED BY PCOEF AND IS A SIMPSON'S RULE NUMERICAL INTEGRATION ROUTINE USED TO DETERMINE THE FOURIER COEFFICIENTS FROM THE DEFLECTION PROFILES BENEATH THE FRONT AND REAR SUSPENSIONS
C
C      DIMENSION X(2),S(2)
C      S(1)=0.
C      S(2)=(5.*X(1)+8.*X(2))*H/12.+S(1)
C      H3=H/3.
C      DO 1 I=3,NPTT
C      1 S(1)=S(1-2)+H3*(X(I-2)+4.*X(I-1)+X(I))
        RETURN
        END

```

T
 X

APPENDIX F
TABLES OF NONDIMENSIONAL SUSPENSION DEFLECTION
FOURIER COEFFICIENTS

Table F.1

Nondimensional Suspension Deflection Fourier Coefficients

Configuration: $V_c = 0.33$, $L_a = 0.5$, $L_p = 0.3$, $E_m = 0.0$

	AF	BF	AR	BR	
0	-0.328		-0.328		
1	0.249	0.123	-0.250	0.123	
2	0.051	-0.000	0.051	0.001	
3	0.002	0.002	-0.002	0.002	Single
4	0.005	-0.000	0.005	0.001	Span
5	0.002	-0.000	-0.002	-0.000	$Y_m = 0.7$
6	0.001	-0.000	0.000	0.000	$H_{cm} = 0.7$
	AF	BF	AR	BR	
0	-0.209		-0.209		
1	-0.043	-0.024	-0.024	-0.043	
2	0.178	0.107	-0.178	0.107	
3	0.026	-0.007	0.007	-0.026	Two
4	0.020	-0.000	0.020	0.000	Span
5	0.006	0.004	0.004	0.006	
6	0.001	0.001	-0.001	0.001	$Y_m = 0.52$
7	0.001	-0.000	0.000	-0.002	$H_{cm} = 0.52$
8	0.001	-0.000	0.000	0.000	
9	0.000	-0.001	-0.000	-0.000	
10	0.000	0.000	-0.001	-0.000	
11	0.001	0.000	0.000	-0.002	
12	0.000	-0.000	0.000	0.000	
	AF	BF	AR	BR	
0	-0.182		-0.182		
1	-0.058	-0.014	-0.042	-0.043	
2	-0.000	-0.010	-0.009	-0.005	
3	0.160	0.102	-0.160	0.101	
4	0.020	-0.001	-0.009	-0.018	
5	0.015	-0.004	0.011	-0.011	
6	0.013	-0.000	0.013	0.000	Three
7	0.006	0.003	0.005	0.004	Span
8	0.002	0.002	0.000	0.003	
9	0.001	0.001	-0.001	-0.000	$Y_m = 0.47$
10	0.001	-0.000	-0.000	-0.001	$H_{cm} = 0.51$
11	0.001	-0.002	-0.000	-0.002	
12	0.000	-0.001	0.001	-0.002	
13	0.000	0.000	0.000	0.000	
14	0.000	0.001	-0.001	-0.000	
15	0.000	0.000	-0.000	-0.000	
16	0.000	0.000	0.000	-0.000	
17	0.000	-0.000	0.000	-0.001	
18	0.000	-0.000	0.000	-0.000	

Table F.2

Non-dimensional Suspension Deflection Fourier Coefficients

Configuration: $V_c = 0.5$, $L_a = 0.5$, $L_p = 0.3$, $E_m = 0.0$

	AF	BF	AR	BR	
0	-0.334		-0.334		
1	0.241	0.129	-0.241	0.161	
2	0.055	0.065	0.055	0.109	
3	0.013	-0.021	-0.014	0.035	Single
4	0.005	-0.004	0.005	-0.007	Span
5	0.003	-0.002	-0.003	0.009	$Y_m = 0.82$
6	0.001	-0.002	0.001	-0.003	$R_{cm} = 0.82$
	AF	BF	AR	BR	
0	-0.214		-0.214		
1	-0.045	-0.022	-0.022	-0.045	
2	0.178	0.113	-0.178	0.113	
3	0.057	-0.012	0.012	-0.057	
4	0.023	-0.005	0.023	0.005	Two
5	-0.023	0.003	0.003	-0.023	Span
6	0.004	0.002	-0.003	0.002	$Y_m = 0.54$
7	0.002	0.000	-0.000	-0.002	$R_{cm} = 0.57$
8	0.002	0.000	0.002	-0.000	
9	0.001	0.000	0.000	0.001	
10	0.001	0.000	-0.001	0.000	
11	0.001	0.000	-0.000	-0.001	
12	0.000	0.000	0.000	-0.000	
	AF	BF	AR	BR	
0	-0.186		-0.186		
1	-0.059	-0.015	-0.042	-0.045	
2	-0.002	-0.011	-0.006	-0.006	
3	0.162	0.103	-0.161	0.109	
4	0.019	-0.016	-0.017	-0.011	
5	0.016	0.008	0.029	0.001	
6	0.016	0.009	0.018	0.013	Three
7	0.009	0.023	-0.007	0.020	Span
8	0.004	-0.007	0.009	0.014	$Y_m = 0.51$
9	0.002	-0.003	-0.001	0.005	$R_{cm} = 0.33$
10	0.002	-0.004	-0.001	0.003	
11	0.001	-0.002	-0.001	-0.001	
12	0.002	-0.000	0.001	-0.001	
13	0.001	-0.000	0.001	0.000	
14	0.001	-0.000	-0.000	0.001	
15	0.001	-0.000	-0.001	0.000	
16	0.000	-0.000	-0.000	-0.000	
17	0.000	-0.000	-0.000	-0.001	
18	0.000	-0.000	0.000	-0.000	

Table F.3

Non-dimensional Suspension Deflection Fourier Coefficients

Configuration: $V_c = 0.66$, $L_a = 0.5$, $L_p = 0.3$, $\xi_m = 0.0$

	AF	BF	AR	BR	
0	-0.337		-0.337		
1	0.310	0.103	-0.310	0.102	Single
2	-0.023	0.033	-0.023	-0.033	Span
3	0.017	0.000	-0.017	-0.000	
4	0.007	0.000	0.007	0.000	$Y_m = 0.68$
5	0.004	0.000	-0.004	-0.000	$H_{cm} = 0.68$
6	0.001	0.000	0.001	0.000	
	AF	BF	AR	BR	
0	-0.219		-0.219		
1	-0.043	-0.018	-0.018	-0.043	
2	0.214	0.092	-0.214	0.092	Two
3	0.024	-0.012	0.007	-0.021	Span
4	-0.008	0.023	-0.009	-0.024	
5	0.004	-0.000	-0.008	0.008	$Y_m = 0.46$
6	-0.000	0.001	-0.002	-0.003	$H_{cm} = 0.50$
7	0.005	0.001	-0.002	-0.004	
8	0.003	0.000	0.002	-0.000	
9	0.002	0.000	0.000	0.002	
10	0.001	0.000	-0.001	0.000	
11	0.001	0.000	-0.000	-0.001	
12	0.001	0.000	0.001	-0.000	
	AF	BF	AR	BR	
0	-0.191		-0.190		
1	-0.060	-0.013	-0.040	-0.045	
2	-0.004	-0.007	-0.002	-0.005	
3	0.182	0.091	-0.183	0.096	Three
4	0.024	0.005	-0.002	-0.024	Span
5	0.028	0.002	0.034	-0.008	
6	0.002	0.021	0.000	-0.007	$Y_m = 0.49$
7	0.004	-0.006	0.006	0.003	$H_{cm} = 0.54$
8	-0.005	-0.007	0.012	0.004	
9	0.001	-0.001	-0.002	0.003	
10	0.002	0.000	-0.002	-0.001	
11	0.001	-0.000	-0.000	-0.001	
12	0.001	-0.000	0.001	-0.001	
13	0.001	0.000	0.001	0.001	
14	0.001	-0.000	-0.000	0.001	
15	0.001	-0.000	-0.001	0.000	
16	0.001	-0.000	-0.000	-0.000	
17	0.000	-0.000	-0.000	-0.000	
18	0.000	-0.000	0.000	-0.000	

Table F.4

Nondimensional Suspension Deflection Fourier Coefficients

Configuration: $V_c = 0.83$, $L_a = 0.5$, $L_p = 0.3$, $E_m = 0.0$

	AF	BF	AR	BR	
0	-0.357		-0.336		Single Span $Y_m = 0.8$ $H_{tm} = 0.3$
1	0.334	0.186	-0.222	0.145	
2	-0.010	-0.020	0.056	0.000	
3	0.010	-0.002	-0.019	-0.000	
4	0.005	-0.001	0.009	0.000	
5	0.003	-0.000	-0.005	-0.000	
6	0.001	-0.000	0.002	0.000	
	AF	BF	AR	BR	
0	-0.229		-0.221		Two Span $Y_m = 0.58$ $H_{tm} = 0.61$
1	-0.055	-0.025	-0.013	-0.016	
2	0.215	0.144	-0.181	0.152	
3	0.037	0.004	0.004	0.013	
4	-0.006	0.003	0.009	0.029	
5	0.005	0.006	-0.011	0.007	
6	0.008	-0.005	-0.013	0.004	
7	0.004	-0.001	-0.001	-0.006	
8	0.003	-0.001	0.004	-0.001	
9	0.002	-0.001	0.000	0.003	
10	0.001	-0.001	-0.002	0.000	
11	0.001	-0.000	-0.000	-0.001	
12	0.000	-0.001	0.001	-0.000	
	AF	BF	AR	BR	
0	-0.197		-0.196		Three Span $Y_m = 0.56$ $H_{tm} = 0.60$
1	-0.060	-0.013	-0.036	-0.042	
2	-0.007	-0.025	-0.018	0.008	
3	0.180	0.133	-0.170	0.122	
4	0.035	0.013	0.006	-0.014	
5	0.020	0.020	-0.000	0.012	
6	0.011	-0.008	0.013	0.004	
7	-0.008	-0.008	0.009	0.005	
8	0.004	-0.003	-0.003	0.008	
9	0.000	-0.000	-0.003	-0.001	
10	0.003	-0.001	-0.004	-0.003	
11	0.002	-0.000	0.001	-0.003	
12	0.001	-0.000	0.002	-0.000	
13	0.001	-0.000	0.001	0.001	
14	0.001	-0.000	-0.000	0.001	
15	0.001	-0.000	-0.001	0.000	
16	0.001	-0.000	-0.001	-0.001	
17	0.000	-0.000	0.000	-0.001	
18	0.000	-0.000	0.000	-0.000	

Table F.5

Nondimensional Suspension Deflection Fourier Coefficients

Configuration: $V_c = 1.0$, $L_a = 0.5$, $L_p = 0.3$, $\xi_m = 0.0$

	AF	BF	AR	BR	
0	-0.363		-0.363		Single Span $Y_m = 0.91$ $H_{cm} = 0.91$
1	0.297	0.258	-0.297	0.257	
2	0.022	-0.028	0.022	0.028	
3	0.013	-0.009	-0.013	-0.009	
4	0.006	-0.002	0.006	0.002	
5	0.009	-0.001	-0.003	-0.001	
6	0.001	-0.001	0.001	0.001	
	AF	BF	AR	BR	
0	-0.239		-0.237		Two Span $Y_m = 0.58$ $H_{cm} = 0.67$
1	-0.042	-0.013	-0.004	-0.032	
2	0.215	0.167	-0.207	0.155	
3	0.033	0.007	0.028	-0.004	
4	0.009	-0.010	0.010	0.009	
5	-0.002	-0.008	0.007	0.011	
6	0.004	-0.001	-0.006	0.000	
7	0.002	-0.000	-0.000	-0.004	
8	0.002	-0.000	0.002	-0.000	
9	0.001	-0.000	0.000	0.002	
10	0.001	-0.000	-0.001	0.000	
11	0.000	-0.000	-0.000	-0.001	
12	0.000	-0.000	0.000	-0.000	
	AF	BF	AR	BR	
0	-0.209		-0.206		Three Span $Y_m = 0.54$ $H_{cm} = 0.58$
1	-0.058	-0.009	-0.034	-0.043	
2	0.014	-0.002	-0.008	-0.007	
3	0.208	0.131	-0.186	0.127	
4	0.020	0.002	-0.003	-0.012	
5	-0.000	-0.010	0.004	-0.015	
6	-0.013	0.006	0.004	-0.003	
7	0.004	-0.001	0.003	0.006	
8	0.001	0.003	-0.004	-0.002	
9	0.005	-0.002	-0.007	0.001	
10	0.003	-0.000	-0.002	-0.003	
11	0.002	-0.000	0.001	-0.002	
12	0.002	-0.000	0.002	-0.000	
13	0.001	-0.000	0.001	0.001	
14	0.001	-0.000	-0.001	0.001	
15	0.001	-0.000	-0.001	0.000	
16	0.001	-0.000	-0.000	-0.001	
17	0.001	-0.000	0.000	-0.001	
18	0.000	-0.000	0.000	-0.000	

APPENDIX G
PARAMETRIC DATA FOR SPRUNG AND UNSPRUNG
MASS INERTIA SUSPENSION FORCES

G.1 COMPUTATION OF VEHICLE SUSPENSION FORCES

In this appendix parametric data is presented to illustrate the regions of operation for which vehicle suspension forces may be approximated as constant and equal to the vehicle weight. To perform the parametric evaluation, the reduced order vehicle model shown in Figure G.1 is used in which the effects of finite vehicle length and finite pressure pad length are neglected. The model includes vehicle sprung m_v and unsprung m_u mass, a primary suspension stiffness k_p and secondary suspension stiffness k_s and damping b_s . The input to the vehicle model is the guideway deflection $y_0(t)$. For this model the suspension force f_s may be expressed in terms of the sprung $\ddot{y}_2$ and unsprung $\ddot{y}_1$ mass accelerations as:

$$f_s = -(m_u + m_v)g + m_u \ddot{y}_1 + m_v \ddot{y}_2 \quad (G.1)$$

or in nondimensional form:

$$\bar{F}_s = -1 + \frac{\kappa M_u}{1 + M_u} \hat{\ddot{Y}}_1 + \frac{\kappa}{1 + M_u} \hat{\ddot{Y}}_2 \quad (G.2)$$

where:

$$\begin{aligned} \bar{F}_s &= \frac{f_s}{(m_u + m_v)g} & M_u &= \frac{m_u}{m_v} \\ \kappa &= \frac{\omega_v^2 y^*}{g} & \omega_v &= 2\pi f_v = \sqrt{\frac{k_s}{m_v}} \\ \hat{\ddot{Y}}_1 &= \ddot{y}_1 / (y^* \omega_v^2) & \hat{\ddot{Y}}_2 &= \ddot{y}_2 / (y^* \omega_v^2) \end{aligned}$$

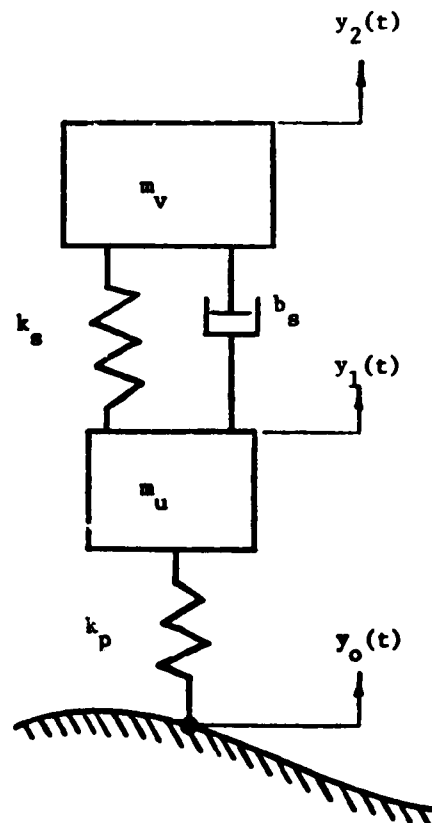


Fig. G-1. Fourth Order Vehicle Model

and where y^* is the deflection of a single span beam due to a single force of $(m_u + m_v)g$ at midspan. The nondimensional suspension force is a function of the unsprung and sprung mass vehicle accelerations. These accelerations may be computed directly from vehicle acceleration to guideway input transfer functions for a specified guideway input Y_o :

$$T_{um} = \frac{\ddot{Y}_1}{Y_o} = \frac{M_u \left[\frac{\omega_u}{\omega_v} \right]^2 (\hat{S}^4 + 2\xi_v \hat{S}^3 + \hat{S}^2)}{CPOL} \quad (G.3)$$

$$T_{sm} = \frac{\ddot{Y}_2}{Y_o} = \frac{\left[\frac{\omega_u}{\omega_v} \right]^2 (2\xi_v \hat{S}^3 + \hat{S}^2)}{CPOL} \quad (G.4)$$

where:

$$CPOL = M_u \hat{S}^4 + 2\xi_v [M_u + 1] \hat{S}^3 + [1 + M_u + \left(\frac{\omega_u}{\omega_v}\right)^2] \hat{S}^2 + 2\xi_v \left(\frac{\omega_u}{\omega_v}\right)^2 \hat{S} + M_u \left(\frac{\omega_u}{\omega_v}\right)^2 \quad (G.5)$$

and where:

T_{sm} = Nondimensional sprung mass transfer function

T_{um} = Nondimensional unsprung mass transfer function

$\hat{S} = S\omega_v$ = Nondimensional LaPlace Operator

$$\xi_v = \frac{b_v}{\sqrt{2m_v k_s}}$$

$$\omega_u = \sqrt{k_p / m_u}$$

The ratio of accelerations is also of interest and may be derived as:

$$\frac{\ddot{\hat{y}}_1}{\ddot{\hat{y}}_2} = \frac{\hat{S}^2 + 2\xi_v \hat{S} + 1}{2\xi_v \hat{S} + 1} \quad (G.6)$$

For a vehicle crossing a uniform beam system at constant speed v , the steady-state guideway deflection y_o beneath the suspension is periodic. It is convenient to represent y_o as a Fourier series and to evaluate vehicle accelerations using transfer function methods. Thus, y_o for a vehicle traveling at constant speed may be represented:

$$y_o = \sum_{i=0}^{\infty} Y_{oi} = \sum_{i=0}^{\infty} [a_i \cos i\omega_e t + b_i \sin i\omega_e t] \quad (G.7)$$

where: Y_{oi} = suspension deflection component at frequency $i\omega_e$

a_i (b_i) = Fourier coefficients determined as described in Chapter 3.

$$\omega_e = 2\pi v / kl_s$$

Using (G.7) and (G.3) - (G.4) with the substitution $\hat{S} = j\hat{\omega}_e$ with $\hat{\omega}_e = \omega_e = \omega_e / \omega_v$, the vehicle accelerations at any given frequency $i\omega_e$ may be determined and, in turn, the accelerations as a function of time may be determined:

$$\hat{\ddot{Y}}_1 = \sum_{i=0}^{\infty} T_{um} (i\omega_e) Y_{oi} \quad (G.8)$$

$$\hat{\ddot{Y}}_2 = \sum_{i=0}^{\infty} T_{sm} (i\omega_e) Y_{oi} \quad (G.9)$$

where the detailed operations implied in (G.8) and (G.9) are identical to those described explicitly in the Fourier analysis of Chapter 3. With the computation of the accelerations, $\hat{\ddot{Y}}_1$ and $\hat{\ddot{Y}}_2$, the suspension force $\hat{F}_s$ may be determined from (G.2).

G.2 VEHICLE CHARACTERISTICS

The acceleration transfer functions of the vehicle model as a function of excitation frequency provide useful information concerning the suspension force $\hat{F}_s$ since for a given input Y_o the force $\hat{F}_s$ depends directly upon the vehicle acceleration transfer functions. The nondimensional magnitudes of the transfer functions are functions only of ξ_v , M_u and (ω_u/ω_v) when plotted versus $\hat{\omega} = \omega/\omega_v$. Plots of transfer function magnitudes for $\xi_v = 0.25$ and parametric variations in M_u and (ω_u/ω_v) are presented in Figures G.2 and G.3. The sprung mass acceleration transfer function shows a low frequency behavior, $\hat{\omega} < 1$, in which the vehicle follows the guideway motion and the acceleration $\hat{\ddot{Y}}_2$ increases proportional to frequency squared until $\hat{\omega} = 1.0$ which corresponds to the sprung mass natural frequency. Relative peaks occur in the transfer function at $\omega = 1.0$ and $\omega = \frac{\omega_u}{\omega_v}$ corresponding to the sprung and unsprung mass natural frequencies. For all the cases plotted, the frequency which excites the maximum acceleration corresponds to the unsprung mass natural frequency.

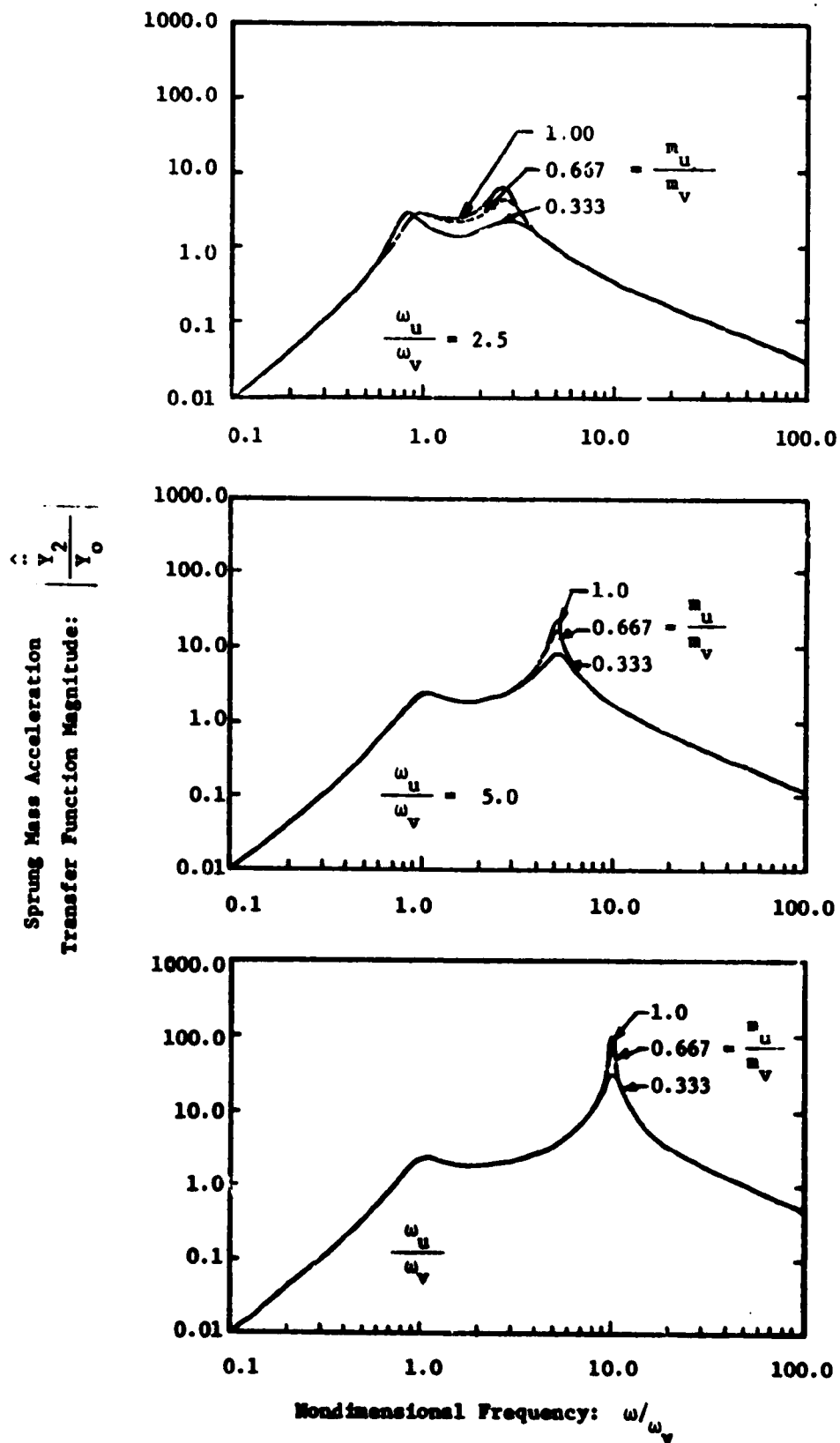


Fig. G-2. Sprung Mass Acceleration Transfer Function

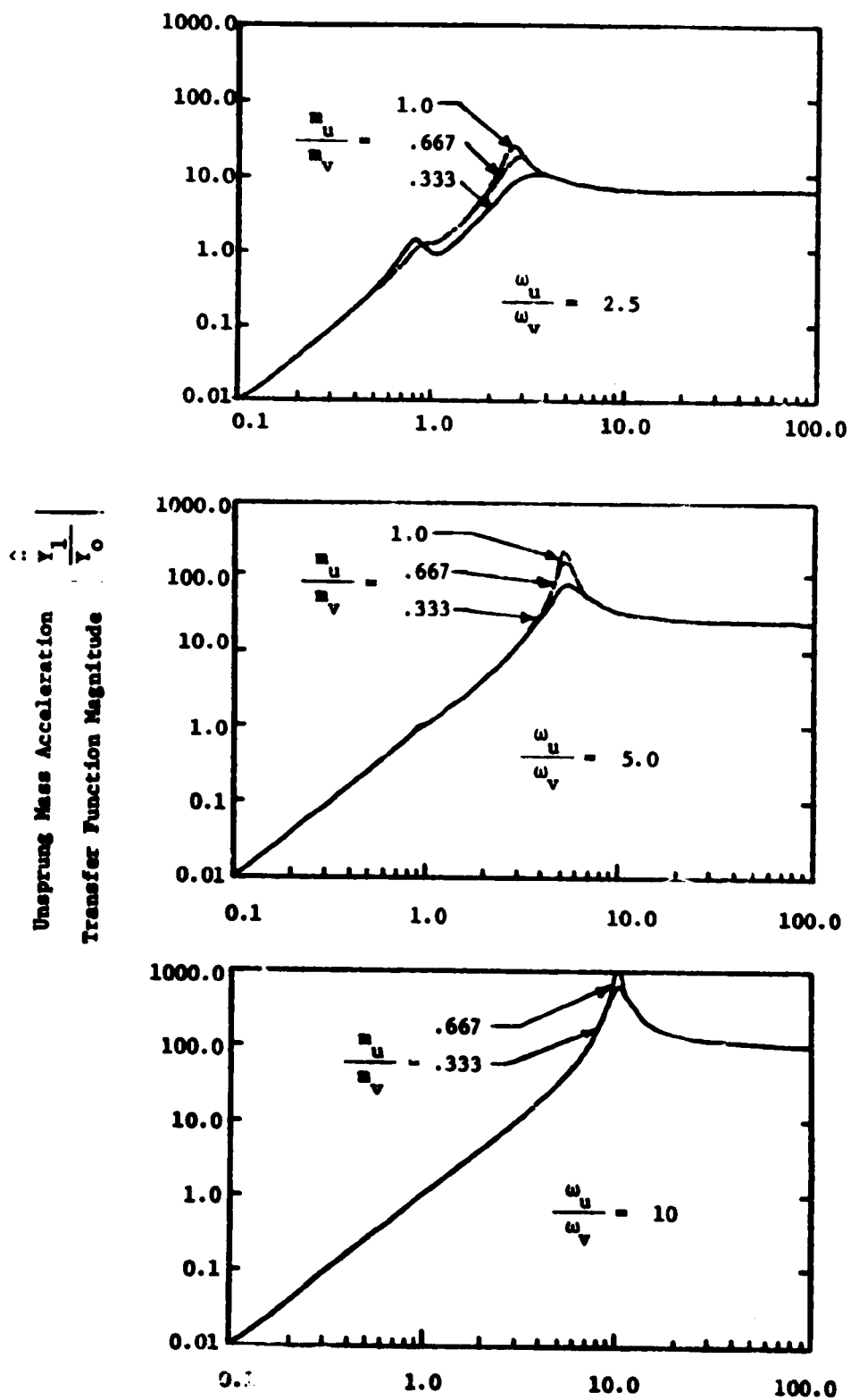


Fig. G-3. Unsprung Mass Acceleration Transfer Function

As ω increases above ω_u , the suspension filtering characteristics reduce the acceleration and the acceleration decreases with a slope of 1 as $\hat{\omega}$ increases.

The unsprung mass acceleration in Fig. G.4 at low frequencies also increases with increasing frequency with a slope of +2, with minor peaking at $\hat{\omega} = 1$, the sprung mass natural frequency, and an absolute maximum at $\hat{\omega} = \omega_u/\omega_v$, the unsprung mass natural frequency. At frequencies above ω_u , the magnitude becomes constant and equal to $(\omega_u/\omega_v)^2$.

Because in advanced transportation systems, the sprung mass accelerations may be directly constrained by passenger comfort requirements, the ratio of unsprung to sprung mass acceleration is of direct interest since the unsprung and sprung mass accelerations determine the suspension force variation. Figure G.4 displays the ratio of unsprung mass acceleration magnitude versus frequency. This ratio is independent of M_u and ω_u/ω_v as shown by (G.6). At low frequencies $\hat{\omega} < 1$, the ratio of unsprung to sprung mass nondimensional acceleration is 1.0, with the sprung and unsprung masses responding in unison to the input. At frequencies above $\hat{\omega} = 1.0$ the ratio of unsprung mass acceleration increases with a slope of +1. Thus, only for guideway inputs which have frequency content greater than the sprung mass natural frequency can unsprung mass accelerations significantly exceed sprung mass accelerations.

*If lower values of damping ξ_v were used, the peaking at $\hat{\omega} = 1.0$ would be more pronounced.

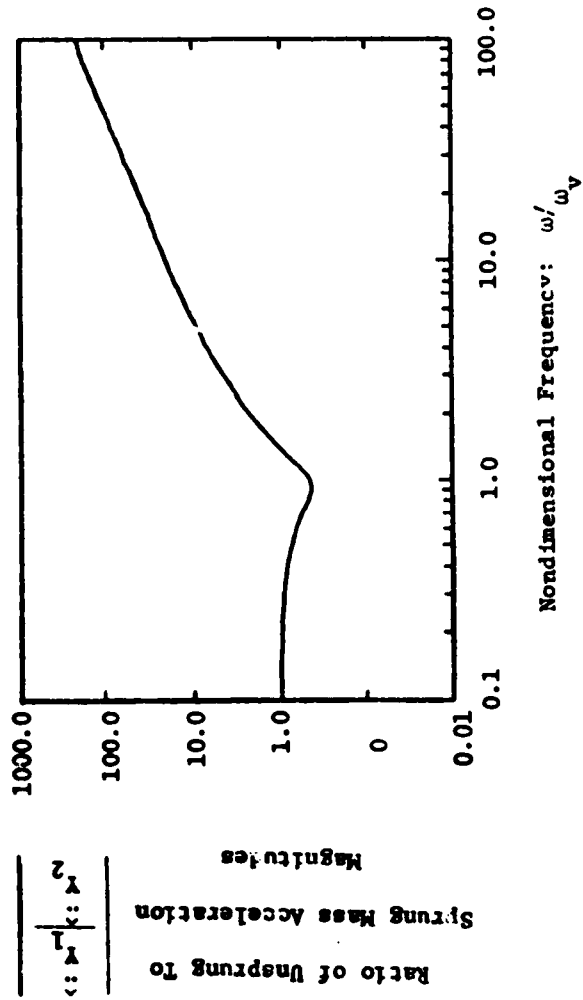


Fig. G-4. Ratio of Unsprung to Sprung Mass Acceleration

G.3 Evaluation of Vehicle Suspension Force Magnitudes

To evaluate vehicle suspension force magnitudes, the vehicle response to the crossing of single spans has been determined. Two types of data are presented below:

- (1) Parametric data illustrating vehicle suspension force variation magnitudes occurring in the partially coupled constant guideway force model.
- (2) Data comparing the partially coupled model and the fully coupled model simulations.

First parametric data is presented based upon the constant guideway force model. The nondimensional deflection of the guideway Y_0 beneath a constant force traversing the guideway is a function of only the span crossing velocity ratio V_c and span damping. Figure G.5 displays Y_0 as a function of nondimensional time vt/l_s for several values of V_c and for zero span damping. At low crossing velocity $V_c=0.15$, the deflection beneath the traveling force is symmetric with a ripple at the span natural frequency caused by the undamped span oscillating with a small amplitude about its quasi-static deflection. As the vehicle speed increases to $V_c > 0.5$ the curves become asymmetric showing the overall influence of the span dynamic effects upon the response, i.e. the span response time is approached and then exceeded as V_c increases. As the crossing frequency exceeds 1.0, the maximum deflection under the force decreases, even though, as shown in Chapter 3, the maximum midspan deflection increases since for $V_c > 1.0$, the force crosses the span midpoint before the maximum span deflection occurs.

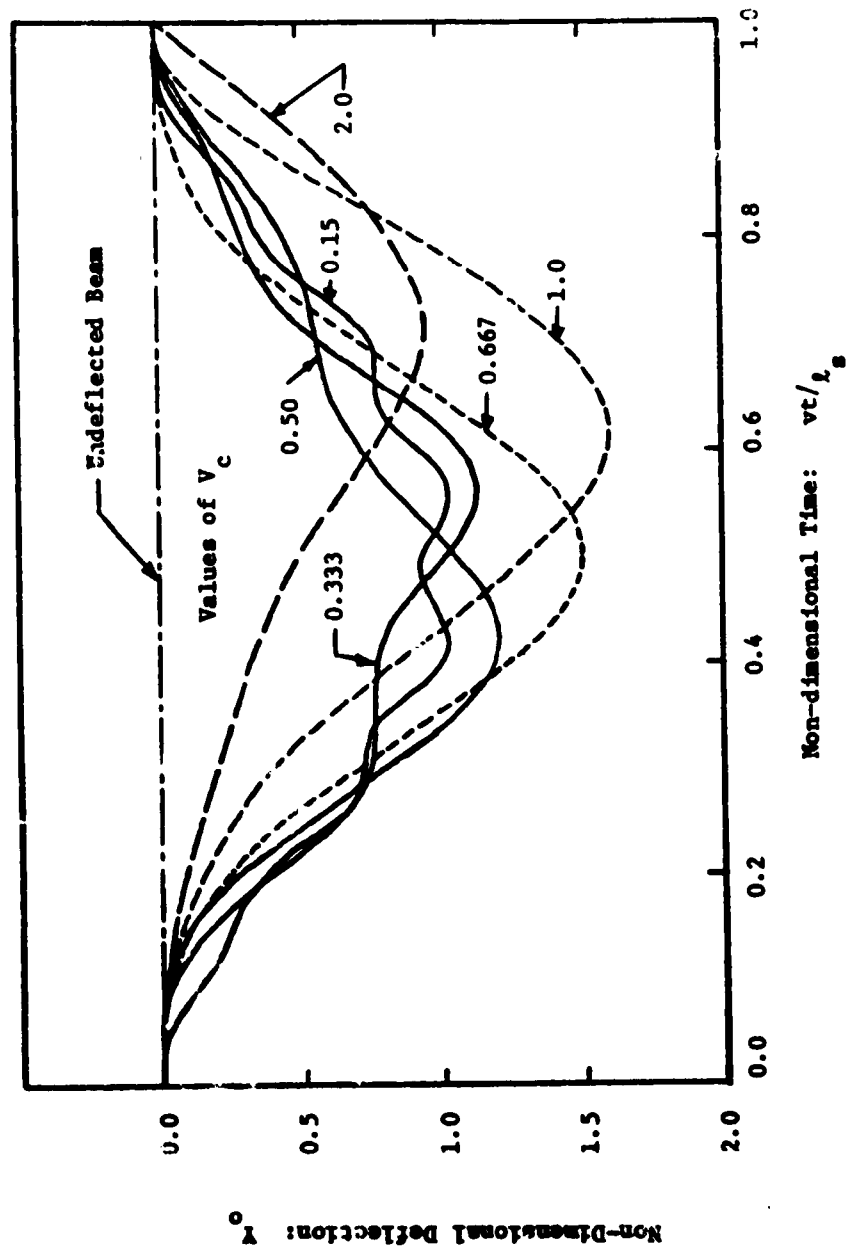


Fig. G-5. Normalized Single Span Guideway Deflection Beneath a Constant Traveling Force for Zero Damping

The guideway deflection beneath the suspension y_o , generated due to the constant force $(m_u + m_v)g$ has been represented in the Fourier series of (G.7) and the vehicle sprung and unsprung mass accelerations computed using (G.8) and (G.9). The suspension force $\bar{F}_s$ resulting from these accelerations has then been determined using (G.2). If the suspension force variation is sufficiently small then the assumption of the partially coupled model that the guideway motion may be determined assuming a constant suspension force is valid. The magnitude of the suspension force variation $\Delta \bar{F}_{sm}$ due to the vehicle accelerations in the partially coupled model has been computed to provide a measure of the partially coupled model accuracy: The variation in suspension force $\Delta \bar{F}_{sm}$ is determined as:

$$\Delta \bar{F}_{sm} = \text{Max. of } |\bar{F}_s + 1| - \text{Max. of } \left| \frac{M_u}{1+M_u} \kappa \hat{Y}_1 + \frac{1}{1+M_u} \kappa \hat{Y}_2 \right| \quad (G.10)$$

which is the maximum absolute variation of the suspension force from a constant value equal to the vehicle weight. This maximum variation is a function of the sprung and unsprung mass accelerations.

The nondimensional accelerations computed from the partially coupled model are functions of:

- (1) Span crossing velocity: $V_c = v / l_s f^*$
- (2) Span damping: ξ_m
- (3) Vehicle unsprung to sprung mass frequency ratio:
 $\Omega_u = \omega_u / \omega_v$
- (4) Vehicle unsprung to sprung mass ratio: M_u

(5) Vehicle suspension damping ratio: ξ_v

(6) Vehicle span encounter to suspension frequency ratio: $\bar{f}_e = v/l_s f_v$

where it is noted that specification of V_c and $\bar{f}_e$ are equivalent to specification of $\Omega = f^*/f_v$, the span to vehicle suspension natural frequency ratio, and either the span crossing velocity V_c or the vehicle span encounter to suspension frequency ratio. In Chapter 2, V_c and Ω are used while in this Appendix it is convenient to use V_c and $\bar{f}_e$.

In order to determine the suspension force variation $\Delta \bar{F}_{sm}$, in addition to the six quantities defined above, a value of $\kappa = \omega_y^2/g$ must be specified. The selection of κ essentially sets the dimensional sprung mass acceleration level.* In the parametric data presented below, the value of κ has been set to represent the class of high levels ride quality vehicles, i.e. κ has been set so that the vehicle sprung mass peak dimensional acceleration is 0.0707 g's. Thus, the data has been specifically prepared for vehicles with the high levels of passenger comfort desired in advanced transportation.

Parametric plots presented in Figs. G.6--G.10 illustrate the maximum suspension force variation $\Delta \bar{F}_{sm}$ as a function of V_c , Ω_u , M_u and $\bar{f}_e$ for fixed values of $\xi_m = 0$, $\xi_v = 0.25$ and for a peak value of $\ddot{y}_2 = 0.0707$ g.

*Selection of κ also sets the vehicle total mass to span mass ratio $M = (m + m_s)/\rho a l$. With the selection of M and the six quantities listed previously, the vehicle-guideway system is completely defined in terms of nondimensional parameters.

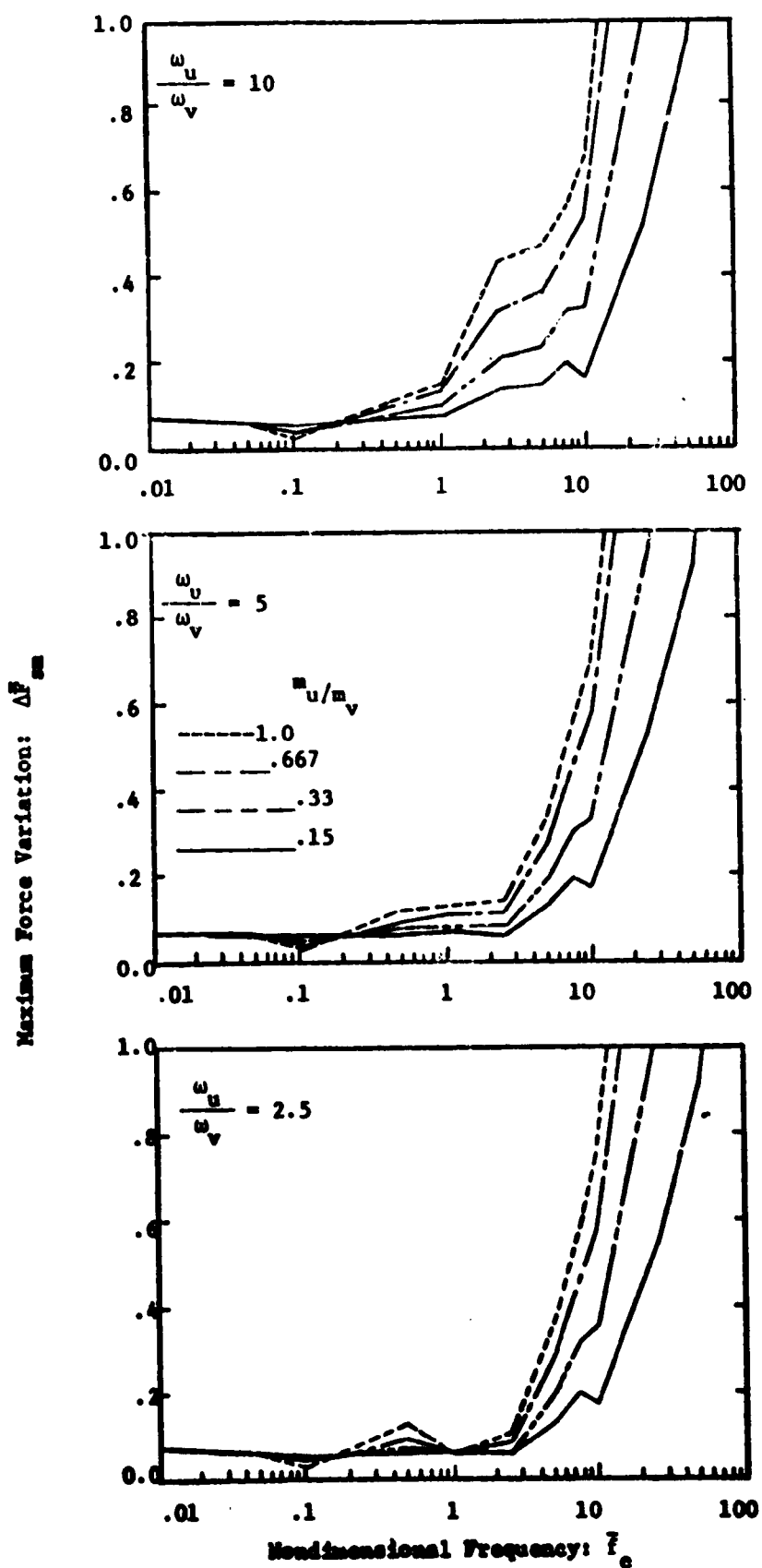


Fig. C-6. Suspension Force Variation for $V_c = 0.15$ and $\bar{y}_2 = 0.0707$ g
C-14

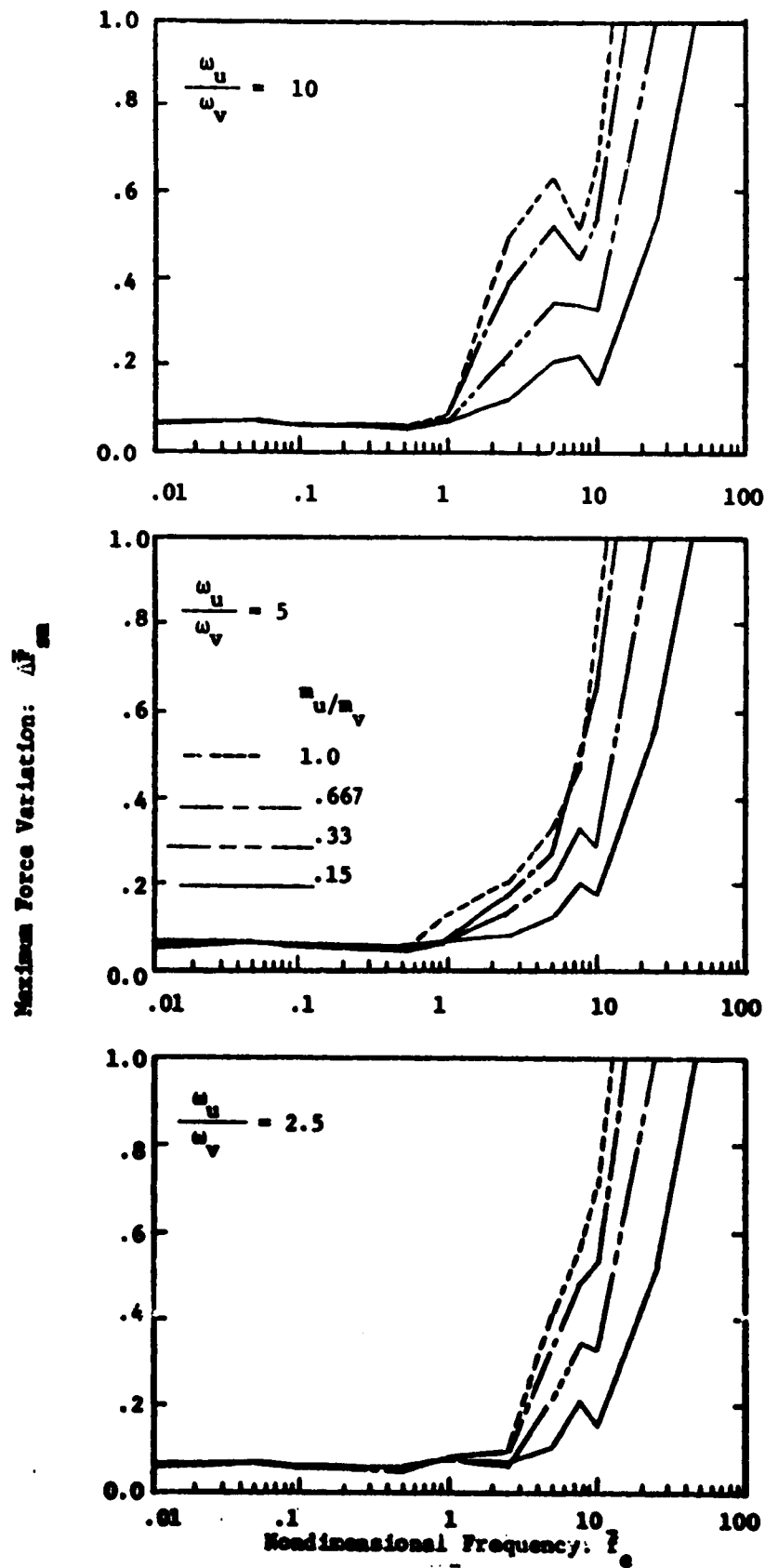


Fig. G-7. Suspension Force Variation for $V_c = 0.33$ and $\ddot{y}_2 = 0.0707 \text{ g}$

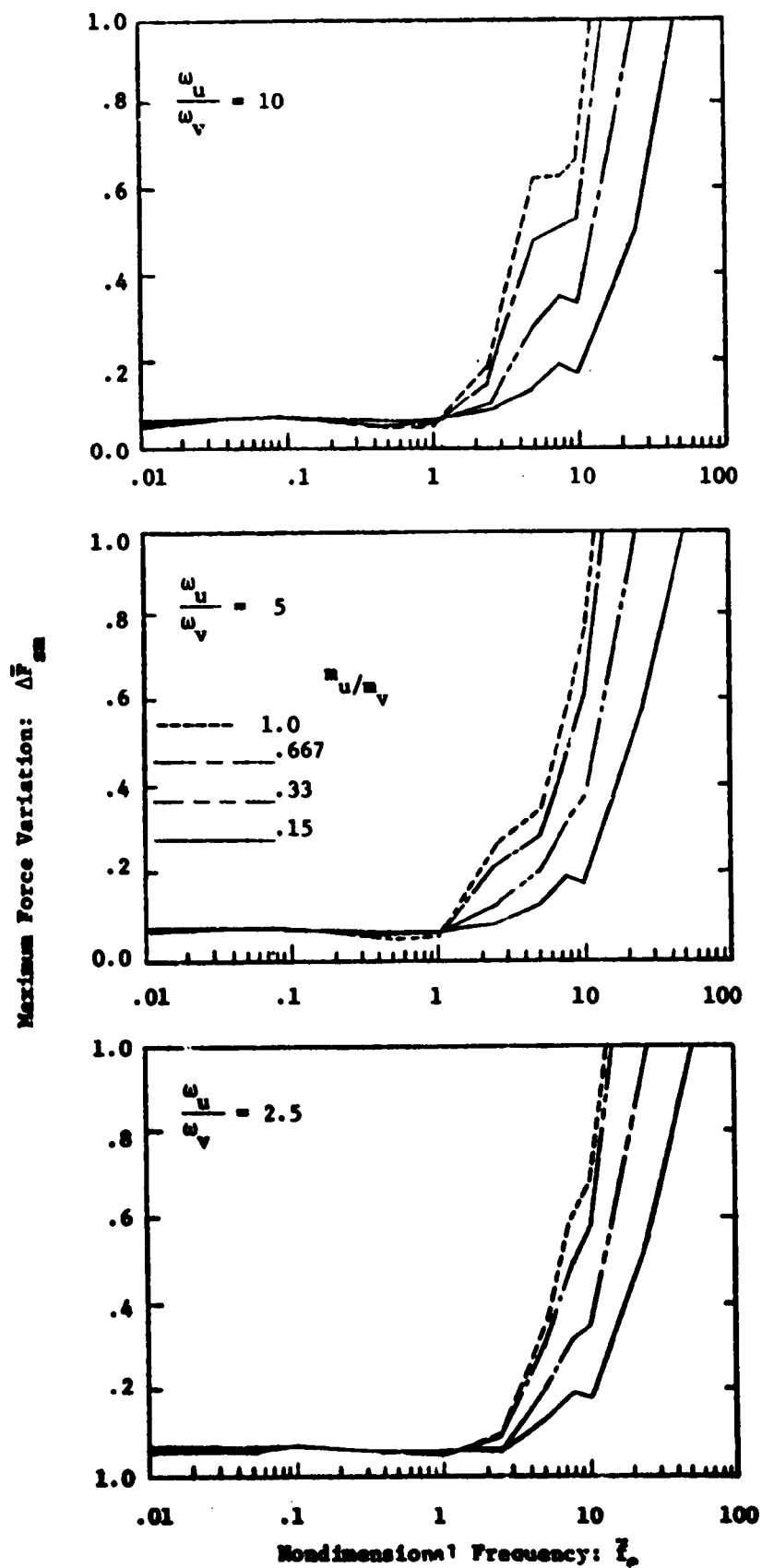


Fig. C-8. Suspension Force Variation for $V_c = 0.33$ and $\bar{y}_2 = 0.0707$ g.

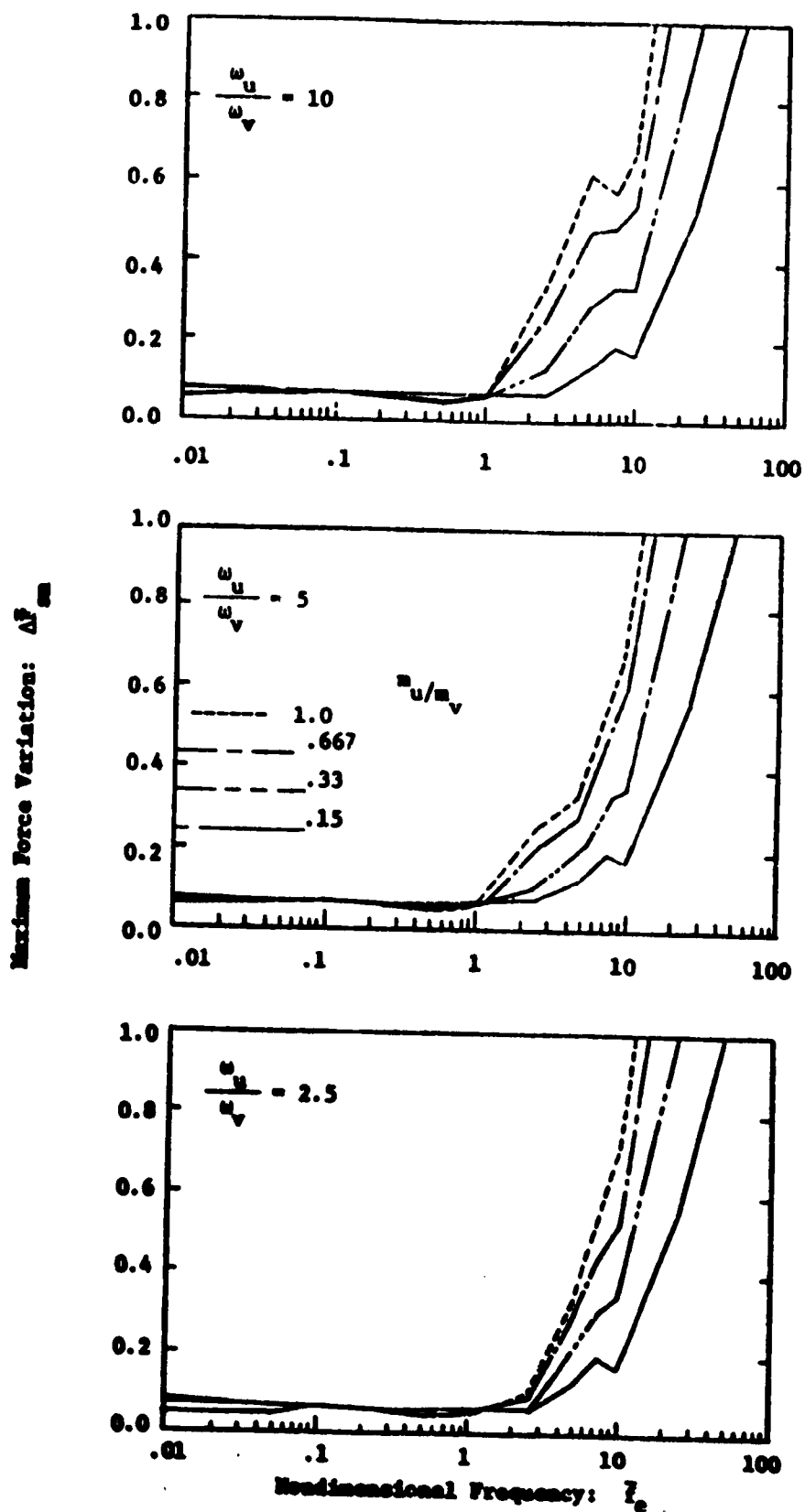


Fig. G-9. Suspension Force Variation for $V_c = 1.0$ and $\bar{y}_2 = 0.0707$ g.

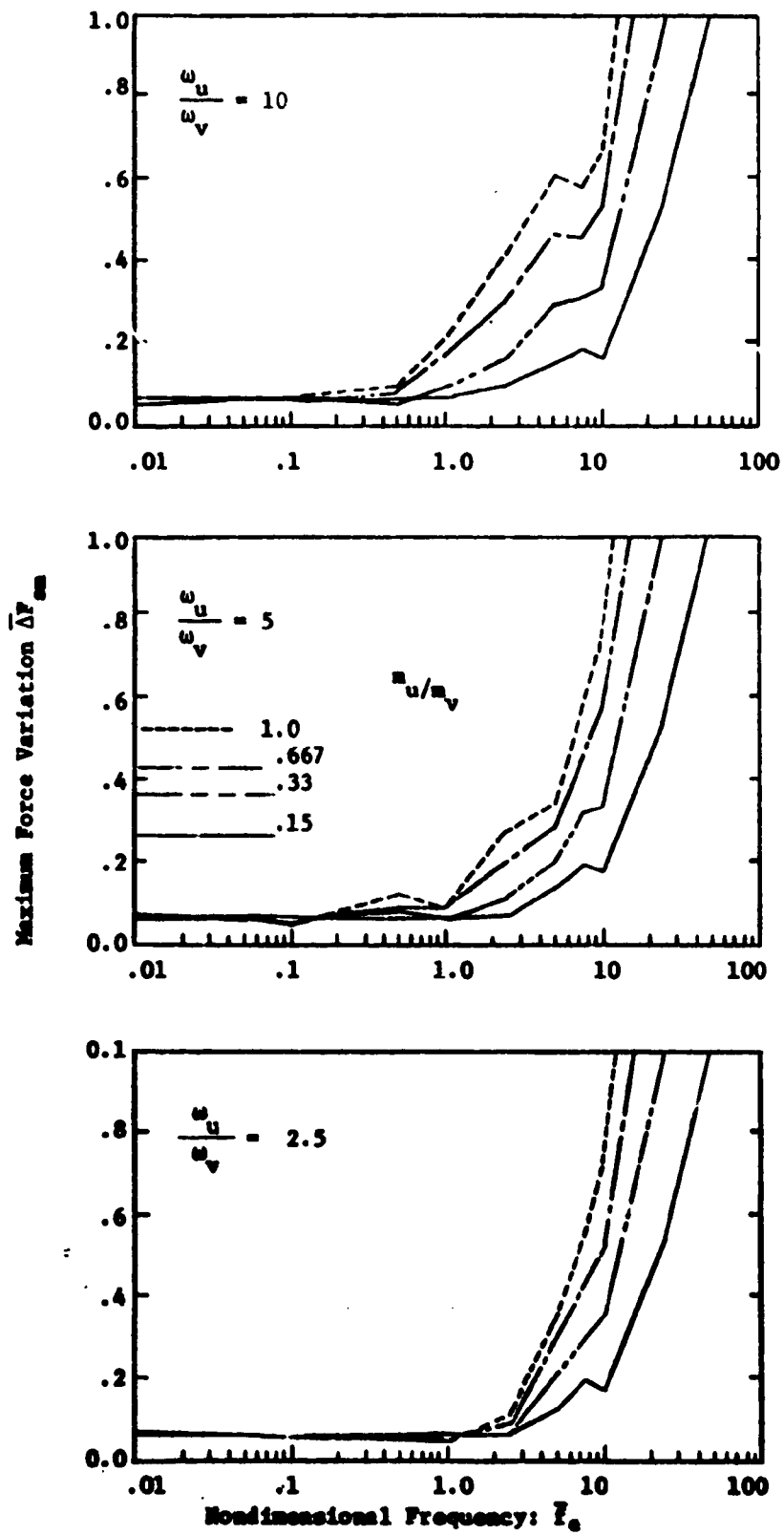


Fig. G-10. Suspension Force Variation for $V_c = 1.5$ and $\ddot{y}_2 = 0.0707 \text{ g}$

For all the data including the full range of V_c , M_u and Ω_u the suspension force variation $\Delta \bar{F}_{sm} < 0.1$ if the vehicle span encounter to suspension frequency ratio $\bar{f}_e < 0.5$ and as $\bar{f}_e$ decreases approaching 0.01, $\Delta \bar{F}_{sm}$ approaches 0.0707. At low values of encounter to suspension frequency ratio the sprung and unsprung mass move in unison with $\ddot{y}_1 = \ddot{y}_2$ and the nondimensional force variation $\Delta \bar{F}_{sm}$ is equal to the value of dimensional acceleration in g's, i.e. at small $\bar{f}_e$, $\Delta \bar{F}_{sm} = \kappa \ddot{Y}_2 = \ddot{y}_{2m}$ where $\ddot{y}_{2m}$ is the peak vehicle acceleration in g's as shown in (G.9). In the range $v/l_s f_v < 0.5$, the suspension force variation is directly related to the level of passenger comfort, $\ddot{y}_{2m}$ and for vehicles with small values of $\ddot{y}_{2m}$ and for vehicles with small values of $\ddot{y}_{2m} \approx 0.0707g$ the suspension force variation is less than 10% as shown even for vehicles with large unsprung mass ratios, $M_u = 1.0$, and the computation of vehicle and span motion and acceleration with the partially coupled model produces data which for all cases studied in this report have been found to be within better than 10% agreement with the fully coupled model. If vehicles have large sprung mass accelerations in this operating range, the force variation $\Delta \bar{F}_{sm}$ is correspondingly large and the partially coupled model is not in close agreement with a fully coupled model.

For the vehicle encounter to suspension frequency ratio range $0.5 < \bar{f}_e < 1.0$, the maximum force variation $\Delta \bar{F}_{sm} < 0.1$ for all the data except for the cases $V_c = 0.15$ and 1.5. For the cases $V_c = .33, .67$ and 1.0, the span deflection beneath the suspension Y_0 is represented principally by the first Fourier component at the

encounter frequency and higher harmonics are not very important. For these cases when $v/l_s < f_v$, Fig. G.4 shows that the unsprung to sprung mass acceleration ratio is 1.0 and the suspension force variation is simply $\Delta \bar{F}_{sm} = \kappa \ddot{Y}_2$ for all $\bar{F}_e < 1.0$, thus, $\Delta \bar{F}_{sm}$ is less than 0.1 for $\bar{F}_e < 1.0$. For the cases $V_c = 0.15$ and 1.5 the deflection profile Y_0 has principal harmonic content at the span encounter frequency v/l_s ; however, significant harmonic content is also contained in the higher harmonics, thus when $\bar{F}_e \approx 0.5$, the harmonics at multiples of $\bar{F}_e$ are sufficiently large so that the unsprung mass is driven to higher accelerations than the sprung mass and $\Delta \bar{F}_{sm}$ increases from 0.0707 to values approaching 0.15-0.2. The harmonic content at multiples of the encounter frequency in the span deflection at $V_c = 0.15$ is shown by the ripple in Y_0 displayed in Fig. G.5, and is present in the high speed case $V_c = 1.5$ because of the asymmetry of the deflection profile. Even though the magnitude of the higher harmonic content in the deflection Y_0 is less than the content at the span encounter frequency, because the ratio of unsprung to sprung mass acceleration increases directly proportional to frequency as shown in Fig. G.4, the higher harmonics in Y_0 can generate unsprung mass accelerations which exceed the sprung mass acceleration. The influence of these higher harmonics results in the increase of the force variation from 0.1 to 0.15-0.2.

Although the higher harmonics influence the force variation, for all cases including $0.15 < M_u = 1.0$, $V_c = 1.5$, $Q_u = 10$, the maximum force variation is less than 15% of the

encounter frequency v/l_s is less than the suspension natural frequency f_v and the partially coupled model may be used to gain a good preliminary estimate of vehicle-guideway motions.

In the operating range $v/l_s > f_v$ ($\bar{f}_e > 1$) the data shows the force variation $\Delta \bar{F}_{sm}$ may be large and is strongly dependent upon M_u , Ω_u and $\bar{f}_e$ at a given value of V_c . The data shows that as $\bar{f}_e$ increases to greater than 5, the force variation increases as $\bar{f}_e$ and M_u increase. In the range $\bar{f}_e > 5$, force variations greater than 0.5, may occur for vehicles with large unsprung mass $M_u > 0.66$. Comparisons of the partially coupled with the fully coupled model for cases with force variations of 0.5 have shown that vehicle accelerations predicted by the two models vary by nearly 50% and for systems with these large force variations the partially coupled model does not agree well with the fully coupled model.

To provide summary information from Figs. G.6--G.10 identifying the regions of parameters for which the maximum force variation $\Delta \bar{F}_g$ is less than 15%, Fig. G.11 has been prepared. In the figure for all values of $0.15 < V_c < 1.5$, the limiting values of mass ratio M_u and encounter to suspension frequency ratio $\bar{f}_e$ have been identified for three values of Ω_u which yield $\Delta \bar{F}_{sm} < 0.15$ when the vehicle maximum sprung mass acceleration is 0.0707 g. The data shows that for a given Ω_u as the unsprung mass ratio increases, the span encounter frequency ratio $\bar{f}_e$ for which force variations are 15% decreases. As the unsprung to

sprung mass frequency decreases, providing stronger coupling between the sprung and unsprung mass motion, the value of $\bar{f}_e$ increases for a given M_u . For example, in the case $M_u = 1.0$, the force variation is 15% if $\bar{f}_e < 1.5$ for $\Omega_u = 5$ or if $\bar{f}_e < 2.5$ even if $M_u = 1.0$ a significant range of speeds $v < 1.5 f_v l_s$ exist for which $\Delta \bar{F}_{sm} < 0.15$.

Simulation results for the partially coupled and fully coupled vehicle-guideway models have been obtained to illustrate the correlation between the variation in the maximum suspension force variation $\Delta \bar{F}_{sm}$ and the agreement between the partially and fully coupled models. The three cases summarized in Table G.1 have been selected for comparison. Case 1 with $\bar{f}_e = 1.0$ corresponds to a condition in which $\Delta \bar{F}_{sm} < 0.1$ and should provide good agreement between the two models even though $M_u = 0.66$. Case 2 with $f_e = 5.0$, $M_u = 0.33$ corresponds to $\Delta \bar{F}_{sm} = 0.3$ which should provide measurable differences in the two models while Case 3 with $f_e = 5$, $M_u = 0.66$ corresponds to $\Delta \bar{F}_{sm} = 0.45$ and should provide even greater difference between the two models.

Simulation results presented in Table G.2 show the models with the difference between all variables 10% or less. Thus, for cases in which $\Delta \bar{F}_{sm}$ is less than 0.1, agreement which is sufficiently good for preliminary design achieved. In Case 2 with $\Delta \bar{F}_{sm} = 0.3$, and $M_u = 0.33$, the differences between the partially and fully coupled models for the sprung and unsprung mass accelerations are respectively 34% and 47% with the partially coupled

TABLE G.1
CASES SELECTED FOR COMPARISON OF
FULLY AND PARTIALLY COUPLED MODEL SIMULATIONS

Case:	V_c	$\bar{f}_e$	M_u	Ω_u	M	ξ_v	ξ_m
1	0.66	1.0	0.66	5.0	0.38	0.25	0.0
2	0.66	5.0	0.33	10.0	0.24	0.25	0.0
3	0.66	5.0	0.66	10.0	0.14	0.25	0.0

All cases designed nominally to have 0.0707 g peak sprung
mass acceleration based on the partially coupled model.

TABLE G.2
COMPARISON OF PARTIALLY AND FULLY

Case	Modeling Coupling	COUPLED VEHICLE MODELS			
		Y_o	$\Delta \bar{F}_{sm}$	$\ddot{y}_2$ (g's)	$\ddot{y}_1$ (g's)
1	Full	-1.48	.075	.074	.065
1	Partial	-1.50	.071	.071	.071
2	Full	-1.40	.25	.053	.85
2	Partial	-1.50	.30	.071	1.15
3	Full	-1.33	.25	.042	.75
3	Partial	-1.50	.45	.071	1.2

model predicting greater accelerations, forces and deflections than the fully coupled model. In cases 2 and 3 for which $\Delta \bar{F}_{gm} > 0.3$, the agreement between the partially and fully coupled models is poor.

For both case 2 and 3, the constant force model predicts greater span deflections, vehicle accelerations and suspension forces than the fully coupled model. In the fully coupled model as the span is deflected downward by the vehicle, the sprung and unsprung masses must move downward also; the acceleration of these masses decreases the force exerted on the guideway span as the vehicle first enters the span and as shown by the data, the maximum guideway deflections beneath the vehicle are less in the fully coupled than the partially coupled model. The reduced guideway deflections in the fully coupled model also result in reduced accelerations and suspension force variations compared to the partially coupled model. For the cases presented the encounter frequencies are less than the unsprung mass natural frequency ω_u and the constant force model predicts accelerations which are significantly greater than fully coupled model accelerations. However, for cases in which the encounter frequency is greater than the unsprung mass natural frequency, i.e. $\bar{f}_e > \Omega_u$, the partially coupled model may predict lower accelerations than the fully coupled model.

The data in this appendix has shown that for vehicles with low levels of sprung mass acceleration, i.e. peak sprung mass

acceleration less than 0.0707 g, the partially and fully coupled models are in close agreement, within 15% when the suspension force variation $\Delta \bar{F}_{sm}$ is small, i.e. less than 15%. Fig. G.11 summarizes operating conditions for $0.15 < V_c < 1.5$, $\xi_v = 0.25$ and $\xi_m = 0$ which correspond to $\Delta \bar{F}_{sm} < 0.15$. For all values of $0.15 < V_c < 1.5$, $M_u < 1.0$, $2.5 < \Omega_u < 10$ except for $M_u = 1.0$, $\Omega_u = 10$, $V_c = 1.5$, if v/l_s is less than f_v , the suspension force variation is limited to 15% while for $v/l_s f_v > 1.0$, the range of $v/l_s f_v$ in which $\Delta \bar{F}_{sm} < 0.15$ decreases as M_u increases and as Ω_u increases. The data shows that even for vehicles with large unsprung mass $M_u = 1.0$, a significant operating range exists for which the partially coupled model yields results in close agreement with the fully coupled model. However, for conditions in which $\Delta \bar{F}_{sm}$ is large, the partially and fully coupled models are in poor agreement. These cases may occur for $v/l_s < f_v$ only if the sprung mass acceleration is large since the sprung and unsprung mass accelerations are similar and the vehicle has poor ride quality. For $v/l_s > f_v$ these cases of large force variation may occur even if the sprung mass acceleration is small because the unsprung mass acceleration may be large as shown in Fig. G.11. The data in Fig. G.11 may be used to identify these regions of large force variation.

Data For Maximum Sprung Mass Acceleration

Limit of $\ddot{y}_2 = 0.0707 \text{ g}$ and $0.15 < v_c < 1.5$

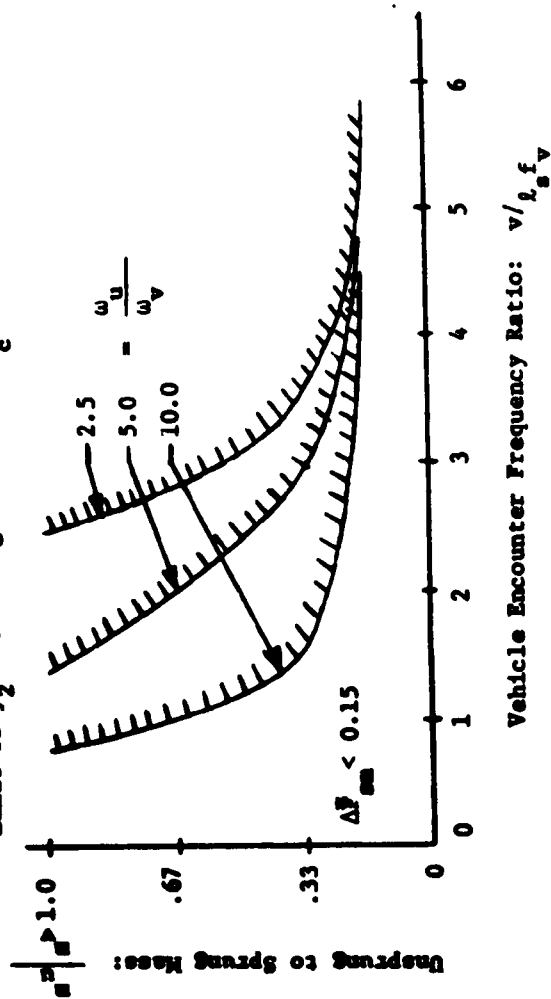


Fig. G-11. Range of System Parameters for 15% Suspension Force Variation

APPENDIX H NOMENCLATURE

a	span cross section area
a_1, a_{f1}, a_{r1}	nondimensional Fourier series coefficient of 1 th cosine term for representation of Y_o, Y_{of}, Y_{or}
a'_m	coefficient in function representing ϕ_m
a_{ms}	coefficient in function representing ϕ_{ms}
A_c	span camber amplitude
$\bar{A}_c$	nondimensional span camber amplitude: A_c/y^*
A_m	time varying m th modal coefficient
b	beam damping per unit length
b_b	secondary suspension damping
b_1, b_{f1}, b_{r1}	nondimensional Fourier series coefficient of the 1 th sine term for representation of Y_o, Y_{of}, Y_{or}
b_f	guideway foundation damping
b_m	damping associated with mode m
b'_m	coefficient in function representing ϕ_m
b_{ms}	coefficient in function representing ϕ_{ms}
b_s	secondary suspension damping, one dimensional vehicle
c	beam dimension defined in Fig. 4.3
c^*	distance from beam centroid axis to stress surface
c_{c1}	coefficient of 1 th order of numerator of T_c transfer function
c'_m	coefficient in function representing ϕ_m
c_{ms}	coefficient in function representing ϕ_{ms}
c_s	$\sqrt{E/\rho}$

c_{s1}	coefficient of 1^{th} order term of numerator of T_s transfer function
d_1	coefficient of 1^{th} order term of denominator of T_s and T_c transfer functions
d'_m	coefficient in function representing ϕ_m
d_{ms}	coefficient in function representing ϕ_{ms}
E	span elastic modulus
f	general force distribution acting on a span
f_p	force acting on guideway pier support
$f^* = f_1$	natural frequency of the first mode of vibration of a simple pinned end beam
f_v	vehicle suspension natural frequency
F_1	net force generated by the 1^{th} suspension pad
F_s, F_{sf}, F_{sr}	net force acting on guideway due to a suspension pad, a front pad, a rear pad
$\bar{F}_1, \bar{F}_s, \bar{F}_{sf}, \bar{F}_{sr}$	nondimensional suspension forces: $F_1/W, F_s/W, F_{sf}/W, F_{sr}/W$
$\bar{F}_p$	nondimensional force acting on pier f_p/W
g	acceleration due to gravity, 32.2 ft./sec. ²
G_s	soil shear modulus
h'	beam dimension defined in Fig. 4.3
h	beam height defined in Fig. 4.3
h_o	guideway pier height
I	span cross-section moment of inertia
I_v	vehicle nondimensional moment of inertia

j	$\sqrt{-1}$
k	number of spans per beam
k_b	secondary suspension stiffness
k_f	guideway foundation stiffness
k_p	primary suspension stiffness, one dimensional vehicle
k_s	secondary suspension stiffness, one dimensional vehicle
k_{sr}	primary suspension stiffness
K	vehicle primary to secondary suspension stiffness ratio
l_a	length between front and rear suspension attachment points
l_p	pad length
l_i	distance between suspension pad i and $i + 1$
l_s	length of span
l_v	$l_a + l_p$, total vehicle length
L	beam length
L_a	vehicle suspension separation length ratio: l_a/l_s
$\bar{l}_i$	nondimensional distance between suspension pads: l_i/l_s
$L_p = \bar{l}_p$	pad length to span length ratio: l_p/l_s
n	mode number
m_c	guideway support column mass
m_f	guideway foundation mass
m_s	guideway span mass
m_v	vehicle sprung mass

m_u	vehicle total unsprung mass
M	vehicle to span mass ratio
M_p	ratio of guideway span to span plus column plus footing mass
M_t	dynamic span bending moment
M_{tm}	maximum midspan moment
M_u	vehicle unsprung to sprung mass ratio
M_t^*	first mode midspan moment in a single discrete span due to a force equal to the vehicle weight concentrated at the midspan
$\bar{M}_t$	ratio of midspan moment to normalizing moment M_t^*
$\bar{M}_{tm}$	maximum normalized midspan moment
n	an integer, also number of terms in Fourier series
p	pier support number
p_i	uniform spatial pressure distribution for i^{th} suspension pad
q	number of pads
r_o	effective guideway footing radius
s	span number
S	Laplace transform operator
$\hat{S}$	nondimensional Laplace transform operator S/ω_v
t	time
$\hat{t}$	nondimensional time: $t\omega_v$
T_c	nondimensional vehicle cross suspension transfer function
T_h	nondimensional vehicle heave transfer function

T_p	nondimensional vehicle pitch transfer function
T_s	nondimensional vehicle self suspension transfer function
T_{sm}	nondimensional vehicle sprung mass transfer function, one dimension vehicle
T_{um}	nondimensional vehicle unsprung mass transfer function, one dimensional vehicle
v	vehicle speed
V_c	crossing velocity frequency ratio: $v/l_s f^*$
V	nondimensional span shear force
w	beam width defined in Fig. 4.3
W	total vehicle weight
x	spatial horizontal displacement
$x_{v1}(\bar{x}_{v1})$	position of center (front) of pad on beam
$x_{s1}(\bar{x}_{s1})$	position of center (front) of pad on span s
$X = \frac{x}{l_s}$	nondimensional horizontal position
X_d	nondimensional horizontal displaced position: $X - L_s$
y	deflection of a point on the beam due to a load
y_c	initial beam camber profile
y_m	maximum dynamic midspan deflection
y_p	pier deflection
y_t	beam total vertical displacement: $y + y_c$
y_o, y_{of}, y_{or}	vertical displacement of guideway directly beneath a pad midpoint, a front pad midpoint, a rear pad midpoint
y_1, y_{1f}, y_{1r}	vertical displacement of a vehicle unsprung mass, a front unsprung mass, a rear unsprung mass

y_2, y_{2f}, y_{2r}

vertical displacement of vehicle sprung mass at a suspension attachment point, a front suspension, a rear suspension

$$y^* = \frac{2(m_u + m_v)gl_s^3}{\pi^4 EI}$$

normalizing deflection defined as the deflection of a single pinned end span of length l_s loaded at midspan by a single concentrated force equal to $(m_u + m_v)g$

$\ddot{y}_{cg}$

vehicle center of mass acceleration

$\ddot{y}_{2d}$

passenger comfort limiting dimensional acceleration in g's

$$Y = \frac{y}{y^*}$$

ratio of vertical displacements to normalizing deflection

Y_m

maximum nondimensional dynamic span deflection

Y_{of}, Y_{or}

nondimensional values: $y_{of}/y^*, y_{or}/y^*$

Y_{ofn}, Y_{orn}

n^{th} order Fourier representations of Y_{of}, Y_{or}

Y_p

nondimensional pier displacement: y_p/y^*

$\ddot{Y}_c$

nondimensional vehicle heave acceleration:

$\hat{\ddot{Y}}_c$

$$\ddot{y}_{cg}/y^* (2\pi f_s)^2$$

$\hat{\ddot{Y}}_c$

nondimensional vehicle heave acceleration:

$\hat{\ddot{Y}}_c$

$$\ddot{y}_{cg}/u_v^2 y^*$$

$\hat{\ddot{Y}}_{c1}$

nondimensional vehicle heave acceleration at frequency θ_1

$\hat{\ddot{Y}}_{2f}, \hat{\ddot{Y}}_{2r}$

nondimensional vehicle body accelerations at front and rear suspension attachment points:

$\hat{\ddot{Y}}_{2f}, \hat{\ddot{Y}}_{2r}$

$$\ddot{y}_{2f}/u_v^2 y^*, \ddot{y}_{2r}/u_v^2 y^*$$

$\hat{\ddot{Y}}_{2f}, \hat{\ddot{Y}}_{2r}, \hat{\ddot{Y}}_{c1}$

rms values of $\hat{\ddot{Y}}_{2f}, \hat{\ddot{Y}}_{2r}, \hat{\ddot{Y}}_{c1}$

$$\alpha_n = \frac{A_n}{y}$$

nondimensional n^{th} modal coefficient

$\hat{\ddot{\delta}}$

vehicle pitch angular acceleration

$\hat{\ddot{\delta}}$

$$\text{nondimensional angular acceleration: } \ddot{\delta}/u_v^2$$

$\hat{\delta}_1$	nondimensional angular acceleration at frequency ω_1
$\tilde{\delta}_1$	rms value of $\hat{\delta}_1$
κ	constant defined in Appendix G
λ_m	m^{th} span eigenvalue
$\bar{\lambda}_m$	$\lambda_m l_s$, m^{th} normalized span eigenvalue
ν_s	poisson ratio of soil
ω_d	frequency at which acceleration passenger comfort specification is just met
ω_m	undamped natural frequency of the m^{th} mode of beam vibration
$\bar{\omega}_m = \frac{\omega_m}{2\pi f_v}$	ratio of m^{th} mode natural frequency to first mode natural frequency of simple pinned end span
ω_p	pier-foundation natural frequency
$\omega_1 = 2\pi i v / k l_s$	1^{th} Fourier frequency
$\hat{\omega}_1 = 2\pi i v / k l_s \omega_v$	1^{th} nondimensional Fourier frequency
$\omega_v = 2\pi f_v$	vehicle suspension natural frequency
Ω	span to vehicle suspension frequency ratio: f^*/f_v
Ω_p	ratio of pier to span natural frequency
π	3.1416
ϕ_m	m^{th} modal space function
$\bar{\phi}_m$	normalized m^{th} modal space function
ϕ_{ms}	m^{th} modal space function of span s
$\bar{\phi}_{ms}$	normalized m^{th} modal space function of span s

ψ_m	m^{th} modal suspension pad effect space function
ψ_{ms}	m^{th} modal suspension pad space on span s
ρ	span mass density
ρ_s	soil mass density
$\bar{\sigma}_t$	ratio of midspan stress to normalizing stress
σ_t	dynamic stress at beam exterior surface
σ_t^*	first mode midspan stress in a single discrete span due to a force equal to the vehicle weight concentrated at the midspan.
$\tau = 2\pi f^* t$	nondimensional time
θ_{h_1}	phase angle of $T_h(j\hat{\omega}_1)$
θ_{p_1}	phase angle of $T_p(j\hat{\omega}_1)$
ξ_m	m^{th} mode span damping ratio
ξ_p	pier foundation damping ratio
ξ_v	vehicle suspension damping ratio

APPENDIX I
REPORT OF INVENTIONS

A diligent review of the work performed under this contract has revealed no new innovation, discovery, improvement or invention.