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# Load and Resistance Factor Design (LRFD) for Highway Bridge Superstructures



## DESIGN EXAMPLES



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<b>16. Abstract</b> <p>This document provides two comprehensive superstructure design examples for the application of Load and Resistance Factor Design (LRFD) to highway bridge design. One design example is a two-span steel plate girder bridge, and the other is a two-span prestressed concrete girder bridge with simple span prestressed girders made continuous for live load. These design examples accompany a four-day training course that presents the theory, methodology, and application for the design and analysis of both steel and concrete highway bridge superstructures. The design examples, as well as the training course, are based on the <i>AASHTO LRFD Bridge Design Specifications</i>, Seventh Edition, 2014, with Interim Revisions through 2015.</p> <p>The first design example includes a series of flowcharts that present the general design steps required for the design of a steel plate girder bridge. The flowcharts include references to the <i>AASHTO LRFD Bridge Design Specifications</i>, as well as accompanying notes to assist the bridge engineer in understanding the various design steps. This design example also includes six major design steps that illustrate the application of these design procedures. The six major design steps included in this design example are general information, deck design, steel girder design, splice design, miscellaneous steel design, and bearing design.</p> <p>The second design example also includes a series of flowcharts that present the general design steps required for the design of prestressed concrete superstructure bridges. This design example includes five major design steps. The design steps included are general information, deck design, prestressed concrete girder design, bearing design, and substructure design (integral abutment and intermediate pier and foundation design). (It should be noted that the substructure design steps are based on a previous version of <i>AASHTO LRFD Bridge Design Specifications</i> and have not been updated.)</p> <p>The design examples include references to <i>AASHTO LRFD Bridge Design Specifications</i>, commentary to assist the bridge engineer in understanding its application, and a variety of figures and tables to supplement the narrative and calculations.</p>					
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**Load and Resistance Factor Design (LRFD)  
for  
Highway Bridge Superstructures**

**STEEL DESIGN  
EXAMPLE**

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## Design Example

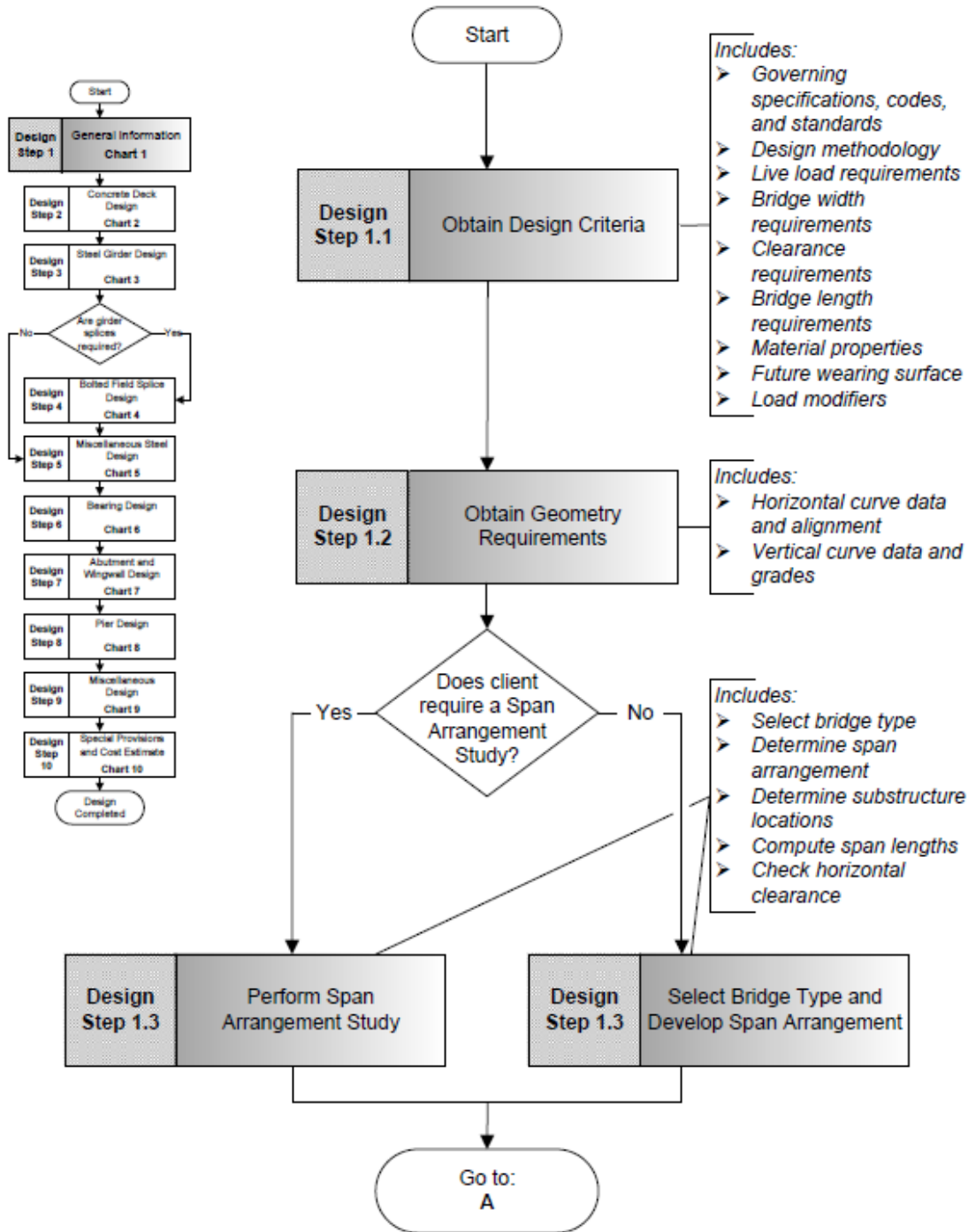
Design Step 1	General Information
Design Step 2	Deck Design
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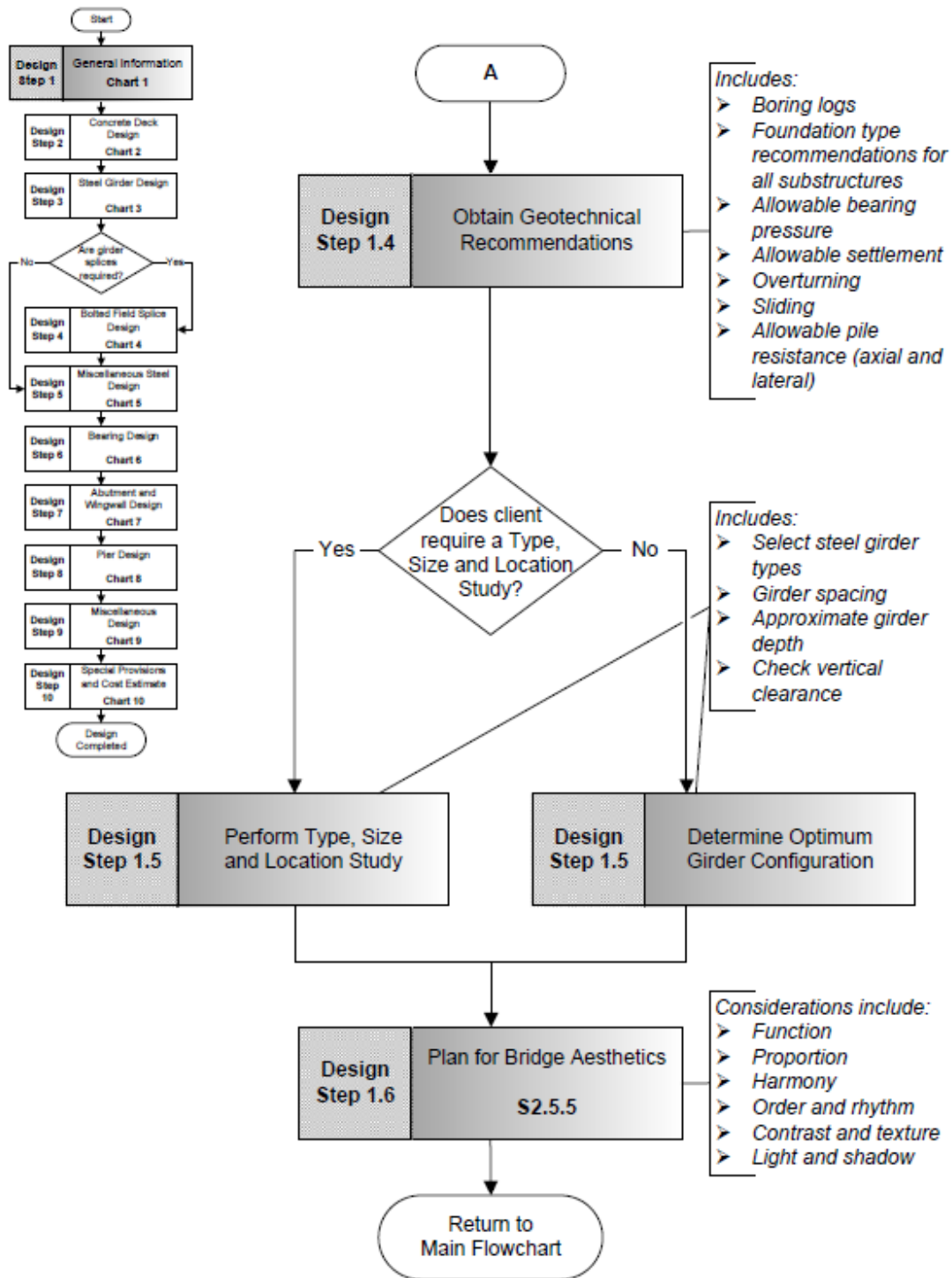
General Information Flowchart

Chart 1



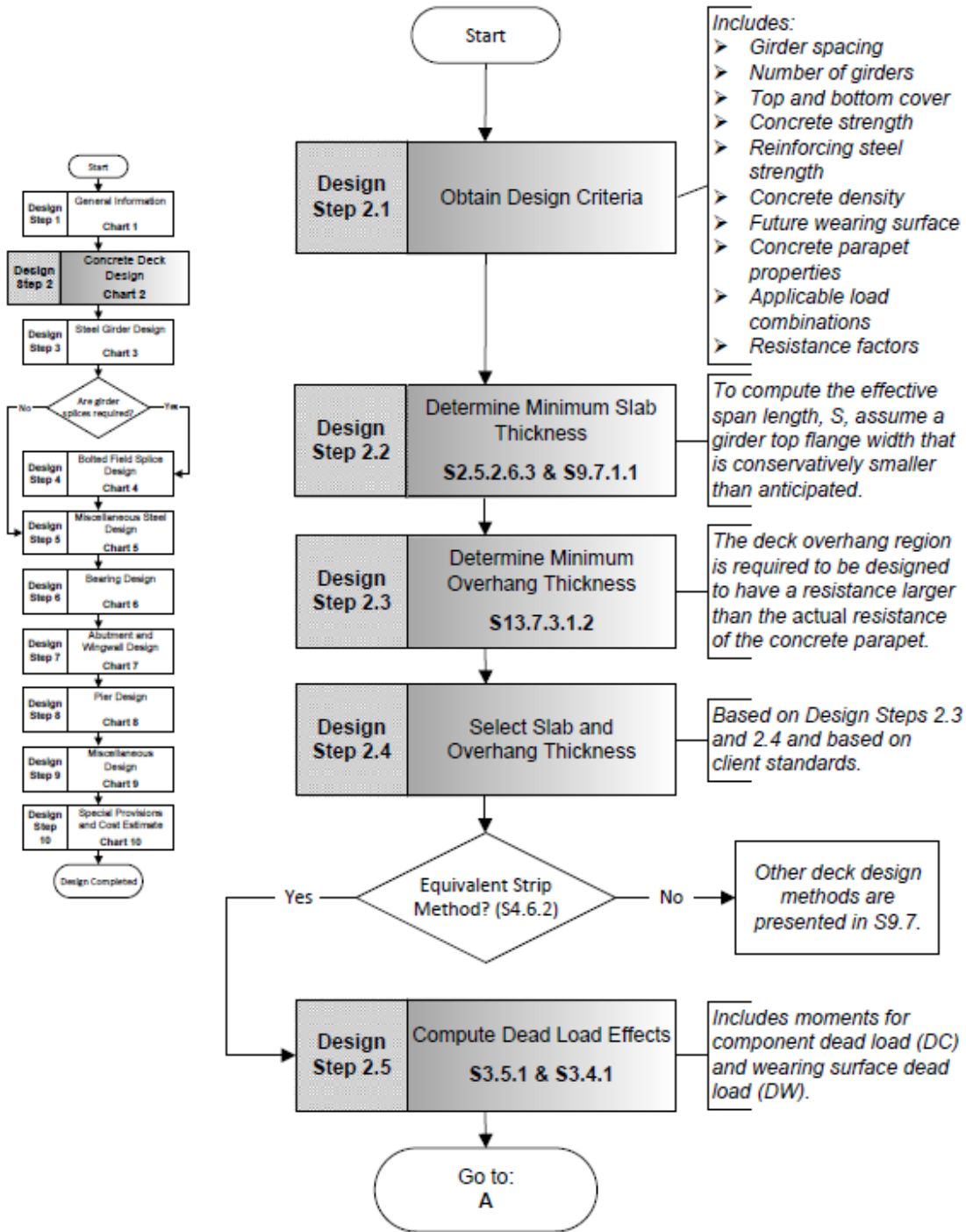
General Information Flowchart (Continued)

Chart 1



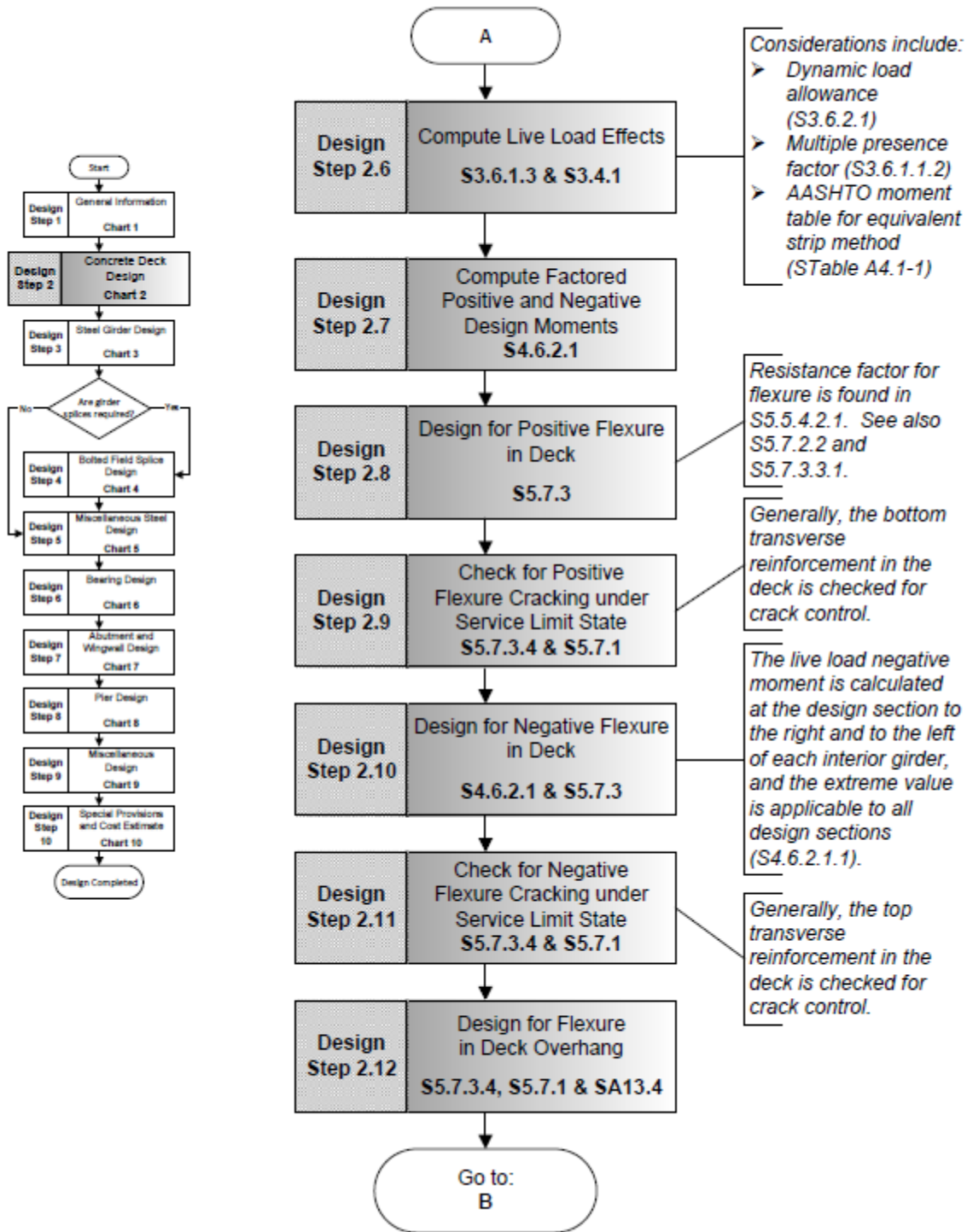
Concrete Deck Design Flowchart

Chart 2



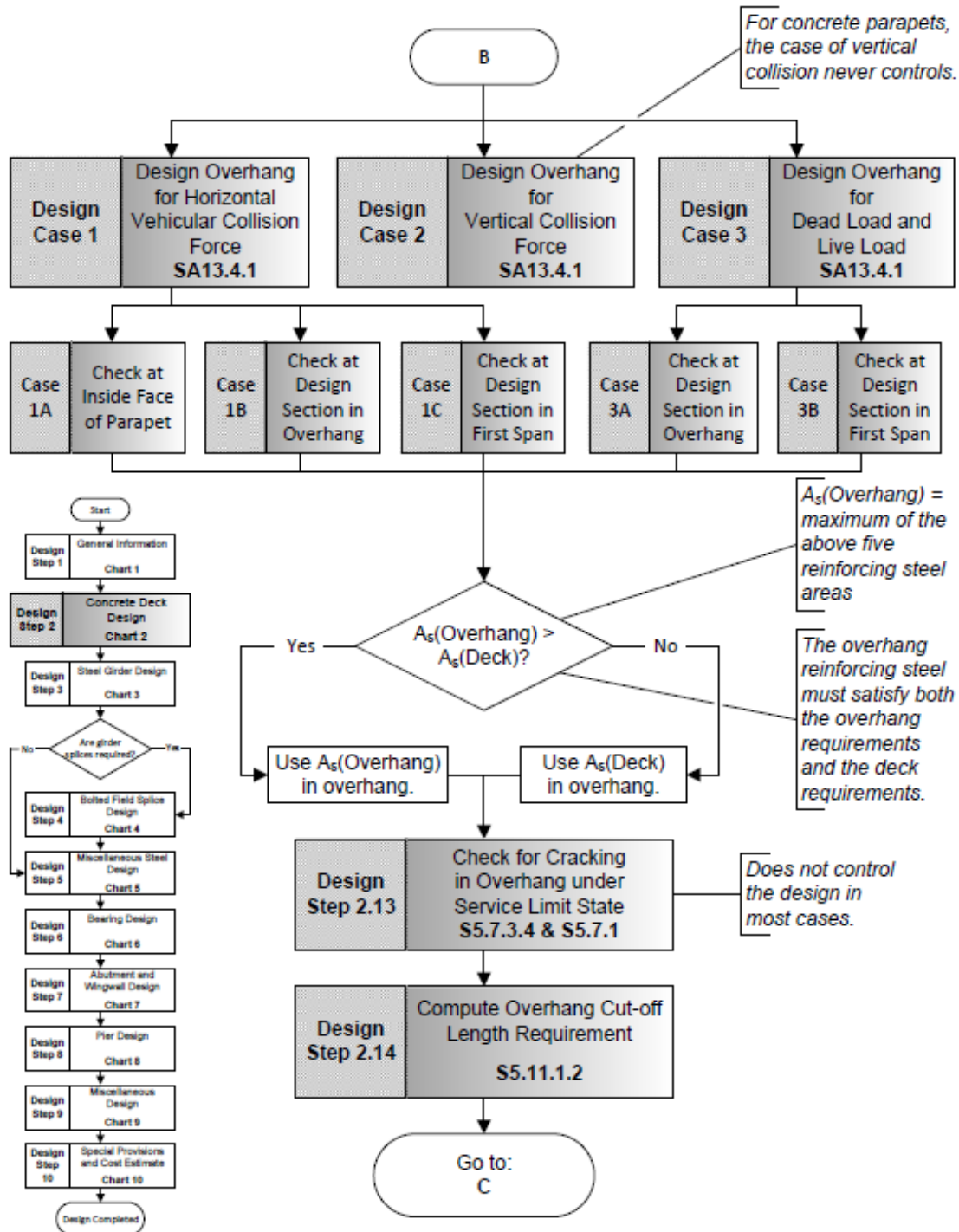
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Chart 2



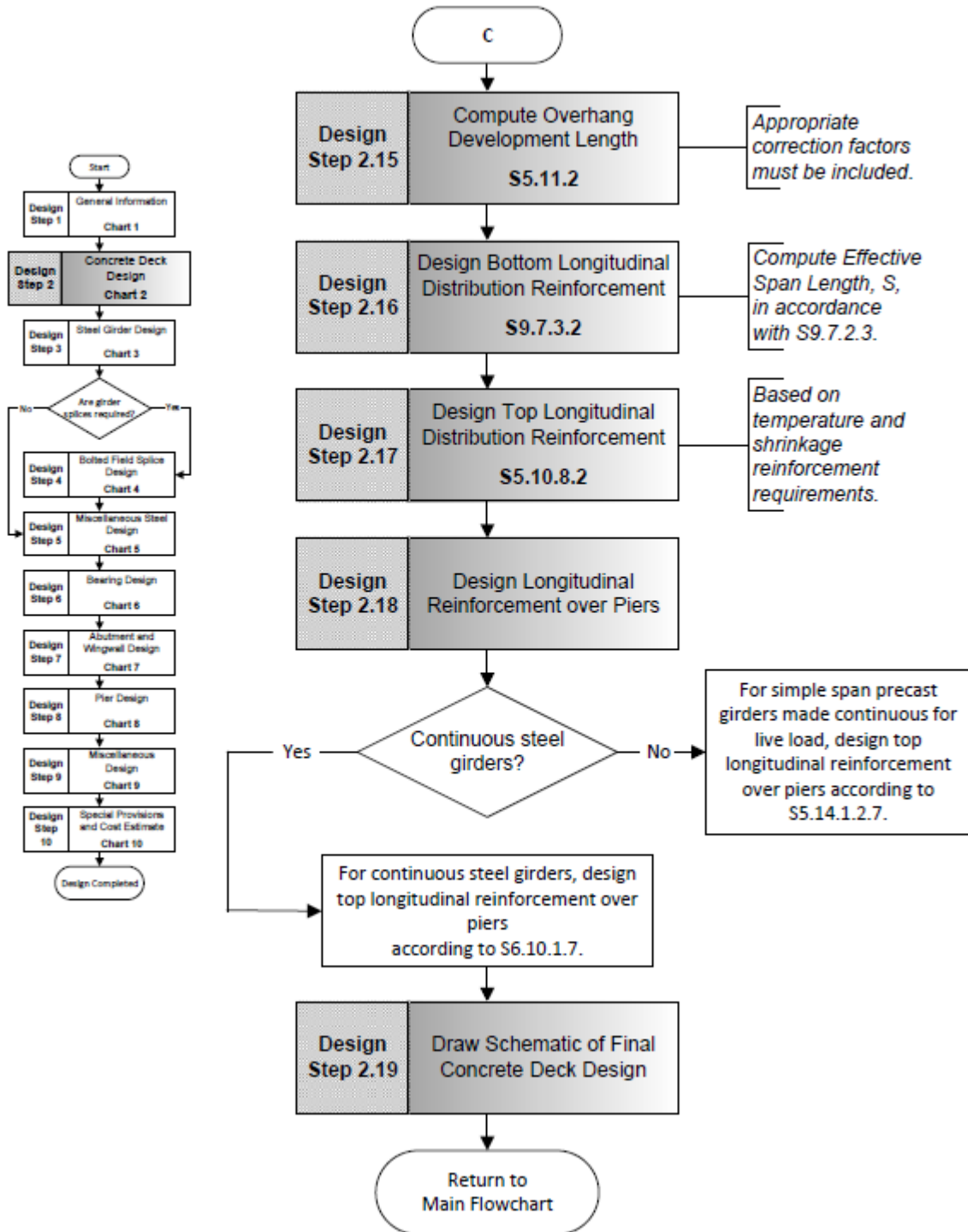
Concrete Deck Design Flowchart (Continued)

Chart 2



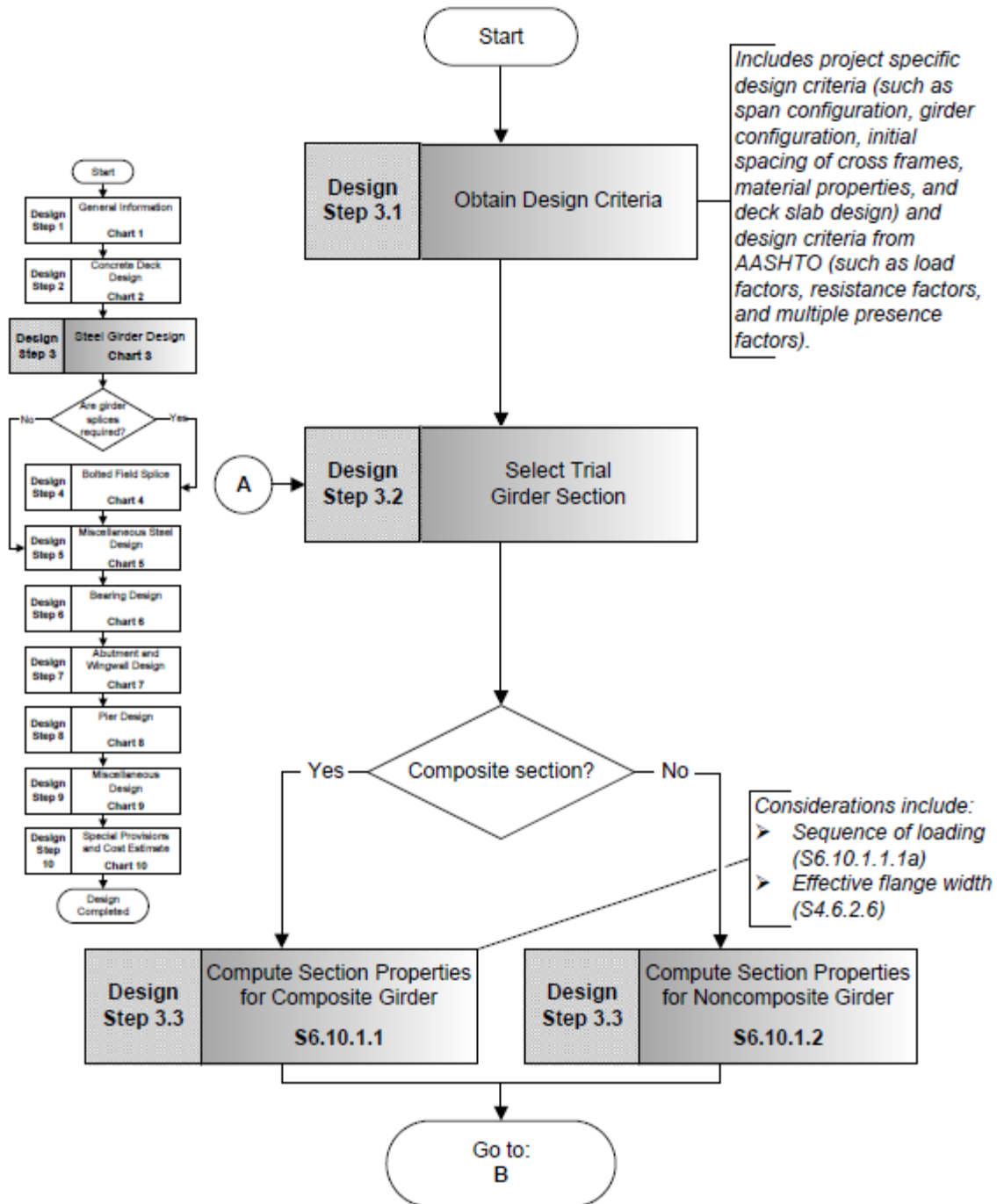
Concrete Deck Design Flowchart (Continued)

Chart 2



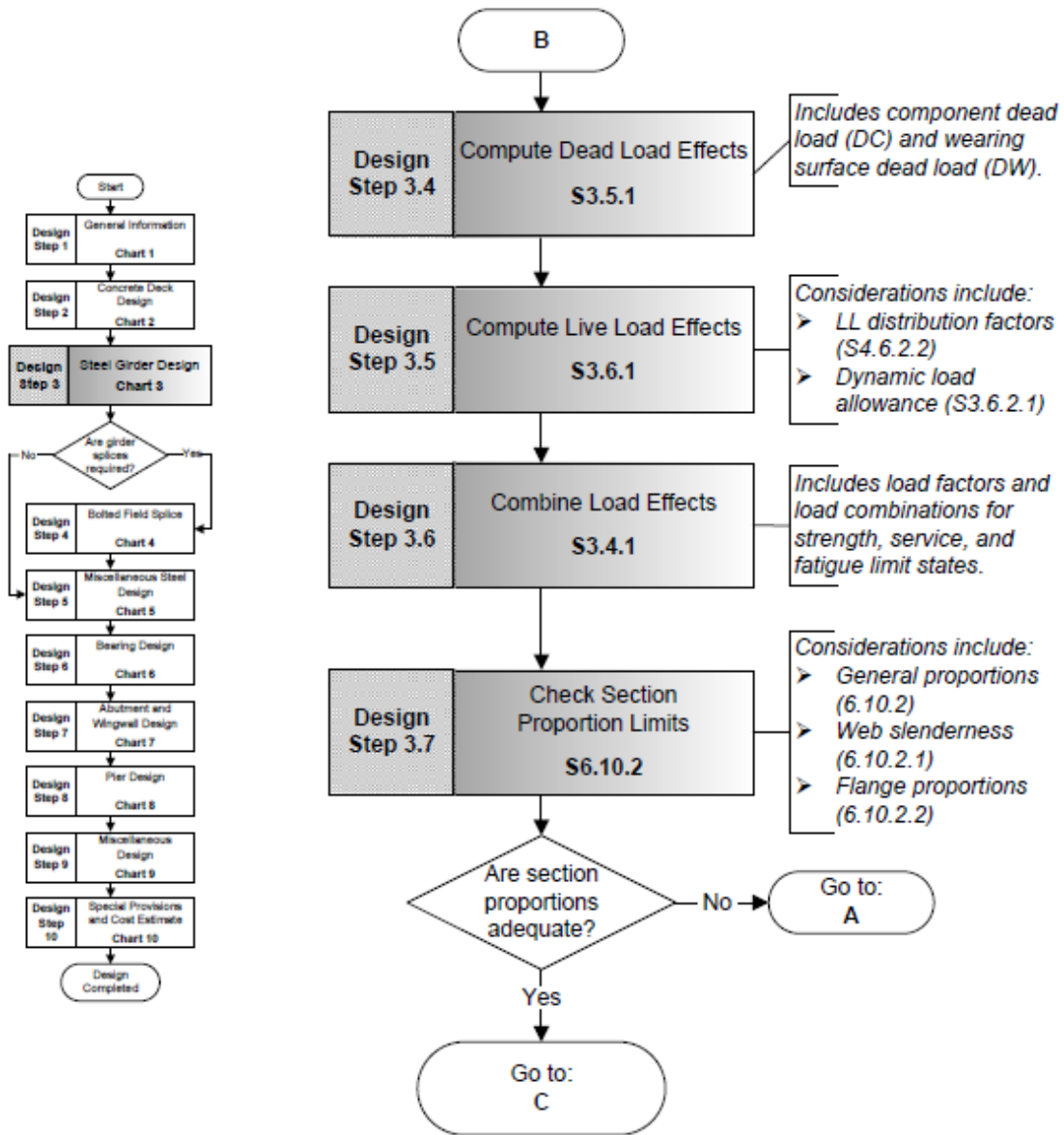
Steel Girder Design Flowchart

Chart 3



Steel Girder Design Flowchart (Continued)

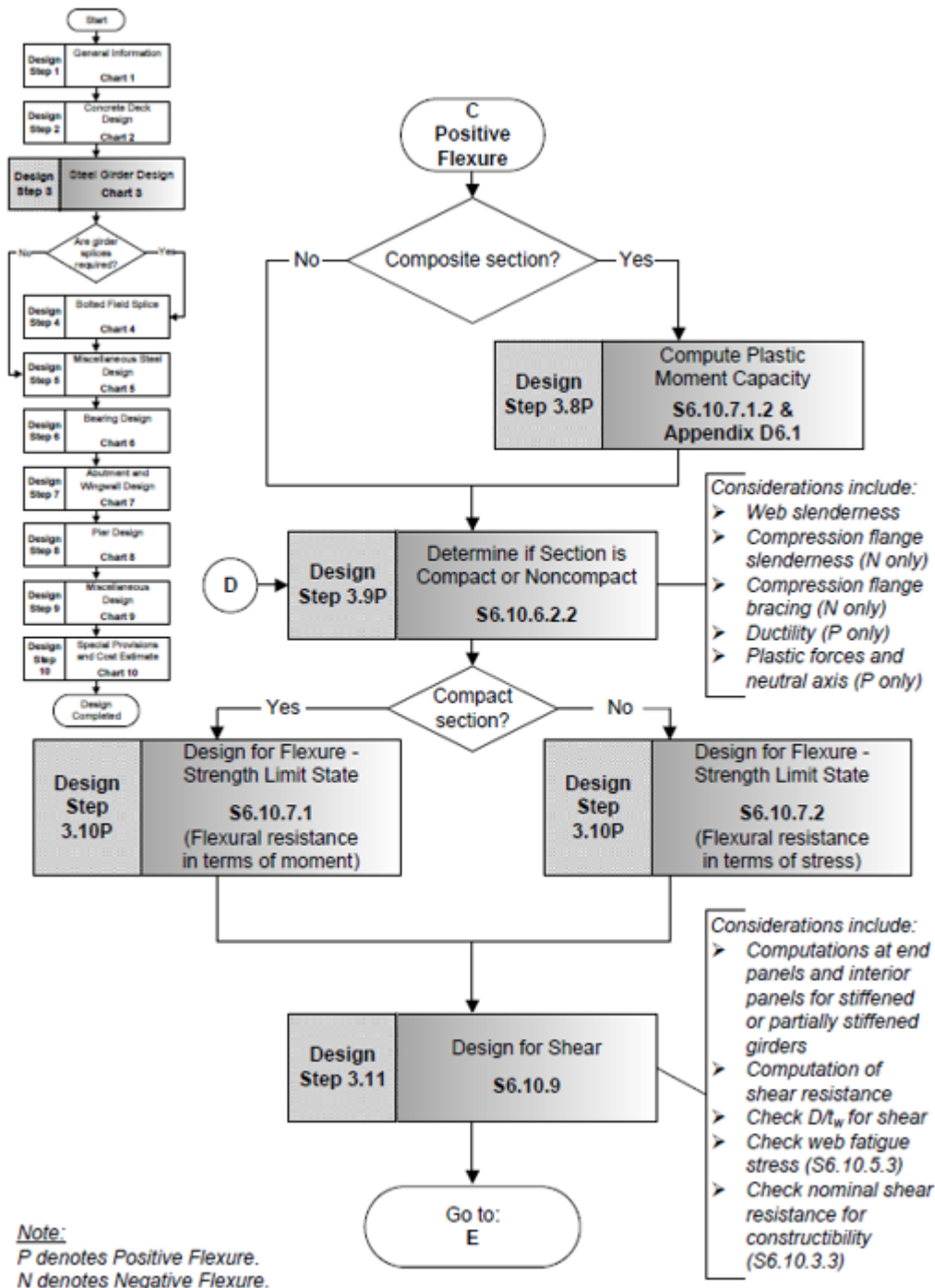
Chart 3





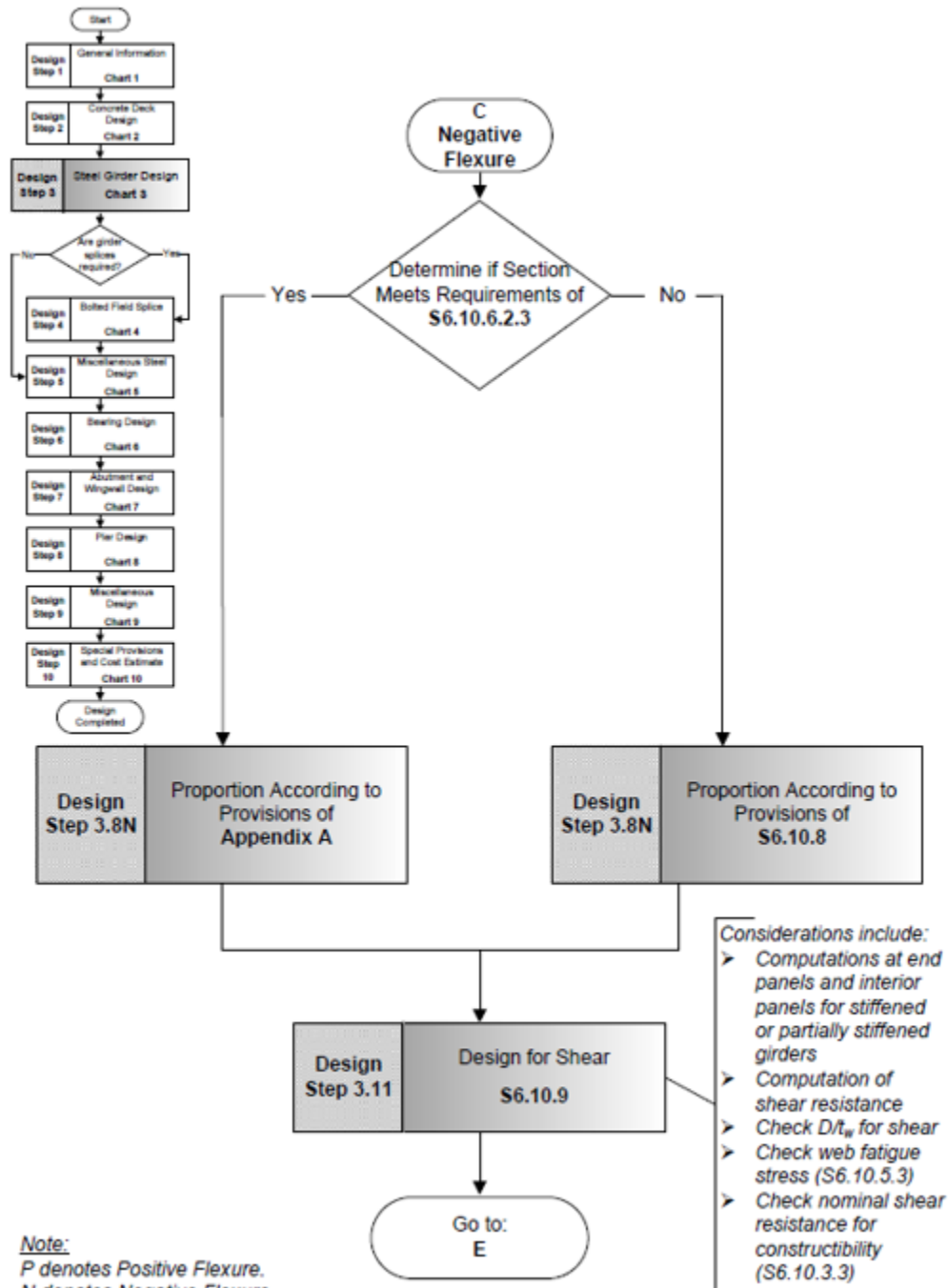
Steel Girder Design Flowchart (Continued)

Chart 3



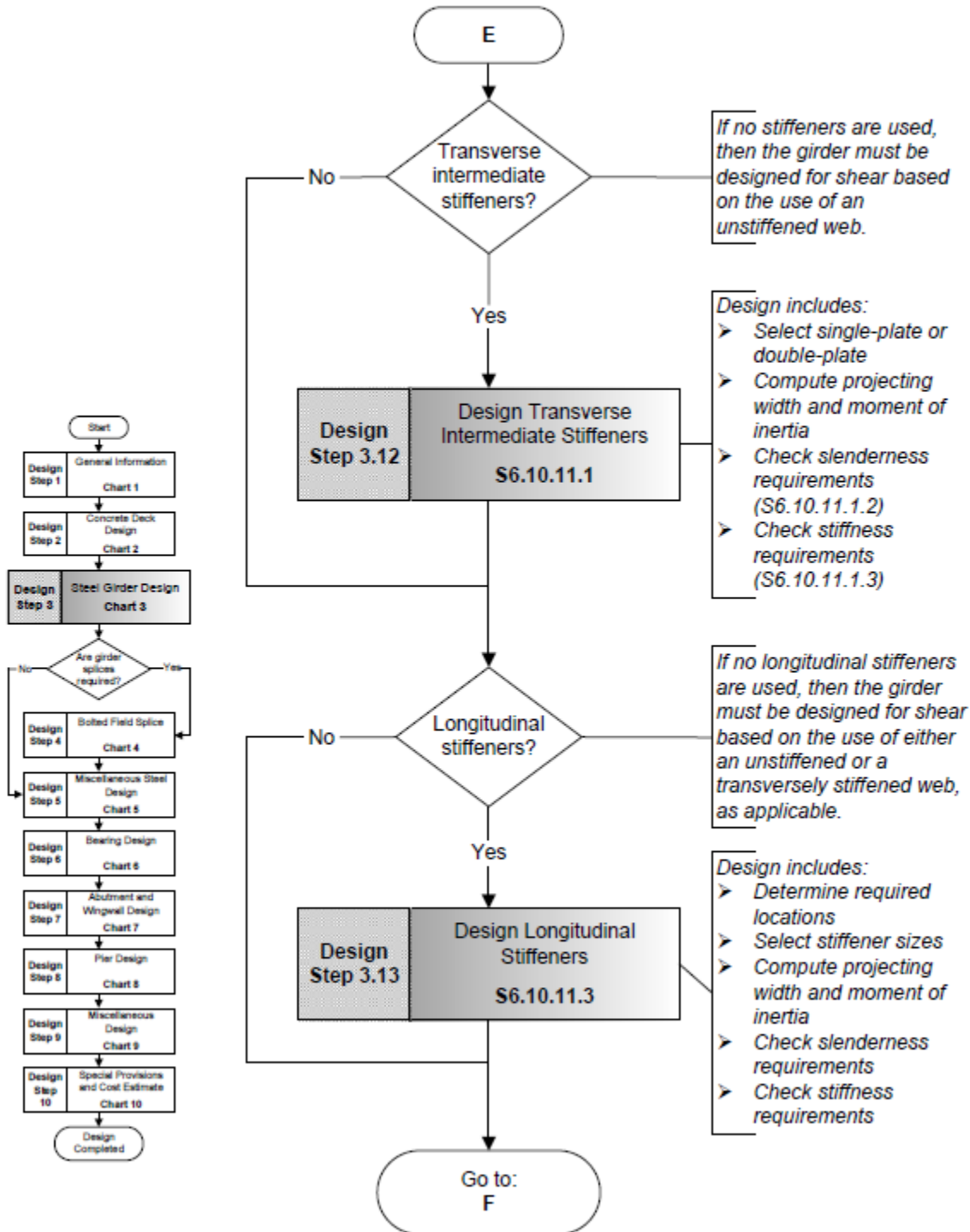
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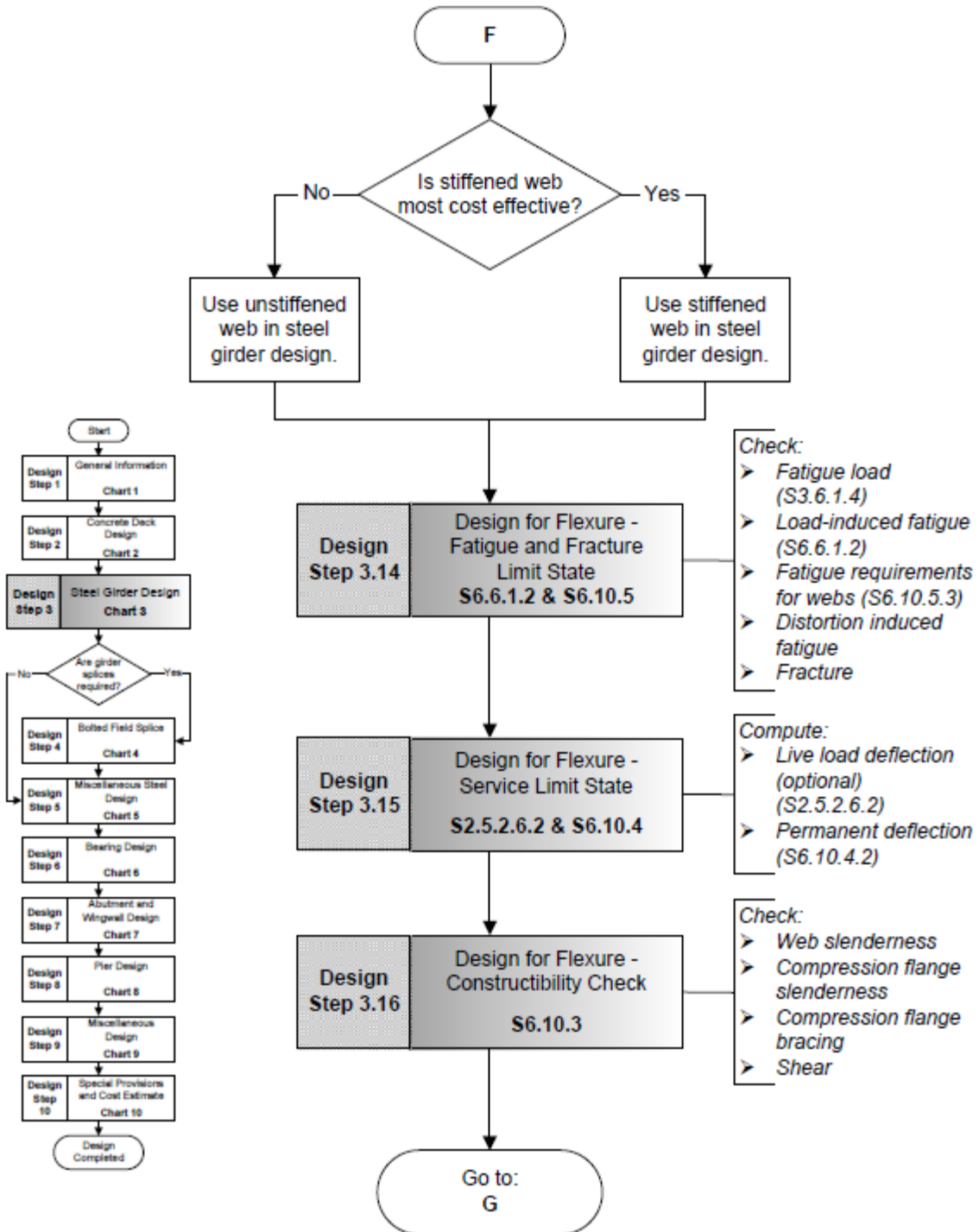
Steel Girder Design Flowchart (Continued)

Chart 3



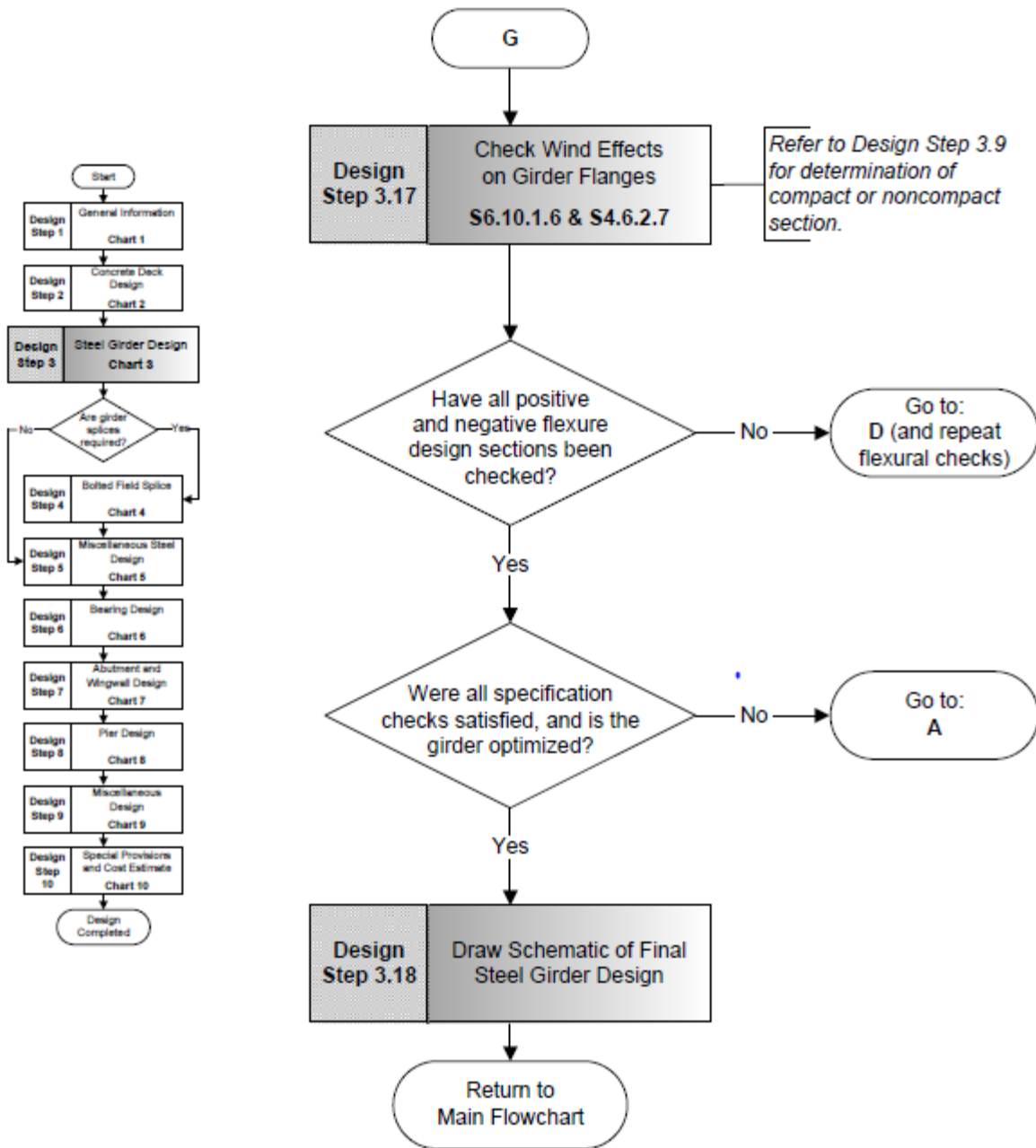
Steel Girder Design Flowchart (Continued)

Chart 3



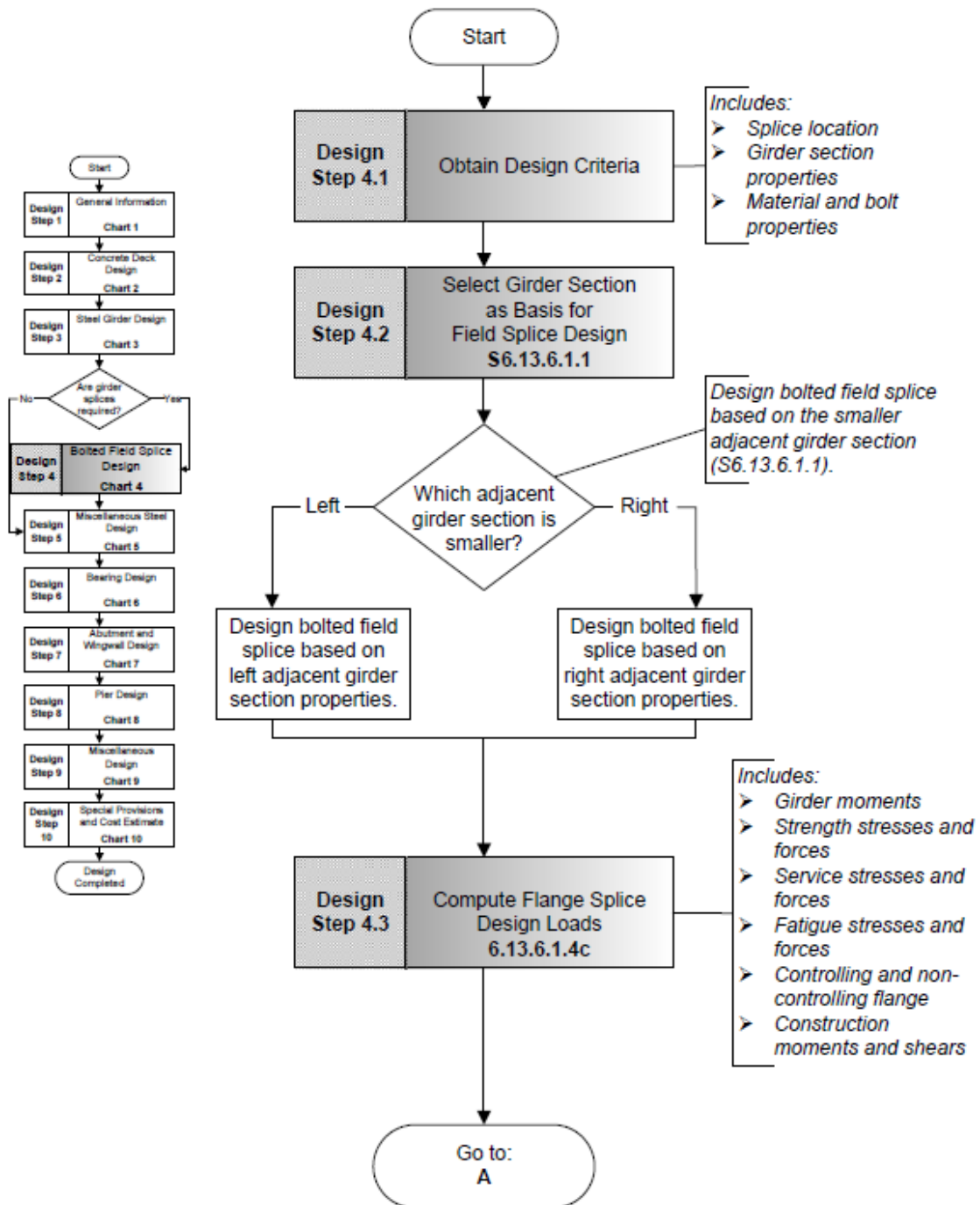
Steel Girder Design Flowchart (Continued)

Chart 3



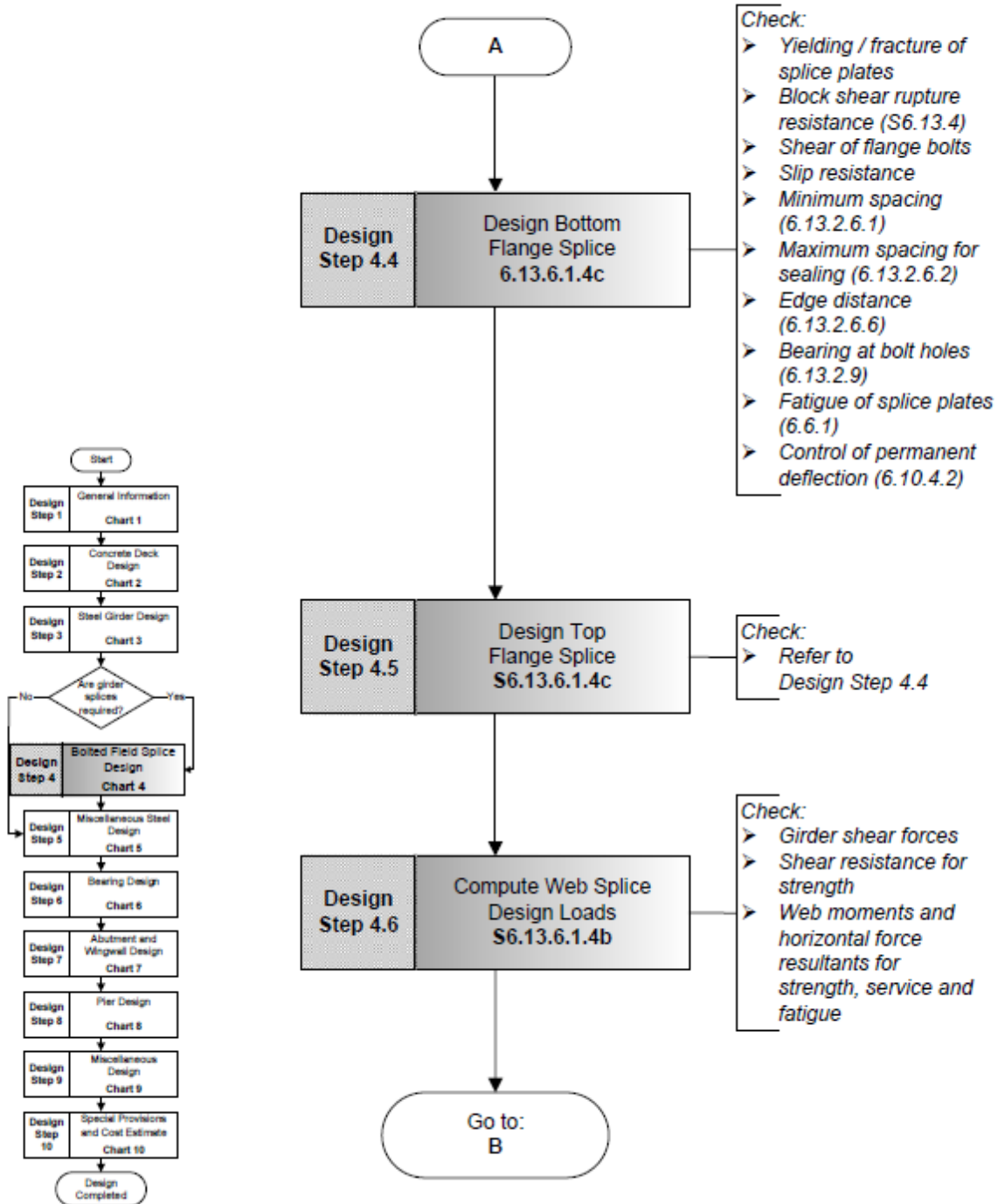
**Bolted Field Splice Design Flowchart**

**Chart 4**



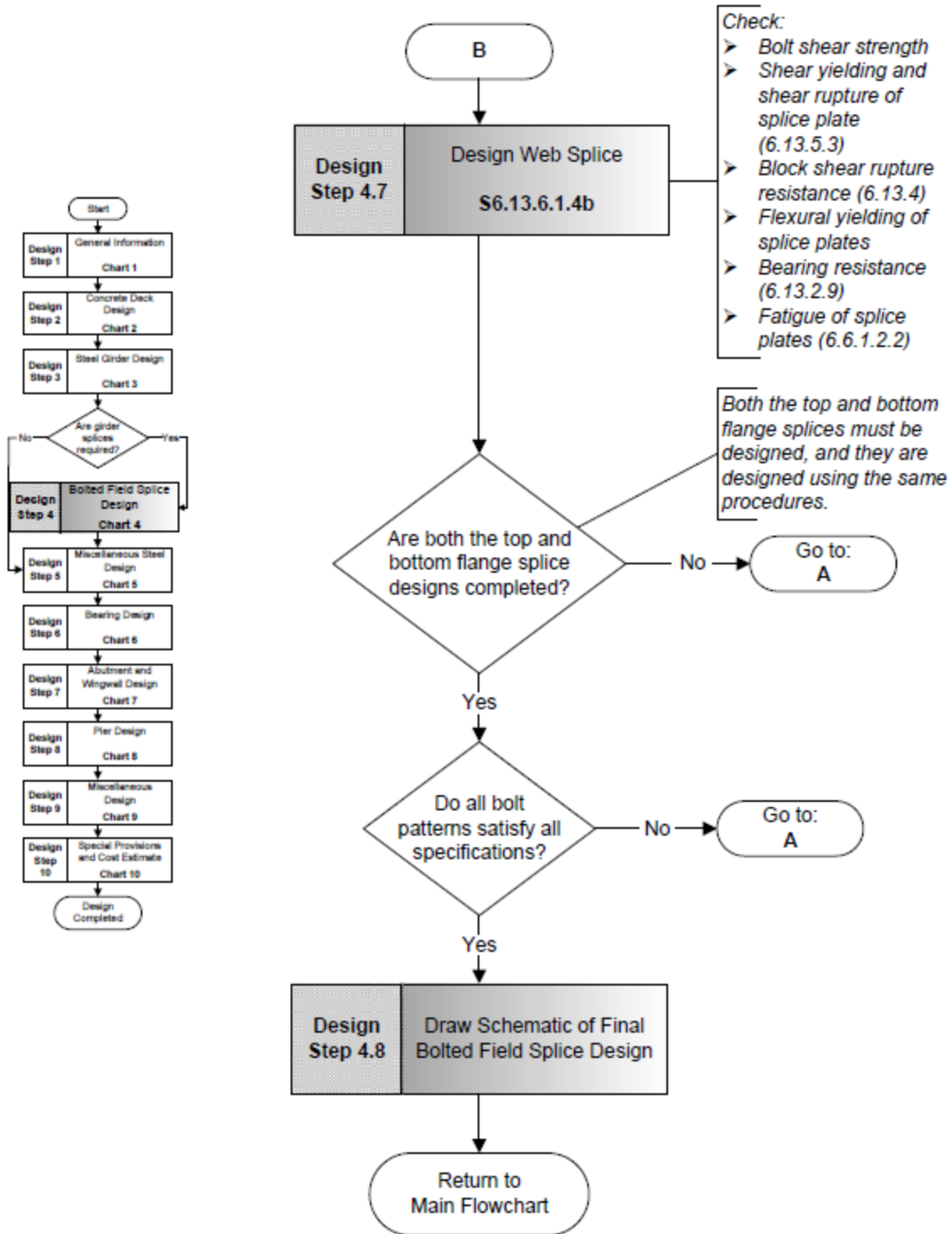
**Bolted Field Splice Design Flowchart (Continued)**

**Chart 4**



**Bolted Field Splice Design Flowchart (Continued)**

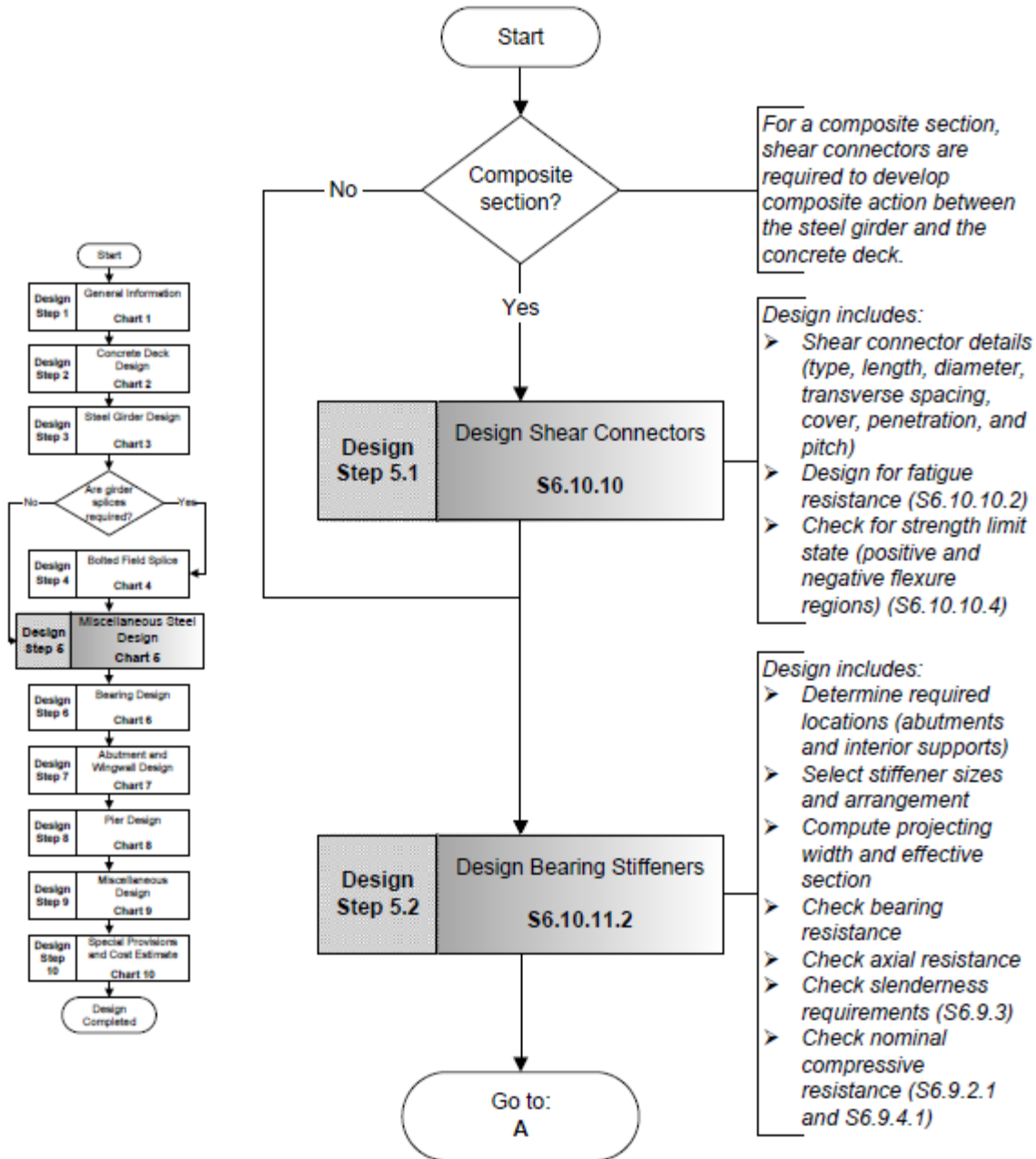
**Chart 4**





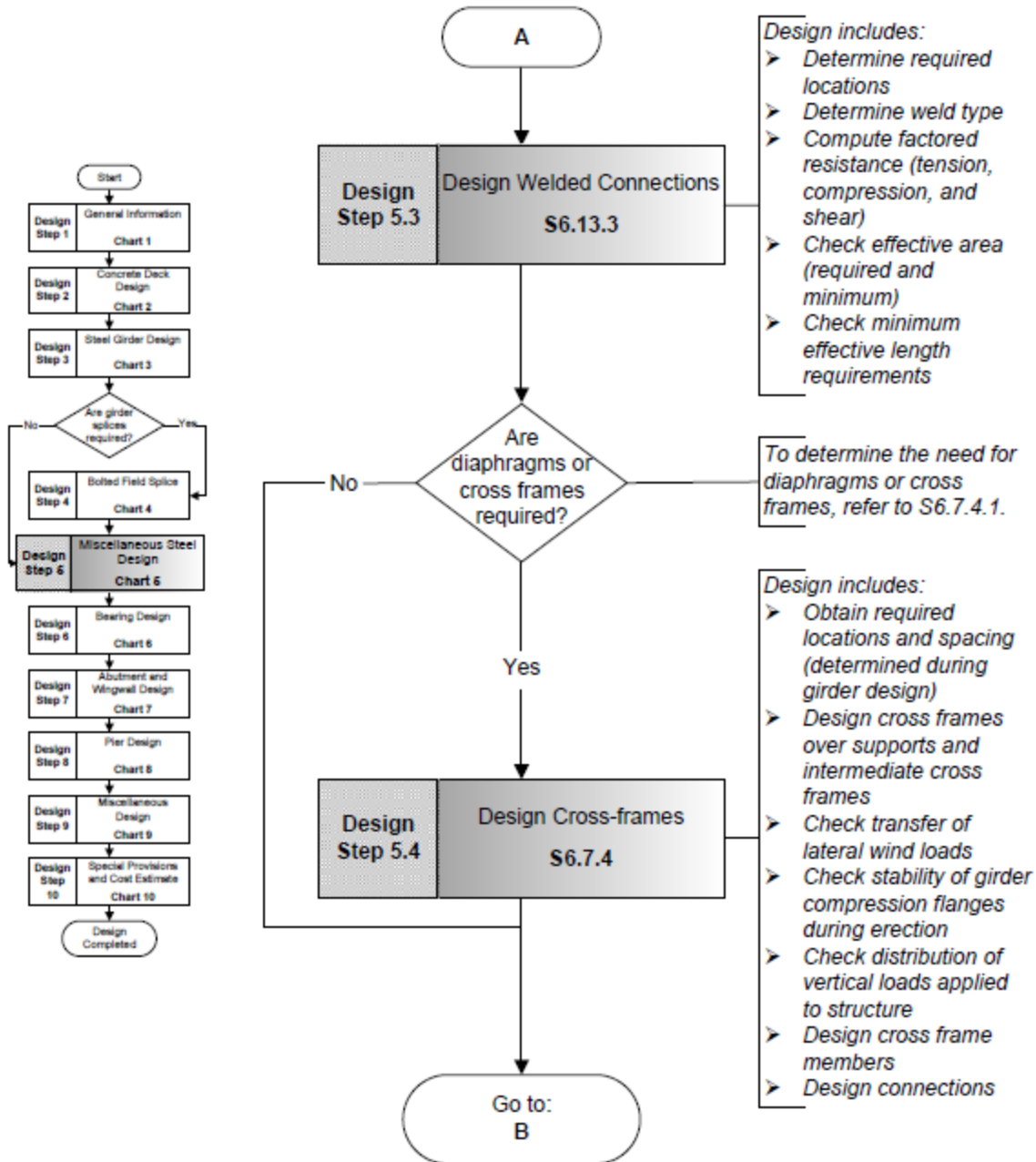
Miscellaneous Steel Design Flowchart

Chart 5



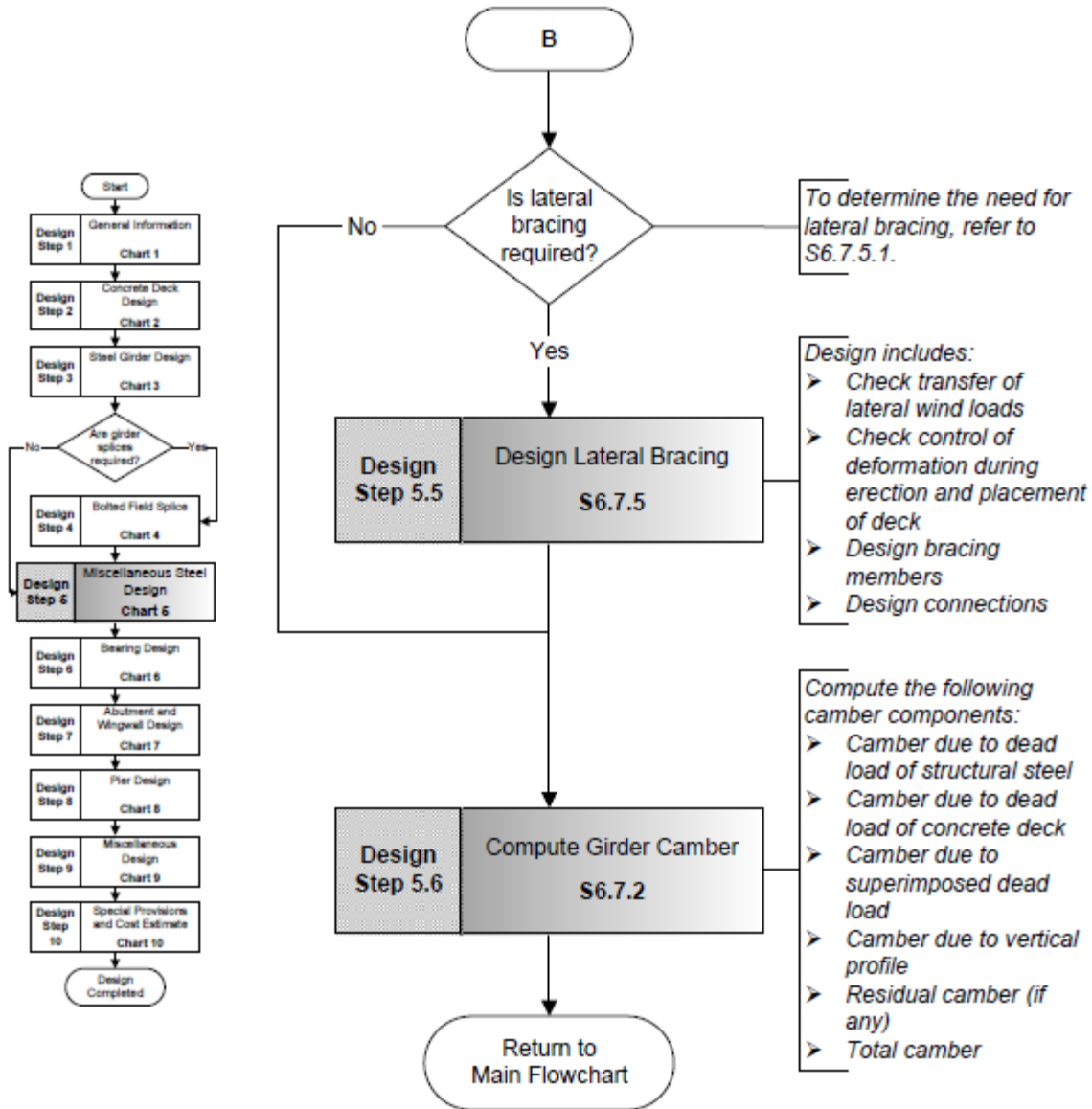
Miscellaneous Steel Design Flowchart (Continued)

Chart 5



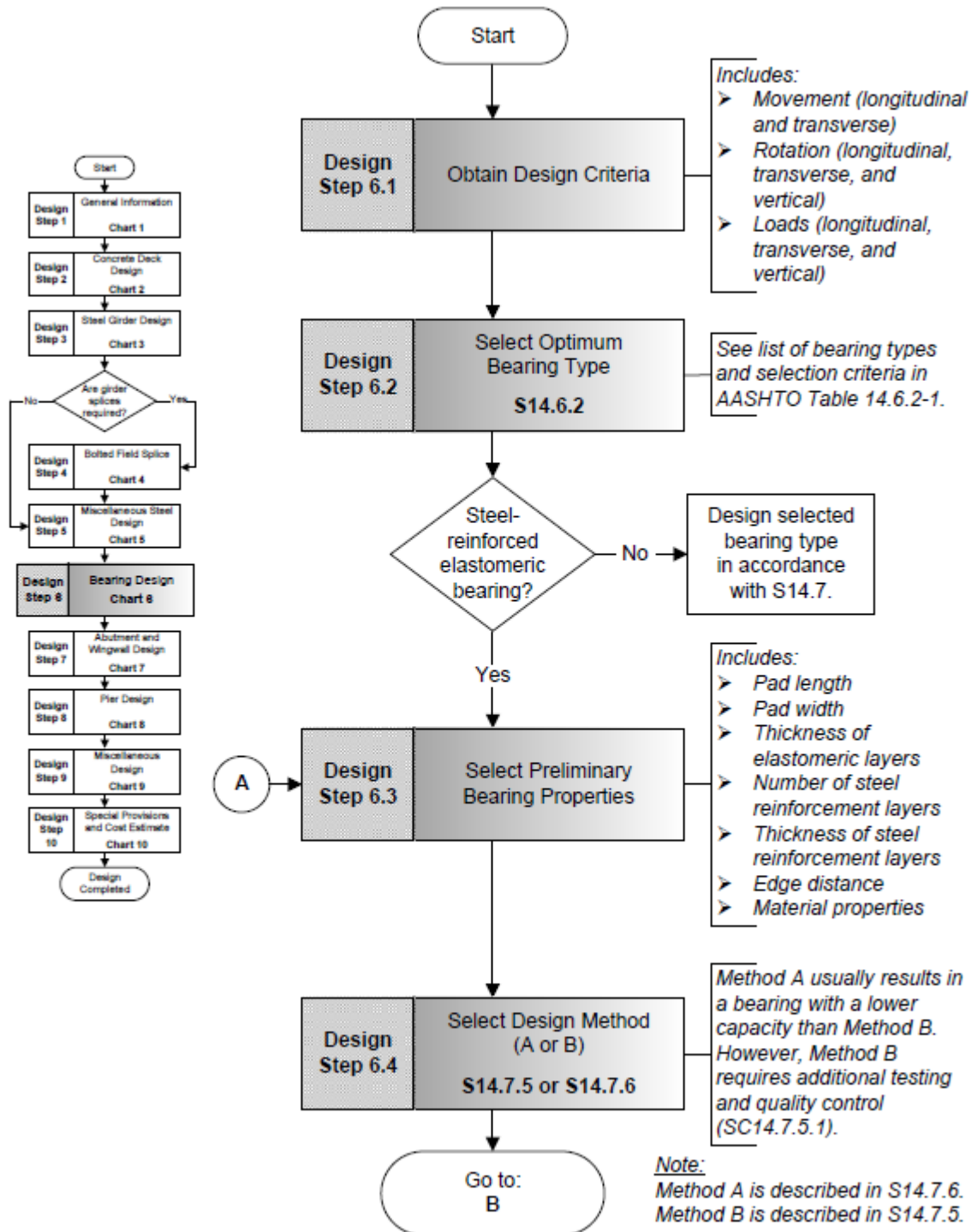
Miscellaneous Steel Design Flowchart (Continued)

Chart 5



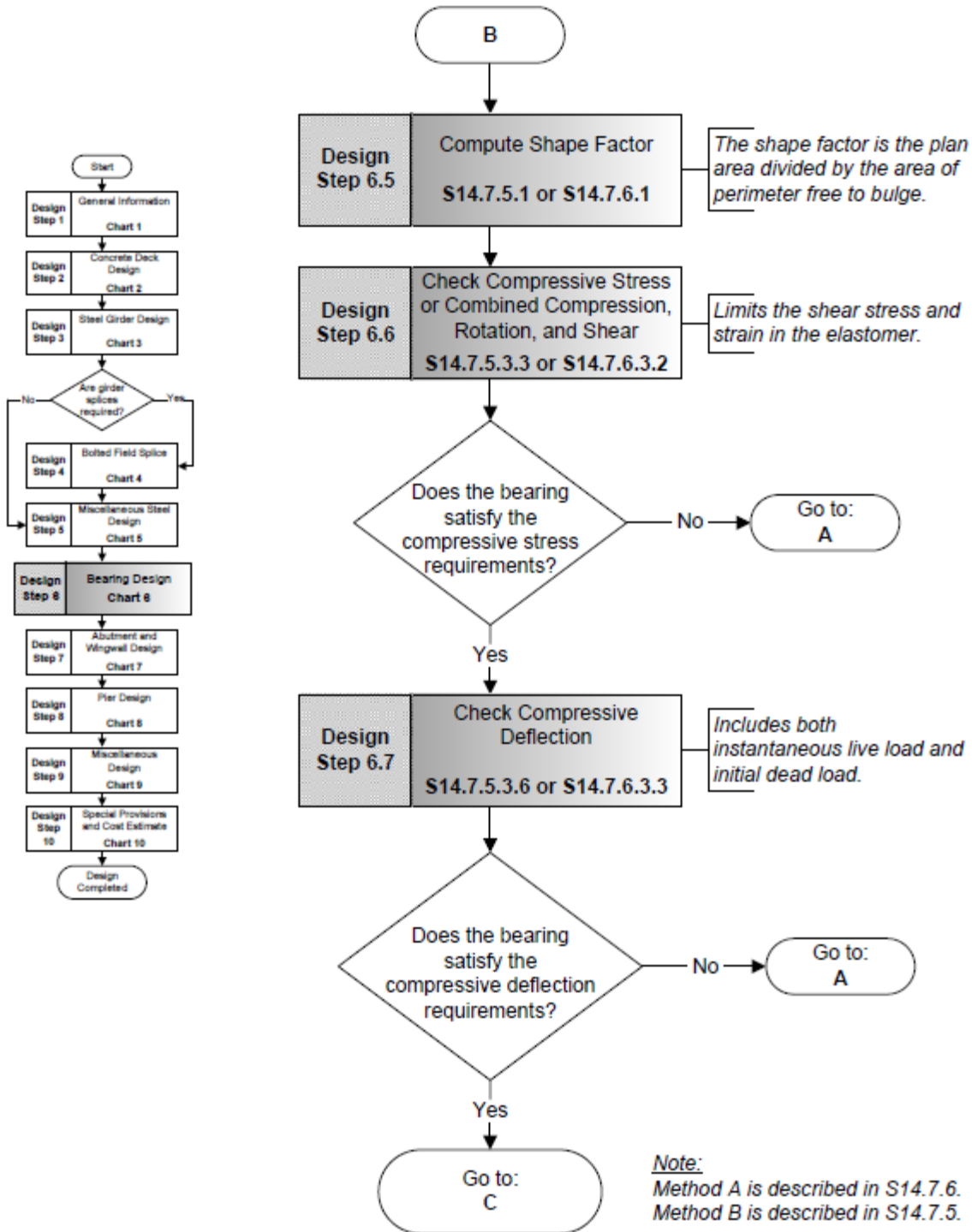
**Bearing Design Flowchart**

**Chart 6**



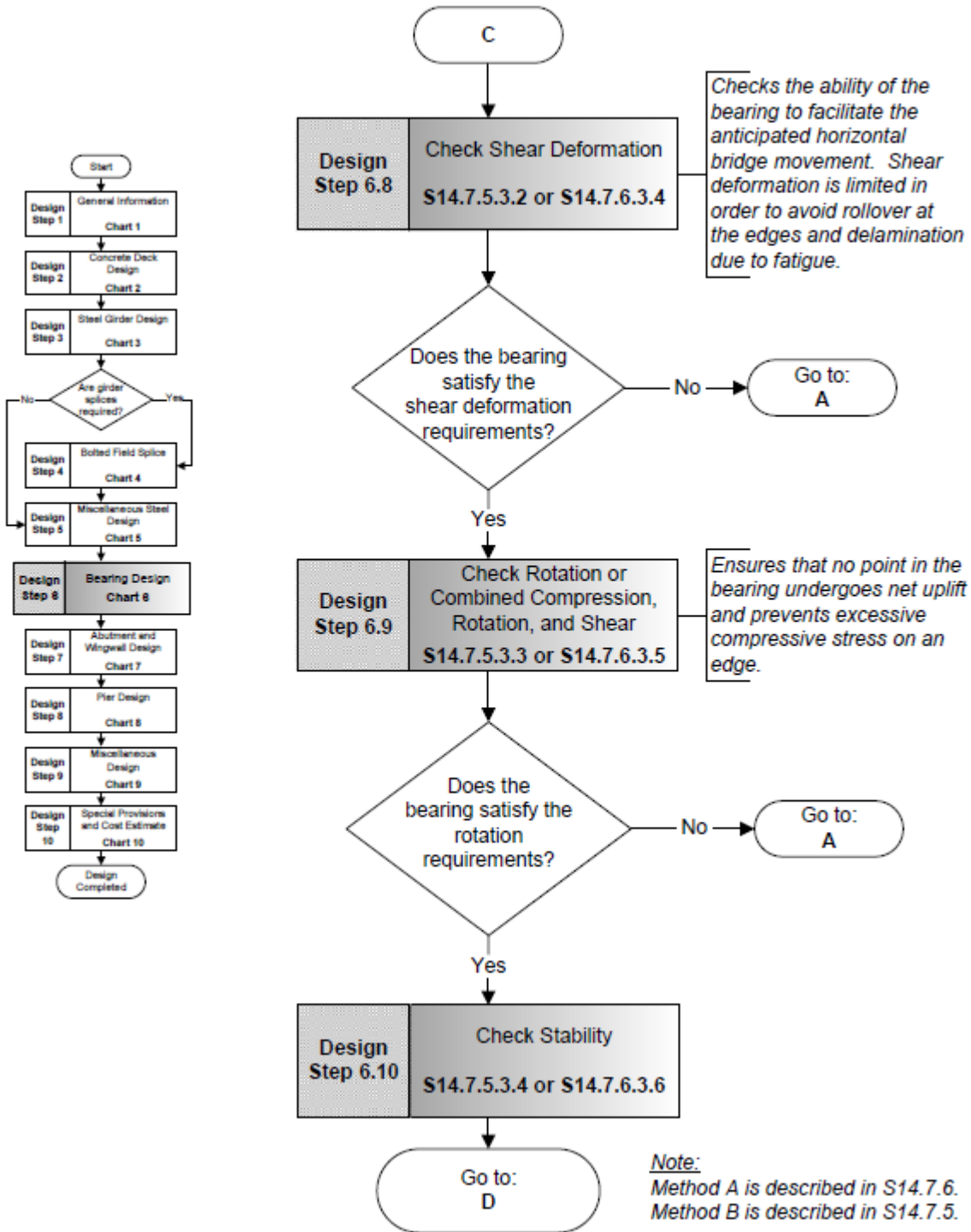
**Bearing Design Flowchart (Continued)**

**Chart 6**



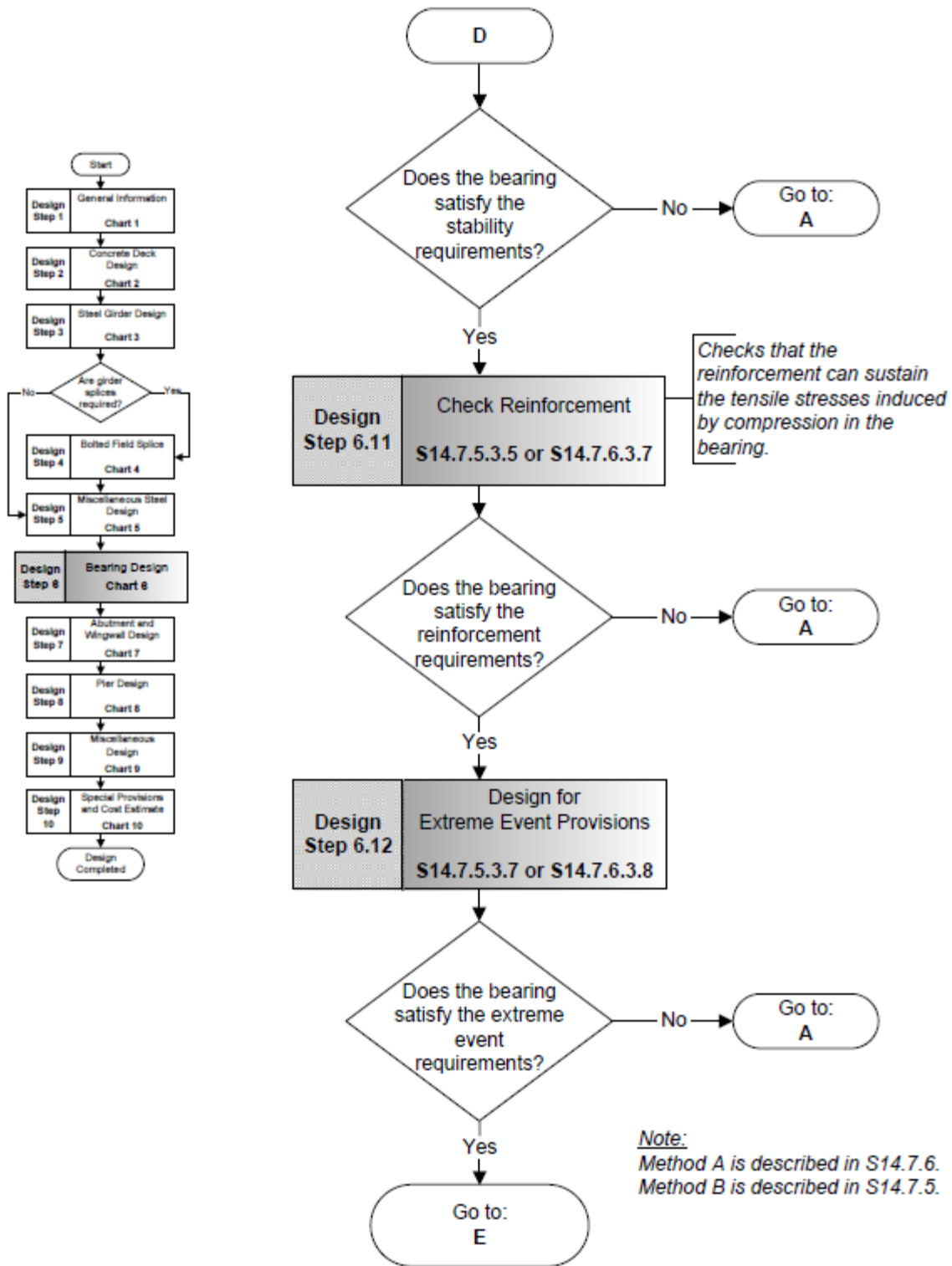
**Bearing Design Flowchart (Continued)**

**Chart 6**



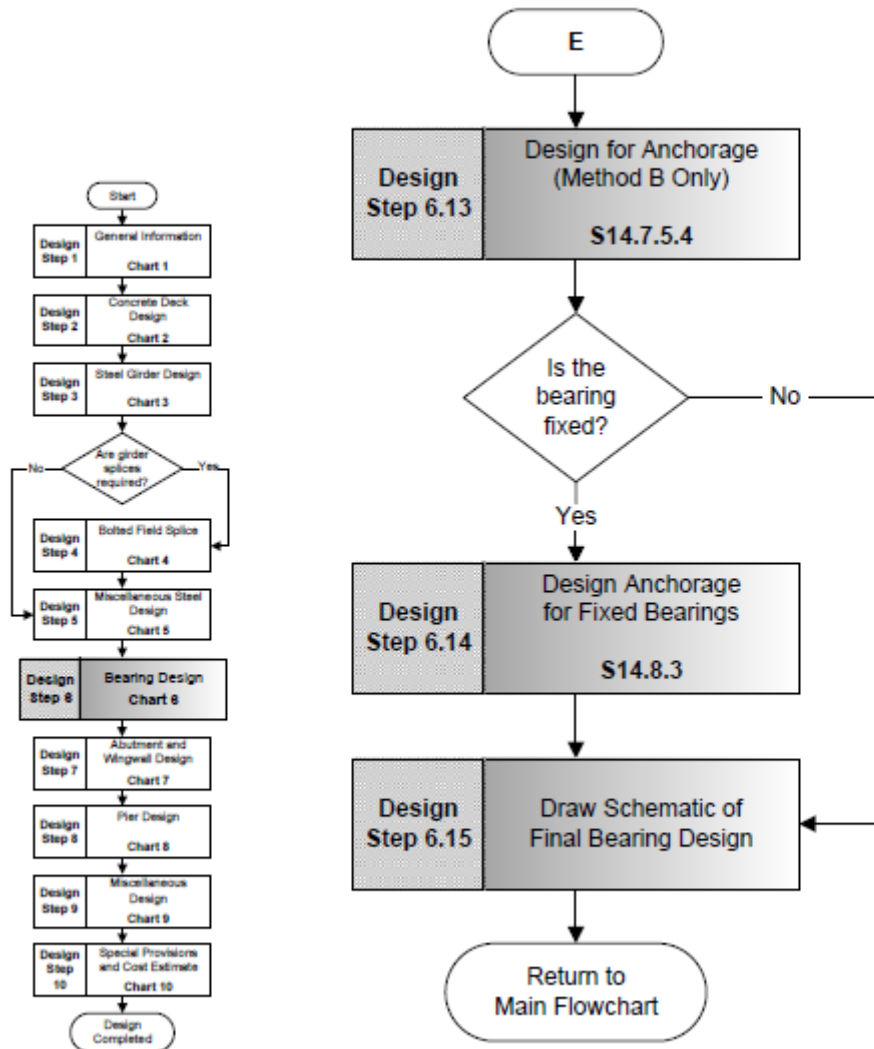
**Bearing Design Flowchart (Continued)**

**Chart 6**



**Bearing Design Flowchart (Continued)**

**Chart 6**





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## Introduction

Design Step 1 is the first of several steps that illustrate the design procedures used for a steel girder bridge. This design step serves as an introduction to this design example and it provides general information about the bridge design.

### Purpose

The purpose of this project is to provide a basic design example for a steel girder bridge as an informational tool for the practicing bridge engineer.

### AASHTO References

For uniformity and simplicity, this design example is based on the *AASHTO LRFD Bridge Design Specifications* (Seventh Edition, 2014), including the 2015 Interim Specifications. References to the *AASHTO LRFD Bridge Design Specifications* are included throughout the design example. AASHTO references are presented in a dedicated column in the right margin of each page, immediately adjacent to the corresponding design procedure. The following abbreviations are used in the AASHTO references:

*S* designates specifications

*S**Table* designates a table within the specifications

*S**Figure* designates a figure within the specifications

*S**Equation* designates an equation within the specifications

*S**Appendix* designates an appendix within the specifications

*C* designates commentary

*C**Table* designates a table within the commentary

*C**Figure* designates a figure within the commentary

*C**Equation* designates an equation within the commentary

State-specific specifications are generally not used in this design example. Any exceptions are clearly noted.

### Design Methodology

This design example is based on Load and Resistance Factor Design (LRFD), as presented in the *AASHTO LRFD Bridge Design Specifications*.

Load and Resistance Factor Design (LRFD) takes into account both the statistical mean resistance and the statistical mean loads. The fundamental LRFD equation includes a load modifier ( $\eta$ ), load factors ( $\gamma$ ), force effects ( $Q$ ), a resistance factor ( $\phi$ ), a nominal resistance ( $R_n$ ), and a factored resistance ( $R_r = \phi R_n$ ). LRFD provides a more uniform level of safety throughout the entire bridge, in which the measure of safety is a function of the variability of the loads and the resistance.

## Detailed Outline and Flowcharts

Each step in this design example is based on a detailed outline and a series of flowcharts that were developed for this project.

The detailed outline and the flowcharts are intended to be comprehensive. They include the primary design steps that would be required for the design of various steel girder bridges.

This design example includes the major steps shown in the detailed outline and flowcharts, but it does not include all design steps. For example, longitudinal stiffener design, girder camber computations, and development of special provisions are included in the detailed outline and the flowcharts. However, their inclusion in the design example is beyond the scope of this project.

## Software

An analysis of the superstructure was performed using AASHTO BrD® software. The design moments, shears, and reactions used in the design example are taken from the BrD output, but their computation is not shown in the design example.

## Organization of Design Example

To make this reference user-friendly, the numbers and titles of the design steps are consistent between the detailed outline, the flowcharts, and the design example.

In addition to design computations, the design example also includes many tables and figures to illustrate the various design procedures and many AASHTO references. It also includes commentary to explain the design logic in a user-friendly way. A figure is generally provided at the end of each design step, summarizing the design results for that particular bridge element.

### **Tip Boxes**



Tip boxes are used throughout the design example computations to present useful information, common practices, and rules of thumb for the bridge designer. Tip boxes are shaded and include a tip icon, just like this. Tips do not explain what must be done based on the design specifications; rather, they present suggested alternatives for the designer to consider.

## Design Step 1.1 - Obtain Design Criteria

The first step for any bridge design is to establish the design criteria. For this design example, the following is a summary of the primary design criteria:

### Design Criteria

Governing specifications:

*AASHTO LRFD Bridge Design Specifications* (Seventh Edition, 2014), including the 2015 Interim Specifications

Design methodology:

Load and Resistance Factor Design (LRFD)

*Live load requirements:* S3.6

HL-93

Deck width:

$$W_{deck} = 45.375 \text{ ft}$$

Roadway width:

$$W_{roadway} = 42.5 \text{ ft}$$

Bridge length:

$$L_{total} = 240 \text{ ft}$$

Skew angle:

$$Skew = 0 \text{ deg}$$

*Structural steel yield strength:* STable 6.4.1-1

$$F_y = 50 \text{ ksi}$$

*Structural steel tensile strength:* STable 6.4.1-1

$$F_u = 65 \text{ ksi}$$

*Concrete 28-day compressive strength:* S5.4.2.1

$$f'_c = 4.0 \text{ ksi}$$

Reinforcement strength: S5.4.3

$$f_y = 60\text{ksi}$$

Steel density: STable 3.5.1-1

$$W_s = 0.490\text{kcf}$$

Concrete density: STable 3.5.1-1

$$W_c = 0.150\text{kcf}$$

Parapet weight (each):

$$W_{par} = 0.53 \frac{K}{ft}$$

Future wearing surface: STable 3.5.1-1

$$W_{fws} = 0.140\text{kcf}$$

Future wearing surface thickness:

$$t_{fws} = 2.50\text{in} \quad (\text{assumed})$$

### Design Factors from AASHTO LRFD Bridge Design Specifications

#### S1.3.2.1

The first set of design factors applies to all force effects and is represented by the Greek letter  $\eta$  (eta) in the Specifications. These factors are related to the ductility, redundancy, and operational importance of the structure. A single, combined eta is required for every structure. When a maximum load factor from STable 3.4.1-2 is used, the factored load is multiplied by eta, and when a minimum load factor is used, the factored load is divided by eta. All other loads, factored in accordance with STable 3.4.1-1, are multiplied by eta if a maximum force effect is desired and are divided by eta if a minimum force effect is desired. In this design example, it is assumed that all eta factors are equal to 1.0.

Ductility:

$$\eta_D = 1.0$$

Redundancy:

$$\eta_R = 1.0$$

Importance:

$$\eta_I = 1.0$$

For loads for which the maximum value of  $\gamma_i$  is appropriate:

$$\eta = \eta_D \cdot \eta_R \cdot \eta_I \quad \text{and} \quad \eta \geq 0.95 \quad \text{SEquation 1.3.2.1-2}$$

For loads for which the minimum value of  $\gamma_i$  is appropriate:

$$\eta = \frac{1}{\eta_D \cdot \eta_R \cdot \eta_I} \quad \text{and} \quad \eta \leq 1.00 \quad \text{SEquation 1.3.2.1-3}$$

Therefore for this design example, use:

$$\eta = 1.00$$

The following is a summary of other design factors from the *AASHTO LRFD Bridge Design Specifications*. Additional information is provided in the Specifications, and specific section references are provided in the right margin of the design example.

Load factors:

STable 3.4.1-1 &  
STable 3.4.1-2

Load Combinations and Load Factors								
Limit State	Load Factors							
	DC		DW		LL	IM	WS	WL
	Max.	Min.	Max.	Min.				
Strength I	1.25	0.90	1.50	0.65	1.75	1.75	-	-
Strength II	1.25	0.90	1.50	0.65	1.35	1.35		
Strength III	1.25	0.90	1.50	0.65	-	-	1.40	-
Strength V	1.25	0.90	1.50	0.65	1.35	1.35	0.40	1.00
Service I	1.00	1.00	1.00	1.00	1.00	1.00	0.30	1.00
Service II	1.00	1.00	1.00	1.00	1.30	1.30	-	-
Fatigue I	-	-	-	-	1.50	1.50	-	-
Fatigue II	-	-	-	-	0.75	0.75	-	-

**Table 1-1 Load Combinations and Load Factors**

S3.4.2.1

The abbreviations used in Table 1-1 are as defined in S3.3.2. Also, S3.4.2.1 states that primary steel superstructure components are to be investigated for maximum force effects during construction for an additional special load combination consisting of the applicable DC loads and any construction loads that are applied to the fully erected

steelwork. The load factor for force effects caused by DC loads and construction loads, including dynamic effects (if applicable), is not to be less than 1.4 for this additional special load combination.

The extreme event limit state (including earthquake load) is not considered in this design example.

S5.5.4.2 & S6.5.4.2

Resistance factors (See S5.7.2.1 for a detailed description of tension and compression controlled concrete sections):

Resistance Factors		
Material	Type of Resistance	Resistance Factor, $\phi$
Structural steel	For flexure	$\phi_f = 1.00$
	For shear	$\phi_v = 1.00$
	For axial compression	$\phi_c = 0.95$
	For bearing	$\phi_b = 1.00$
Reinforced concrete	Tension controlled	$\phi = 0.90$
	For shear and torsion	$\phi = 0.90$
	Compression controlled	$\phi = 0.75$
	Bearing on concrete	$\phi = 0.70$

**Table 1-2 Resistance Factors**

Multiple presence factors:

STable 3.6.1.1.2-1

Multiple Presence Factors	
Number of Lanes Loaded	Multiple Presence Factor, m
1	1.20
2	1.00
3	0.85
>3	0.65

**Table 1-3 Multiple Presence Factors**

Dynamic load allowance:

STable 3.6.2.1-1

Dynamic Load Allowance	
Limit State	Dynamic Load Allowance, IM
Deck Joints - All Limit States	75%
Fatigue and Fracture Limit State	15%
All Other Limit States	33%

**Table 1-4 Dynamic Load Allowance**

### Design Step 1.2 - Obtain Geometry Requirements

Geometry requirements for the bridge components are defined by the bridge site and by the highway geometry. Highway geometry constraints include horizontal alignment and vertical alignment.

Horizontal alignment can be tangent, curved, spiral, or a combination of these three geometries.

Vertical alignment can be straight sloped, crest, sag, or a combination of these three geometries.

For this design example, it is assumed that the horizontal alignment geometry is tangent and the vertical alignment geometry is straight sloped.

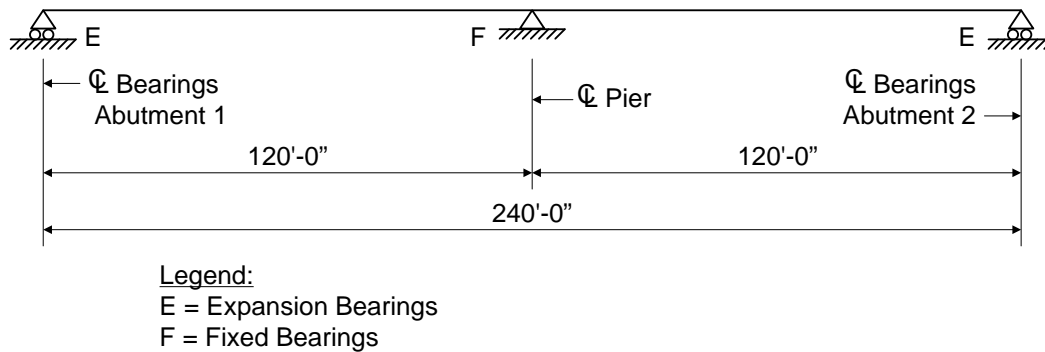
### Design Step 1.3 - Perform Span Arrangement Study

Some clients require a Span Arrangement Study. The Span Arrangement Study includes selecting the bridge type, determining the span arrangement, determining substructure locations, computing span lengths, and checking horizontal clearance for the purpose of approval.

Although a Span Arrangement Study may not be required by the client, these determinations must still be made by the engineer before proceeding to the next design step.

For this design example, the span arrangement is presented in Figure 1-1. This span arrangement was selected to illustrate various design criteria and the established geometry constraints identified for this example.





**Figure 1-1 Span Arrangement**

### Design Step 1.4 - Obtain Geotechnical Recommendations

The subsurface conditions must be determined to develop geotechnical recommendations.

Subsurface conditions are commonly determined by taking core borings at the bridge site. The borings provide a wealth of information about the subsurface conditions, all of which is recorded in the boring logs.

It is important to note that the boring log reveals the subsurface conditions for a finite location and not necessarily for the entire bridge site. Therefore, several borings are usually taken at each proposed substructure location. This improves their reliability as a reflection of subsurface conditions at the bridge site, and it allows the engineer to compensate for significant variations in the subsurface profile.

After the subsurface conditions have been explored and documented, a geotechnical engineer must develop foundation type recommendations for all substructures. Foundations can be spread footings, pile foundations, or drilled shafts. Geotechnical recommendations typically include allowable bearing pressure, allowable settlement, and allowable pile resistances (axial and lateral), as well as required safety factors for overturning and sliding.

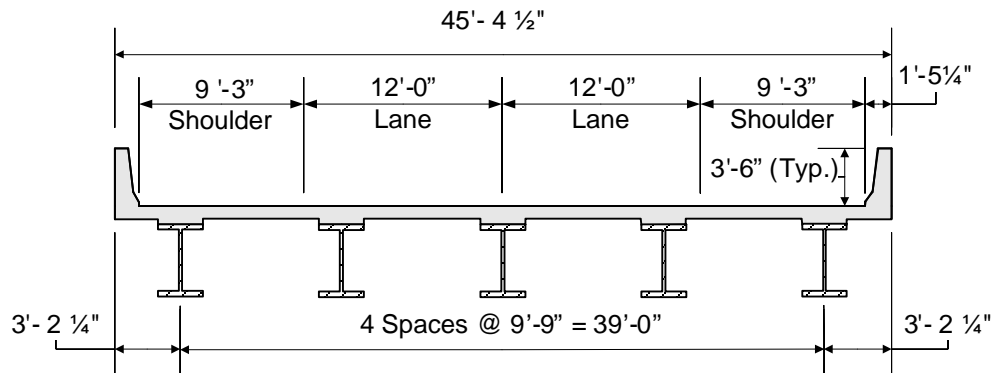
For this design example, pile foundations are used for all substructure units.

### Design Step 1.5 - Perform Type, Size and Location Study

Some clients require a Type, Size and Location study for the purpose of approval. The Type, Size and Location study includes preliminary configurations for the superstructure and substructure components relative to highway geometry constraints and site conditions. Details of this study for the superstructure include selecting the girder types, determining the girder spacing, computing the approximate required girder span and depth, and checking vertical clearance.

Although a Type, Size and Location study may not be required by the client, these determinations must still be made by the engineer before proceeding to the next design step.

For this design example, the superstructure cross section is presented in Figure 1-2. This superstructure cross section was selected to illustrate selected design criteria and the established geometry constraints. When selecting the girder spacing, consideration was given to half-width deck replacement.



**Figure 1-2 Superstructure Cross Section**

### Design Step 1.6 - Plan for Bridge Aesthetics

Finally, the bridge engineer must consider bridge aesthetics throughout the design process. Special attention to aesthetics should be made during the preliminary stages of the bridge design, before the bridge layout and appearance has been fully determined.

To plan an aesthetic bridge design, the engineer must consider the following parameters:

- Function: Aesthetics is generally enhanced when form follows function.
- Proportion: Provide balanced proportions for members and span lengths.
- Harmony: The parts of the bridge must usually complement each other, and the bridge must usually complement its surroundings.
- Order and rhythm: All members must be tied together in an orderly manner.
- Contrast and texture: Use textured surfaces to reduce visual mass.
- Light and shadow: Careful use of shadow can give the bridge a more slender appearance.

## Concrete Deck Design Example Design Step 2

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### **Design Step 2.1 - Obtain Design Criteria**

The first design step for a concrete bridge deck is to choose the correct design criteria. The following concrete deck design criteria are obtained from the typical superstructure cross section shown in Figure 2-1 and from the referenced articles and tables in the *AASHTO LRFD Bridge Design Specifications, Seventh Edition (2014)*, including the 2015 interims.

Refer to Design Step 1 for introductory information about this design example. Additional information is presented about the design assumptions, methodology, and criteria for the entire bridge, including the concrete deck.

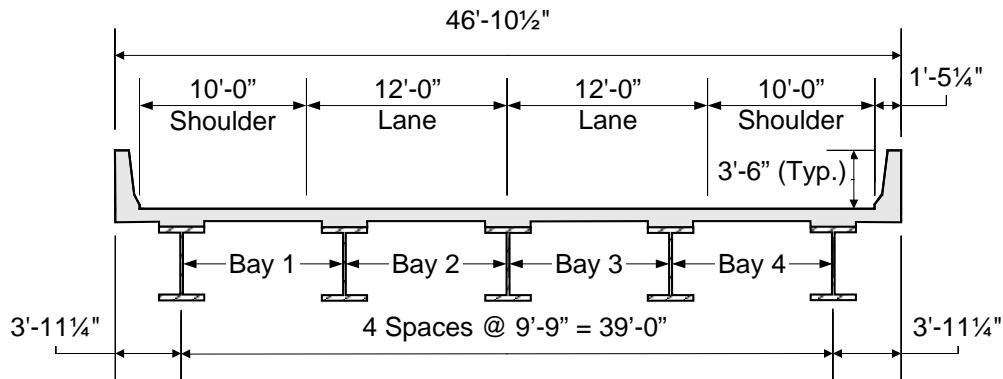
*S4.6.2 and S9.7.2*

The next step is to decide which deck design method will be used. In this example, the equivalent strip method will be used. For the equivalent strip method analysis, the girders act as supports, and the deck acts as a simple or continuous beam spanning from support to support. The empirical method could be used for the positive and negative moment interior regions since the cross section meets all the requirements given in *S9.7.2.4*. However, the empirical method could not be used to design the overhang as stated in *S9.7.2.2*.

#### **Overhang Width**



The overhang width is generally determined such that the moments and shears in the exterior girder are similar to those in the interior girder. In addition, the overhang is set such that the positive and negative moments in the deck slab are balanced. A common rule of thumb is to make the overhang approximately 0.28 to 0.35 times the girder spacing.



**Figure 2-1 Superstructure Cross Section**

The following units are defined for use in this design example:

$$K = 1000lb \quad kcf = \frac{K}{ft^3} \quad ksi = \frac{K}{in^2}$$

Deck Properties

Girder spacing:	S = 9.75ft	STable 5.12.3-1
Number of girders:	N = 5	STable 5.12.3-1
Deck top cover:	Cover <sub>t</sub> = 2.5in	
Deck bottom cover:	Cover <sub>b</sub> = 1.0in	
Reinforced Concrete density:	W <sub>c</sub> = 0.150kcf	
Concrete 28-day compressive strength (Type A (AE)):	f' <sub>c</sub> = 4.0ksi	STable 5.4.2.1-1
Reinforcement strength:	f <sub>y</sub> = 60ksi	S5.4.3
Future wearing surface:	W <sub>fws</sub> = 0.140kcf	STable 3.5.1-1

Parapet properties:

Weight per foot:

$$W_{\text{par}} = 0.53 \frac{\text{K}}{\text{ft}}$$

Width at base:

$$W_{\text{base}} = 1.4375\text{ft}$$

Moment capacity at base\*:

$$M_{\text{co}} = 28.21 \frac{\text{K}\cdot\text{ft}}{\text{ft}}$$

Parapet Height:

$$H_{\text{par}} = 3.5\text{ft}$$

Critical length of yield line failure pattern\*:

$$L_c = 12.84\text{ft} \quad (\text{calculated in Design Step 2.12})$$

Total transverse resistance of the parapet:

$$R_w = 117.40\text{K} \quad (\text{calculated in Design Step 2.12})$$

\* Based on parapet properties not included in this design example. See Publication Number FHWA HI-95-017, Load and Resistance Factor Design for Highway Bridges, Participant Notebook, Volume II (Version 3.01), for the method used to compute the parapet properties.

*S*Table 5.12.3-1

**Deck top cover** - The concrete top cover is set at 2.5 inches since the bridge deck may be exposed to deicing salts and/or tire stud or chain wear. This includes the 1/2 inch integral wearing surface that is required.

*S*Table 5.12.3-1

**Deck bottom cover** - The concrete bottom cover is set at 1.0 inch since the bridge deck will use reinforcement that is smaller than a #11 bar.

*S*5.4.2.1-1 and *S*Table C5.4.2.1-1

**Concrete 28-day compressive strength** – The compressive strength for decks shall

not be less than 4.0 KSI. Also, type “AE” concrete should be specified when the deck will be exposed to deicing salts or the freeze-thaw cycle. Class A (AE) concrete has a compressive strength of 4.0 KSI.

*S*Table 3.5.1-1

**Future wearing surface density** - The future wearing surface density is 0.140 KCF. A 2.5 inch thickness will be assumed.

### **Design Step 2.2 - Determine Minimum Slab Thickness**

*S*9.1.1.1

The concrete deck depth cannot be less than 7.0 inches, excluding any provision for grinding, grooving, and sacrificial surface.

### **Design Step 2.3 - Determine Minimum Overhang Thickness**

*S*13.7.3.1.2

For concrete deck overhangs supporting concrete parapets or barriers, the minimum deck overhang thickness is:  $t_o = 8.0\text{in}$

### **Design Step 2.4 – Select Slab and Overhang Thickness**

Once the minimum slab and overhang thicknesses are computed, they can be increased as needed based on client standards and design computations. The following slab and overhang thicknesses will be assumed for this design example:

$$t_s = 8.5\text{in} \text{ and } t_o = 9.0\text{in}$$

### **Design Step 2.5 - Compute Dead Load Effects**

*S*Table 3.5.1-1

The next step is to compute the dead load moments. The dead load moments for the deck slab, parapets, and future wearing surface are tabulated in Table 2-1. The tabulated moments are presented for tenth points for Bays 1 through 4 for a 1-foot strip. The tenth points are based on the equivalent span and not the center-to-center beam spacing.

*S*Table 3.4.1-2

After the dead load moments are computed for the slab, parapets, and future wearing surface, the correct load factors must be identified. The load factors for dead loads are:



For slab and parapet:

Maximum:  $\gamma_{pDCmax} = 1.25$

Minimum:  $\gamma_{pDCmin} = 0.90$

For future wearing surface:

Maximum:  $\gamma_{pDWmax} = 1.50$

Minimum:  $\gamma_{pDWmin} = 0.65$

	DISTANCE	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
SLAB DEAD LOAD	BAY 1	-0.74	-0.33	-0.01	0.22	0.36	0.41	0.37	0.24	0.01	-0.30	-0.71
	BAY 2	-0.71	-0.30	0.02	0.24	0.38	0.42	0.38	0.24	0.01	-0.31	-0.72
	BAY 3	-0.72	-0.31	0.01	0.24	0.38	0.42	0.38	0.24	0.02	-0.30	-0.71
	BAY 4	-0.71	-0.30	0.01	0.24	0.37	0.41	0.36	0.22	-0.01	-0.33	-0.74
PARAPET DEAD LOAD	BAY 1	-1.66	-1.45	-1.24	-1.03	-0.82	-0.61	-0.40	-0.19	0.02	0.22	0.43
	BAY 2	0.47	0.40	0.33	0.26	0.19	0.12	0.05	-0.02	-0.09	-0.16	-0.23
	BAY 3	-0.23	-0.16	-0.09	-0.02	0.05	0.12	0.19	0.26	0.33	0.40	0.47
	BAY 4	0.43	0.22	0.02	-0.19	-0.40	-0.61	-0.82	-1.03	-1.24	-1.45	-1.66
FWS DEAD LOAD	BAY 1	-0.06	0.04	0.11	0.15	0.17	0.17	0.14	0.08	0.00	-0.11	-0.24
	BAY 2	-0.24	-0.12	-0.02	0.05	0.09	0.11	0.10	0.07	0.01	-0.07	-0.18
	BAY 3	-0.18	-0.07	0.01	0.07	0.10	0.11	0.09	0.05	-0.02	-0.12	-0.24
	BAY 4	-0.24	-0.11	0.00	0.08	0.14	0.17	0.17	0.15	0.11	0.04	-0.06

**Table 2-1 Unfactored Dead Load Moments (K-FT/FT)**

**Design Step 2.6 - Compute Live Load Effects**

Before the live load effects can be computed, the following basic parameters must be defined:

S3.6.1.3.1

The minimum distance from the center of design vehicle wheel to the inside face of parapet = 1 foot

S3.6.1.3.1

The minimum distance between the wheels of two adjacent design vehicles = 4 feet

STable 3.6.2.1-1

Dynamic load allowance, IM  $IM = 0.33$

STable 3.4.1-1

Load factor for live load – Strength I  $\gamma_{LL} = 1.75$

Multiple presence factor, m:

STable 3.6.1.1.2-1

With one lane loaded, m = 1.20

With two lanes loaded, m = 1.00

With three lanes loaded, m = 0.85

S9.5.3 & S5.5.3.1

Fatigue does not need to be investigated for concrete deck design.

Resistance factors for flexure:

Strength limit state  $\phi_{str} = 0.90$   
S5.5.4.2

Service limit state  $\phi_{serv} = 1.00$   
S1.3.2.1

Extreme even limit state  $\phi_{ext} = 1.00$   
S1.3.2.1

Based on the above information and based on S4.6.2.1, the live load effects for one and two trucks are tabulated in Table 2-2. The live load effects are given for tenth points for Bays 1 through 4. Multiple presence factors are included, but dynamic load allowance is excluded.

		DISTANCE	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
SINGLE TRUCK (MULTIPLE PRESENCE FACTOR OF 1.20 INCLUDED)	MAX. MOMENT	BAY 1	5.62	22.94	30.53	36.44	36.76	31.20	32.48	27.13	18.10
		BAY 2	4.07	7.73	20.88	25.56	29.09	28.26	28.00	27.20	18.35
		BAY 3	6.04	6.22	18.22	27.37	28.14	28.14	29.28	25.39	20.68
		BAY 4	4.55	5.56	11.66	16.19	26.14	31.10	36.62	36.64	30.43
	MIN. MOMENT	BAY 1	-25.75	-14.45	-3.70	-5.33	-6.96	-8.59	-10.22	-11.84	-13.48
		BAY 2	-28.38	-19.81	-17.00	-14.18	-11.38	-8.57	-9.20	-11.41	-13.63
		BAY 3	-27.13	-15.85	-13.63	-11.42	-9.20	-8.27	-10.97	-13.68	-16.39
		BAY 4	-28.51	-15.11	-13.48	-11.84	-10.22	-8.59	-6.96	-5.33	-3.70
TWO TRUCKS (MULTIPLE PRESENCE FACTOR OF 1.00 INCLUDED)	MAX. MOMENT	BAY 1	4.36	17.44	22.36	26.35	25.93	21.01	21.21	14.00	4.50
		BAY 2	2.04	7.98	17.10	19.26	21.19	21.72	19.58	20.48	15.14
		BAY 3	-2.92	7.32	7.73	16.71	19.41	21.64	21.30	19.15	16.98
		BAY 4	2.55	2.30	4.59	10.48	17.96	20.93	25.84	26.46	22.32
	MIN. MOMENT	BAY 1	-21.47	-12.09	-2.71	-3.24	-4.23	-5.21	-6.20	-7.19	-8.18
		BAY 2	-29.36	-16.92	-8.53	-1.29	-2.40	-3.51	-4.62	-5.74	-8.01
		BAY 3	-27.92	-17.38	-8.02	-6.41	-5.17	-3.92	-2.68	-1.44	-2.10
		BAY 4	-29.40	-18.33	-8.18	-7.19	-6.20	-5.21	-4.22	-3.24	-2.25

Table 2-2 Unfactored Live Load Moments (Excluding Dynamic Load Allowance) (K-FT)

**Design Step 2.7 - Compute Factored Positive and Negative Design Moments**

For this example, the design moments will be computed two different ways.

For Method A, the live load portion of the factored design moments will be computed based on the values presented in Table 2-2. Table 2-2 represents a continuous beam analysis of the example deck using a finite element analysis program.

*S*Table A4-1

For Method B, the live load portion of the factored design moments will be computed using *S*Table A4-1. In *S*Table A4-1, moments per unit width include dynamic load allowance and multiple presence factors. The values are tabulated using the equivalent strip method for various bridge cross sections. The values in *S*Table A4-1 maybe slightly higher than the values from a deck analysis based on the actual number of beams and the actual overhang length. The maximum live load moment is obtained from the table based on the girder spacing. For girder spacing between the values listed in the table, interpolation can be used to get the moment.

S1.3.2.1

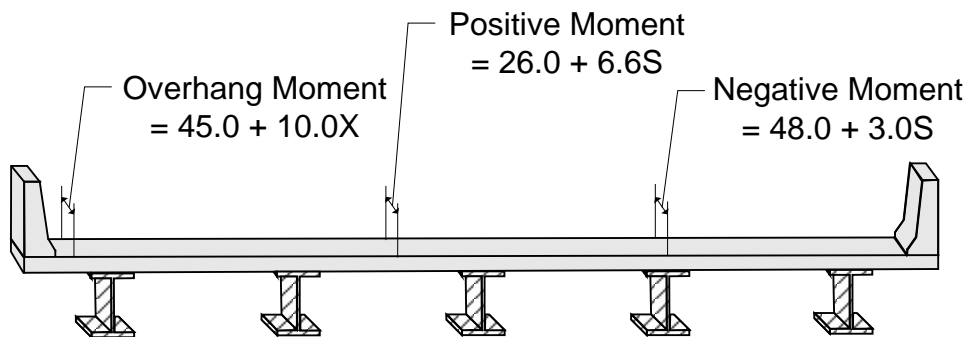
Based on Design Step 1, the load modifier eta ( $\eta$ ) is 1.0 and will not be shown throughout the design example. Refer to Design Step 1 for a discussion of eta.

**Factored Positive Design Moment Using Table 2-2 - Method A**

Factored positive live load moment:

The positive, negative, and overhang moment equivalent strip equations are presented in Figure 2-2 below.

*S*Table 4.6.2.1.3-1



**Figure 2-2 Equivalent Strip Equations for Various Parts of the Deck**

*S*Table 4.6.2.1.3-1

The width of the equivalent strip for positive moment is:

$$W_{\text{posstripa}} = 26.0 + 6.6S$$

For  $S = 9.75 \text{ ft}$

$$W_{\text{posstripa}} = 90.35 \text{ in} \quad \text{or} \quad W_{\text{posstripa}} = 7.53 \cdot \text{ft}$$

Based on Table 2-2, the maximum unfactored positive live load moment is 36.76 K-ft, located at 0.4S in Bay 1 for a single truck. The maximum factored positive live load moment is:

$$M_{U_{\text{posliveA}}} = \gamma_{LL} \cdot (1 + IM) \cdot \frac{36.76 \text{ K} \cdot \text{ft}}{W_{\text{posstripa}}}$$

$$M_{U_{\text{posliveA}}} = 11.36 \cdot \frac{\text{K} \cdot \text{ft}}{\text{ft}}$$

Factored positive dead load moment:

Based on Table 2-1, the maximum unfactored slab, parapet, and future wearing surface positive dead load moment occurs in Bay 2 at a distance of 0.4S. The maximum factored positive dead load moment is as follows:

$$M_{U_{\text{posdead}}} = \gamma_{pDC_{\text{max}}} \cdot \left( 0.38 \cdot \frac{\text{K} \cdot \text{ft}}{\text{ft}} \right) + \gamma_{pDC_{\text{max}}} \cdot \left( 0.19 \cdot \frac{\text{K} \cdot \text{ft}}{\text{ft}} \right) \dots$$

$$+ \gamma_{pDW_{\text{max}}} \cdot \left( 0.09 \cdot \frac{\text{K} \cdot \text{ft}}{\text{ft}} \right)$$

$$M_{U_{\text{posdead}}} = 0.85 \cdot \frac{\text{K} \cdot \text{ft}}{\text{ft}}$$

The total factored positive design moment for Method A is:

$$M_{U_{\text{postotalA}}} = M_{U_{\text{posliveA}}} + M_{U_{\text{posdead}}}$$

$$M_{U_{\text{postotalA}}} = 12.21 \cdot \frac{\text{K} \cdot \text{ft}}{\text{ft}}$$

It should be noted that the total maximum factored positive moment is comprised of the maximum factored positive live load moment in Bay 1 at 0.4S and the maximum

factored positive dead load moment in Bay 2 at 0.4S. Summing the factored moments indifferent bays gives a conservative result. The exact way to compute the maximum total factored design moment is by summing the dead and live load moments at each tenth point per bay. However, the method presented here is a simpler and slightly conservative method of finding the maximum total factored moment.

### Factored Positive Design Moment Using STable A4-1 - Method B

Factored positive live load moment:

*S*Table A4-1

For a girder spacing of 9'-9", the maximum unfactored positive live load moment is 6.74 K-ft/ft.

This moment is on a per foot basis and includes dynamic load allowance. The maximum factored positive live load moment is:

$$M_{u_{\text{positiveB}}} = \gamma_{LL} \cdot 6.74 \frac{\text{K}\cdot\text{ft}}{\text{ft}}$$

$$M_{u_{\text{positiveB}}} = 11.80 \cdot \frac{\text{K}\cdot\text{ft}}{\text{ft}}$$

Factored positive dead load moment:

The factored positive dead load moment for Method B is the same as that for Method A:

$$M_{u_{\text{posdead}}} = 0.85 \cdot \frac{\text{K}\cdot\text{ft}}{\text{ft}}$$

The total factored positive design moment for Method B is:

$$M_{u_{\text{postotalB}}} = M_{u_{\text{positiveB}}} + M_{u_{\text{posdead}}}$$

$$M_{u_{\text{postotalB}}} = 12.64 \cdot \frac{\text{K}\cdot\text{ft}}{\text{ft}}$$

Comparing Methods A and B, the difference between the total factored design moment for the two methods is:

$$\frac{M_{u_{\text{postotalB}}} - M_{u_{\text{postotalA}}}}{M_{u_{\text{postotalB}}}} = 3.4\%$$



**Method A or Method B**

It can be seen that the tabulated values based on STable A4-1 (Method B) are slightly greater than the computed live load values using a finite element analysis program (Method A). For real world deck design, Method B would be preferred over Method A due to the amount of time that would be saved by not having to develop a finite element model. Since the time was spent to develop the finite element model for this deck design, the Method A values will be used.

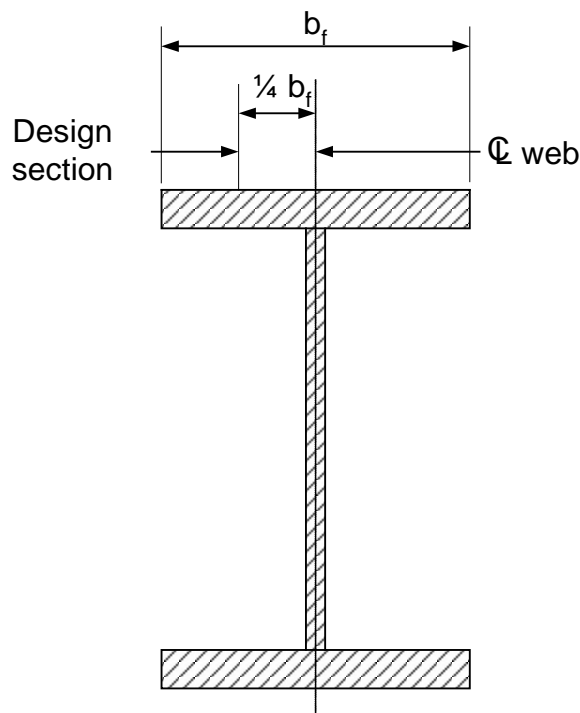
**Factored Negative Design Moment Using Table 2-2 - Method A**

Factored negative live load moment:

S4.6.2.1.6

The deck design section for a steel beam for negative moments and shear forces is taken as one-quarter of the top flange width from the centerline of the web.

S4.6.2.1.6



**Figure 2-3 Location of Design Section**

Assume  $b_f = 1.0\text{ft}$

$$\frac{1}{4}b_f = 0.25\text{ft}$$

*S*Table 4.6.2.1.3-1

The width of the equivalent strip for negative moment is:

$$W_{\text{negstripa}} = 48.0 + 3.0S$$

$$W_{\text{negstripa}} = 77.25\text{in or } W_{\text{negstripa}} = 6.44\text{ft}$$

Based on Table 2-2, the maximum unfactored negative live load moment is -29.40 K-ft, located at 0.0S in Bay 4 for two trucks. The maximum factored negative live load moment is:

$$M_{\text{negliveA}} = \gamma_{\text{LL}} \cdot (1 + \text{IM}) \cdot \frac{-29.40\text{K}\cdot\text{ft}}{W_{\text{negstripa}}}$$

$$M_{\text{negliveA}} = -10.63 \cdot \frac{\text{K}\cdot\text{ft}}{\text{ft}}$$

Factored negative dead load moment:

From Table 2-1, the maximum unfactored negative dead load moment occurs in Bay 4 at a distance of 1.0S. The maximum factored negative dead load moment is as follows:

$$\begin{aligned} M_{\text{negdead}} = & \gamma_{\text{pDCmax}} \cdot \left( -0.74 \cdot \frac{\text{K}\cdot\text{ft}}{\text{ft}} \right) \dots \\ & + \gamma_{\text{pDCmax}} \cdot \left( -1.66 \cdot \frac{\text{K}\cdot\text{ft}}{\text{ft}} \right) \dots \\ & + \gamma_{\text{pDWmax}} \cdot \left( -0.06 \cdot \frac{\text{K}\cdot\text{ft}}{\text{ft}} \right) \end{aligned}$$

$$M_{\text{negdead}} = -3.09 \cdot \frac{\text{K}\cdot\text{ft}}{\text{ft}}$$

The total factored negative design moment for Method A is:

$$M_{u_{negtotalA}} = M_{u_{negliveA}} + M_{u_{negdead}}$$

$$M_{u_{negtotalA}} = -13.72 \cdot \frac{\text{K}\cdot\text{ft}}{\text{ft}}$$

### Factored Negative Design Moment Using STable A4-1 - Method B

Factored negative live load moment:

*S*Table A4-1

For a girder spacing of 9'-9" and a 3" distance from the centerline of girder to the design section, the maximum unfactored negative live load moment is 6.65 K-ft/ft.

If the distance from the centerline of the girder to the design section does not match one of the distances given in the table, the design moment can be obtained by interpolation. As stated earlier, these moments are on a per foot basis and include dynamic load allowance.

The maximum factored negative live load moment is:

$$M_{u_{negliveB}} = \gamma_{LL} \cdot -6.65 \frac{\text{K}\cdot\text{ft}}{\text{ft}}$$

$$M_{u_{negliveB}} = -11.64 \cdot \frac{\text{K}\cdot\text{ft}}{\text{ft}}$$

Factored negative dead load moment:

The factored negative dead load moment for Method B is the same as that for Method A:

$$M_{u_{negdead}} = -3.09 \cdot \frac{\text{K}\cdot\text{ft}}{\text{ft}}$$

The total factored negative design moment for Method B is:

$$M_{u_{negtotalB}} = M_{u_{negliveB}} + M_{u_{negdead}}$$

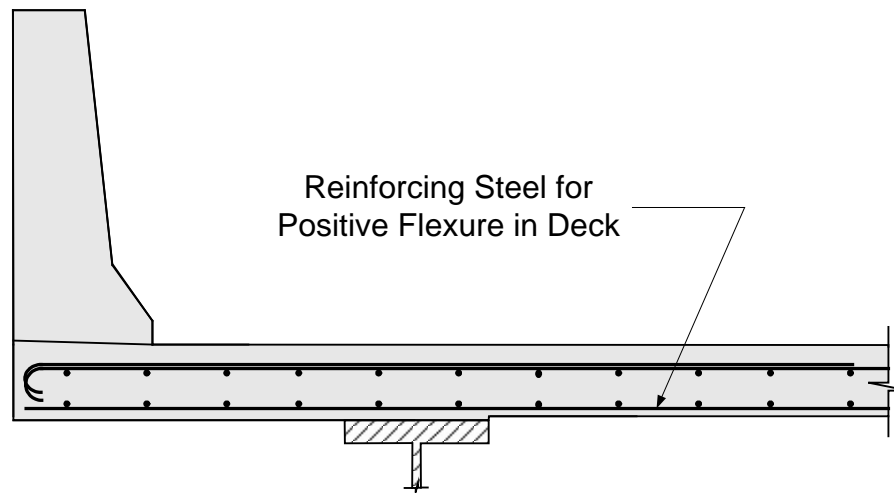
$$M_{u_{negtotalB}} = -14.73 \cdot \frac{\text{K}\cdot\text{ft}}{\text{ft}}$$

Comparing Methods A and B, the difference between the total factored design moment for the two methods is:

$$\frac{M_{u_{\text{negtotalB}}} - M_{u_{\text{negtotalA}}}}{M_{u_{\text{negtotalB}}}} = 6.8\%$$

### **Design Step 2.8 - Design for Positive Flexure in Deck**

The first step in designing the positive flexure steel is to assume a bar size. From this bar size, the required area of steel ( $A_s$ ) can be calculated. Once the required area of steel is known, the required bar spacing can be calculated.



**Figure 2-4 Reinforcing Steel for Positive Flexure in Deck**

Assume #5 bars:            bar\_diam = 0.625in

Bar\_area = 0.31in<sup>2</sup>

Effective depth,  $d_e$  = total slab thickness - bottom cover - 1/2 bar diameter - top integral wearing surface

$$d_e = t_s - \text{Cover}_b - \frac{\text{bar\_diam}}{2} - 0.5\text{in}$$

$$d_e = 6.69\text{in}$$

Solve for the required amount of reinforcing steel, as follows:

S5.5.4.2.1

$$\phi_f = 0.90$$

$$b = 12\text{in}$$

$$R_n = \frac{M_{u_{\text{posttotal}}} \cdot 12\text{in}}{(\phi_f \cdot b \cdot d_e^2)}$$

$$R_n = 0.30 \cdot \frac{K}{\text{in}^2}$$

$$\rho = 0.85 \left( \frac{f_c}{f_y} \right) \cdot \left[ 1.0 - \sqrt{1.0 - \frac{(2 \cdot R_n)}{(0.85 \cdot f_c)}} \right]$$

$$\rho = 0.00530$$

Note: The above two equations are derived formulas that can be found in most reinforced concrete textbooks.

$$A_s = \rho \cdot \frac{b}{\text{ft}} \cdot d_e \quad A_s = 0.43 \cdot \frac{\text{in}^2}{\text{ft}}$$

$$\text{Required bar spacing} = \frac{\text{bar\_area}}{A_s} = 8.7 \cdot \text{in}$$

Use #5 bars @ bar\_space = 8.0in

### **Design Step 2.9 - Check for Positive Flexure Cracking under Service Limit State**

S5.7.3.4

The control of cracking by distribution of reinforcement must be checked (Applicable only when slab is not designed with empirical method).

The spacing of mild steel reinforcement in the layer closest to the tension face shall satisfy:

$$s_{\text{cracking}} \leq \frac{(700 \cdot \gamma_e)}{\beta_s \cdot f_{ss}} - 2 \cdot d_c$$

SEquation 5.7.3.4-1

Thickness of concrete cover measured from extreme tension fiber to center of flexural reinforcement:

$$d_c = 1\text{in} + \frac{\text{bar\_diam}}{2}$$

$$d_c = 1.31 \cdot \text{in}$$

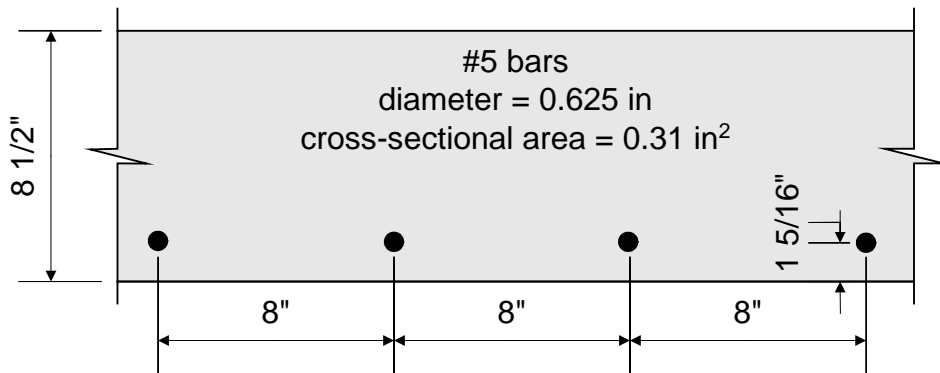
For Class 1 exposure condition:  $\gamma_e = 1.0$

Overall thickness or depth of component:  $h = 8.5\text{in}$

Overall Calculated tensile stress in mild steel reinforcement at service limit state not to exceed:

$$f_{ss} = 0.6 \cdot f_y = 36.00 \cdot \text{ksi}$$

$$\frac{(700 \cdot \gamma_e)}{\beta_s \cdot f_{ss}} - 2 \cdot d_c = 12.81\text{in} > 8\text{in} \quad \text{OK}$$



**Figure 2-5 Bottom Transverse Reinforcement**

$$E_s = 29000\text{ksi} \quad \text{S5.4.3.2}$$

$$K_1 = 1.0$$

$$w_c = 0.145\text{kcf} \quad \text{S5.4.2.4}$$

$$E_c = 33000 \cdot K_1 \cdot w_c^{1.5} \cdot \sqrt{f_c} \quad \text{SEquation C5.4.2.4-2}$$

$$E_c = 3644\text{ksi}$$

$$n = \frac{E_s}{E_c} \quad n = 7.96$$

Use  $\underline{n} = 8$

Service positive live load moment:

Based on Table 2-2, the maximum unfactored positive live load moment is 36.76 K-ft, located at 0.4S in Bay 1 for a single truck. The maximum service positive live load moment is computed as follows:

$$\gamma_{LL} = 1.0$$

$$M_{u_{positiveA}} = \gamma_{LL} \cdot (1 + IM) \cdot \frac{36.76 \text{ K} \cdot \text{ft}}{W_{posstripa}}$$

$$M_{u_{positiveA}} = 6.49 \cdot \frac{\text{K} \cdot \text{ft}}{\text{ft}}$$

Service positive dead load moment:

From Table 2-1, the maximum unfactored slab, parapet, and future wearing surface positive dead load moment occurs in Bay 2 at a distance of 0.4S. The maximum service positive dead load moment is computed as follows:

$$\gamma_{pDCserv} = 1.0$$

STable 3.4.1-1

$$\gamma_{pDWserv} = 1.0$$

STable 3.4.1-1

$$M_{u_{posdead}} = \gamma_{pDCserv} \cdot \left( 0.38 \cdot \frac{\text{K} \cdot \text{ft}}{\text{ft}} \right) + \gamma_{pDCserv} \cdot \left( 0.19 \cdot \frac{\text{K} \cdot \text{ft}}{\text{ft}} \right) \dots \\ + \gamma_{pDWserv} \cdot \left( 0.09 \cdot \frac{\text{K} \cdot \text{ft}}{\text{ft}} \right)$$

$$M_{u_{posdead}} = 0.66 \cdot \frac{\text{K} \cdot \text{ft}}{\text{ft}}$$

The total service positive design moment is:

$$M_{u_{postotalA}} = M_{u_{positiveA}} + M_{u_{posdead}}$$

$$M_{u_{postotalA}} = 7.15 \cdot \frac{\text{K} \cdot \text{ft}}{\text{ft}}$$

To solve for the actual stress in the reinforcement, the transformed moment of inertia and the distance from the neutral axis to the centroid of the reinforcement must be computed:

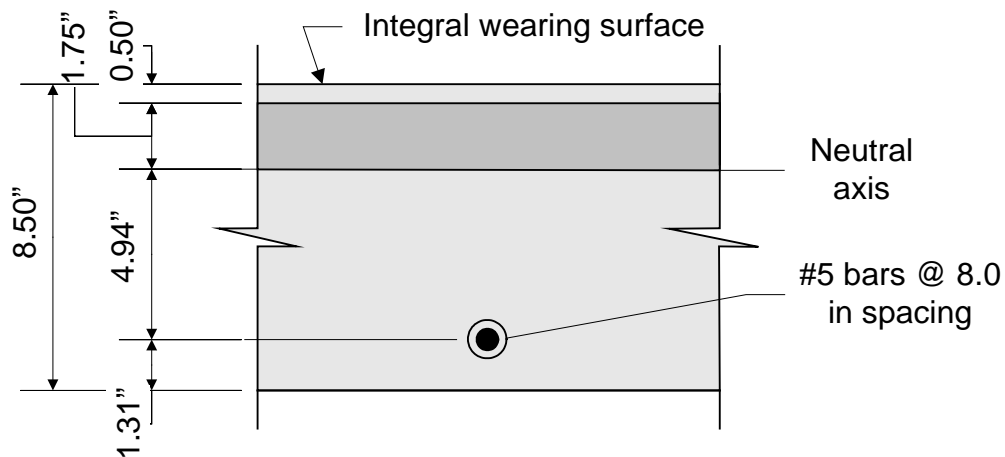
$$d_e = 6.69\text{in} \quad A_s = 0.465 \frac{\text{in}^2}{\text{ft}} \quad n = 8$$

$$\rho = \frac{A_s}{\frac{b}{\text{ft}} \cdot d_e} \quad \rho = 0.00579$$

$$k = \sqrt{(\rho \cdot n)^2 + (2 \cdot \rho \cdot n)} - \rho \cdot n$$

$$k = 0.262$$

$$k \cdot d_e = 1.75 \cdot \text{in}$$



**Figure 2-6 Crack Control for Positive Reinforcement under Live Loads**

Once  $k d_e$  is known, the transformed moment of inertia can be computed:

$$d_e = 6.69\text{in}$$

$$A_s = 0.465 \frac{\text{in}^2}{\text{ft}}$$

$$I_t = \frac{1}{3} \cdot \left( 12 \frac{\text{in}}{\text{ft}} \right) \cdot (k \cdot d_e)^3 + n \cdot A_s \cdot (d_e - k \cdot d_e)^2$$



$$I_t = 112.22 \cdot \frac{\text{in}^4}{\text{ft}}$$

Now, the actual stress in the reinforcement can be computed:

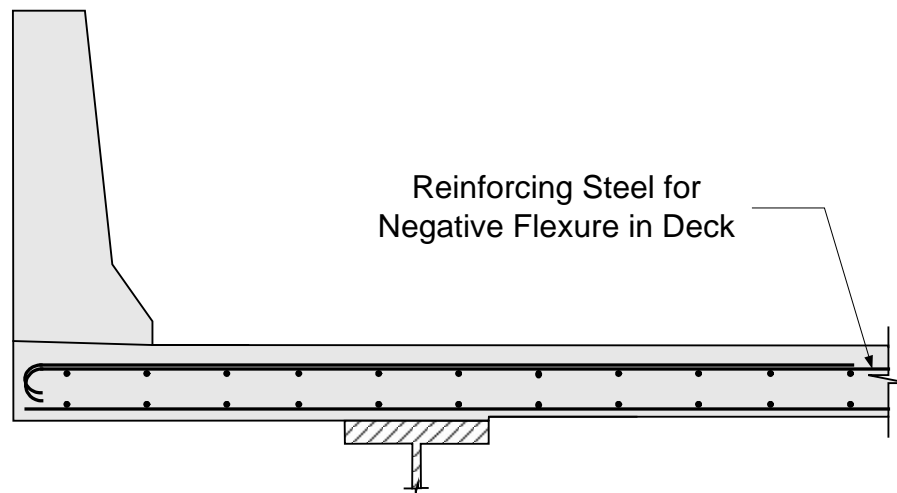
$$M_{u_{\text{posttotalA}}} = 7.15 \cdot K \cdot \frac{\text{ft}}{\text{ft}} \quad y = d_e - k \cdot d_e \quad y = 4.94 \cdot \text{in}$$

$$f_s = \frac{n \cdot \left( M_{u_{\text{posttotalA}}} \cdot 12 \frac{\text{in}}{\text{ft}} \cdot y \right)}{I_t}$$

$$f_s = 30.23 \cdot \text{ksi} \quad f_{sa} > f_s \quad \text{OK}$$

### **Design Step 2.10 - Design for Negative Flexure in Deck**

The negative flexure reinforcing steel design is similar to the positive flexure reinforcing steel design.



**Figure 2-7 Reinforcing Steel for Negative Flexure in Deck**

Assume #5 bars: bar\_diam = 0.625in

Bar\_area = 0.31in<sup>2</sup>

Effective depth,  $d_e$  = total slab thickness - top cover - 1/2 bar diameter

$$d_e = t_s - \text{Cover}_t - \frac{\text{bar\_diam}}{2} \quad d_e = 5.69 \text{ in}$$

Solve for the required amount of reinforcing steel, as follows:

S5.5.4.2.1

$$\phi_f = 0.90$$

$$b = 12 \text{ in}$$

$$R_n = \frac{-M_{\text{unegttotalA}} \cdot 12 \text{ in}}{(\phi_f \cdot b \cdot d_e^2)} \quad R_n = 0.47 \cdot \frac{\text{K}}{\text{in}^2}$$

$$\rho = 0.85 \left( \frac{f_c}{f_y} \right) \cdot \left[ 1.0 - \sqrt{1.0 - \frac{(2 \cdot R_n)}{(0.85 \cdot f_c)}} \right]$$

$$\rho = 0.00849$$

$$A_s = \rho \cdot \frac{b}{\text{ft}} \cdot d_e \quad A_s = 0.58 \cdot \frac{\text{in}^2}{\text{ft}}$$

$$\text{Required bar spacing} = \frac{\text{bar\_area}}{A_s} = 6.4 \text{ in}$$

Use #5 bars @ bar\_space = 6.0in

### **Design Step 2.11 - Check for Negative Flexure Cracking under Service Limit State**

S5.7.3.4

The control of cracking by distribution of reinforcement must be checked (Applicable only when slab is not designed with empirical method).

The spacing of mild steel reinforcement in the layer closest to the tension face shall satisfy:

$$s_{\text{cracking}} \leq \frac{(700 \cdot \gamma_e)}{\beta_s \cdot f_{ss}} - 2 \cdot d_c$$

SEquation 5.7.3.4-1

Thickness of concrete cover measured from extreme tension fiber to center of flexural reinforcement (1/2" wearing surface not included):

$$d_c = 2\text{in} + \frac{\text{bar\_diam}}{2}$$

$$d_c = 2.31\text{in}$$

For Class 1 exposure condition:  $\gamma_e = 1.0$

Overall thickness or depth of component:  $h = 8.5\text{in}$

Overall Calculated tensile stress in mild steel reinforcement at service limit state not to exceed:

$$f_{ss} = 0.6 \cdot f_y = 36.00\text{ksi}$$

$$\beta_s = 1 + \frac{d_c}{0.7 \cdot (h - d_c)} = 1.53$$

$$\frac{(700 \cdot \gamma_e)}{\beta_s \cdot f_{ss}} - 2 \cdot d_c = 8.09\text{in} > 6\text{in} \quad \text{OK}$$

Service negative live load moment:

From Table 2-2, the maximum unfactored negative live load moment is -29.40 K-ft, located at 0.0S in Bay 4 for two trucks. The maximum service negative live load moment is:

STable 3.4.1-1

$$\gamma_{LL} = 1.0$$

$$M_{U_{\text{negliveA}}} = \gamma_{LL} \cdot (1 + IM) \cdot \frac{-29.40\text{K}\cdot\text{ft}}{W_{\text{negstripa}}}$$

$$M_{U_{\text{negliveA}}} = -6.07 \cdot \frac{\text{K}\cdot\text{ft}}{\text{ft}}$$

Service negative dead load moment:

From Table 2-1, the maximum unfactored negative dead load moment occurs in Bay

4 at a distance of 1.0S. The maximum service negative dead load moment is computed as follows:

STable 3.4.1-1

$$\gamma_{pDCservice} = 1.0 \quad \gamma_{pDWservice} = 1.0$$

$$\begin{aligned} \underline{M_{Unegdead}} &= \gamma_{pDCservice} \cdot \left( -0.74 \cdot \frac{K \cdot ft}{ft} - 1.66 \frac{K \cdot ft}{ft} \right) \dots \\ &\quad + \gamma_{pDWservice} \cdot \left( -0.06 \cdot \frac{K \cdot ft}{ft} \right) \end{aligned}$$

$$M_{Unegdead} = -2.46 \cdot \frac{K \cdot ft}{ft}$$

The total service negative design moment is:

$$\underline{M_{UnegtotalA}} = M_{UnegliveA} + M_{Unegdead}$$

$$M_{UnegtotalA} = -8.53 \cdot \frac{K \cdot ft}{ft}$$

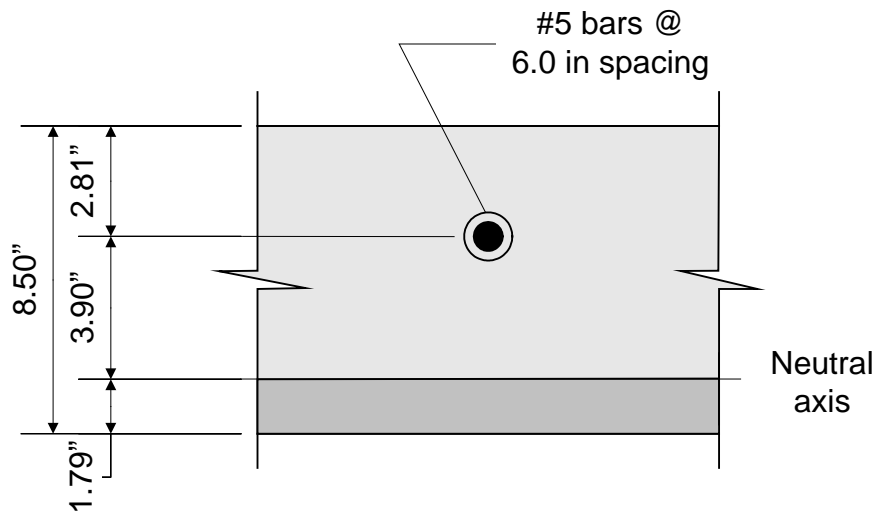
$$\underline{d_e} = 5.69in \quad \underline{A_s} = 0.62 \frac{in^2}{ft} \quad n = 8$$

$$\underline{\rho} = \frac{A_s}{\frac{b}{ft} \cdot d_e} \quad \rho = 0.00908$$

$$\underline{k} = \sqrt{(\rho \cdot n)^2 + (2 \cdot \rho \cdot n)} - \rho \cdot n$$

$$k = 0.315$$

$$k \cdot d_e = 1.79 \cdot in$$



**Figure 2-8 Crack Control for Negative Reinforcement under Live Loads**

Once  $k d_e$  is known, the transformed moment of inertia can be computed:

$$d_e = 5.69 \text{ in}$$

$$A_s = 0.62 \frac{\text{in}^2}{\text{ft}}$$

$$I_t = \frac{1}{3} \cdot \left( 12 \frac{\text{in}}{\text{ft}} \right) \cdot (k \cdot d_e)^3 + n \cdot A_s \cdot (d_e - k \cdot d_e)^2$$

$$I_t = 98.38 \cdot \frac{\text{in}^4}{\text{ft}}$$

Now, the actual stress in the reinforcement can be computed:

$$M_{u_{\text{negtotalA}}} = -8.53 \cdot K \cdot \frac{\text{ft}}{\text{ft}} \quad y_w = d_e - k \cdot d_e \quad y = 3.90 \cdot \text{in}$$

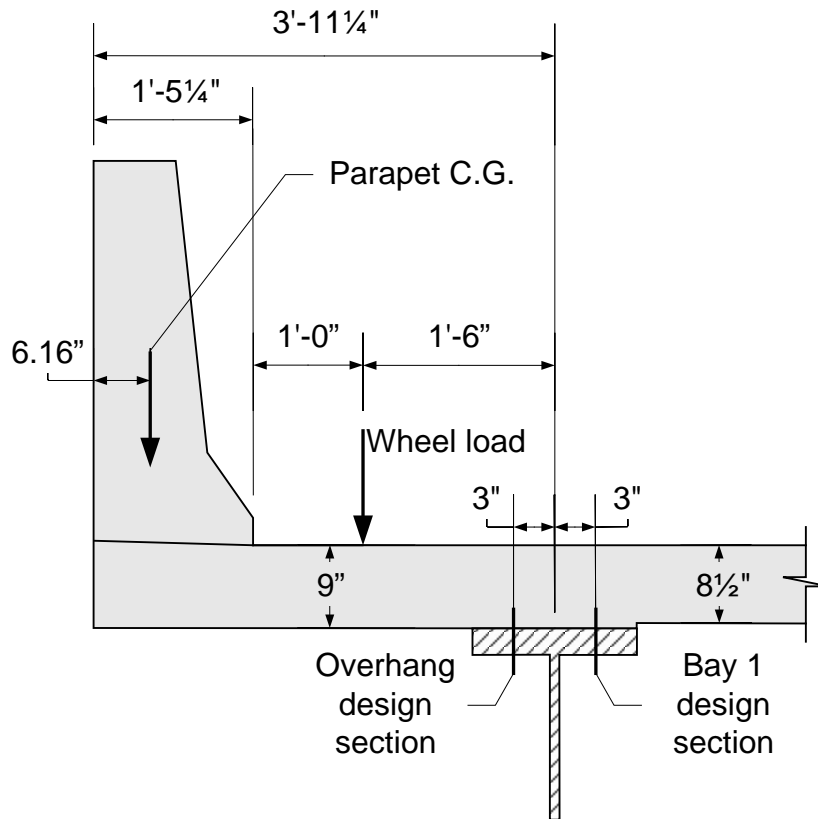
$$f_{sa} = \frac{n \cdot \left( -M_{u_{\text{negtotalA}}} \cdot 12 \frac{\text{in}}{\text{ft}} \cdot y \right)}{I_t}$$

$$f_s = 32.44 \cdot \text{ksi} \quad f_{sa} > f_s \quad \text{OK}$$

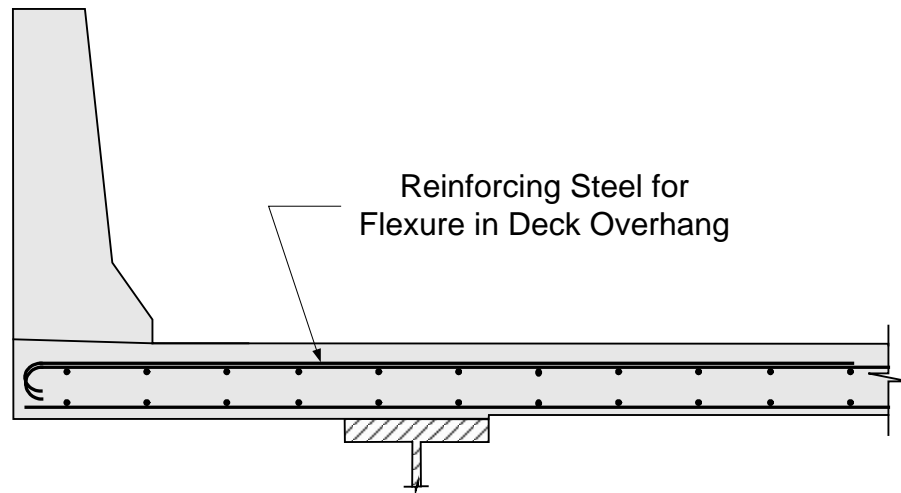
**Design Step 2.12 - Design for Flexure in Deck Overhang**

*SA13.4.1 and CA13.4.2*

Bridge deck overhangs must be designed to satisfy three different design cases. In the first design case, the overhang must be designed for horizontal (transverse and longitudinal) vehicular collision forces. For the second design case, the overhang must be designed to resist the vertical collision force. Finally, for the third design case, the overhang must be designed for dead and live loads. For Design Cases 1 and 2, the design forces are for the extreme event limit state. For Design Case 3, the design forces are for the strength limit state. Also, the deck overhang region must be designed to have a resistance larger than the actual resistance of the concrete parapet.



**Figure 2-9 Deck Overhang Dimensions and Live Loading**



**Figure 2-10 Reinforcing Steel for Flexure in Deck Overhang**

### Design Case 1 - Design Overhang for Horizontal Vehicular Collision Force

#### SA13.4.1

The horizontal vehicular collision force must be checked at the inside face of the parapet, at the design section in the overhang, and at the design section in the first bay.

#### Case 1A - Check at Inside Face of Parapet

The overhang must be designed for the vehicular collision plus dead load moment acting concurrently with the axial tension force from vehicular collision.

For the extreme event limit state:

$$\phi_{ext} = 1.0$$

S1.3.2.1

$$\gamma_{pDC} = 1.25$$

STable 3.4.1-2

$$M_{co} = 28.21 \cdot K \cdot \frac{\text{ft}}{\text{ft}} \quad (\text{see parapet properties})$$

$$M_{DCdeck} = \gamma_{pDC} \cdot \frac{\left[ \left( \frac{9 \text{ in}}{12 \frac{\text{in}}{\text{ft}}} \right) \cdot (0.150 \text{ kcf}) \cdot (1.4375 \text{ ft})^2 \right]}{2}$$

$$M_{DCdeck} = 0.15 \cdot \frac{K \cdot \text{ft}}{\text{ft}}$$

$$M_{DCpar} = \gamma_{pDC} \cdot W_{par} \cdot \left( 1.4375 \text{ ft} - \frac{6.16 \text{ in}}{12 \frac{\text{in}}{\text{ft}}} \right)$$

$$M_{DCpar} = 0.61 \cdot \frac{K \cdot \text{ft}}{\text{ft}}$$

$$M_{Utotal} = M_{co} + M_{DCdeck} + M_{DCpar}$$

$$M_{Utotal} = 28.97 \cdot \frac{K \cdot \text{ft}}{\text{ft}}$$

The axial tensile force is:

$$T = \frac{R_w}{L_c + 2H_{par}}$$

SA13.4.2

Before the axial tensile force can be calculated, the terms  $L_c$  and  $R_w$  need to be defined.

SA13.3.1

$L_c$  is the critical wall length over which the yield line mechanism occurs:

$$L_c = \frac{L_t}{2} + \sqrt{\left( \frac{L_t}{2} \right)^2 + \frac{8 \cdot H \cdot (M_b + M_w)}{M_c}}$$



Since the parapet is not designed in this design example, the variables involved in this calculation are given below:

STable A13.2-1

$L_t = 4$ ft	longitudinal length of distribution of impact force $F_t$
$M_b = 0$ K·ft *	additional flexural resistance of beam in addition to $M_w$ , if any, at top of wall
$M_c = 16.00$ $\frac{\text{K}\cdot\text{ft}}{\text{ft}}$ *	flexural resistance of the wall about an axis parallel to the longitudinal axis of the bridge
$M_w = 18.52$ K·ft *	flexural resistance of the wall about its vertical axis
$H = 3.50$ ft	height of parapet

\* Based on parapet properties not included in this design example. See Publication Number FHWA HI-95-017, Load and Resistance Factor Design for Highway Bridges, Participant Notebook, Volume II (Version 3.01), for the method used to compute the parapet properties.

$$L_c \text{ is then: } L_{cw} = \frac{L_t}{2} + \sqrt{\left(\frac{L_t}{2}\right)^2 + \frac{8 \cdot H \cdot (M_b + M_w)}{M_c}} \quad L_c = 8.03 \text{ ft}$$

SA13.3.1

$R_w$  is the total transverse resistance of the railing and is calculated using the following equation for impacts within a wall segment:

$$R_{ww} = \left(\frac{2}{2 \cdot L_c - L_t}\right) \cdot \left(8 \cdot M_b + 8M_w + \frac{M_c \cdot L_c^2}{H}\right)$$

$$R_w = 73.45 \text{ K}$$

$$\text{use } R_{ww} = 117.40\text{K}$$

Now, the axial tensile force is:

$$T = 5.92 \cdot \frac{K}{ft}$$

The overhang slab thickness is:  $t_o=9.0in$

For #5 bars:  $bar\_diam = 0.625in$

$$d_e = t_o - Cover_t - \frac{bar\_diam}{2} \quad d_e = 6.19 \cdot in$$

The required area of reinforcing steel is computed as follows:

$$b = 12in$$

$$R_n = \frac{Mu_{total} \cdot 12in}{(\phi_{ext} \cdot b \cdot d_e^2)} \quad R_n = 0.76 \cdot \frac{K}{in^2}$$

$$\rho = 0.85 \left( \frac{f_c}{f_y} \right) \cdot \left[ 1.0 - \sqrt{1.0 - \frac{(2 \cdot R_n)}{(0.85 \cdot f_c)}} \right]$$

$$\rho = 0.0145$$

$$A_s = \rho \cdot \frac{b}{ft} \cdot d_e \quad A_s = 1.07 \cdot \frac{in^2}{ft}$$

$$Use A_s = 1.24 \frac{in^2}{ft} \quad (2 - \#5 \text{ bars bundled at } 6.0 \text{ in})$$

Once the required area of steel is known, the moment resistance must be checked, along with the tension strain limits.

$$T_a = A_s \cdot f_y \quad T_a = 74.40 \cdot \frac{K}{ft}$$

$$C = T_a - T \quad C = 68.48 \cdot \frac{K}{ft} \quad Use \quad C = 68.48K$$

$$a = \frac{C}{0.85 \cdot f_c \cdot b} \quad a = 1.68 \cdot \text{in}$$

$$M_n = T_a \cdot \left( d_e - \frac{a}{2} \right) - T \cdot \left( \frac{d_e}{2} - \frac{a}{2} \right) \quad M_n = 32.05 \cdot \frac{\text{K} \cdot \text{ft}}{\text{ft}}$$

$$M_r = \phi_{\text{ext}} \cdot M_n \quad M_r = 32.05 \cdot \frac{\text{K} \cdot \text{ft}}{\text{ft}}$$

$$M_r \geq M_{u_{\text{total}}} \quad \text{OK}$$

A5.7.2.1

$$\text{Tensile Strain Limit:} \quad \epsilon_{tl} = 0.005$$

$$\text{Actual Tensile Strain:} \quad \frac{T_a}{\frac{A_s}{E_s}} = 0.002$$

$$0.002 < 0.005 \quad \text{OK}$$

### Case 1B - Check at Design Section in Overhang

The collision forces are distributed over a distance  $L_c$  for moment and  $L_c + 2H$  for axial force. When the design section is moved to  $1/4b_f$  away from the girder centerline in the overhang, the distribution length will increase. This example assumes a distribution length increase based on a 30 degree angle from the face of the parapet.

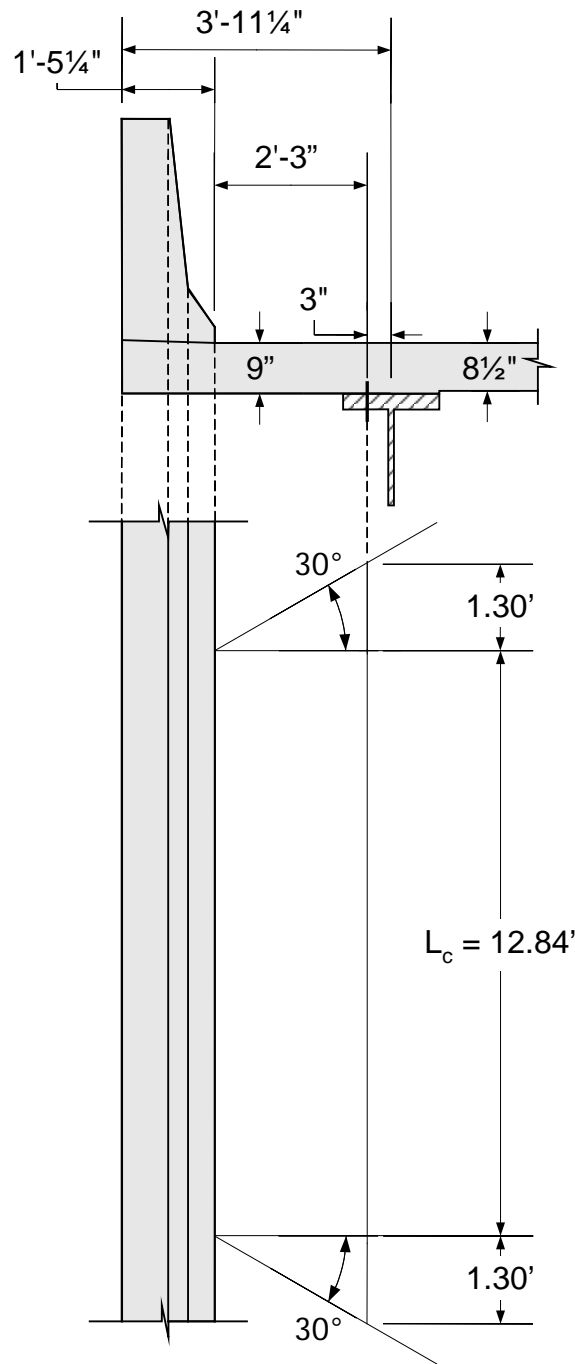


Figure 2-11 Assume Distribution of Collision Moment Load in the Overhang

For the extreme event limit state:

$$\phi_{ext} = 1.0 \quad \text{S1.3.2.1}$$

$$\gamma_{pDC} = 1.25 \quad \text{STable 3.4.1-2}$$

$$\gamma_{pDW} = 1.50 \quad \text{STable 3.4.1-2}$$

$$L_c = 12.84\text{ft} \quad (\text{see parapet properties})$$

$$M_{co} = 28.21 \cdot \frac{\text{K}\cdot\text{ft}}{\text{ft}} \quad (\text{see parapet properties})$$

$$M_{cB} = \frac{M_{co} \cdot L_c}{L_c + 2 \cdot 1.30\text{ft}} \quad M_{cB} = 23.46 \cdot \frac{\text{K}\cdot\text{ft}}{\text{ft}}$$

Factored dead load moment:

$$M_{DCdeck} = \gamma_{pDC} \cdot \frac{\left[ \left( \frac{9.0\text{in}}{12 \frac{\text{in}}{\text{ft}}} \right) \cdot (W_c) \cdot (3.6875\text{ft})^2 \right]}{2}$$

$$M_{DCdeck} = 0.96 \cdot \frac{\text{K}\cdot\text{ft}}{\text{ft}}$$

$$M_{DCpar} = \gamma_{pDC} \cdot W_{par} \cdot \left( 3.6875\text{ft} - \frac{6.16\text{in}}{12 \frac{\text{in}}{\text{ft}}} \right)$$

$$M_{DCpar} = 2.10 \cdot \frac{\text{K}\cdot\text{ft}}{\text{ft}}$$

$$M_{DWfws} = \gamma_{pDW} \cdot \frac{\left[ \left( \frac{2.5\text{in}}{12 \frac{\text{in}}{\text{ft}}} \right) \cdot (W_{fws}) \cdot (3.6875\text{ft} - 1.4375\text{ft})^2 \right]}{2}$$

$$M_{DWfws} = 0.11 \cdot \frac{\text{K}\cdot\text{ft}}{\text{ft}}$$

$$M_{u_{total}} = M_{cB} + M_{DCdeck} + M_{DCpar} + M_{DWfws}$$

$$M_{u_{total}} = 26.63 \cdot K \cdot \frac{ft}{ft}$$

SA13.4.2

The axial tensile force is:

$$T = \frac{R_w}{L_c + 2H_{par} + 2 \cdot (1.30ft)}$$

$$T = 5.23 \cdot \frac{K}{ft}$$

The overhang slab thickness is:  $t_o = 9.0in$

For #5 bars:  $bar\_diam = 0.625in$

$$d_e = t_o - Cover_t - \frac{bar\_diam}{2} \quad d_e = 6.19 \cdot in$$

The required area of reinforcing steel is computed as follows:

$$b = 12in$$

$$R_n = \frac{M_{u_{total}} \cdot 12in}{(\phi_{ext} \cdot b \cdot d_e^2)} \quad R_n = 0.70 \cdot \frac{K}{in^2}$$

$$\rho = 0.85 \left( \frac{f_c}{f_y} \right) \cdot \left[ 1.0 - \sqrt{1.0 - \frac{(2 \cdot R_n)}{(0.85 \cdot f_c)}} \right]$$

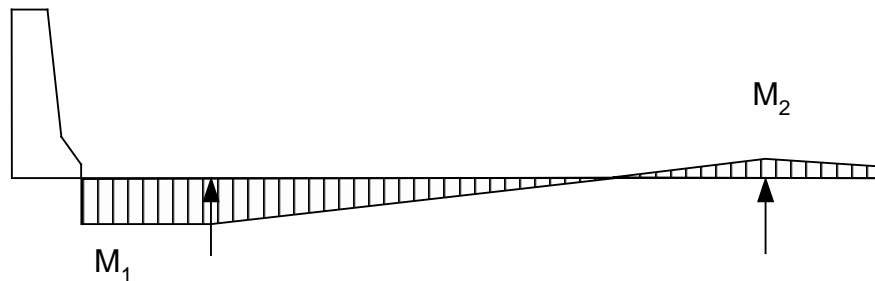
$$\rho = 0.0131$$

$$A_s = \rho \cdot \frac{b}{ft} \cdot d_e \quad A_s = 0.97 \cdot \frac{in^2}{ft}$$

The above required reinforcing steel is less than the reinforcing steel required for Case 1A.

### Case 1C - Check at Design Section in First Span

The total collision moment can be treated as shown in Figure 2-12. The moment ratio,  $M_2/M_1$ , can be calculated for the design strip. One way to approximate this moment is to set it equal to the ratio of the moments produced by the parapet self-weight at the 0.0S points of the first and second bay. The collision moment per unit width can then be determined by using the increased distribution length based on the 30 degree angle distribution (see Figure 2-11). The dead load moments at this section can be obtained directly from Table 2-1.



**Figure 2-12 Assumed Distribution of the Collision Moment Across the Width of the Deck**

Collision moment at exterior girder:

$$M_{co} = -28.21 \frac{\text{K}\cdot\text{ft}}{\text{ft}} \quad M_1 = M_{co}$$

Parapet self-weight moment at Girder 1 (0.0S in Bay 1):

$$Par_1 = -1.66 \frac{\text{K}\cdot\text{ft}}{\text{ft}}$$

Parapet self-weight moment at Girder 2 (0.0S in Bay 2):

$$Par_2 = 0.47 \frac{\text{K}\cdot\text{ft}}{\text{ft}}$$

Collision moment at 1/4bf in Bay 1:

$$M_2 = M_1 \cdot \left( \frac{Par_2}{Par_1} \right) \quad M_2 = 7.99 \cdot \frac{\text{K}\cdot\text{ft}}{\text{ft}}$$

By interpolation for a design section at 1/4bf in Bay 1, the total collision moment is:

$$M_{cM2M1} = M_{co} + 0.25ft \cdot \frac{(-M_{co} + M_2)}{9.75ft}$$

$$M_{cM2M1} = -27.28 \cdot \frac{K \cdot ft}{ft}$$

As in Case 1B, the 30 degree angle distribution will be used:

$$\phi_{ext} = 1.0 \quad \text{S1.3.2.1}$$

$$\gamma_{pDC} = 1.25 \quad \text{STable 3.4.1-2}$$

$$\gamma_{pDW} = 1.50 \quad \text{STable 3.4.1-2}$$

$$M_{cM2M1} = -27.28 \cdot \frac{K \cdot ft}{ft}$$

$$M_{cC} = \frac{M_{cM2M1} \cdot L_c}{L_c + 2 \cdot (1.59ft)} \quad M_{cC} = -21.87 \cdot \frac{K \cdot ft}{ft}$$

Factored dead load moment (from Table 2-1):

$$M_{DCdeck} = \gamma_{pDC} \cdot \left( -0.74 \frac{K \cdot ft}{ft} \right)$$

$$M_{DCdeck} = -0.93 \cdot K \cdot \frac{ft}{ft}$$

$$M_{DCpar} = \gamma_{pDC} \cdot \left( -1.66 \frac{K \cdot ft}{ft} \right)$$

$$M_{DCpar} = -2.07 \cdot \frac{K \cdot ft}{ft}$$

$$M_{DWfs} = \gamma_{pDW} \cdot \left( -0.06 \frac{K \cdot ft}{ft} \right)$$



$$M_{DWfws} = -0.09 \cdot \frac{K \cdot ft}{ft}$$

$$M_{u_{total}} = M_{cC} + M_{DCdeck} + M_{DCpar} + M_{DWfws}$$

$$M_{u_{total}} = -24.96 \cdot \frac{K \cdot ft}{ft}$$

SA13.4.2

The axial tensile force is:

$$T = \frac{R_w}{L_c + 2H_{par} + 2 \cdot (1.59ft)}$$

$$T = 5.10 \cdot \frac{K}{ft}$$

Use a slab thickness equal to:  $t_s = 8.50in$

For #5 bars:  $bar\_diam = 0.625in$

$$d_e = t_s - Cover_t - \frac{bar\_diam}{2} \quad d_e = 5.69 \cdot in$$

The required area of reinforcing steel is computed as follows

$$b = 12in$$

$$R_n = \frac{-M_{u_{total}} \cdot 12in}{(\phi_{ext} \cdot b \cdot d_e^2)} \quad R_n = 0.77 \cdot \frac{K}{in^2}$$

$$\rho = 0.85 \left( \frac{f_c}{f_y} \right) \cdot \left[ 1.0 - \sqrt{1.0 - \frac{(2 \cdot R_n)}{(0.85 \cdot f_c)}} \right]$$

$$\rho = 0.0148$$

$$A_s = \rho \cdot \frac{b}{ft} \cdot d_e \quad A_s = 1.01 \cdot \frac{in^2}{ft}$$

The above required reinforcing steel is less than the reinforcing steel required for Case 1A.

### Design Case 2 - Design Overhang for Vertical Collision Force

SA13.4.1

For concrete parapets, the case of vertical collision force never controls. Therefore, this procedure does not need to be considered in this design example.

### Design Case 3 - Design Overhang for Dead Load and Live Load

SA13.4.1

#### Case 3A - Check at Design Section in Overhang

S5.5.4.2.1

The resistance factor for the strength limit state for flexure and tension in concrete is:

$$\phi_{str} = 0.90$$

STable 4.6.2.1.3-1

The equivalent strip for live load on an overhang is:

$$W_{overstrip} = 45.0 + 10.0 \cdot X$$

$$\text{For } X = 1.25 \text{ ft}$$

$$W_{overstrip} = 45.0 + 10.0X$$

$$W_{overstrip} = 57.50 \text{ in} \quad \text{or} \quad W_{overstrip} = 4.79 \text{ ft}$$

STable 3.6.1.1.2-1

Use a multiple presence factor of 1.20 for one lane loaded.

STable 3.6.2.1-1

Use a dynamic load allowance of 0.33.

Design factored overhang moment:

$$\gamma_{LL} = 1.75$$

STable 3.4.1-1

$$\gamma_{pDC} = 1.25$$

STable 3.4.1-2

$$\gamma_{pDW} = 1.50$$

STable 3.4.1-2

$$M_{DCdeck} = \gamma_{pDC} \cdot \frac{\left[ \left( \frac{9.0 \text{ in}}{12 \frac{\text{in}}{\text{ft}}} \right) \cdot (W_c) \cdot (3.6875 \text{ ft})^2 \right]}{2}$$

$$M_{DCdeck} = 0.96 \cdot \frac{\text{K} \cdot \text{ft}}{\text{ft}}$$

$$M_{DCpar} = \gamma_{pDC} \cdot W_{par} \cdot \left( 3.6875 \text{ ft} - \frac{6.16 \text{ in}}{12 \frac{\text{in}}{\text{ft}}} \right)$$

$$M_{DCpar} = 2.10 \cdot \frac{\text{K} \cdot \text{ft}}{\text{ft}}$$

$$M_{DWfws} = \gamma_{pDW} \cdot W_{fws} \cdot \frac{\left( \frac{2.5 \cdot \text{in}}{12 \cdot \frac{\text{in}}{\text{ft}}} \right) \cdot (3.6875 \cdot \text{ft} - 1.4375 \cdot \text{ft})^2}{2}$$

$$M_{DWfws} = 0.11 \cdot \frac{\text{K} \cdot \text{ft}}{\text{ft}}$$

$$M_{LL} = \gamma_{LL} \cdot (1 + IM) \cdot (1.20) \cdot \left( \frac{16 \text{ K}}{W_{\text{overstrip}}} \right) \cdot 1.25 \text{ ft}$$

$$M_{LL} = 11.66 \cdot \frac{\text{K} \cdot \text{ft}}{\text{ft}}$$

$$M_{Utotal} = M_{DCdeck} + M_{DCpar} + M_{DWfws} + M_{LL}$$

$$M_{Utotal} = 14.83 \cdot \frac{\text{K} \cdot \text{ft}}{\text{ft}}$$

Calculate the required area of steel:

For #5 bars:  $\text{bar\_diam} = 0.625\text{in}$

$$d_e = t_o - \text{Cover}_t - \frac{\text{bar\_diam}}{2}$$

$$d_e = 6.19\text{in}$$

$$b = 12\text{in}$$

$$R_n = \frac{M_{u\text{total}} \cdot 12\text{in}}{(\phi_{\text{str}} \cdot b \cdot d_e^2)} \quad R_n = 0.43 \cdot \frac{\text{K}}{\text{in}^2}$$

$$\rho = 0.85 \left( \frac{f_c}{f_y} \right) \cdot \left[ 1.0 - \sqrt{1.0 - \frac{(2 \cdot R_n)}{(0.85 \cdot f_c)}} \right]$$

$$\rho = 0.00770$$

$$A_s = \rho \cdot \frac{b}{\text{ft}} \cdot d_e \quad A_s = 0.57 \cdot \frac{\text{in}^2}{\text{ft}}$$

The above required reinforcing steel is less than the reinforcing steel required for Cases 1A, 1B, and 1C.

### Case 3B - Check at Design Section in First Span

Use a slab thickness equal to:  $t_s = 8.5\text{in}$

The dead and live load moments are taken from Tables 2-1 and 2-2. The maximum negative live load moment occurs in Bay 4. Since the negative live load moment is produced by a load on the overhang, compute the equivalent strip based on a moment arm to the centerline of girder.

Design factored moment:

$$\gamma_{LL} = 1.75$$

S Table 3.4.1-1

$$\gamma_{DC} = 1.25$$

S Table 3.4.1-2

$$\gamma_{pDW} = 1.50$$

STabel 3.4.1-2

$$W_{\text{overstrip}} = 45.0 + 10.0 \cdot X$$

$$\text{For } X = 1.50 \text{ ft}$$

$$W_{\text{overstrip}} = 45.0 + 10.0X$$

$$W_{\text{overstrip}} = 60.00 \text{ in} \quad \text{or} \quad W_{\text{overstrip}} = 5.00 \text{ ft}$$

$$M_{\text{DCdeck}} = \gamma_{pDC} \cdot \left( -0.74 \frac{\text{K} \cdot \text{ft}}{\text{ft}} \right)$$

$$M_{\text{DCdeck}} = -0.93 \cdot \frac{\text{K} \cdot \text{ft}}{\text{ft}}$$

$$M_{\text{DCpar}} = \gamma_{pDC} \cdot \left( -1.66 \frac{\text{K} \cdot \text{ft}}{\text{ft}} \right)$$

$$M_{\text{DCpar}} = -2.07 \cdot \frac{\text{K} \cdot \text{ft}}{\text{ft}}$$

$$M_{\text{DWfws}} = \gamma_{pDW} \cdot \left( -0.06 \frac{\text{K} \cdot \text{ft}}{\text{ft}} \right)$$

$$M_{\text{DWfws}} = -0.09 \cdot \frac{\text{K} \cdot \text{ft}}{\text{ft}}$$

$$M_{\text{LL}} = \gamma_{\text{LL}} \cdot (1 + \text{IM}) \cdot \frac{(-29.40 \text{ K} \cdot \text{ft})}{W_{\text{overstrip}}}$$

$$M_{\text{LL}} = -13.69 \cdot \frac{\text{K} \cdot \text{ft}}{\text{ft}}$$

$$M_{\text{total}} = M_{\text{DCdeck}} + M_{\text{DCpar}} + M_{\text{DWfws}} + M_{\text{LL}}$$

$$M_{\text{total}} = -16.78 \cdot \frac{\text{K} \cdot \text{ft}}{\text{ft}}$$

Calculate the required area of steel:

For #5 bars: bar\_diam = 0.625in

$$d_e = t_s - \text{Cover}_t - \frac{\text{bar\_diam}}{2}$$

$$d_e = 5.69 \cdot \text{in}$$

$$b = 12 \text{in}$$

$$R_n = \frac{-M_{u\text{total}} \cdot 12 \text{in}}{(\phi_{\text{str}} \cdot b \cdot d_e^2)} \quad R_n = 0.58 \cdot \frac{\text{K}}{\text{in}^2}$$

$$\rho = 0.85 \left( \frac{f_c}{f_y} \right) \cdot \left[ 1.0 - \sqrt{1.0 - \frac{(2 \cdot R_n)}{(0.85 \cdot f_c)}} \right]$$

$$\rho = 0.0106$$

$$A_s = \rho \cdot \frac{b}{\text{ft}} \cdot d_e \quad A_s = 0.72 \cdot \frac{\text{in}^2}{\text{ft}}$$

The above required reinforcing steel is less than the reinforcing steel required for Cases 1A, 1B, and 1C.

The required area of reinforcing steel in the overhang is the largest of that required for Cases 1A, 1B, 1C, 3A, and 3B.

Case 1A controls with:

$$A_s = 1.24 \frac{\text{in}^2}{\text{ft}}$$

The negative flexure reinforcement provided from the design in Steps 2.10 and 2.11 is:

#5 bars at 6.0 inches: bar\_diam = 0.625in

$$\text{Bar\_area} = 0.31 \text{in}^2$$

$$A_{s\text{neg}} = \frac{\text{bar\_area}}{\text{ft}} \cdot \left( \frac{12\text{in}}{6\text{in}} \right)$$

$$A_{s\text{neg}} = 0.62 \cdot \frac{\text{in}^2}{\text{ft}}$$

$$0.62 \frac{\text{in}^2}{\text{ft}} < 1.24 \frac{\text{in}^2}{\text{ft}}$$

Since the area of reinforcing steel required in the overhang is greater than the area of reinforcing steel required in the negative moment regions, reinforcement must be added in the overhang area to satisfy the design requirements.

Bundle one #5 bar to each negative flexure reinforcing bar in the overhang area.

The new area of reinforcing steel is now:

$$A_s = 2 \cdot \left( 0.31 \cdot \frac{\text{in}^2}{\text{ft}} \right) \cdot \left( \frac{12\text{in}}{6\text{in}} \right)$$

$$A_s = 1.24 \cdot \frac{\text{in}^2}{\text{ft}}$$

Once the required area of reinforcing steel is known, the tension strain limits must be checked

$$d_{\text{emin}} = t_s - \text{Cover}_t - \frac{\text{bar\_diam}}{2}$$

$$d_{\text{emin}} = 5.69 \cdot \text{in}$$

$$T = A_s \cdot f_y \quad T = 74.40 \cdot \frac{\text{K}}{\text{ft}} \quad \text{Use} \quad T = 74.40\text{K}$$

$$a = \frac{T}{0.85 \cdot f_c \cdot b} \quad a = 1.82 \cdot \text{in}$$

A5.7.2.1

Tensile Strain Limit:  $\epsilon_{\text{tl}} = 0.005$

$$\frac{T}{A_s E_s} = 0.002$$

Actual Tension Strain:

$$0.002 < 0.005$$

OK

### **Design Step 2.13 - Check for Cracking in Overhang under Service Limit State**

Cracking in the overhang must be checked for the controlling service load (similar to Design Steps 2.9 and 2.11). In most deck overhang design cases, cracking does not control. Therefore, the computations for the cracking check are not shown in this deck overhang design example.

### **Design Step 2.14 - Compute Overhang Cut-off Length Requirement**

The next step is to compute the cut-off location of the additional #5bars in the first bay. This is done by determining the location where both the dead and live load moments, as well as the dead and collision load moments, are less than or equal to the resistance provided by #5 bars at 6 inch spacing (negative flexure steel design reinforcement).

Compute the nominal negative moment resistance based on #5bars at 6 inch spacing:

$$\text{Bar\_diam} = 0.625\text{in}$$

$$\text{Bar\_area} = 0.31\text{in}^2$$

$$A_s = \frac{\text{bar\_area}}{\text{ft}} \cdot \left( \frac{12\text{in}}{6\text{in}} \right)$$

$$A_s = 0.62 \cdot \frac{\text{in}^2}{\text{ft}}$$

$$d_e = t_s - \text{Cover}_t - \frac{\text{bar\_diam}}{2}$$

$$d_e = 5.69\text{in}$$

$$T = A_s \cdot f_y \quad T = 37.20 \cdot \frac{\text{K}}{\text{ft}} \quad \text{Use} \quad T = 37.20\text{K}$$

$$a = \frac{T}{0.85 \cdot f_c \cdot b} \quad a = 0.91\text{in}$$

$$M_n = A_s \cdot f_y \cdot \left( d_e - \frac{a}{2} \right)$$



$$M_n = 16.22 \cdot \frac{\text{K}\cdot\text{ft}}{\text{ft}}$$

Compute the nominal flexural resistance for negative flexure, as follows:

$$M_r = \phi_f \cdot M_n$$

$$M_r = 14.60 \cdot \frac{\text{K}\cdot\text{ft}}{\text{ft}}$$

Based on the nominal flexural resistance and on interpolation of the factored design moments, the theoretical cut-off point for the additional #5 bar is 3.75 feet from the centerline of the fascia girder.

#### S5.11.1.2

The additional cut-off length (or the distance the reinforcement must extend beyond the theoretical cut-off point) is the maximum of:

The effective depth of the member:  $d_e = 5.69\text{in}$

15 times the nominal bar diameter:  $15 \times 0.625\text{in} = 9.38\text{in}$

1/20 of the clear span:  $\frac{1}{20} \cdot \left( 9.75\text{ft} \cdot 12 \frac{\text{in}}{\text{ft}} \right) = 5.85\text{in}$

Use cut\_off = 9.5in

The total required length past the centerline of the fascia girder into the first bay is:

$$\text{cut\_off}_{\text{total}} = 3.75\text{ft} \cdot 12 \frac{\text{in}}{\text{ft}} + \text{cut\_off}$$

$$\text{cut\_off}_{\text{total}} = 54.50\text{in}$$

### **Design Step 2.15 - Compute Overhang Development Length**

$$d_b = 0.625 \text{ in}$$

$$A_b = 0.31 \text{ in}^2$$

$$f'_c = 4.0 \text{ ksi}$$

$$f_y = 60 \text{ ksi}$$

## S5.11.2.1.1

The basic development length is the larger of the following:

$$\frac{1.25 \cdot A_b \cdot f_y}{\sqrt{f_c}} = 11.63 \text{ in} \quad \text{or} \quad 0.4 \cdot d_b \cdot f_y = 15.00 \text{ in} \quad \text{or} \quad 12 \text{ in}$$

Use  $l_d = 15.00 \text{ in}$

## S5.11.2

The following modification factors must be applied:

Epoxy coated bars: 1.2 S5.11.2.1.2

Bundled bars: 1.2 S5.11.2.3

## S5.11.2.1.3

Spacing > 6 inches with more than 3 inches of clear cover in direction of spacing: 0.8

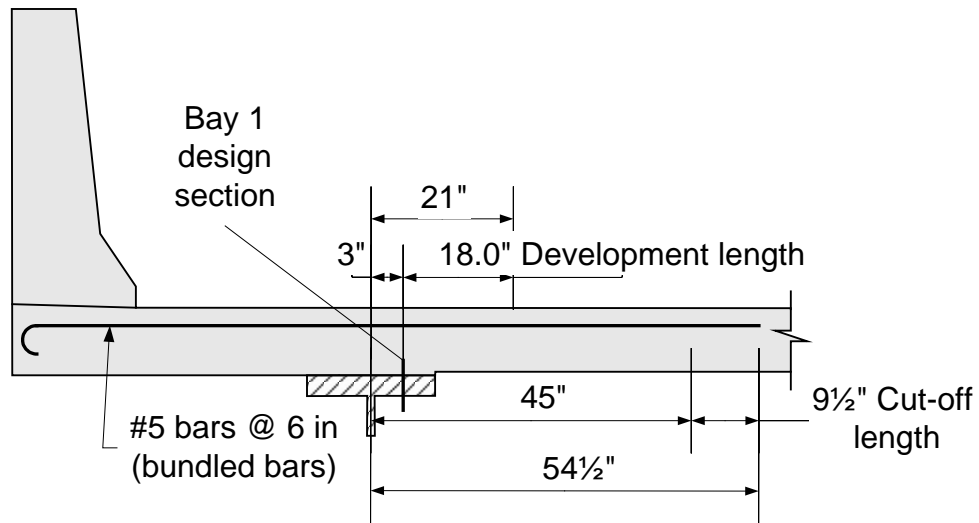
$$l_d = 15.00 \text{ in} \cdot (1.2) \cdot (1.2) \cdot (0.8)$$

$$l_d = 17.28 \text{ in} \quad \text{Use} \quad l_d = 18.00 \text{ in}$$

The required length past the centerline of the fascia girder is:

$$3.0 \text{ in} + l_d = 21.00 \text{ in}$$

$$21.00 \text{ in} < 54.50 \text{ in} \quad \text{provided}$$

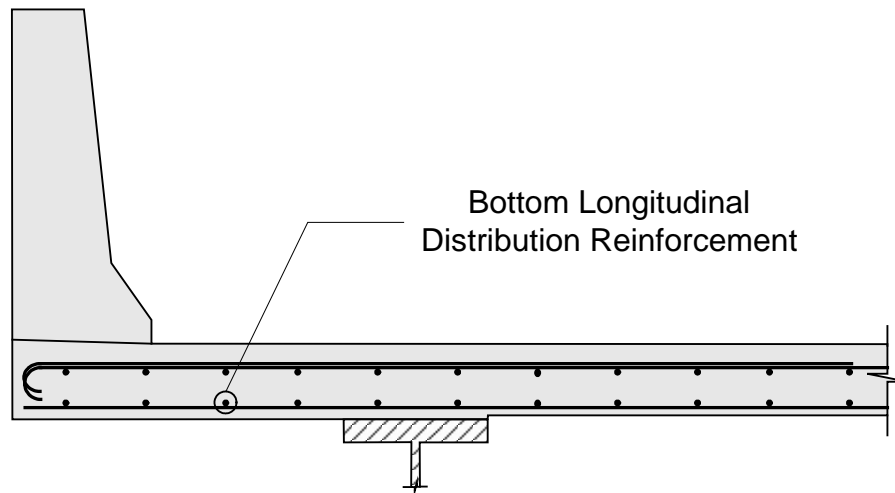


**Figure 2-13 Length of Overhang Negative Moment Reinforcement**

**Design Step 2.16 - Design Bottom Longitudinal Distribution Reinforcement**

S9.7.3.2

The bottom longitudinal distribution reinforcement is calculated based on whether the primary reinforcement is parallel or perpendicular to traffic.



**Figure 2-14 Bottom Longitudinal Distribution Reinforcement**

For this design example, the primary reinforcement is perpendicular to traffic.

$$S_e = 9.25 \text{ ft}$$

$$A_{s\text{botpercent}} = \frac{220}{\sqrt{S_e}} \quad \text{where} \quad A_{s\text{botlong}} \leq 67\%$$

$$A_{s\text{botpercent}} = 72.3 \quad \%$$

$$\text{Use } A_{s\text{botpercent}} = 67\%$$

For this design example, #5 bars at 8 inches were used to resist the primary positive moment.

$$\text{Bar\_diam} = 0.625\text{in}$$

$$\text{Bar\_area} = 0.31\text{in}^2$$

$$A_{s\_ft} = \text{bar\_area} \cdot \left( \frac{12\text{in}}{8\text{in}} \right)$$

$$A_{s\_ft} = 0.465 \frac{\text{in}^2}{\text{ft}}$$

$$A_{s\text{botlong}} = A_{s\text{botpercent}} \cdot A_{s\_ft}$$

$$A_{s\text{botlong}} = 0.31 \cdot \frac{\text{in}^2}{\text{ft}}$$

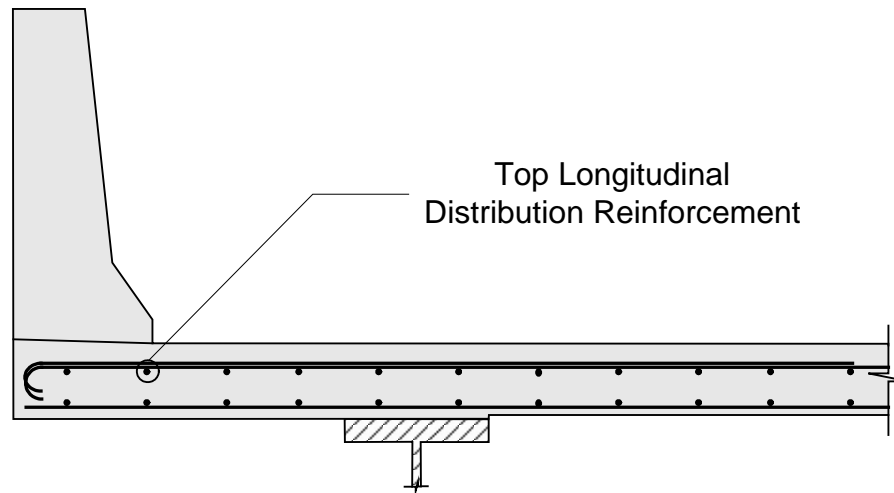
Calculate the required spacing using #5 bars:

$$\text{spacing} = \frac{\text{bar\_area}}{A_{s\text{botlong}}}$$

$$\text{spacing} = 1.00 \cdot \text{ft} \quad \text{or} \quad \text{spacing} = 11.94 \cdot \text{in}$$

$$\text{Use } \text{spacing} = 10\text{in}$$

Use #5 bars at 10 inch spacing for the bottom longitudinal reinforcement.

**Design Step 2.17 - Design Top Longitudinal Distribution Reinforcement****Figure 2-15 Top Longitudinal Distribution Reinforcement**

S5.10.8

The top longitudinal temperature and shrinkage reinforcement must satisfy:

$$A_s \geq \frac{1.30 \cdot b \cdot t}{2 \cdot (b + t) \cdot f_y}$$

And

$$0.11 \leq A_s \leq 0.60$$

When using the above equation, the calculated area of reinforcing steel must be equally distributed on both concrete faces. In addition, the maximum spacing of the temperature and shrinkage reinforcement must be the smaller of 3.0 times the deck thickness or 18.0 inches.

The amount of steel required for the top longitudinal reinforcement is:

$$A_{sreq} = \frac{(1.3 \cdot 12 \cdot 8.5)}{[2 \cdot (12 + 8.5) \cdot f_y]} \quad A_{sreq} = 0.054 \cdot \frac{\text{in}^2}{\text{ft}}$$

Check #4 bars at 10 inch spacing:

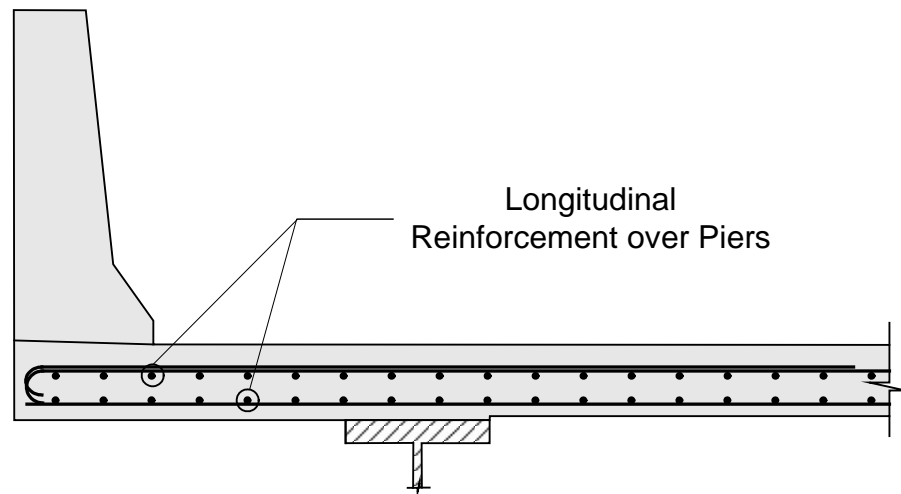
$$A_{sact} = 0.20 \cdot \frac{\text{in}^2}{\text{ft}} \cdot \left( \frac{12 \text{in}}{10 \text{in}} \right) \quad A_{sact} = 0.24 \cdot \frac{\text{in}^2}{\text{ft}}$$

$$0.24 \frac{\text{in}^2}{\text{ft}} > 0.054 \frac{\text{in}^2}{\text{ft}} \quad \text{And} \quad 0.11 \leq 0.24 \leq 0.60 \quad \text{OK}$$

Use #4 bars at 10 inch spacing for the top longitudinal temperature and shrinkage reinforcement.

### **Design Step 2.18 - Design Longitudinal Reinforcement over Piers**

If the superstructure is comprised of simple span precast girders made continuous for live load, the top longitudinal reinforcement should be designed according to S5.14.1.3. For continuous steel girder superstructures, design the top longitudinal reinforcement according to S6.10.1.7. For this design example, continuous steel girders are used.



**Figure 2-16 Longitudinal Reinforcement over Piers**

S6.10.1.7

The total longitudinal reinforcement should not be less than 1 percent of the total slab cross-sectional area. These bars must have a specified minimum yield strength of at least 60 ksi. Also, the bar size cannot be larger than a #6 bar.

Deck cross section:

$$A_{\text{deck}} = \frac{8.5\text{in} \cdot 12\text{in}}{\text{ft}}$$

$$A_{\text{deck}} = 102.00 \cdot \frac{\text{in}^2}{\text{ft}}$$

$$A_{s\_1\_percent} = 0.01 \cdot A_{\text{deck}}$$

$$A_{s\_1\_percent} = 1.02 \cdot \frac{\text{in}^2}{\text{ft}}$$

## S6.10.1.7

Two-thirds of the required longitudinal reinforcement should be placed uniformly in the top layer of the deck, and the remaining portion should be placed uniformly in the bottom layer. For both rows, the spacing should not exceed 12 inches.

$$\left(\frac{2}{3}\right) \cdot A_{s\_1\_percent} = 0.68 \cdot \frac{\text{in}^2}{\text{ft}} \quad \left(\frac{1}{3}\right) \cdot A_{s\_1\_percent} = 0.34 \cdot \frac{\text{in}^2}{\text{ft}}$$

Use #5 bars at 5 inch spacing in the top layer.

$$A_{s\text{provided}} = 0.31 \frac{\text{in}^2}{\text{ft}} \cdot \left(\frac{12\text{in}}{5\text{in}}\right)$$

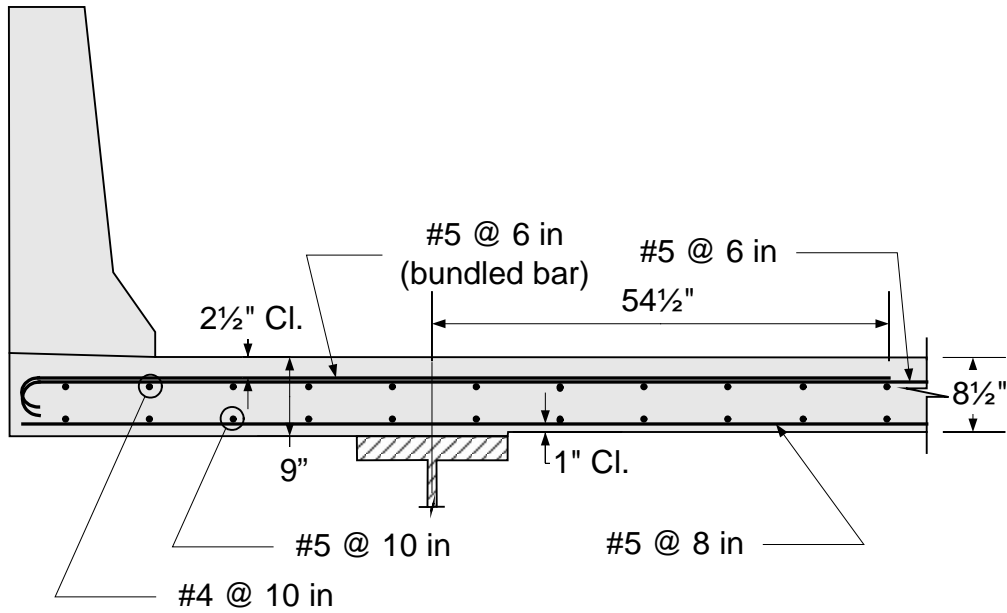
$$A_{s\text{provided}} = 0.74 \cdot \frac{\text{in}^2}{\text{ft}} > 0.68 \frac{\text{in}^2}{\text{ft}} \quad \text{OK}$$

Use #5 bars at 5 inch spacing in the bottom layer to satisfy the maximum spacing requirement of 12 inches.

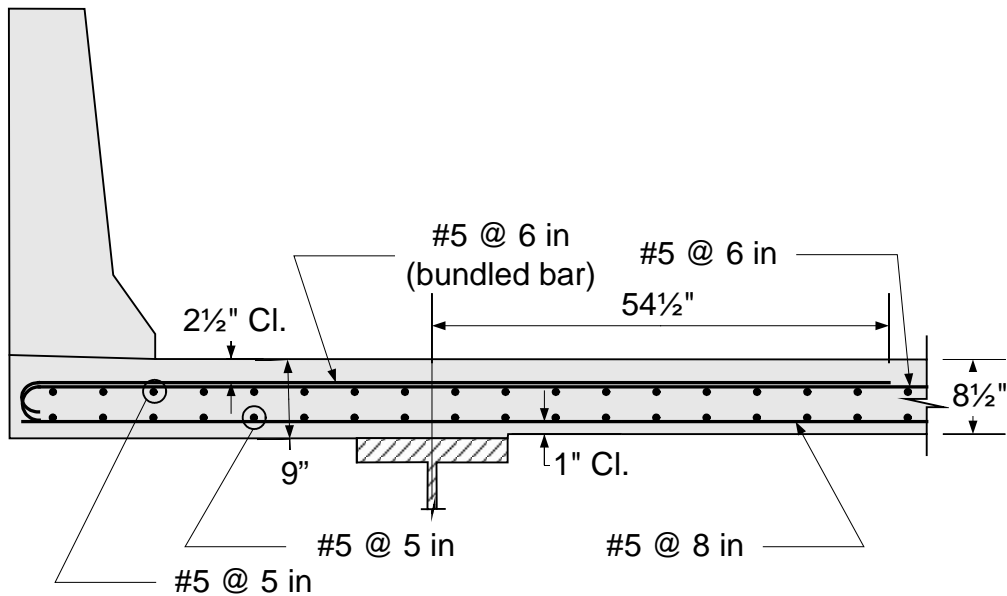
$$A_{s\text{provided}} = 0.31 \frac{\text{in}^2}{\text{ft}} \cdot \left(\frac{12\text{in}}{5\text{in}}\right)$$

$$A_{s\text{provided}} = 0.74 \cdot \frac{\text{in}^2}{\text{ft}} > 0.34 \frac{\text{in}^2}{\text{ft}} \quad \text{OK}$$

**Design Step 2.19 - Draw Schematic of Final Concrete Deck Design**



**Figure 2-17 Superstructure Positive Moment Deck Reinforcement**



**Figure 2-18 Superstructure Negative Moment Deck Reinforcement**



## Steel Girder Design Example

### Design Step 3

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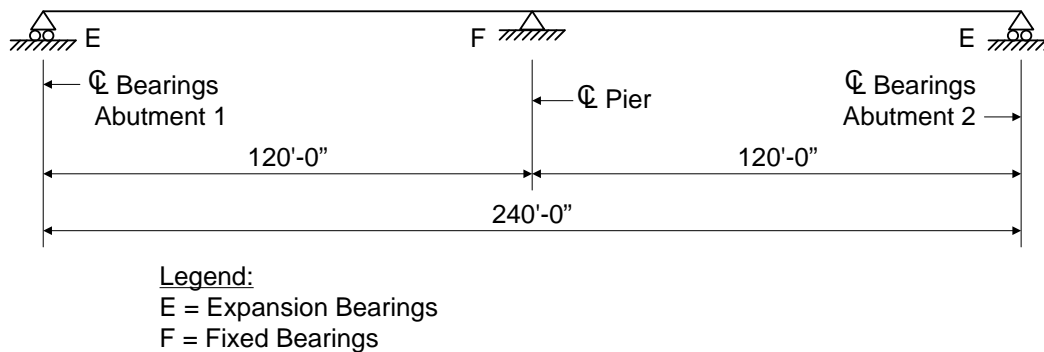
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### Design Step 3.1 - Obtain Design Criteria

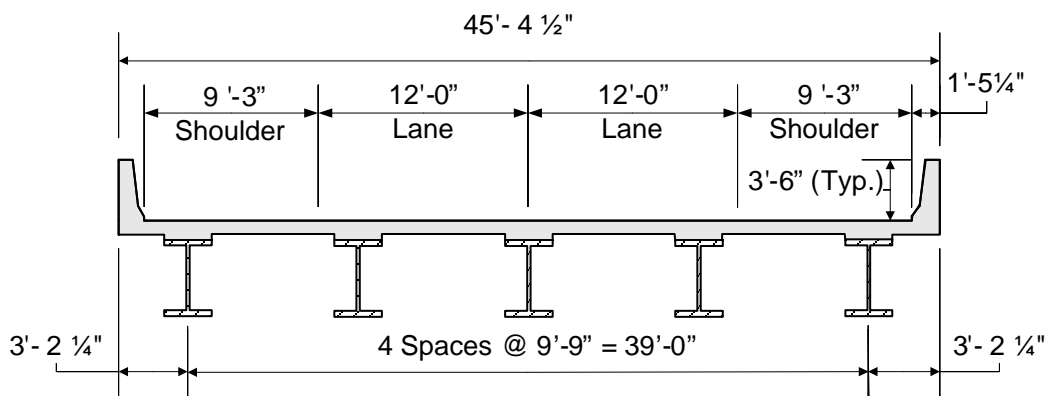
The first design step for a steel girder is to choose the appropriate design criteria.

The steel girder design criteria are obtained from Figures 3-1 through 3-3 (shown below), from the concrete deck design example (FHWA Publication NHI-04-041, December 2003, Design Step 2), and from the referenced articles and tables in the *AASHTO LRFD Bridge Design Specifications, Seventh Edition*. For this steel girder design example, a plate girder will be designed for an HL-93 live load. The girder is assumed to be composite throughout.

Refer to Design Step 1 for introductory information about this design example. Additional information is presented about the design assumptions, methodology, and criteria for the entire bridge, including the steel girder.



**Figure 3-1 Span Configuration**



**Figure 3-2 Superstructure Cross Section**

### Girder Spacing

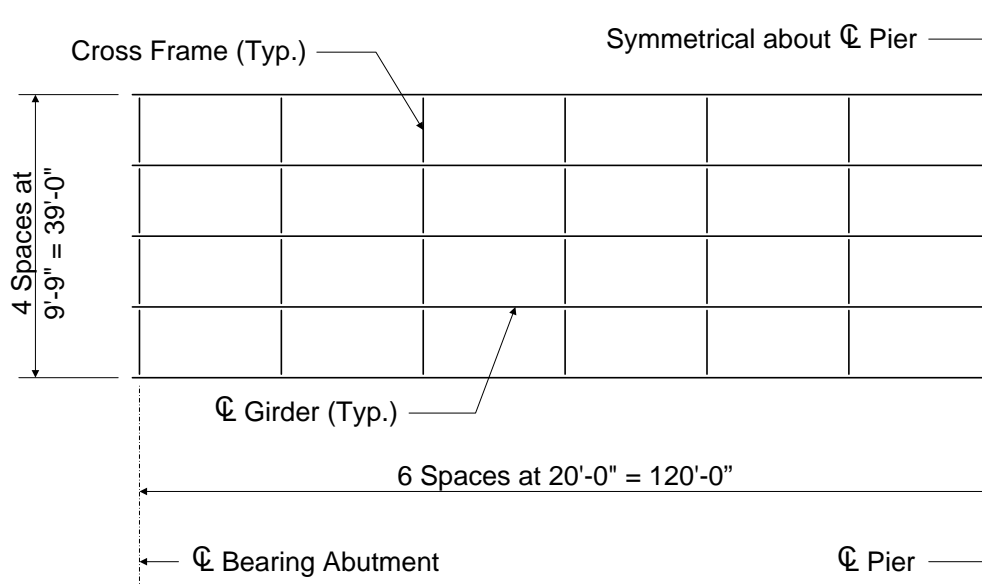


Where depth or deflection limitations do not control the design, it is generally more cost-effective to use a wider girder spacing. For this design example, the girder spacing shown in Figure 3-2 was developed as a reasonable value for all limit states. Four girders are generally considered to be the minimum, and five girders are desirable to facilitate future redecking. Further optimization of the superstructure could be achieved by revising the girder spacing.

### Overhang Width



The overhang width is generally determined such that the total dead plus live load moments and shears in the exterior girder are similar to those in the interior girder. In addition, the overhang is set such that the positive and negative moments in the deck slab are reasonably balanced. A common rule of thumb is to make the overhang approximately 0.28 to 0.35 times the girder spacing.



**Figure 3-3 Framing Plan**

### Cross-frame Spacing



The long-standing requirement limiting the maximum cross-frame spacing in I-girder bridges to 25 feet has been removed. The present provisions do not limit the cross-frame spacing for straight I-girder bridges.

For this design example, a cross-frame spacing of 20 feet is used. Refer to RM Section 6.3.2.9.3 for a detailed discussion on how to layout the cross-frame spacing.

This spacing also affects constructibility checks for stability before the deck is cured. Currently, stay-in-place forms should not be considered to provide adequate bracing to the top flange.

The following units are defined for use in this design example:

$$K = 1000lb \quad kcf = \frac{K}{ft^3}$$

$$ksi = \frac{K}{in^2} \quad ksf = \frac{K}{ft^2}$$

#### Design criteria:

Number of spans:

$$N_{spans} = 2$$

Span length:

$$L_{span} = 120 ft$$

Skew angle:

$$Skew = 0 deg$$

Number of girders:

$$N_{girders} = 5$$

Girder spacing:

$$S = 9.75 \text{ ft}$$

Deck overhang:

$$S_{\text{overhang}} = 3.1875 \text{ ft}$$

Cross-frame spacing:

S6.7.4

$$L_b = 20 \text{ ft}$$

Web yield strength:

S6.4.1-1

$$F_{yw} = 50 \text{ ksi}$$

Flange yield strength:

S6.4.1-1

$$F_y = 50 \text{ ksi}$$

Concrete 28-day compressive strength:

S5.4.2.1 &  
S6.4.2.1-1

$$f'_c = 4.0 \text{ ksi}$$

Reinforcement yield strength:

S5.4.3 & S6.10.1.7

$$f_y = 60 \text{ ksi}$$

Modulus of elasticity:

S6.4.1

$$E = 29000 \text{ ksi}$$

Total structural deck thickness:

$$t_{\text{deck}} = 8.5 \text{ in}$$

Effective deck thickness:

$$t_{\text{effdeck}} = 8.0 \text{ in}$$

Total overhang thickness:

$$t_{\text{overhang}} = 9.0 \text{ in}$$

Effective overhang thickness:

$$t_{\text{effoverhang}} = 8.5\text{in}$$

Steel density:

*STable 3.5.1-1*

$$W_s = 0.490\text{kcf}$$

Concrete density:

*STable 3.5.1-1 &*

$$W_c = 0.150\text{kcf}$$

*C3.5.1*

Additional miscellaneous dead load (per girder):

$$W_{\text{misc}} = 0.015 \frac{\text{K}}{\text{ft}}$$

Stay-in-place deck form weight:

$$W_{\text{deckforms}} = 0.015\text{kssf}$$

Parapet weight (each):

$$W_{\text{par}} = 0.53 \frac{\text{K}}{\text{ft}}$$

Future wearing surface:

*STable 3.5.1-1*

$$W_{\text{fws}} = 0.140\text{kcf}$$

Future wearing surface thickness:

$$t_{\text{fws}} = 2.5\text{in}$$

Deck width:

$$W_{\text{deck}} = 45.375\text{ft}$$

Roadway width:

$$W_{\text{roadway}} = 42.5\text{ft}$$

Haunch depth (from top of web):

$$d_{\text{haunch}} = 3.5\text{in}$$

Average Daily Truck Traffic (Single-Lane):

$$ADTT_{SL} = 3000$$

For this design example, transverse stiffeners will be designed in Step 3.12. In addition, a bolted field splice will be designed in Step 4, shear connectors will be designed in Step 5.1, bearing stiffeners will be designed in Step 5.2, welded connections will be designed in Step 5.3, cross-frames are described in Step 5.4, and an elastomeric bearing will be designed in Step 6. Longitudinal stiffeners are not used, and a deck pouring sequence will not be considered in this design example. Refer to RM Section 6.5.3.3 for a discussion on the impact of the deck placement sequence.

Design factors from AASHTO LRFD Bridge Design Specifications:

Load factors:

STable 3.4.1-1 &  
STable 3.4.1-2

Load Combinations and Load Factors							
Limit State	Load Factors						
	DC	DW	LL	IM	WS	WL	EQ
Strength I	1.25	1.50	1.75	1.75	-	-	-
Service II	1.00	1.00	1.30	1.30	-	-	-
Fatigue I	-	-	1.50	1.50	-	-	-
Fatigue II	-	-	0.75	0.75	-	-	-

**Table 3-1 Load Combinations and Load Factors**

S3.4.2.1

The abbreviations used in Table 3-1 are as defined in S3.3.2. Also, S3.4.2.1 states that primary steel superstructure components are to be investigated for maximum force effects during construction for an additional special load combination consisting of the applicable DC loads and any construction loads that are applied to the fully erected steelwork. The load factor for force effects caused by DC loads and construction loads, including dynamic effects (if applicable), is not to be less than 1.4 for this additional special load combination.

It should be noted that AASHTO may revise the fatigue load combinations and load factors at some point in the future.

The extreme event limit state (including earthquake load) is generally not considered for a steel girder design.



Structural steel resistance factors:

S6.5.4.2

Structural Steel Resistance Factors	
Type of Resistance	Resistance Factor, $\phi$
For flexure	$\phi_f = 1.00$
For shear	$\phi_v = 1.00$
For axial compression	$\phi_c = 0.95$

**Table 3-2 Structural Steel Resistance Factors****Multiple Presence Factors**

Multiple presence factors are described in S3.6.1.1.2. They are already included in the computation of live load distribution factors, as presented in S4.6.2.2. An exception, however, is that they must be included when the live load distribution factor for an exterior girder is computed using the lever rule or assuming that the cross section deflects and rotates as a rigid cross section, as presented in S4.6.2.2.2d. The factors must also be applied whenever refined methods of analysis are employed.

Since S3.6.1.1.2 states that the effects of the multiple presence factor are not to be applied to the fatigue limit state, all empirically determined distribution factors for one-lane loaded that are applied to the single fatigue truck must be divided by 1.20 (that is, the multiple presence factor for one lane loaded). In addition, for distribution factors computed using the lever rule or based on S4.6.2.2.2d, the 1.20 factor should not be included when computing the distribution factor for one-lane loaded for the fatigue limit state. It should also be noted that the multiple presence factor still applies to the distribution factors for one-lane loaded at the strength limit state.

Dynamic load allowance:

STable 3.6.2.1-1

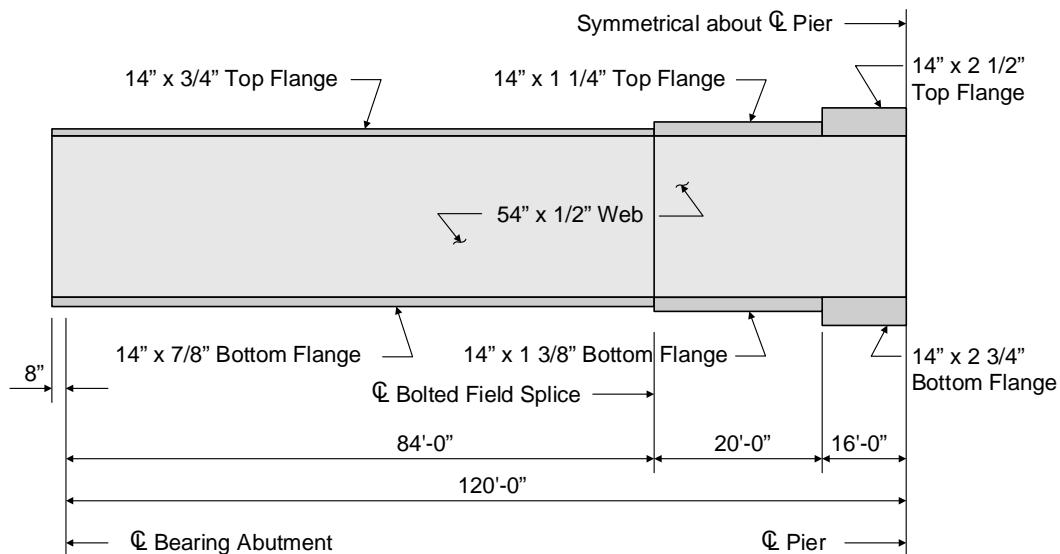
Dynamic Load Allowance	
Limit State	Dynamic Load Allowance, IM
Fatigue and Fracture Limit State	15%
All Other Limit States	33%

**Table 3-3 Dynamic Load Allowance**

Dynamic load allowance is the same as impact. The term "impact" was used in previous editions of the AASHTO Specifications. However, the term "dynamic load allowance" is used in the *AASHTO LRFD Bridge Design Specifications*. Refer to RM Section 3.4.8 for additional discussion on dynamic load allowance.

**Design Step 3.2 - Select Trial Girder Section**

Before the dead load effects can be computed, a trial girder section must be selected. This trial girder section is selected based on previous experience and based on preliminary design (see RM Sections 6.3.3 and 6.3.4). For this design example, the trial girder section presented in Figure 3-4 will be used. Based on this trial girder section, section properties and dead load effects will be computed. Then specification checks will be performed to determine if the trial girder section successfully resists the applied loads. If the trial girder section does not pass all specification checks or if the girder optimization is not acceptable, then a new trial girder section must be selected and the design process must be repeated.



**Figure 3-4 Plate Girder Elevation**

It should be noted that most state requirements and most steel fabricators call for a 3/4" minimum flange thickness, although the requirements of S6.7.3 show a smaller permitted minimum steel thickness. In addition, the AASHTO/NSBA Steel Bridge Collaboration Document "Guidelines for Design for Constructibility" recommends a 3/4" minimum flange thickness.

### Girder Depth



The minimum girder depth is specified in *S*Table 2.5.2.6.3-1. An estimate of the optimum girder depth, or the depth that provides the minimum cost girder, can be obtained from trial runs using readily available design software. In many cases, the optimum depth will be greater than the minimum depth specified in *S*Table 2.5.2.6.3-1. Fortunately, the efficiency of girders does not vary greatly when near the optimum depth.

### Web Thickness



A "partially stiffened" web (approximately 1/16 inch thinner than "unstiffened") will generally provide the least cost alternative or very close to it. However, for web depths of approximately 50 inches or less, unstiffened webs may be more economical.

### Plate Transitions



A common rule of thumb is to use no more than three plates (two shop splices) in the top or bottom flange of field sections up to 130 feet long. In some cases, a single flange plate size can be carried through the full length of the field section.

### Flange Widths



Flange widths should remain constant within field sections. The use of constant flange widths simplifies construction of the deck, and allows the fabricator to potentially utilize slab welding (see RM Section 6.3.4.4.5). The unsupported length in compression of the shipping piece divided by the minimum width of the compression flange in that piece should be less than approximately 85.

### Flange Plate Transitions



It is good design practice to reduce the flange cross-sectional area by no more than approximately one-half of the area of the heavier flange plate at welded flange shop splices. This reduces the stress concentration and ensures a smooth transition of stress across the splice.

The above tips are presented to help bridge designers in developing an economical steel girder for most steel girder designs. Other design tips are available in various publications from the National Steel Bridge Alliance (NSBA) and from the AASHTO/NSBA Steel Bridge Collaboration. Refer to RM Chapter 6 for a list of references and further discussion.

## Design Step 3.3 - Compute Section Properties

### *S6.10.1.1*

Since the superstructure is composite, several sets of section properties must be computed. The initial dead loads (or the noncomposite dead loads) are applied to the girder-only section. For permanent loads assumed to be applied to the long-term composite section, the long-term modular ratio of  $3n$  is used to transform the concrete deck area to account in an approximate fashion for the effects of concrete creep (*S6.10.1.1.1b*). For transient loads assumed applied to the short-term composite section, the short-term modular ratio of  $n$  is used to transform the concrete deck area.

### *S6.6.1.2.1*

For girders with shear connectors provided throughout their entire length and with slab reinforcement satisfying the provisions of *S6.10.1.7*, dead load and live load stresses and live load stress ranges at all sections in the member due to loads applied to the composite section for the fatigue limit state may be computed using the short-term

composite section assuming the concrete slab to be fully effective for both positive and negative flexure.

#### S6.10.4.2.1

For girders with shear connectors provided throughout their entire length that also satisfy the provisions of S6.10.1.7, and with maximum longitudinal tensile stresses in the concrete deck at the section under consideration caused by the Service II loads smaller than  $2f_r$ , flexural stresses caused by Service II loads applied to the composite section may be computed using the short-term or long-term composite section, as appropriate, assuming the concrete deck is effective for both positive and negative flexure.  $f_r$  is the modulus of rupture of the concrete specified in S6.10.1.7.

In general, both the exterior and interior girders must be considered, and the controlling design is used for all girders, both interior and exterior.

Although the exterior girder typically controls the design of the girders, for this particular design example, only the interior girder design is presented. However, for the exterior girder, the computation of the live load distribution factors and the moment and shear envelopes are also presented. These computations, along with the subsequent discussion on the computation of girder dead load effects, illustrate why the exterior girder most often controls the girder design and why it is important to limit the deck overhang to somewhere between approximately 0.28 to 0.35 times the girder spacing to ensure a reasonable girder moment balance. Note however that the shear design is most often controlled by the interior girder for straight I-girder bridges with no skew.

The modular ratio is computed as follows:

$$W_c = 0.150kcf \quad \text{STable 3.5.1-1 \& C3.5.1}$$

$$f'_c = 4.0ksi \quad \text{S5.4.2.1 \& STable C5.4.2.1-1}$$

$$K_1 = 1 \quad \text{S5.4.2.4}$$

$$E_c = 120,000 \cdot K_1 \cdot (W_c^{2.0}) \cdot f'_c^{0.33} = 4266ksi \quad \text{S5.4.2.4}$$

However, based on concrete lab tests for this bridge, use  $E_c = 3834ksi$

$$E_s = 29000ksi \quad \text{S6.4.1}$$

$$n = \frac{E_s}{E_c} \quad n = 7.6 \quad \text{S6.10.1.1.1b}$$

Therefore, use  $n = 8$ .

## C6.10.1.1.1b

In lieu of the above computations, the modular ratio can also be obtained from C6.10.1.1.1b. The above computations are presented simply to illustrate the process. Both the above computations and C6.10.1.1.1b result in a modular ratio of 8.

## S4.6.2.6

The effective flange width of the concrete deck is computed in the majority of cases as the tributary width of the deck perpendicular to the axis of the member for determining the cross-section stiffnesses for the analysis and for determining flexural resistances. Exceptions are noted in S4.6.2.6.

For this design example, since the girder spacing is 9.75 feet, the effective flange width for the interior girder is 9.75 feet or 117 inches. However, it should be noted that in this design example, the effective flange width computed using previous AASHTO specifications is 103 inches, and this previous value is used in determining the composite stiffness for the analysis. This is not anticipated to have a significant effect on the analysis results. The effective flange width based on the most current AASHTO specifications is 117 inches, and this value is used in this example for the calculation of the section properties for design.

Based on the concrete deck design example (FHWA Publication NHI-04-041, December 2003, Design Step 2), the total area of longitudinal deck reinforcing steel in the negative moment region is computed as follows (# 5 bars at 5 inch spacing in the top and bottom mat running longitudinally):

$$A_{deck\ inf} = 2 \cdot 0.31in^2 \cdot \frac{W_{eff\ flange}}{5in}$$

$$A_{deck\ inf} = 14.508in^2$$

### Slab Haunch



For this design example, the slab haunch is 3.5 inches throughout the length of the bridge. That is, the bottom of the slab is located 3.5 inches above the top of the web. For this design example, this distance is used in computing the location of the centroid of the slab. However, the area of the haunch is not considered in the section properties.

Some states and agencies assume that the slab haunch is zero when computing the section properties.

If the haunch depth is not known, it is conservative to assume that the haunch is zero. If the haunch varies, it is reasonable to use either the minimum value or an average value.

Based on the trial plate sizes shown in Figure 3-4 and based on an effective flange width of the concrete deck of 117 inches, the noncomposite and composite section properties for the positive moment region are computed as shown in the following table. The distance to the centroid is measured from the bottom of the girder.

Positive Moment Region Section Properties						
Section	Area, A (Inches <sup>2</sup> )	Centroid, d (Inches)	A*d (Inches <sup>3</sup> )	I <sub>o</sub> (Inches <sup>4</sup> )	A*y <sup>2</sup> (Inches <sup>4</sup> )	I <sub>total</sub> (Inches <sup>4</sup> )
Girder only:						
Top flange	10.500	55.250	580.1	0.5	8441.1	8441.6
Web	27.000	27.875	752.6	6561.0	25.8	6586.8
Bottom flange	12.250	0.438	5.4	0.8	8576.1	8576.9
Total	49.750	26.897	1338.1	6562.3	17043.0	23605.3
Composite (3n):						
Girder	49.750	26.897	1338.1	23605.3	12092.4	35697.6
Slab	39.000	62.375	2432.6	208.0	15425.5	15633.5
Total	88.750	42.487	3770.7	23813.3	27517.9	51331.2
Composite (n):						
Girder	49.750	26.897	1338.1	23605.3	30829.0	54434.3
Slab	117.000	62.375	7297.9	624.0	13108.9	13732.9
Total	166.750	51.790	8636.0	24229.3	43937.9	68167.2
Section	Y <sub>botgdr</sub> (Inches)	Y <sub>topgdr</sub> (Inches)	Y <sub>topslab</sub> (Inches)	S <sub>botgdr</sub> (Inches <sup>3</sup> )	S <sub>topgdr</sub> (Inches <sup>3</sup> )	S <sub>topslab</sub> (Inches <sup>3</sup> )
Girder only	26.897	28.728	---	877.6	821.7	---
Composite (3n)	42.487	13.138	23.888	1208.2	3907.1	2148.8
Composite (n)	51.790	3.835	14.585	1316.2	17775.1	4673.8

**Table 3-4 Positive Moment Region Section Properties**

Similarly, the noncomposite and composite section properties for the negative moment region are computed as shown in the following table. The distance to the centroid is measured from the bottom of the girder.

For the strength limit state, since the deck concrete is in tension in the negative moment region, the deck reinforcing steel contributes to the composite section properties and the deck concrete does not.

#### S6.6.1.2.1 & S6.10.4.2.1

As previously explained, for this design example, the concrete slab will be assumed to be fully effective for both positive and negative flexure for computing stresses and stress ranges at all sections in the member due to loads applied to the composite section at the fatigue limit state. The appropriate checks will be made, as necessary, to determine if the concrete slab can be assumed to be fully effective for both positive and negative flexure for computing these stresses at the service limit state.



Negative Moment Region Section Properties						
Section	Area, A (Inches <sup>2</sup> )	Centroid, d (Inches)	A*d (Inches <sup>3</sup> )	I <sub>o</sub> (Inches <sup>4</sup> )	A*y <sup>2</sup> (Inches <sup>4</sup> )	I <sub>total</sub> (Inches <sup>4</sup> )
Girder only:						
Top flange	35.000	58.000	2030.0	18.2	30009.7	30027.9
Web	27.000	29.750	803.3	6561.0	28.7	6589.7
Bottom flange	38.500	1.375	52.9	24.3	28784.7	28809.0
Total	100.500	28.718	2886.2	6603.5	58823.1	65426.6
Composite (deck concrete using 3n):						
Girder	100.500	28.718	2886.2	65426.6	9917.0	75343.6
Slab	39.000	64.250	2505.8	208.0	25555.3	25763.3
Total	139.500	38.652	5391.9	65634.6	35472.3	101106.9
Composite (deck concrete using n):						
Girder	100.500	28.718	2886.2	65426.6	36715.7	102142.3
Slab	117.000	64.250	7517.3	624.0	31537.8	32161.8
Total	217.500	47.832	10403.4	66050.6	68253.5	134304.1
Composite (deck reinforcement only):						
Girder	100.500	28.718	2886.2	65426.6	1962.7	67389.3
Deck reinf.	14.508	63.750	924.9	0.0	13595.9	13595.9
Total	115.008	33.137	3811.1	65426.6	15558.5	80985.1
Section	Y <sub>botgdr</sub> (Inches)	Y <sub>topgdr</sub> (Inches)	Y <sub>deck</sub> (Inches)	S <sub>botgdr</sub> (Inches <sup>3</sup> )	S <sub>topgdr</sub> (Inches <sup>3</sup> )	S <sub>deck</sub> (Inches <sup>3</sup> )
Girder only	28.718	30.532	---	2278.2	2142.9	---
Composite (3n)	38.652	20.598	29.598	2615.8	4908.5	3416.0
Composite (n)	47.832	11.418	20.418	2807.8	11762.4	6577.7
Composite (rebar)	33.137	26.113	30.613	2443.9	3101.4	2645.5

**Table 3-5 Negative Moment Region Section Properties**

### Design Step 3.4 - Compute Dead Load Effects

The girder must be designed to resist the dead load effects, as well as the other load effects. The dead load components consist of some dead loads that are resisted by the noncomposite section, as well as other dead loads that are resisted by the composite section. In addition, some dead loads are factored with the DC load factor and other dead loads are factored with the DW load factor. The following table summarizes the various dead load components that must be included in the design of a steel girder.

Dead Load Components		
Resisted by	Type of Load Factor	
	DC	DW
Noncomposite section	<ul style="list-style-type: none"> <li>• Steel girder</li> <li>• Concrete deck</li> <li>• Concrete haunch</li> <li>• Stay-in-place deck forms</li> <li>• Miscellaneous dead load (including cross-frames, stiffeners, etc.)</li> </ul>	
Composite section	<ul style="list-style-type: none"> <li>• Concrete parapets</li> </ul>	<ul style="list-style-type: none"> <li>• Future wearing surface</li> </ul>

**Table 3-6 Dead Load Components**

For the steel girder, the dead load per unit length varies due to the change in plate sizes. The moments and shears due to the weight of the steel girder can be computed using readily available analysis software. Since the actual plate sizes are entered as input, the moments and shears are computed based on the actual, varying plate sizes.

For the concrete deck, the dead load per unit length for an interior girder is computed as follows:

$$W_c = 0.150 \frac{K}{ft^3} \quad S = 9.75 ft \quad t_{deck} = 8.5 in$$

$$DL_{deck} = W_c \cdot S \cdot \frac{t_{deck}}{12 \frac{in}{ft}} \quad DL_{deck} = 1.036 \frac{K}{ft}$$

For the concrete haunch, the dead load per unit length varies due to the change in top flange plate sizes. The moments and shears due to the weight of the concrete haunch can be computed using readily available analysis software. Since the top flange plate sizes are entered as input, the moments and shears due to the concrete haunch are computed based on the actual, varying haunch thickness.

For the stay-in-place forms, the dead load per unit length is computed as follows:

$$W_{deckforms} = 0.015 ksf \quad S = 9.75 ft \quad W_{topflange} = 14 in$$

$$DL_{deckforms} = W_{deckforms} \cdot \left( S - \frac{W_{topflange}}{12} \right)$$

$$DL_{deckforms} = 0.129 \frac{K}{ft}$$

For the miscellaneous dead load (including cross-frames, stiffeners, and other miscellaneous structural steel), the dead load per unit length is assumed to be as follows:

$$DL_{misc} = 0.015 \frac{K}{ft}$$

#### S4.6.2.2.1

Although not done in this particular example, for bridges with moderate deck overhangs and approximately equal girder stiffnesses at points of connection of the cross-frames, it is reasonable to assume that all girders in the cross-section will resist the dead loads acting on the noncomposite section (e.g. steel weight, deck, deck haunch, deck forms, etc.) equally. This assumption neglects any effects of elastic shortening in the cross-frame members, and will generally result in the exterior girder carrying more of the noncomposite dead load than is often assumed. Such conditions exist in straight bridges with approximately equal-size girders and bearing lines skewed not more than approximately 10 degrees from normal (where intermediate cross-frames are placed in collinear lines normal to the girders) or where intermediate cross-frames are placed in collinear skewed lines parallel to the skewed supports (which is permitted for skew angles up to 20 degrees). Under these conditions where the composite deck is not present, the cross-frames provide the necessary restoring forces to resist the differential deflections between the girders, which tends to equalize the load the girders carry regardless of how the localized loads are applied. The assumption of equal vertical girder deflections under the noncomposite dead loads in these cases has been borne out analytically and in the field.

For the concrete parapets, the dead load per unit length is computed as follows, assuming that the superimposed dead load of the two parapets is distributed uniformly among all of the girders:


$$W_{par} = 0.5 \frac{K}{ft} \quad N_{girders} = 5$$

$$DL_{par} = W_{par} \cdot \frac{2}{N_{girders}} \quad DL_{par} = 0.212 \frac{K}{ft} \quad S4.6.2.2.1$$

S4.6.2.2.1 specifies that permanent loads of and on the deck may be distributed uniformly among the beams. This provision dates back to when concrete deck overhangs were much smaller and lighter curbs and railings were typically used (see RM Section 4.3.3). For the future wearing surface load (see below), this remains a reasonable assumption and has been the customary practice. However, this

assumption is often still applied to the concrete parapet loads, as illustrated above. These loads, generally applied at the edges of the deck overhangs, usually result in the computed portion of the load resisted by the exterior girders (from a refined analysis) to be significantly larger than an equal distribution assumption would indicate.

**Distribution of Parapet Loads**



To better simulate the actual distribution of parapet loads when line-girder analyses are employed, at least one State DOT requires that the parapet load be equally distributed to an exterior girder and the adjacent interior girder. Other State DOT's assign 60 percent of the barrier weight to the exterior girder and 40 percent to the adjacent interior girder, or other percentages along those lines. Although the traditional equal distribution assumption has been employed in this example, it is recommended that consideration be given to recognizing the concentrated effect of these heavy edge loads in actual design practice, regardless of the analysis approach that is employed.

For the future wearing surface, the dead load per unit length is computed as follows, assuming that the superimposed dead load of the future wearing surface is distributed uniformly among all of the girders:

$$W_{fws} = 0.140kcf \quad t_{fws} = 2.5in$$

$$w_{roadway} = 42.5ft \quad N_{girders} = 5$$

$$DL_{fws} = \frac{W_{fws} \cdot \frac{t_{fws}}{12 \frac{in}{ft}} \cdot w_{roadway}}{N_{girders}} \quad DL_{fws} = 0.248 \frac{K}{ft}$$

Since the plate girder and its section properties are not uniform over the entire length of the bridge, an analysis must be performed to compute the dead load moments and shears. Such an analysis can be performed using one of various computer programs.

### Need for Revised Analysis



It should be noted that during the optimization process, minor adjustments can be made to the plate sizes and transition locations without needing to recompute the analysis results. However, if significant adjustments are made, such that the moments and shears would change significantly, then a revised analysis is required.

The following two tables present the unfactored dead load moments and shears in an interior girder, as computed by an analysis computer program (AASHTO BrD software). Since the bridge is symmetrical, the moments and shears in Span 2 are symmetrical to those in Span 1.

Dead Load Moments (Kip-feet)											
Dead Load Component	Location in Span 1										
	0.0L	0.1L	0.2L	0.3L	0.4L	0.5L	0.6L	0.7L	0.8L	0.9L	1.0L
Steel girder	0.0	75.4	125.5	150.3	150.0	124.4	73.6	-2.5	-107.2	-244.0	-421.5
Concrete deck & haunches	0.0	467.4	776.9	928.6	922.4	758.4	436.6	-43.1	-679.7	-1472.0	-2418.3
Other dead loads acting on girder alone	0.0	68.8	114.3	136.7	135.8	111.7	64.4	-6.2	-99.9	-216.9	-357.1
Concrete parapets	0.0	93.9	157.2	189.9	192.2	163.8	104.9	15.5	-104.5	-255.0	-436.1
Future wearing surface	0.0	113.7	190.4	230.1	232.7	198.4	127.1	18.8	-126.6	-308.9	-528.2

Table 3-7 Dead Load Moments – Interior Girder

Dead Load Shears (Kips)											
Dead Load Component	Location in Span 1										
	0.0L	0.1L	0.2L	0.3L	0.4L	0.5L	0.6L	0.7L	0.8L	0.9L	1.0L
Steel girder	7.33	5.23	3.12	1.02	-1.08	-3.18	-5.29	-7.39	-10.06	-12.74	-16.84
Concrete deck & haunches	45.53	32.37	19.22	6.06	-7.09	-20.24	-33.40	-46.55	-59.54	-72.52	-85.18
Other dead loads acting on girder alone	6.70	4.76	2.83	0.89	-1.04	-2.98	-4.91	-6.85	-8.78	-10.72	-12.65
Concrete parapets	9.10	6.55	4.00	1.46	-1.09	-3.63	-6.18	-8.73	-11.27	-13.82	-16.36
Future wearing surface	11.02	7.93	4.85	1.77	-1.32	-4.40	-7.49	-10.57	-13.65	-16.74	-19.82

Table 3-8 Dead Load Shears – Interior Girder

## Design Step 3.5 - Compute Live Load Effects

### LRFD Live Load



In LRFD, the basic live load designation is HL-93.

The HL-93 load consists of a design truck or tandem, combined with a lane load.

In LRFD, 90% of the effect of two design trucks at a specified minimum headway of 50 feet combined with 90% of the lane load is considered when computing the maximum negative live load moment between points of permanent-load contraflexure and the interior-pier reactions.

In LRFD, the term "dynamic load allowance" is used instead of "impact."

In LRFD, dynamic load allowance is applied only to the design truck or design tandem.

For additional information about the live load used in LRFD, refer to S3.6 and C3.6.

S3.6.1.2

The girder must also be designed to resist the live load effects. The live load consists of an HL-93 loading. Similar to the dead load, the live load moments and shears for an HL-93 loading can be obtained from an analysis computer program.

S3.6.2.1

Based on Table 3-3, for all limit states other than fatigue and fracture, the dynamic load allowance,  $IM$ , is as follows:

$$IM = 0.33$$

S4.6.2.2.2

The live load distribution factors for moment for an interior girder are computed as follows:

S4.6.2.2.1

First, the longitudinal stiffness parameter,  $K_G$ , must be computed:



$$K_g = n \cdot (I + A \cdot e_g^2)$$

Longitudinal Stiffness Parameter, $K_g$				
	Region A (Pos. Mom.)	Region B (Intermediate)	Region C (At Pier)	Weighted Average *
Length (Feet)	84	24	12	
n	8	8	8	
I (Inches <sup>4</sup> )	23,605.3	34,639.8	65,426.6	
A (Inches <sup>2</sup> )	49.750	63.750	100.500	
$e_g$ (Inches)	35.478	35.277	35.532	
$K_g$ (Inches <sup>4</sup> )	689,800	911,796	1,538,481	819,068

\*Weighted average is estimated based on length of each region

**Table 3-9 Longitudinal Stiffness Parameter**

After the longitudinal stiffness parameter is computed, *S*Table 4.6.2.2.1-1 is used to find the letter corresponding with the superstructure cross section. The letter corresponding with the superstructure cross section in this design example is "a."

S4.6.2.2.1

If the superstructure cross section does not correspond with any of the cross sections illustrated in *S*Table 4.6.2.2.1-1, then the bridge should be analyzed as presented in S4.6.3.

Based on cross section "a," *S*Tables 4.6.2.2.2b-1 and 4.6.2.2.2.3a-1 are used to compute the distribution factors for moment and shear, respectively.

*S*Table 4.6.2.2.2b-1

Check the range of applicability as follows:

$$3.5 \leq S \leq 16.5$$

$$S = 9.75 \text{ ft}$$

$$4.5 \leq t_s \leq 12.0$$

$$t_s = 8.0 \text{ in (structural depth)}$$

$$20 \leq L \leq 240$$

$$L = 120 \text{ ft}$$

$$N_b \geq 4$$

$$N_b = 5$$

$$10000 \leq K_g \leq 7000000$$

$$K_g = 819068 \text{ in}^4$$

*STable 4.6.2.2.2b-1*

For one design lane loaded, the distribution of live load per lane for moment in interior beams is as follows:

$$g_{int\_moment\_1} = 0.06 + \left(\frac{S}{14}\right)^{0.4} \cdot \left(\frac{S}{L}\right)^{0.3} \cdot \left(\frac{K_g}{12.0 \cdot L \cdot t_s^3}\right)^{0.1}$$

$$g_{int\_moment\_1} = 0.472 \text{ lanes}$$

*STable 4.6.2.2.2b-1*

For two or more design lanes loaded, the distribution of live load per lane for moment in interior beams is as follows:

$$g_{int\_moment\_2} = 0.075 + \left(\frac{S}{9.5}\right)^{0.6} \cdot \left(\frac{S}{L}\right)^{0.2} \cdot \left(\frac{K_g}{12.0 \cdot L \cdot t_s^3}\right)^{0.1}$$

$$g_{int\_moment\_2} = 0.696 \text{ lanes}$$

*STable 4.6.2.2.3a-1*

The live load distribution factors for shear for an interior girder are computed in a similar manner. The range of applicability is similar to that for moment.

*STable 4.6.2.2.3a-1*

For one design lane loaded, the distribution of live load per lane for shear in interior beams is as follows:

$$g_{int\_shear\_1} = 0.36 + \frac{S}{25.0}$$

$$g_{int\_shear\_1} = 0.750 \text{ lanes}$$

*S*Table 4.6.2.2.3a-1

For two or more design lanes loaded, the distribution of live load per lane for shear in interior beams is as follows:

$$g_{int\_shear\_2} = 0.2 + \frac{S}{12} - \left( \frac{S}{35} \right)^{2.0}$$

$$g_{int\_shear\_2} = 0.935 \text{ lanes}$$

S4.6.2.2.2e, S4.6.2.2.3c

Since this bridge has no skew, the skew correction factor does not need to be considered for this design example.

The distribution of live load for the fatigue limit state is similar to the above for moment and shear with one lane loaded. The only difference is that the factor 1.20 from *S*Table 3.6.1.1.2-1 for one lane loaded has already been included in the approximate equations and should be removed for the purpose of fatigue investigations.

S4.6.2.2.2

This design example is based on an interior girder. However, for illustrative purposes, the live load distribution factors for an exterior girder are computed below, as follows:

The distance,  $d_e$ , is defined as the distance between the exterior web of the exterior girder and the interior edge of the curb. For this design example use the distance from the centerline of web to interior edge of the curb for simplicity, based on Figure 3-2:

$$d_e = 1.75 \text{ ft}$$

*S*Table 4.6.2.2.2d-1

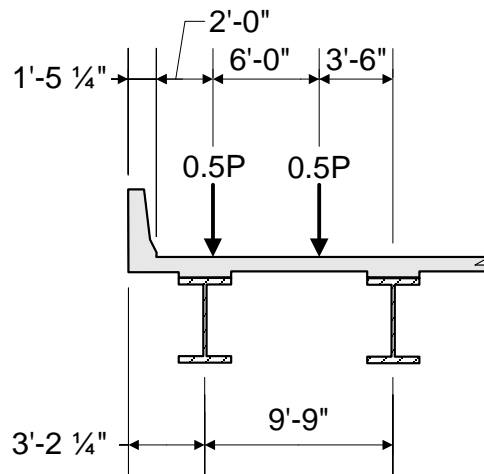
Check the range of applicability as follows:

$$-1.0 \leq d_e \leq 5.5$$

$$d_e = 1.75 \text{ ft OK}$$

*S*Table 4.6.2.2.2d-1

For one design lane loaded, the distribution of live load per lane for moment in exterior beams is computed using the lever rule, as follows:



**Figure 3-5 Lever Rule**

$$g_{ext\_moment\_1} = \frac{(0.5)(3.5\text{ ft}) + (0.5)(9.5\text{ ft})}{9.75\text{ ft}}$$

$$g_{ext\_moment\_1} = 0.667 \text{ lanes}$$

$$Multiple\_presence\_factor = 1.20$$

$$g_{ext\_moment\_1} = g_{ext\_moment\_1} \cdot Multiple\_presence\_factor$$

$$g_{ext\_moment\_1} = 0.800 \text{ lanes (for strength limit state)}$$

*S*Table 4.6.2.2.2d-1

For two or more design lanes loaded, the distribution of live load per lane for moment in exterior beams is as follows:

$$g_{ext\_moment\_2} = e \cdot g_{int\_moment\_2}$$

$$e = 0.77 + \frac{d_e}{9.1} \quad e = 0.962$$

$$g_{ext\_moment\_2} = 0.670 \text{ lanes}$$

*S*Table 4.6.2.2.3b-1

The live load distribution factors for shear for an exterior girder are computed in a similar manner. The range of applicability is similar to that for moment.

*S*Table 4.6.2.2.3b-1

For one design lane loaded, the distribution of live load per lane for shear in exterior beams is computed using the lever rule, as illustrated in Figure 3-5 and as follows:

$$g_{ext\_shear\_1} = \frac{(0.5)(3.5\text{ ft}) + (0.5)(9.5\text{ ft})}{9.75\text{ ft}}$$

$$g_{ext\_shear\_1} = 0.667 \text{ lanes}$$

$$\text{Multiple\_presence\_factor} = 1.20$$

$$g_{ext\_shear\_1} = g_{ext\_shear\_1} \cdot \text{Multiple\_presence\_factor}$$

$$g_{ext\_shear\_1} = 0.800 \text{ lanes (for strength limit state)}$$

*S*Table 4.6.2.2.3b-1

For two or more design lanes loaded, the distribution of live load per lane for shear in exterior beams is as follows:

$$g_{ext\_shear\_2} = e \cdot g_{int\_shear\_2}$$

$$e = 0.6 + \frac{d_e}{10} \quad e = 0.775$$

$$g_{ext\_shear\_2} = 0.725 \text{ lanes}$$

*S*4.6.2.2.2d

In steel beam-slab bridge cross-sections with diaphragms or cross-frames, the distribution factor for the exterior beam cannot be taken to be less than that which would be obtained by assuming that the cross-section deflects and rotates as a rigid cross-section. *CEquation 4.6.2.2.2d-1* provides one approximate approach to satisfy this requirement. The multiple presence factor provisions of *S*3.6.1.1.2 must be applied when this equation is used.

*S*4.6.2.2.2e, *S*4.6.2.2.3c

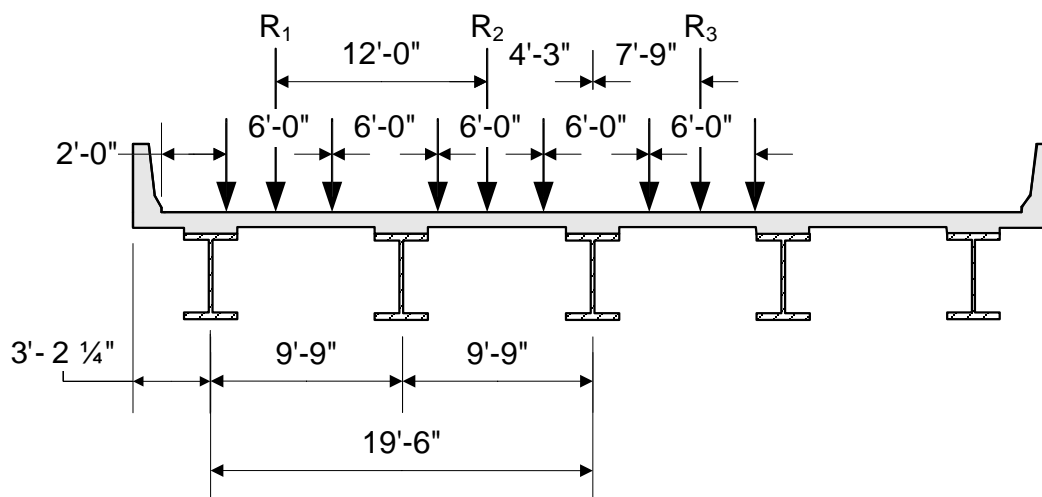


Figure 3-6 Live Load Placement used for Rigid Cross-Section Equation (CEquation 4.6.2.2d-1)

$$R = \frac{N_L}{N_b} + \frac{X_{ext} \sum_{N_L} e}{\sum_{N_b} x^2}$$

For three lanes loaded:

$$g_{ext} = \frac{3}{5} + \frac{19.5(16.25 + 4.25 - 7.75)}{2(19.5^2 + 9.75^2)} = 0.862 \text{ lanes}$$

$$m_3 = 0.85 \therefore g_{ext} = 0.85(0.862) = 0.733 \text{ lanes}$$

For two lanes loaded:

$$g_{ext} = \frac{2}{5} + \frac{19.5(16.25 + 4.25)}{2(19.5^2 + 9.75^2)} = 0.821 \text{ lanes}$$

$$m_3 = 1.00 \therefore g_{ext} = 1.00(0.821) = 0.821 \text{ lanes}$$

For one lane loaded:

$$g_{ext} = \frac{1}{5} + \frac{19.5(16.25)}{2(19.5^2 + 9.75^2)} = 0.533 \text{ lanes}$$

$$m_3 = 1.20 \therefore g_{ext} = 1.20(0.533) = 0.640 \text{ lanes}$$

As can be seen from the above computations, the exterior girder distribution factors are larger than the interior girder. Although the exterior girder typically controls the design of the girders, for this particular design example, only the interior girder design is presented.

Since this bridge has no skew, the skew correction factor does not need to be considered for this design example.

The following table presents the unfactored maximum positive and negative live load moments and shears for HL-93 live loading for an interior girder, as computed using an analysis computer program. These values include the live load distribution factor, and they also include dynamic load allowance. Since the bridge is symmetrical, the moments and shears in Span 2 are symmetrical to those in Span 1.

*S3.6.1, S3.6.2, S4.6.2.2*

The design live load values for HL-93 loading, as presented in the following table, are computed based on the product of the live load effect per lane and live load distribution factor. These values also include the effects of dynamic load allowance. However, it is important to note that the dynamic load allowance is applied only to the load effects due to the design truck or tandem. The dynamic load allowance is not applied to load effects due to pedestrian loads or to the design lane load.

Live Load Effects (for Interior Beams)											
Live Load Effect	Location in Span 1										
	0.0L	0.1L	0.2L	0.3L	0.4L	0.5L	0.6L	0.7L	0.8L	0.9L	1.0L
Maximum positive moment (K-ft)	0	836	1422	1766	1908	1857	1628	1318	746	277	0
Maximum negative moment (K-ft)	0	-128	-256	-384	-512	-640	-768	-966	-1097	-1593	-2450
Maximum positive shear (kips)	110.5	93.7	76.6	61.0	49.6	42.5	23.6	14.5	7.5	2.9	0.0
Maximum negative shear (kips)	-14.7	-15.1	-20.9	-36.4	-47.8	-62.2	-76.7	-91.1	-105.1	-118.5	-131.4

Table 3-10 Live Load Effects – Interior Girder



### Design Step 3.6 - Combine Load Effects

After the load factors and load combinations have been established (see Design Step 3.1), the section properties have been computed (see Design Step 3.3), and all of the load effects have been computed (see Design Steps 3.4 and 3.5), the force effects must be combined for each of the applicable limit states.

S1.3

For this design example,  $\eta$  equals 1.00. (For more detailed information about  $\eta$ , refer to Design Step 1.)

S3.4.1

Based on the previous design steps, the maximum positive moment (located at 0.4L) for the Strength I load combination is computed as follows:

$$LF_{DC} = 1.25$$

$$M_{DC} = 150.0K \cdot ft + 922.4K \cdot ft + 135.8K \cdot ft + 192.2K \cdot ft$$

$$M_{DC} = 1400.4K \cdot ft$$

$$LF_{DW} = 1.50$$

$$M_{DW} = 232.7K \cdot ft$$

$$LF_{LL} = 1.75$$

$$M_{LL} = 1908K \cdot ft$$

$$M_{total} = LF_{DC} \cdot M_{DC} + LF_{DW} \cdot M_{DW} + LF_{LL} \cdot M_{LL}$$

$$M_{total} = 5439K \cdot ft$$

Similarly, the maximum stress in the top of the girder due to positive moment (located at 0.4L) for the Strength I load combination is computed as follows:

Noncomposite dead load:

$$M_{noncompDL} = 150.0K \cdot ft + 922.4K \cdot ft + 135.8K \cdot ft = 1208.2K \cdot ft$$

$$S_{topgdr} = 821.7in^3$$

$$f_{noncompDL} = \frac{-M_{noncompDL} \left( \frac{12in}{ft} \right)}{S_{topgdr}}$$

$$f_{noncompDL} = -17.64ksi$$

Parapet dead load (composite):

$$M_{parapet} = 192.2K \cdot ft \quad S_{topgdr} = 3907.1in^3$$

$$f_{parapet} = \frac{-M_{parapet} \left( \frac{12in}{ft} \right)}{S_{topgdr}}$$

$$f_{parapet} = -0.59ksi$$

Future wearing surface dead load (composite):

$$M_{fws} = 232.7K \cdot ft \quad S_{topgdr} = 3907.1in^3$$

$$f_{fws} = \frac{-M_{fws} \cdot \left( \frac{12in}{ft} \right)}{S_{topgdr}}$$

$$f_{fws} = -0.71ksi$$

Live load (HL-93) and dynamic load allowance:

$$M_{LL} = 1908K \cdot ft$$

$$S_{topgdr} = 17775.1in^3$$

$$f_{LL} = \frac{-M_{LL} \cdot \left( \frac{12in}{ft} \right)}{S_{topgdr}}$$

$$f_{LL} = -1.29ksi$$

S3.4.1

Multiplying the above stresses by their respective load factors and adding the products results in the following combined stress for the Strength I load combination:

$$f_{Str} = (LF_{DC} \cdot f_{noncompDL}) + (LF_{DC} \cdot f_{parapet}) + (LF_{DW} \cdot f_{fws}) + (LF_{LL} \cdot f_{LL})$$

$$f_{Str} = -26.12 \text{ksi}$$

Similarly, all of the combined moments, shears, and flexural stresses can be computed at the controlling locations. A summary of those combined load effects for an interior girder is presented in the following three tables, summarizing the results obtained using the procedures demonstrated in the above computations.

Combined Effects at Location of Maximum Positive Moment				
Summary of Unfactored Values:				
Loading	Moment (K-ft)	$f_{botgdr}$ (ksi)	$f_{topgdr}$ (ksi)	$f_{topslab}$ (ksi)
Noncomposite DL	1208	16.52	-17.64	0.00
Parapet DL	192	1.91	-0.59	-0.06
FWS DL	233	2.31	-0.71	-0.07
LL+IM - HL-93	1908	17.40	-1.29	-0.61
LL+IM-Fatigue	822	7.49	-0.55	-0.26
Summary of Factored Values:				
Limit State	Moment (K-ft)	$f_{botgdr}$ (ksi)	$f_{topgdr}$ (ksi)	$f_{topslab}$ (ksi)
Strength I	5439	56.95	-26.12	-1.26
Service II	4114	43.35	-20.62	-0.93
Fatigue I	1233	11.24	-0.83	-0.40
Fatigue II	616	5.62	-0.42	-0.20

**Table 3-11 Combined Effects at Location of Maximum Positive Moment**

As shown in the above table, the Strength I load combination elastic stress in the bottom of the girder exceeds the girder yield stress. However, for this design example, this value is of no concern because of the local yielding that is permitted to occur at this section (see Design Steps 3.8 through 3.10).

Combined Effects at Location of Maximum Negative Moment				
Summary of Unfactored Values (Assuming Concrete Not Effective):				
Loading	Moment (K-ft)	$f_{botgdr}$ (ksi)	$f_{topgdr}$ (ksi)	$f_{deck}$ (ksi)
Noncomposite DL	-3197	-16.84	17.90	0.00
Parapet DL	-436	-2.14	1.69	1.98
FWS DL	-528	-2.59	2.04	2.40
LL+IM - HL-93	-2450	-12.03	9.48	11.11
Summary of Unfactored Values (Assuming Concrete Effective):				
Loading	Moment (K-ft)	$f_{botgdr}$ (ksi)	$f_{topgdr}$ (ksi)	$f_{deck}$ (ksi)
Noncomposite DL	-3197	-16.84	17.90	0.00
Parapet DL	-436	-2.00	1.07	0.10
FWS DL	-528	-2.42	1.29	0.12
LL+IM - HL-93	-2450	-10.47	2.50	0.56
LL+IM-Fatigue	-406	-1.74	0.41	0.09
Summary of Factored Values:				
Limit State	Moment (K-ft)	$f_{botgdr}$ (ksi)	$f_{topgdr}$ (ksi)	$f_{deck}$ (ksi)
Strength I *	-9621	-48.67	44.14	25.51
Service II **	-7346	-34.87	23.51	0.95
Fatigue I **	-609	-2.60	0.62	0.14
Fatigue II **	-305	-1.30	0.31	0.07

Legend:

- \* Strength I Limit State stresses are based on section properties assuming the deck concrete is not effective, and  $f_{deck}$  is the stress in the deck reinforcing steel.
- \*\*  $f_{deck}$  is the stress in the deck concrete. Since  $f_{deck}$  under the Service II loads does not exceed  $2f_r$  at this section (= 0.96 ksi), Service II stresses due to loads applied to the composite section are computed assuming the deck concrete is effective. Fatigue I and II stresses are based on section properties assuming the deck concrete is effective.

**Table 3-12 Combined Effects at Location of Maximum Negative Moment – Interior Girder**

Combined Effects at Location of Maximum Shear	
Summary of Unfactored Values:	
Loading	Shear (kips)
Noncomposite DL	114.7
Parapet DL	16.4
FWS DL	19.8
LL+IM - HL-93	131.4
LL+IM - Fatigue	46.5
Summary of Factored Values:	
Limit State	Shear (kips)
Strength I	423.5
Service II	321.7
Fatigue I	69.6
Fatigue II	34.8

**Table 3-13 Combined Effects at Location of Maximum Shear – Interior Girder**

Envelopes of the factored Strength I moments and shears in an interior girder are presented in the following two figures. Maximum and minimum values are presented. As mentioned previously, all remaining design computations in this example are based on the interior girder. The basic approach illustrated in the subsequent design calculations applies equally to the exterior and interior girders (with some exceptions noted) once the load effects in each girder have been determined.

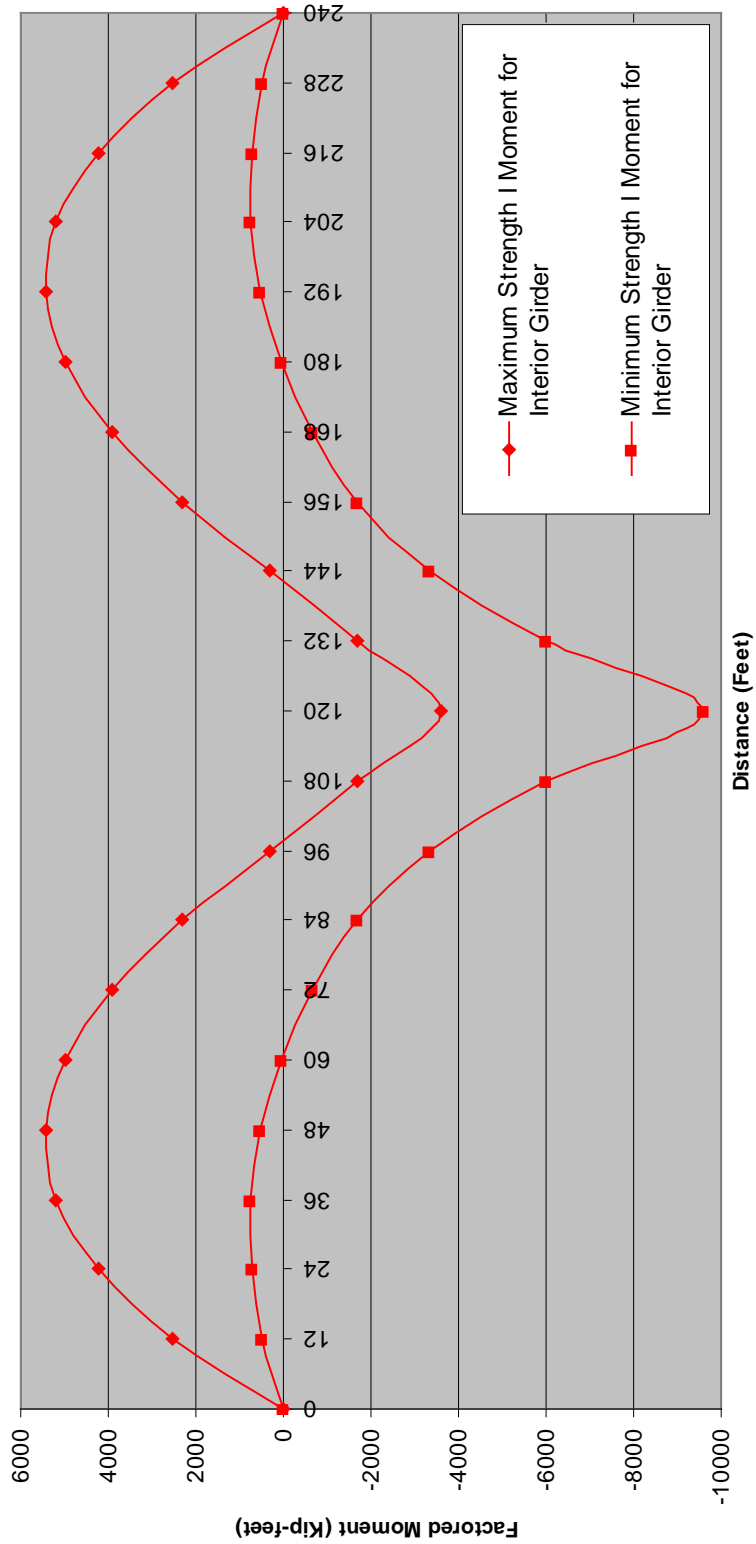


Figure 3-6 Envelope of Strength I Moments

Figure 3-7 Envelope of Strength I Moments – Interior Girder

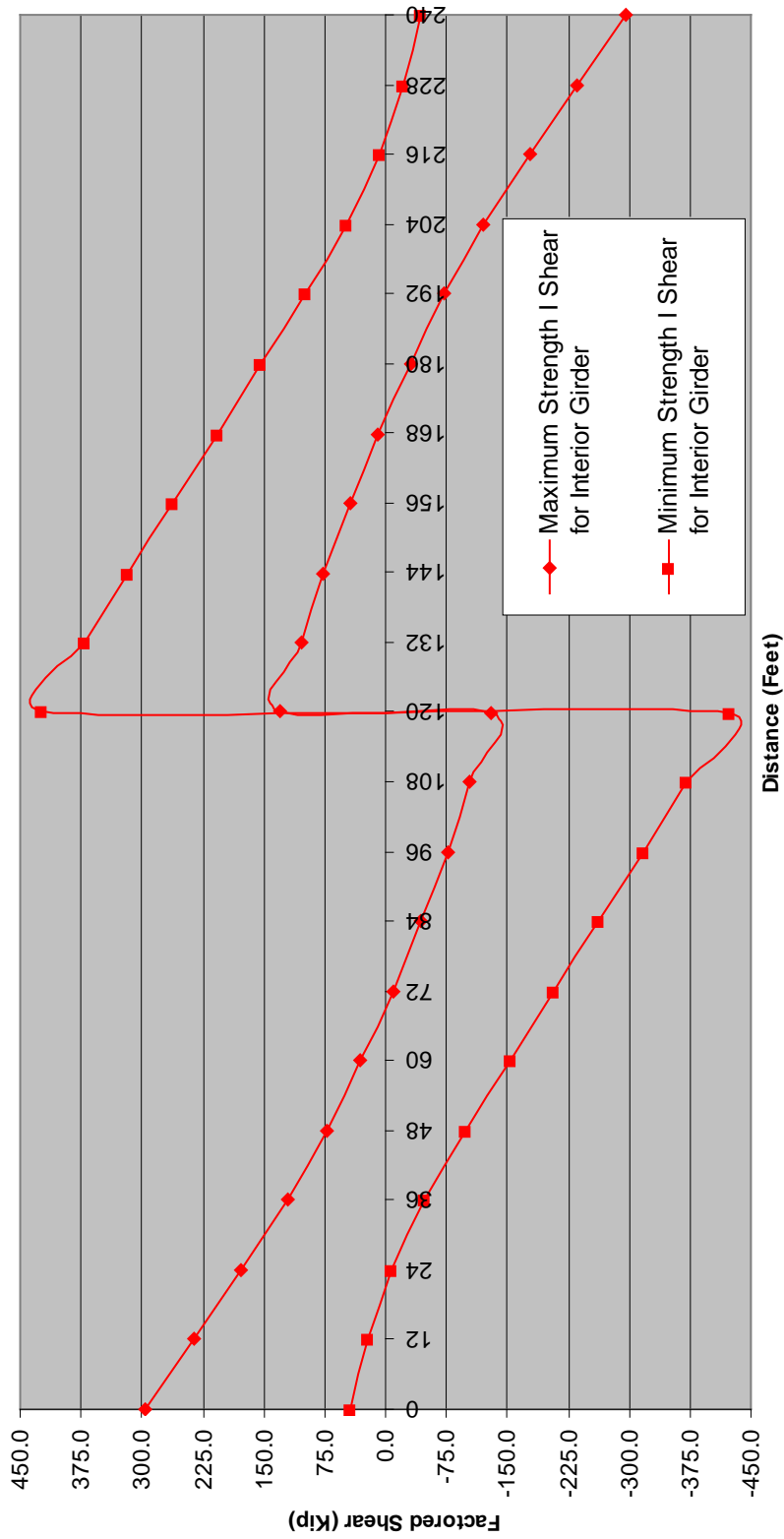


Figure 3-8 Envelope of Strength I Shears – Interior Girder

Figure 3-7 Envelope of Strength I Shears

Design Steps 3.7 through 3.17 consist of verifying the structural adequacy of critical beam locations using appropriate sections of the Specifications.

For this design example, two design sections will be checked for illustrative purposes. First, all specification checks for Design Steps 3.7 through 3.17 will be performed for the location of maximum positive moment, which is at  $0.4L$  in Span 1. Second, all specification checks for these same design steps will be performed for the location of maximum negative moment and maximum shear, which is at the pier.

### **Specification Check Locations**



For steel girder designs, specification checks are generally performed using a computer program at the following locations:

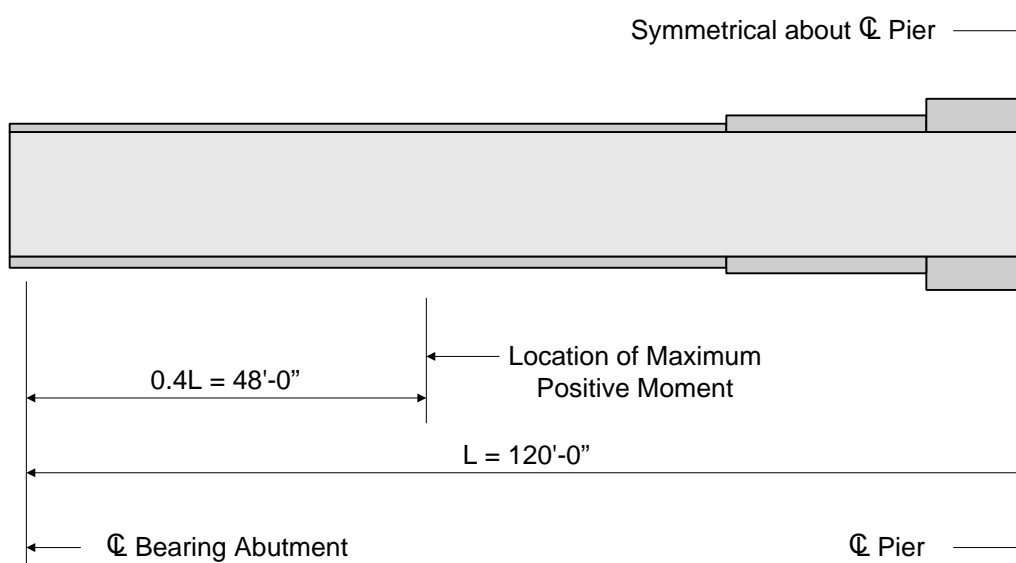
- Span tenth points
- Locations of plate transitions
- Locations of stiffener spacing transitions

However, it should be noted that the maximum moment within a span may not necessarily occur at any of the above locations.

For additional information about the live load used in LRFD, refer to S3.6 and C3.6.

The following specification checks are for the location of maximum positive moment, which is at  $0.4L$  in Span 1, as shown in Figure 3-9.





**Figure 3-9 Location of Maximum Positive Moment**

### Design Step 3.7 - Check Section Proportion Limits - Positive Moment Region

S6.10.2

Several checks are required to ensure that the proportions of the trial girder section are within specified limits.

S6.10.2.1

The first section proportion check relates to the web slenderness. For a section without longitudinal stiffeners, the web must be proportioned such that:

$$\frac{D}{t_w} \leq 150$$

$$D = 54in$$

$$t_w = 0.50in$$

$$\frac{D}{t_w} = 108.0 \quad \text{OK}$$

S6.10.2.2

The second set of section proportion checks relate to the general proportions of the section. The compression and tension flanges must be proportioned such that:

$$\frac{b_f}{2t_f} \leq 12.0 \quad b_f = 14 \quad t_f = 0.75 \quad \frac{b_f}{2t_f} \leq 9.3 \quad \text{OK}$$

$$b_f \geq \frac{D}{6} \quad \frac{D}{6} = 9.0in \quad \text{OK}$$

$$t_f \geq 1.1 \cdot t_w \quad 1.1 \cdot t_w = 0.550in \quad \text{OK}$$

$$0.1 \leq \frac{I_{yc}}{I_{yt}} \leq 10$$

$$I_{yc} = \frac{0.75in \cdot (14in)^3}{12} \quad I_{yc} = 171.5in^4$$

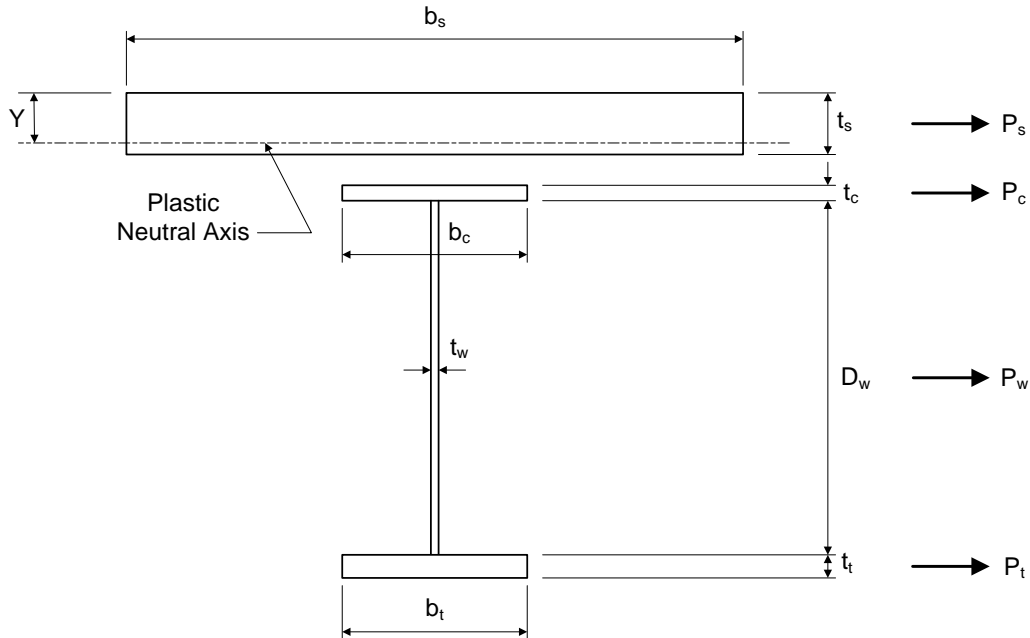
$$I_{yt} = \frac{0.875in \cdot (14in)^3}{12} \quad I_{yt} = 200.1in^4$$

$$\frac{I_{yc}}{I_{yt}} = 0.857 \quad \text{OK}$$

### Design Step 3.8 - Compute Plastic Moment Capacity - Positive Moment Region

*SAppendix D6.1*

For composite sections, the plastic moment,  $M_p$ , is calculated as the first moment of plastic forces about the plastic neutral axis.



**Figure 3-10 Computation of Plastic Moment Capacity for Positive Bending Sections**

*SAppendix D6.1*

For the tension flange:

$$F_{yt} = 50ksi \quad b_t = 14in \quad t_t = 0.875in$$

$$P_t = F_{yt} \cdot b_t \cdot t_t \quad P_t = 613K$$

For the web:

$$F_{yw} = 50ksi \quad D_w = 54in \quad t_w = 0.50in$$

$$P_w = F_{yw} \cdot D_w \cdot t_w \quad P_w = 1350K$$

For the compression flange:

$$F_{yc} = 50ksi \quad b_c = 14in \quad t_c = 0.75in$$

$$P_c = F_{yc} \cdot b_c \cdot t_c \quad P_c = 525K$$

For the slab:

$$f'_c = 4.0ksi \quad b_s = 117in \quad t_s = 8.0in$$

$$P_s = 0.85 \cdot f'_c \cdot b_s \cdot t_s \qquad P_s = 3182K$$

*SAppendix D6.1*

The forces in the longitudinal reinforcement may be conservatively neglected in regions of positive flexure.

*SAppendix D6.1*

Check the location of the plastic neutral axis, as follows:

$$P_t + P_w = 1963K \qquad P_c + P_s = 3707K$$

$$P_t + P_w + P_c = 2488K \qquad P_s = 3182K$$

Therefore, the plastic neutral axis is located within the slab.

$$Y = (t_s) \left( \frac{P_c + P_w + P_t}{P_s} \right) \qquad Y = 6.26in \qquad \text{STable D6.1-1 Case III}$$

Check that the position of the plastic neutral axis, as computed above, results in an equilibrium condition in which there is no net axial force.

$$\text{Compression} = 0.85 f'_c \cdot b_s \cdot Y$$

$$\text{Compression} = 2488K$$

$$\text{Tension} = P_t + P_w + P_c$$

$$\text{Tension} = 2488K \qquad \text{OK}$$

*STable D6.1-1*

The plastic moment,  $M_p$ , is computed as follows, where  $d$  is the distance from an element force (or element neutral axis) to the plastic neutral axis (note that the distance from the top of web to bottom of slab is taken as 3.5 inches as discussed earlier):

$$d_c = \frac{-t_c}{2} + 3.5in + t_s - Y \qquad d_c = 4.87in$$

$$d_w = \frac{D_w}{2} + 3.5in + t_s - Y \qquad d_w = 32.24in$$

$$d_t = \frac{t_t}{2} + D_w + 3.5in + t - Y \qquad d_t = 59.68in$$

$$M_p = \frac{Y^2 \cdot P_s}{2 \cdot t_s} + (P_c \cdot d_c + P_w \cdot d_w + P_t \cdot d_t) \quad M_p = 7538K \cdot ft$$

### Design Step 3.9 - Determine if Section is Compact or Noncompact - Positive Moment Region

Since the section is in a straight bridge, the next step in the design process is to determine if the section is compact or noncompact. This, in turn, will determine which formula should be used to compute the flexural resistance of the girder.

S6.10.6.2.2

If the specified minimum yield strengths of the flanges do not exceed 70.0 ksi and the girder does not have longitudinal stiffeners, then the first step is to check the compact-section web slenderness provisions, as follows:

$$\frac{2 \cdot D_{cp}}{t_w} \leq 3.76 \cdot \sqrt{\frac{E}{F_{yc}}} \quad S6.10.6.2.2$$

Since the plastic neutral axis is located within the slab,

$$D_{cp} = 0in$$

S6.10.6.2.2

Therefore the web is deemed compact. Since this is a composite section in positive flexure and there are no holes in the tension flange at this section, the flexural resistance is computed as defined by the composite compact-section positive flexural resistance provisions of S6.10.7.1.2.

### Design Step 3.10 - Design for Flexure - Strength Limit State - Positive Moment Region

Since the section was determined to be compact, and since it is a composite section in the positive moment region with no holes in the tension flange, the flexural resistance is computed in accordance with the provisions of S6.10.7.1.2.

All design sections of this girder are homogenous. That is, the same structural steel is used for the top flange, the web, and the bottom flange. Therefore, the hybrid factor,  $R_h$ , is as follows:

$$R_h = 1.0 \quad S6.10.1.10.1$$

SAppendix D6.2.2

The yield moment,  $M_y$ , is computed as follows:

$$F_y = \frac{M_{D1}}{S_{NC}} + \frac{M_{D2}}{S_{LT}} + \frac{M_{AD}}{S_{ST}}$$

$$M_y = M_{D1} + M_{D2} + M_{AD}$$

$$F_y = 50 \text{ ksi}$$

$$M_{D1} = (1.25 \cdot 1208 \text{ K} \cdot \text{ft}) = 1510 \text{ K} \cdot \text{ft}$$

$$M_{D2} = (1.25 \cdot 192 \text{ K} \cdot \text{ft}) + (1.50 \cdot 233 \text{ K} \cdot \text{ft}) = 590 \text{ K} \cdot \text{ft}$$

For the bottom flange:

$$S_{NC} = 877.6 \text{ in}^3$$

$$S_{LT} = 1208.2 \text{ in}^3$$

$$S_{ST} = 1316.2 \text{ in}^3$$

$$M_{AD} = \left[ S_{ST} \cdot \left( F_y - \frac{M_{D1}}{S_{NC}} - \frac{M_{D2}}{S_{LT}} \right) \right] \cdot \left( \frac{1 \text{ ft}}{12 \text{ in}} \right)$$

$$M_{AD} = 2577 \text{ K} \cdot \text{ft}$$

$$M_{ybot} = M_{D1} + M_{D2} + M_{AD} = 4677 \text{ K} \cdot \text{ft}$$

For the top flange:

$$S_{NC} = 821.7 \text{ in}^3$$

$$S_{LT} = 3907.1 \text{ in}^3$$

$$S_{ST} = 17775.1 \text{ in}^3$$

$$M_{AD} = S_{ST} \cdot \left( F_y - \frac{M_{D1}}{S_{NC}} - \frac{M_{D2}}{S_{LT}} \right)$$

$$M_{AD} = 38714 \text{ K} \cdot \text{ft}$$

$$M_{ytop} = M_{D1} + M_{D2} + M_{AD} = 40814 \text{ K} \cdot \text{ft}$$

## SAppendix D6.2.2

The yield moment,  $M_y$ , is the lesser value computed for both flanges. Therefore,  $M_y$  is determined as follows:

$$M_y = \min( M_{ybot}, M_{ytop} )$$

$$M_y = 4677K \cdot ft$$

## S6.10.7.1.2

Therefore, for the positive moment region of this design example, the nominal flexural resistance is computed as follows (refer to RM Section 6.5.6.3 for a detailed discussion of the nominal flexural resistance for sections in positive flexure):

$$D_p \leq 0.1D_t$$

$$D_p = Y = 6.26in$$

$$D_t = 0.875in + 54in + 0.75in + 8in = 63.625in$$

$$0.1D_t = 6.36in > 6.26in$$

Therefore:

$$M_n = M_p = 7538K \cdot ft$$

Since this is neither a simple span nor a continuous span where the span and the sections in the negative-flexure region over the interior supports satisfy the special conditions outlined at the end of S6.10.7.1.2, the nominal flexural resistance of the section must not exceed the following:

$$M_n = 1.3 \cdot R_h \cdot M_y$$

$$M_n = 6080K \cdot ft < 7538K \cdot ft \quad (\text{controls})$$

Use:

$$M_n = 6080K \cdot ft$$

## S6.10.7.3

The ductility requirement is checked as follows:

$$D_p \leq 0.42D_t$$

$$0.42D_t = 26.7in > 6.26in \quad \text{OK}$$

## S6.10.7.1.1

The factored flexural resistance,  $M_r$ , is computed as follows (note that since there is no curvature, no skew and wind load is not considered under the Strength I load combination, the flange lateral bending stress is taken as zero in this case):

$$M_u + \frac{1}{3}(0) \leq \phi_f M_n$$

$$\phi_f = 1.00 \quad \text{S6.5.4.2}$$

$$M_r = \phi_f M_n$$

$$M_r = 6080K \cdot ft$$

## S1.3.2.1

The positive flexural resistance at this design section is checked as follows:

$$\sum \eta_i \cdot \gamma_i \cdot Q_i \leq R_r$$

or in this case:

$$\sum \eta \cdot \gamma \cdot M_u \leq M_r$$

For this design example,

$$\eta = 1.00$$

As computed in Design Step 3.6,

$$\sum \gamma \cdot M_u = 5439K \cdot ft$$

Therefore

$$\sum \eta \cdot \gamma \cdot M_u = 5439K \cdot ft$$

$$M_r = 6080K \cdot ft > 5439K \cdot ft \quad \text{OK}$$

Based on the above computations, the flexural resistance is approximately 12% greater than the factored design moment. The bottom flange thickness cannot be reduced any further because such a reduction would result in a specification check failure at the fatigue limit state (see Design Step 14).



### Plate Thickness Increments



Available plate thicknesses can be obtained from steel fabricators. As a rule of thumb, the following plate thickness increments are generally recommended by steel fabricators:

3/16" to 3/4" - increments of 1/16"

3/4" to 2 1/2" - increments of 1/8"

2 1/2" to 4" - increments of 1/4"

### **Design Step 3.11 - Design for Shear - Positive Moment Region**

*S6.10.9*

Shear must be checked at each section of the girder. However, shear is minimal at the location of maximum positive moment, and it is maximum at the pier.

Therefore, for this design example, the required shear design computations will be presented later for the girder design section at the pier.

*S6.10.9.3.3c*

It should be noted that in end panels, the shear is limited to either the shear yielding or shear buckling resistance in order to provide an anchor for the tension field in adjacent interior panels. Tension field is not allowed in end panels. The design procedure for shear in the end panel is presented in *S6.10.9.3.3c*.

### **Design Step 3.12 - Design Transverse Intermediate Stiffeners - Positive Moment Region**

*S6.10.11.1*

The girder in this design example has transverse intermediate stiffeners. Transverse intermediate stiffeners are used to increase the shear resistance of the girder. They are also used as connection plates for diaphragms or cross-frames.

As stated above, shear is minimal at the location of maximum positive moment but is maximum at the pier for this example. Therefore, the required design computations for transverse intermediate stiffeners will be presented later for the girder design section at the pier.

It should be noted that, based on the steel girder design flowchart, Design Step 3.13 involves the design of longitudinal stiffeners. Since longitudinal stiffeners are not being used for the girder in this design example, Design Step 3.13 is excluded from this design example.

### **Design Step 3.14 - Design for Flexure - Fatigue and Fracture Limit State - Positive Moment Region**

S6.6.1

Load-induced fatigue must be considered in a plate girder design. Fatigue considerations for plate girders may include base metal at:

1. Welds connecting the shear studs to the girder.
2. Welds connecting the flanges and the web.
3. Welds connecting the transverse intermediate stiffeners or other details to the girder.

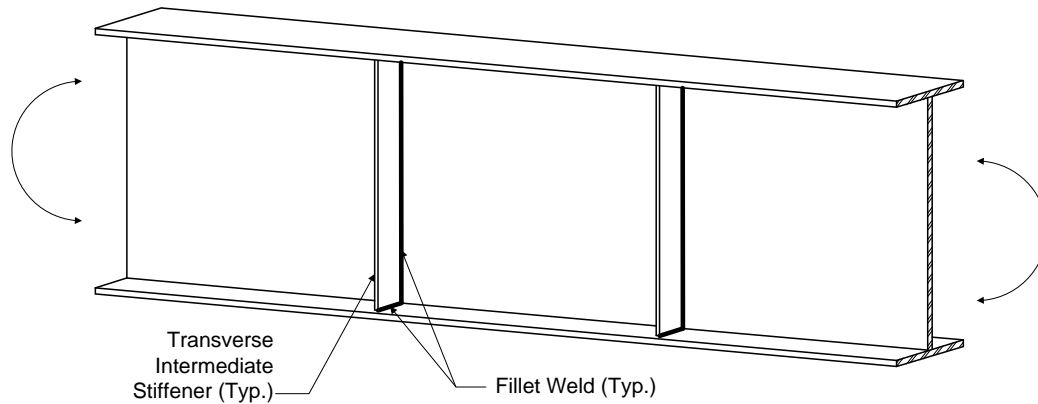
*S*Table 6.6.1.2.3-1

Fatigue considerations may also include the attachment metal. The specific fatigue considerations depend on the unique characteristics of the girder design. Specific fatigue details and detail categories are explained and illustrated in *S*Table 6.6.1.2.3-1.

For this design example, fatigue will be checked for the fillet-welded connection of a transverse intermediate stiffener serving as a cross-frame connection plate to the girder. This detail corresponds to Condition 4.1 in *S*Table 6.6.1.2.3-1, and it is classified as Detail Category C' in *S*Table 6.6.1.2.3-1.

For this design example, this connection will be conservatively checked at the location of maximum positive moment (the cross-frame is actually located 8 feet to the left of this section which would produce a smaller stress range). The fatigue detail at the inner fiber of the tension flange (bottom flange), where the transverse intermediate stiffener is welded to the flange is subject to a net tensile stress since this is a positive moment region. However, for simplicity, the computations will conservatively compute the fatigue stress at the outer fiber of the tension flange.

The fatigue detail being investigated in this design example is illustrated in the following figure:



**Figure 3-11 Load-Induced Fatigue Detail**

$$n = 1.0$$

$$ADTT_{SL} = 3000$$

S6.6.1.2.3

From *S*Table 6.6.1.2.3-2, since  $ADTT_{SL}$  exceeds 745 trucks per day for a Category C' detail, the nominal fatigue resistance is computed for the Fatigue I load combination and infinite life, as follows:

$$\Delta F_n = \Delta F_{TH}$$

$$\Delta F_{TH} = 12.0ksi$$

*S*Table 6.6.1.2.5-3

$$\Delta F_n = 12.00ksi$$

As previously explained, for this design example, the concrete slab is assumed to be fully effective for both positive and negative flexure for fatigue limit states. This is permissible because the provisions of S6.10.1.7 were satisfied in the deck design.

The factored fatigue stress range in the base metal at the weld at the location of maximum positive moment was previously computed in Design Step 3.6, Table 3-11, as follows:

$$fbotgdr = 11.24ksi$$

$$fbotgdr \leq \Delta F_n \quad \text{OK}$$

S6.10.6

In addition to the above fatigue detail check, a special fatigue requirement for webs must also be checked. This calculation will be presented later for the girder design section at the pier.

### Design Step 3.15 - Design for Flexure - Service Limit State - Positive Moment Region

S6.10.4.2

The girder must be checked for service limit state control of permanent deformation. This check is intended to prevent objectionable permanent deflections due to expected severe traffic loadings that would impair rideability. The Service II load combination is used for this check.

The stresses for steel flanges of composite sections must satisfy the following requirements:

Top flange:

$$f_f \leq 0.95R_h \cdot F_{yf} \quad \text{S6.10.4.2.2}$$

Bottom flange:

$$f_f + \frac{f_l}{2} \leq 0.95R_h \cdot F_{yf}$$

Since there is no curvature and no skew,  $f_l$  is taken equal to zero at the service limit state in this case. The factored Service II flexural stress was previously computed in Table 3-11 as follows:

$$f_{botgdr} = 43.35ksi \quad f_{topgdr} = -20.62ksi$$

$$F_{yf} = 50ksi$$

$$0.95R_h \cdot F_{yf} = 47.50ksi$$

Top flange:

$$|-20.62ksi| < 47.50ksi \quad \text{OK}$$

Bottom flange:

$$43.35ksi + \frac{1}{2}(0) < 47.50ksi \quad \text{OK}$$

As indicated in S6.10.4.2.2, the web bend buckling check at the service limit state given by *SEquation 6.10.4.2.2-4* is not required for composite sections in positive flexure without longitudinal stiffeners (i.e. with  $D/t_w \leq 150$ ) (S2.5.2.6.2).

In addition to the check for service limit state control of permanent deformation, the girder can also be checked for live load deflection. Although this check is optional for a concrete deck on steel girders, it is included in this design example (many states still require the deflection check).

Using an analysis computer program, the maximum live load deflection is computed to be the following:

$$\Delta_{max} = 1.43in$$

S2.5.2.6.2

This maximum live load deflection is computed based on the following:

1. All design lanes are loaded.
2. All supporting components are assumed to deflect equally.
3. For composite design, the design cross section includes the entire width of the roadway.
4. The number and position of loaded lanes is selected to provide the worst effect.
5. The live load portion of Service I Limit State is used.
6. Dynamic load allowance is included.
7. The live load is taken from S3.6.1.3.2.

S2.5.2.6.2

In the absence of other criteria, assuming vehicular traffic only, the deflection limit is as follows:

$$Span = 120ft$$

$$\Delta_{allowable} = \left( \frac{Span}{800} \right) \cdot \left( \frac{12in}{ft} \right) = 1.80in \quad \text{OK}$$

### Design Step 3.16 - Design for Flexure - Constructibility Check - Positive Moment Region

S6.10.3.2

The girder must also be checked for flexure during construction. The girder has already been checked in its final condition when it behaves as a composite section. The

constructibility must also be checked for the girder prior to the hardening of the concrete deck when the girder behaves as a noncomposite section.

As previously stated, a deck pouring sequence will not be considered in this design example. However, it is required to consider the effects of the deck pouring sequence in an actual design because it will often control the design of the top flange and the cross-frame spacing in the positive moment regions of composite girders. The calculations illustrated below are instead based on the final noncomposite dead load moments after the sequential placement is complete. For an exterior girder, deck overhang effects must also be considered according to *S6.10.3.4*. Since an interior girder is designed in this example, those effects are not considered here.

*SFigure C6.4.1-1 S6.10.3.2.1 & S6.10.3.2.2*

Based on the flowchart for constructibility checks in *SAppendix C6*, nominal yielding of both flanges must be checked as well as the flexural resistance of the compression flange. For discretely braced flanges (note  $f_l$  is taken as 0 since this is an interior girder and there are no curvature, skew, deck overhang or wind load effects considered):

$$f_{bu} + f_l \leq \phi \cdot R_h \cdot F_{yf}$$

The flange stress,  $f_{bu}$ , is taken from Table 3-11 since no deck placement analysis was performed. By inspection, since lateral flange bending is not considered and no live load effects are considered, the special load combination specified in *S3.4.1.2* (described in Design Step 1.1) is the controlling load combination and the compression flange is the controlling flange.

$$f_{bu} = 1.4 \cdot 17.64 \text{ksi} = 24.70 \text{ksi}$$

$$\phi \cdot R_h \cdot F_{yf} = 50 \text{ksi}$$

$$24.70 \text{ksi} + 0 < 50 \text{ksi} \quad \text{OK}$$

*S6.10.3.2*

The flexural resistance calculation ensures that the compression flange has sufficient strength with respect to lateral torsional and flange local buckling based limit states, including the consideration of flange lateral bending where these effects are judged to be significant. The equation is:

$$f_{bu} + \frac{1}{3} \cdot f_l \leq \phi_f \cdot F_{nc}$$

For straight I-girder bridges with compact or noncompact webs, the nominal resistance may be calculated from Article A6.3.3 which includes the beneficial contribution of the St. Venant constant,  $J$ , in the calculation of the lateral torsional buckling resistance.

This may be useful for sections in such bridges with compact or noncompact webs having larger unbraced lengths, if additional lateral torsional buckling resistance is required beyond that calculated based on the provisions of S6.10.8.2.

$$\frac{2D_c}{t_w} = \frac{2(28.73 - 0.75)}{0.5} = 111.9 < \lambda_{rw} = 5.7 \sqrt{\frac{E}{F_{yc}}} = 137.3 \quad \text{S6.10.6.2.3}$$

Although the noncomposite section has a nonslender web according to *SEquation 6.10.6.2.3-1*, for this example, the beneficial effects of Article A6.3.3 will conservatively not be utilized. The preceding flexural resistance equation must be checked for both local buckling and lateral torsional buckling using the appropriate value of the nominal resistance calculated according to S6.10.8.2.

Local buckling resistance:

$$b_{fc} = 14in \quad (\text{see Figure 3-4})$$

$$t_{fc} = 0.75in \quad (\text{see Figure 3-4}) \quad \text{S6.10.8.2.2}$$

$$\lambda_f = \frac{b_{fc}}{2 \cdot t_{fc}} = 9.3$$

$$\lambda_{pf} = 0.38 \cdot \sqrt{\frac{E}{F_{yc}}} = 9.2$$

Since  $\lambda_f > \lambda_{pf}$ ,  $F_{nc}$  must be calculated by the following equation:

$$F_{yr} = \max(\min(0.7 \cdot F_{yc}, F_{yw}), 0.5 \cdot F_{yc}) = 35.0ksi$$

$$\lambda_{rf} = 0.56 \cdot \sqrt{\frac{E}{F_{yr}}} = 16.1$$

$$R_b = 1.0$$

$$F_{nc} = \left[ 1 - \left( 1 - \frac{F_{yr}}{R_h \cdot F_{yc}} \right) \cdot \left( \frac{\lambda_f - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}} \right) \right] \cdot R_b \cdot R_h \cdot F_{yc} = 49.6ksi$$


$$f_{bu} + \frac{1}{3} \cdot f_l = 24.70ksi + \frac{1}{3}(0) = 24.70ksi$$

$$\phi_f \cdot F_{nc} = (1.00)(49.6ksi) = 49.6ksi \quad \text{OK}$$

## S6.10.8.2.3

Lateral torsional buckling resistance:

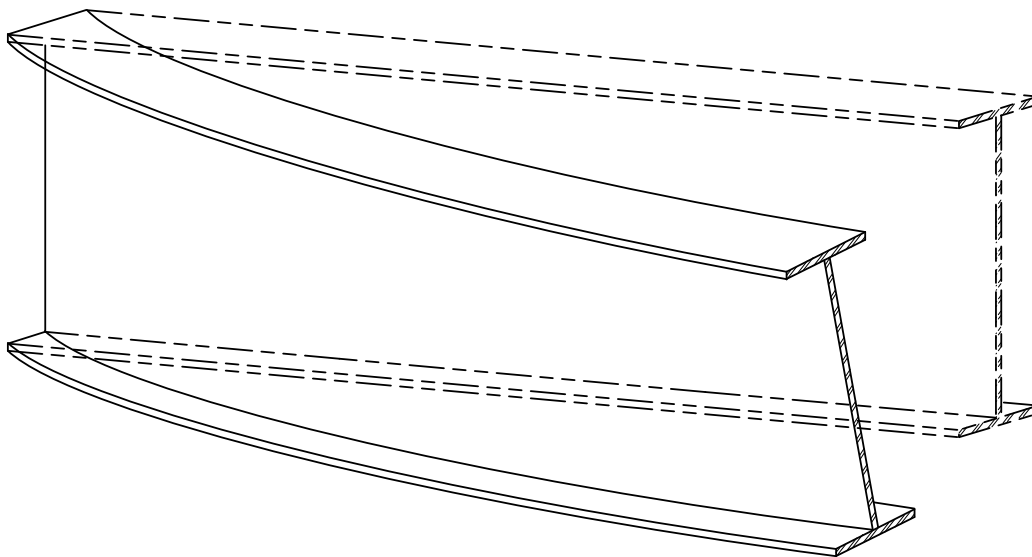
**Lateral Torsional Buckling**



Lateral torsional buckling can occur when the compression flange does not have adequate lateral support such that the member may deflect laterally in a torsional mode. The laterally unsupported compression flange tends to buckle out-of-plane between the points of lateral support. Because the tension flange is kept in line, the girder section must twist when it moves laterally. This behavior is commonly referred to as lateral torsional buckling.

During construction, lateral torsional buckling is generally most critical for the moments induced in the top compression flange during the deck pouring sequence.

Lateral torsional buckling is illustrated in the figure below.



**Figure 3-12 Lateral Torsional Buckling**

For the noncomposite loads during construction:

$$Depth_{comp} = 55.625in - 26.897in = 28.73in$$

(see Figure 3-4 and Table 3-4)



$$t_{f_f} = 0.75in$$

$$D_c = Depth_{comp} - t_{f_f} = 27.98$$

$$b_{f_c} = 14.0in$$

$$r_t = \frac{b_{f_c}}{\sqrt{12 \cdot \left(1 + \frac{1}{3} \cdot \frac{D_c \cdot t_w}{b_{f_c} \cdot t_{f_c}}\right)}} = 3.36in$$

$$E = 29000ksi \quad F_{yc} = 50ksi$$

$$L_p = 1.0 \cdot r_t \cdot \sqrt{\frac{E}{F_{yc}}} = 6.75ft$$

$$L_r = \pi \cdot r_t \cdot \sqrt{\frac{E}{F_{yc}}} = 21.20ft$$

$$L_b = 20.00ft$$

The moment gradient correction factor,  $C_b$ , is computed as follows:

Note since  $f_{mid}$  is greater than  $f_2$  at the location of maximum positive moment, use  $C_b = 1.0$  according to S6.10.8.2.3.

$$C_b = 1.00$$

Therefore:

$$F_{nc} = C_b \cdot \left[ 1 - \left( 1 - \frac{F_{yr}}{R_h \cdot F_{yc}} \right) \cdot \left( \frac{L_b - L_p}{L_r - L_p} \right) \right] \cdot R_b \cdot R_h \cdot F_{yc} = 36.2ksi$$

Use:

$$F_{nc} = 36.2ksi$$

$$\phi_f \cdot F_{nc} = 36.2ksi$$

$$f_{bu} = 24.70ksi$$

$$24.70ksi + \frac{1}{3}(0) < 36.2ksi \quad \text{OK}$$

Web bend-buckling during construction must also be checked according to *SEquation 6.10.3.2.1-3*. However, since the noncomposite section has previously been shown to have a nonslender web, web bend-buckling need not be checked in this case according to *S6.10.3.2.1*.

#### *S6.10.3.2.3*

In addition to checking the nominal flexural resistance during construction, the nominal shear resistance must also be checked. However, shear is minimal at the location of maximum positive moment, and it is maximum at the pier in this case.

Therefore, for this design example, the nominal shear resistance for constructibility will be presented later for the girder design section at the pier.

### **Design Step 3.17 - Check Wind Effects on Girder Flanges - Positive Moment Region**

#### *S6.10.1*

As stated previously, for this design example, the interior girder is being designed.

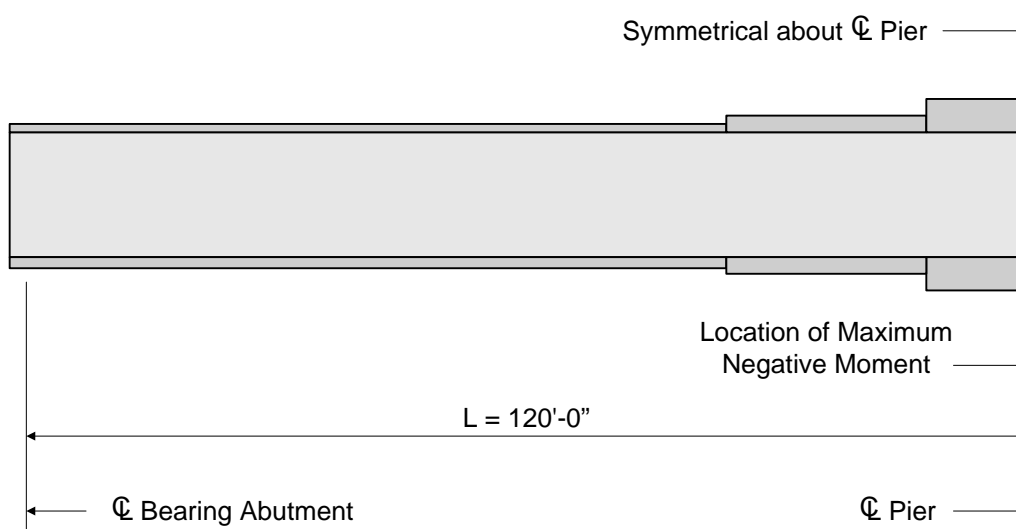
#### *C6.10.1.6 & C4.6.2.7.1*

Wind effects generally do not control a steel girder design, and they are generally considered for the exterior girders only. However, for this design example, wind effects will be presented later for the girder design section at the pier for illustration only.

Specification checks have been completed for the location of maximum positive moment, which is at 0.4L in Span 1.

Now the specification checks are repeated for the location of maximum negative moment, which is at the pier, as shown in Figure 3-13. This is also the location of maximum shear in this case.

Since the design step numbers correspond with the design step numbers presented in the steel girder design flowchart, the design step numbers are repeated for the negative flexure design checks.



**Figure 3-13 Location of Maximum Negative Moment**

### Design Step 3.7 - Check Section Proportion Limits - Negative Moment Region

S6.10.2

Several checks are required to ensure that the proportions of the trial girder section are within specified limits. They include:

1. Web slenderness check (S6.10.2.1)
2. General proportion checks (S6.10.2.2)

S6.10.2.1

The first section proportion check relates to the web slenderness. For a section without longitudinal stiffeners, the web must be proportioned such that:

$$\frac{D}{t_w} \leq 150$$

$$D = 54in$$

$$t_w = 0.50in$$

$$\frac{D}{t_w} = 108.0 \quad \text{OK}$$

S6.10.2.2

The second set of section proportion checks relate to the general proportions of the section. The compression and tension flanges must be proportioned such that:

$$\frac{b_f}{2t_f} \leq 12.0 \quad b_f = 14 \quad t_f = 2.50 \quad \frac{b_f}{2t_f} = 2.8 \quad \text{OK}$$

$$b_f \geq \frac{D}{6} \quad \frac{D}{6} = 9.0in \quad \text{OK}$$

$$t_f \geq 1.1 \cdot t_w \quad 1.1 \cdot t_w = 0.550in \quad \text{OK}$$

Note that only the top flange is checked above. Since the top and bottom flanges are the same width and the bottom flange is thicker than the top, the above checks are satisfied for the bottom flange as well.

$$0.1 \leq \frac{I_{yc}}{I_{yt}} \leq 10$$

$$I_{yc} = \frac{2.75in \cdot (14in)^3}{12} \quad I_{yc} = 628.8in^4$$

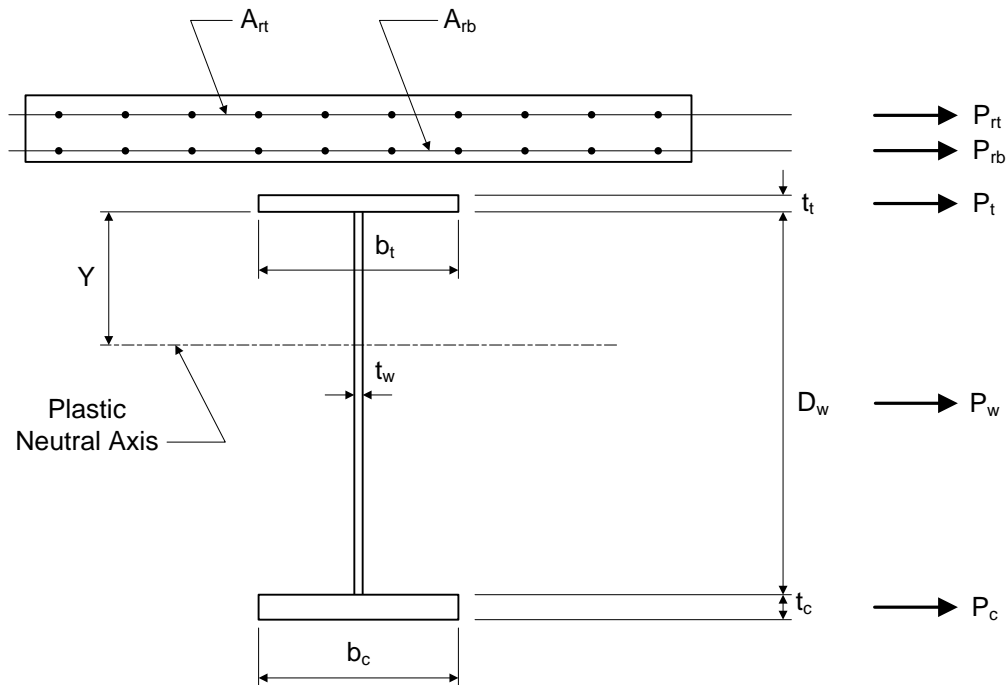
$$I_{yt} = \frac{2.50in \cdot (14in)^3}{12} \quad I_{yt} = 571.7in^4$$

$$\frac{I_{yc}}{I_{yt}} = 1.1 \quad \text{OK}$$

### Design Step 3.8 - Compute Plastic Moment Capacity - Negative Moment Region

*SAppendix D6.1*

For composite sections, the plastic moment,  $M_p$ , is calculated as the first moment of plastic forces about the plastic neutral axis. For composite sections in negative flexure, the concrete deck is ignored and the longitudinal deck reinforcement is included in the computation of  $M_p$ .



**Figure 3-14 Computation of Plastic Moment Capacity for Negative Bending Sections**

*SAppendix D6.1*

For the tension flange:

$$F_{yt} = 50ksi \quad b_t = 14in \quad t_t = 2.50in$$

$$P_t = F_{yt} \cdot b_t \cdot t_t = 1750K$$

For the web:

$$F_{yw} = 50ksi \quad D_w = 54in \quad t_w = 0.50in$$

$$P_w = F_{yw} \cdot D_w \cdot t_w = 1350K$$

For the compression flange:

$$F_{yc} = 50ksi \quad b_c = 14in \quad t_c = 2.75in$$

$$P_c = F_{yc} \cdot b_c \cdot t_c = 1925K$$

For the longitudinal reinforcing steel in the top layer of the slab at the pier:

$$F_{yrt} = 60ksi$$

$$A_{rt} = 0.31in^2 \cdot \left( \frac{117in}{5in} \right) = 7.25in^2$$

$$P_{rt} = F_{yrt} \cdot A_{rt} = 435K$$

For the longitudinal reinforcing steel in the bottom layer of the slab at the pier:

$$F_{yrb} = 60ksi$$

$$A_{rb} = 0.31in^2 \cdot \left( \frac{117in}{5in} \right) = 7.25in^2$$

$$P_{rb} = F_{yrb} \cdot A_{rb} = 435K$$

*SAppendix D6.1*

Check the location of the plastic neutral axis, as follows:

$$P_c + P_w = 3275K \quad P_t + P_{rb} + P_{rt} = 2620K$$

$$P_c + P_w + P_t = 5025K \quad P_{rb} + P_{rt} = 870K$$

Therefore the plastic neutral axis is located within the web.

$$Y = \left( \frac{D}{2} \right) \cdot \left( \frac{P_c - P_t - P_{rt} - P_{rb}}{P_w} + 1 \right) = 13.10in \quad \text{STable D6.1-2}$$

Although it will be shown in the next design step that this section qualifies as a nonslender web section at the strength limit state, the optional provisions of *SAppendix A* to LRFD Section 6 are not employed in this example. Thus, the plastic moment is not used to compute the flexural resistance and therefore does not need to be computed.

### **Design Step 3.9 - Determine if Section is a Compact-Web, Noncompact-Web, or Slender-Web Section - Negative Moment Region**

Since the section is in a straight bridge, the next step in the design process is to determine if the section is a compact-web, noncompact-web, or slender-web section. This, in turn, will determine which formulae should be used to compute the flexural resistance of the girder.

*S6.10.6.2.3*

Where the specified minimum yield strengths of the flanges do not exceed 70.0 ksi and the girder does not have longitudinal stiffeners, then the first step is to check the noncompact web slenderness limit, as follows:

$$\frac{2 \cdot D_c}{t_w} \leq 5.7 \cdot \sqrt{\frac{E}{F_{yc}}} \quad \text{S6.10.6.2.3}$$

*SAppendix D6.3.1*

At sections in negative flexure,  $D_c$  of the composite section consisting of the steel section plus the longitudinal reinforcement is to be used at the strength limit state (see Table 3-5).

$$D_c = 33.137in - 2.75in = 30.39in$$

(see Figure 3-4 and Table 3-5)

$$\frac{2 \cdot D_c}{t_w} = 121.5 \quad 5.7 \cdot \sqrt{\frac{E}{F_{yc}}} = 137.3$$

The section is a nonslender web section (i.e. either a compact-web or noncompact-web section). Next, check:

$$\frac{I_{yc}}{I_{yt}} > 0.3$$

$$I_{yc} = \frac{2.75in \cdot (14in)^3}{12} \quad I_{yc} = 628.8in^4$$

$$I_{yt} = \frac{2.50in \cdot (14in)^3}{12} \quad I_{yt} = 571.7in^4$$

$$\frac{I_{yc}}{I_{yt}} = 1.1 \quad > 0.3 \quad \text{OK}$$

*S6.10.8 & A6.1*

Therefore, the web qualifies to use the optional provisions of *SAppendix A6* to compute the flexural resistance. However, since the web slenderness is closer to the noncompact web slenderness limit than the compact web slenderness limit in this case (*S6.10.8.1.1*), the simpler equations of *S6.10.8*, which assume slender-web behavior and limit the resistance to  $F_{yc}$  or below, will conservatively be applied in this example to compute the flexural resistance at the strength limit state. The investigation proceeds by calculating the flexural resistance of the discretely braced compression flange.

### Design Step 3.10 - Design for Flexure - Strength Limit State - Negative Moment Region

S6.10.8.2.2 & S6.10.8.2.3

Calculate the nominal flexural resistance of the compression flange (bottom flange) based on local buckling and lateral torsional buckling.

Local buckling resistance:

$$b_{fc} = 14in \quad (\text{see Figure 3-4}) \quad S6.10.8.2.2$$

$$t_{fc} = 2.75in \quad (\text{see Figure 3-4})$$

$$\lambda_f = \frac{b_{fc}}{2 \cdot t_{fc}} = 2.5$$

$$\lambda_{pf} = 0.38 \cdot \sqrt{\frac{E}{F_{yc}}} = 9.2$$

Since  $\lambda_f < \lambda_{pf}$ ,  $F_{nc}$  must be calculated by the following equation:

$$F_{nc} = R_b \cdot R_h \cdot F_{yc}$$

S6.10.1.10.2

Since  $2D_c/t_w$  is less than  $\lambda_{rw}$  (calculated above),  $R_b$  is taken as 1.0.

$$R_b = 1.0 \quad R_h = 1.0$$

$$F_{nc} = R_b \cdot R_h \cdot F_{yc} = 50.0ksi$$

S6.10.8.2.3

Since the flange transition is located within 20 percent of the unbraced length away from the brace point with the smaller moment and the lateral moment of inertia of the flanges of the smaller section is equal to or larger than one-half of the corresponding value in the larger section, the lateral torsional buckling resistance is calculated as follows:

$$D_c = 30.39in \quad b_{fc} = 14.0in \quad t_{fc} = 2.75in$$



$$r_t = \frac{b_{fc}}{\sqrt{12 \cdot \left(1 + \frac{1}{3} \cdot \frac{D_c \cdot t_w}{b_{fc} \cdot t_{fc}}\right)}} = 3.80 \text{ in}$$

$$E = 29000 \text{ ksi} \quad F_{yc} = 50 \text{ ksi}$$

$$L_p = 1.0 \cdot r_t \cdot \sqrt{\frac{E}{F_{yc}}} = 7.63 \text{ ft}$$

$$L_r = \pi \cdot r_t \cdot \sqrt{\frac{E}{F_{yc}}} = 23.98 \text{ ft}$$

$$L_b = 20.00 \text{ ft}$$

Since  $L_p < L_b < L_r$ , use *SEquation 6.10.8.2.3-2*. The moment gradient correction factor,  $C_b$ , is computed as follows:

Where the variation in the moment along the entire length between brace points is concave in shape, which is the case here,  $f_1 = f_0 = 24.80$  ksi (based on preliminary design). This should be updated based on the final moments but will be used here to illustrate the computation of  $C_b$ .

$$f_0 = +24.80 \text{ ksi}$$

$$f_2 = +48.67 \text{ ksi}$$

Therefore:

$$\frac{f_1}{f_2} = 0.51$$

$$C_b = 1.75 - 1.05 \cdot \left(\frac{f_1}{f_2}\right) + 0.3 \left(\frac{f_1}{f_2}\right)^2 \leq 2.3$$

$$1.75 - 1.05 \cdot \left(\frac{f_1}{f_2}\right) + 0.3 \left(\frac{f_1}{f_2}\right)^2 = 1.294$$

$$C_b = 1.294$$

If  $C_b$  is greater than 1.0, this indicates the presence of a significant beneficial moment gradient effect. In this case, the lateral torsional buckling resistances may be calculated by the equivalent procedures specified in *SAppendix D6.4.1*. The

equations in both *S6.10.8.2.3* and *SAppendix D6.4.1* permit  $F_{max}$  to be reached at larger unbraced lengths when  $C_b$  is greater than 1.0. The procedures in *SAppendix D6.4.1* allow the Engineer to focus directly on the maximum unbraced length at which the flexural resistance is equal to  $F_{max}$ . AASHTO strongly recommends the use of these equivalent procedures when  $C_b$  values greater than 1.0 are utilized in the design.

Therefore, since  $C_b$  is greater than 1.0 in this design example, the equations of *SAppendix D6.4.1* will be used to calculate the lateral torsional buckling resistance:

$$L_p + \frac{\left(1 - \frac{1}{C_b}\right)}{\left(1 - \frac{F_{yr}}{R_h F_{yc}}\right)} (L_r - L_p) = 20.02 \text{ ft} > 20.0 \text{ ft} = L_b$$

Therefore:

$$F_{nc} = R_b \cdot R_h \cdot F_{yc} = 50 \text{ ksi}$$

The following equation in *S6.10.8.1.1* must be satisfied for both local buckling and lateral torsional buckling:

$$f_{bu} + \frac{1}{3} f_l \leq \phi_f F_{nc} \quad \text{S6.10.8.1.1 and}$$

For local buckling: S6.10.8.2.1

$$\phi_f \cdot F_{nc} = 50.0 \text{ ksi}$$

For lateral torsional buckling:

$$\phi_f \cdot F_{nc} = 50.0 \text{ ksi}$$

From Design Step 3.6,

$$f_{bu} = 48.67 \text{ ksi}$$

Since there are no curvature or skew effects and wind is not considered under the Strength I load combination,  $f_l$  is taken equal to zero. Therefore:

For local buckling:

$$48.67 + \frac{1}{3}(0) < 50.0 \text{ ksi} \quad \text{OK}$$

For lateral torsional buckling:

S6.10.8.1.3 & S6.10.8.3

$$48.67 + \frac{1}{3}(0) < 50.0 \text{ksi} \quad \text{OK}$$

The investigation proceeds by calculating the flexural resistance of the continuously braced tension flange.

$$f_{bu} \leq \phi_f \cdot R_h \cdot F_{yf}$$

$$f_{bu} = 44.14 \text{ksi} \quad (\text{Design Step 3.6})$$

$$\phi_f \cdot R_h \cdot F_{yf} = 50.0 \text{ksi} > 44.14 \text{ksi} \quad \text{OK}$$

### Design Step 3.11 - Design for Shear - Negative Moment Region

S6.10.9

Shear must be checked along the entire length of the girder. For this design example, shear is maximum at the pier. Example calculations are presented below for the girder section at the pier.

S6.10.9.2

The first step in the design for shear is to check if the web must be stiffened. The nominal shear resistance of unstiffened webs of hybrid and homogeneous girders is:

$$V_n = C \cdot V_p$$

$$k = 5.0$$

$$\frac{D}{t_w} = 108.0 \quad \text{S6.10.9.3.2}$$

$$1.12 \cdot \sqrt{\frac{E \cdot k}{F_{yw}}} = 60.3$$

$$1.40 \cdot \sqrt{\frac{E \cdot k}{F_{yw}}} = 75.4$$

Therefore,

$$\frac{D}{t_w} \geq 1.40 \cdot \sqrt{\frac{E \cdot k}{F_{yw}}}$$

$$C = \frac{1.57}{\left(\frac{D}{t_w}\right)^2} \cdot \left(\frac{E \cdot k}{F_{yw}}\right) = 0.390$$

$$F_{yw} = 50 \text{ ksi} \quad D = 54 \text{ in} \quad t_w = 0.5 \text{ in}$$

$$V_p = 0.58 \cdot F_{yw} \cdot D \cdot t_w = 783.0 \text{ K} \quad \text{S6.10.9.2}$$

$$V_n = C \cdot V_p = 305.6 \text{ K}$$

S6.10.9.1

The factored shear resistance,  $V_r$ , is computed as follows:

$$\phi_f = 1.00 \quad \text{S6.5.4.2}$$

$$V_r = \phi_f \cdot V_n$$

$$V_r = 305.6 \text{ K}$$

S1.3.2.1

The shear resistance at this design section is checked as follows:

$$\sum \eta_i \cdot \gamma_i \cdot Q_i \leq R_r$$

or in this case:

$$\sum \eta_i \cdot \gamma_i \cdot V_i \leq V_r$$

For this design example,

$$\eta_i = 1.00$$

As computed in Design Step 3.6, the factored Strength I shear at the pier is as follows:

$$\sum \eta_i \cdot \gamma_i \cdot V_i = 423.5 \text{ K}$$

The factored Strength I shear at the pier exceeds the factored shear resistance for an unstiffened web at the pier. Therefore, transverse stiffeners are required in the region near the pier.

### Partially Stiffened Webs



As previously explained, a "partially stiffened" web (approximately 1/16 inch thinner than "unstiffened") will generally provide the least cost alternative or very close to it. However, for web depths of approximately 50 inches or less, unstiffened webs may be more economical. A useful guideline for determining the trade-off between adding more stiffeners versus increasing the thickness of web material is that approximately 10 pounds of web material should be saved for every one pound of stiffener material added (S6.10.9.1).

For this design example, transverse intermediate stiffeners are used and longitudinal stiffeners are not used. The transverse intermediate stiffener spacing in this design example is 120 inches. Therefore, the spacing of the transverse intermediate stiffeners does not exceed  $3D = (3)(54 \text{ in}) = 162$  inches. Therefore, the design section can be considered stiffened and the provisions of S6.10.9.3 apply.

### Stiffener Spacing



The spacing of the transverse intermediate stiffeners is determined such that it satisfies all spacing requirements in S6.10.9 and such that the shear resistance of the stiffened web is sufficient to resist the applied factored shear.

S6.10.9.3.2

Next, the web area to average flange area proportion limit for interior web panels must be checked to determine if the nominal shear resistance of the web panel adjacent to the interior pier can be computed using the full postbuckling tension-field resistance or a reduced postbuckling tension-field resistance.

$$\frac{2 \cdot D \cdot t_w}{b_{fc} \cdot t_{fc} + b_{ft} \cdot t_{ft}} \leq 2.5$$

$$D = 54.0 \text{ in} \quad b_{fc} = 14.0 \text{ in} \quad b_{ft} = 14.0 \text{ in}$$

$$t_w = 0.50 \text{ in} \quad t_{fc} = 2.75 \text{ in} \quad t_{ft} = 2.50 \text{ in}$$

$$\frac{2 \cdot D \cdot t_w}{b_{fc} \cdot t_{fc} + b_{ft} \cdot t_{ft}} = 0.7 \quad \text{OK}$$

The nominal shear resistance of the interior web panel at the pier is therefore based on the full postbuckling tension-field resistance, as follows:

$$V_n = \left[ V_p \cdot \left[ C + \frac{0.87 \cdot (1 - C)}{\sqrt{1 + \left(\frac{d_o}{D}\right)^2}} \right] \right]$$

$$d_o = 120 \text{ in}$$

$$k = 5 + \frac{5}{\left(\frac{d_o}{D}\right)^2} = 6.0$$

$$\frac{D}{t_w} = 108.0$$

$$1.12 \cdot \sqrt{\frac{E \cdot k}{F_{yw}}} = 66.1$$

$$1.40 \cdot \sqrt{\frac{E \cdot k}{F_{yw}}} = 82.7$$

Therefore,

$$\frac{D}{t_w} \geq 1.40 \cdot \sqrt{\frac{E \cdot k}{F_{yw}}}$$

$$C = \frac{1.57}{\left(\frac{D}{t_w}\right)^2} \cdot \left(\frac{E \cdot k}{F_{yw}}\right) = 0.469$$

$$V_p = 0.58 \cdot F_{yw} \cdot D \cdot t_w = 783.0 \text{ K}$$

$$V_n = \left[ V_p \cdot \left[ C + \frac{0.87 \cdot (1 - C)}{\sqrt{1 + \left(\frac{d_o}{D}\right)^2}} \right] \right] = 515.8 \text{ K}$$

## S6.10.9.1

The factored shear resistance,  $V_r$ , is computed as follows:

$$\phi_v = 1.00 \quad \text{S6.5.4.2}$$

$$V_r = \phi_v \cdot V_n = 515.8K$$

As previously computed, for this design example:

$$\sum \eta_i \cdot \gamma_i \cdot V_i = 423.5K < 515.8K \quad \text{OK}$$

Therefore, the girder design section at the pier satisfies the shear resistance requirements for the web.

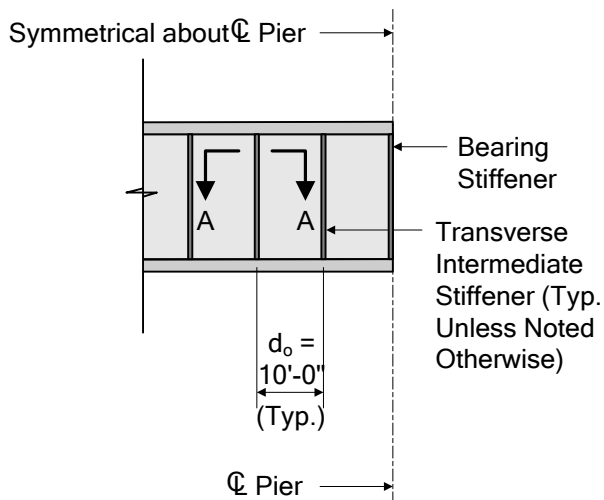
### Design Step 3.12 - Design Transverse Intermediate Stiffeners - Negative Moment Region

## S6.10.11.1

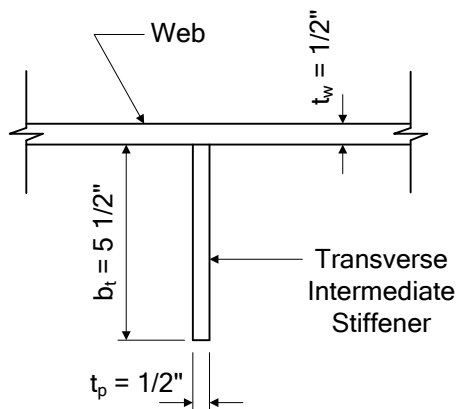
The girder in this design example has transverse intermediate stiffeners. Transverse intermediate stiffeners are used to increase the shear resistance of the girder. They are also used as connection plates for diaphragms or cross-frames. The shear resistance computations shown in the previous design step were based on a stiffener spacing of 120 inches (S6.10.11.1.1).

In this design example, it is assumed that the transverse intermediate stiffeners consist of plates welded to one side of the web except at cross-frame locations where they would be on both sides for an interior girder. The required interface between the transverse intermediate stiffeners and the top and bottom flanges is described in S6.10.11.1.1.

The transverse intermediate stiffener configuration is assumed to be as presented in the following figure.



Partial Girder Elevation at Pier



Section A-A

Figure 3-15 Transverse Intermediate Stiffener

S6.10.11.1.2

The first specification check is for the projecting width of the transverse intermediate stiffener. The width,  $b_t$ , of each projecting stiffener element must satisfy the following:

$$b_t \geq 2.0 + \frac{D}{30.0} \quad \text{and} \quad 16.0t_p \geq b_t \geq 0.25b_f$$

$$b_t = 5.5in$$

$$D = 54in$$



$$t_p = 0.50in$$

$$b_f = 14in$$

$$2.0 + \frac{D}{30.0} = 3.80in$$

Therefore,

$$b_t \geq 2.0 + \frac{D}{30.0} \quad \text{OK}$$

$$16.0t_p = 8.0in$$

$$0.25b_f = 3.5in$$

Therefore,

$$16.0t_p \geq b_t \geq 0.25b_f \quad \text{OK}$$

### S6.10.11.1.3

The second specification check is for the moment of inertia of the strong axis of the transverse intermediate stiffener. This requirement is intended to ensure sufficient rigidity. For transverse stiffeners adjacent to web panels subjected to postbuckling tension-field action, the moment of inertia of the transverse stiffeners must satisfy the following:

- If  $I_{t2} > I_{t1}$ , then:

$$I_t \geq I_{t1} + (I_{t2} - I_{t1})\rho_w$$

- Otherwise:

$$I_t \geq I_{t2}$$

For this design example:

$$V_u = 423.5 \text{ kips}$$

$$\phi_v V_{cr} = 367 \text{ kips}$$

$$\phi_v V_n = 516 \text{ kips}$$

$$d_o = 10'-0'' = 120 \text{ in.}$$

$$I_{t1} = bt_w^3 J$$

$$J = \frac{2.5}{\left(\frac{d_o}{D}\right)^2} - 2.0 \geq 0.5$$

$$\frac{2.5}{\left(\frac{120}{54}\right)^2} - 2.0 = -1.49$$

Therefore, use:  $J = 0.5$

$b$  is taken equal to the smaller of  $D$  and  $d_o$ . In this case,  $b$  is equal to  $D = 54$  in. Therefore:

$$I_{t1} = (54)(0.5)^3(0.5) = 3.38 \text{ in}^4$$

$$I_{t2} = \frac{D^4 \rho_t^{1.3}}{40} \left(\frac{F_{yw}}{E}\right)^{1.5}$$

The local buckling stress,  $F_{crs}$ , for the stiffener is calculated as follows:

$$F_{crs} = \frac{0.31E}{\left(\frac{b_t}{t_p}\right)^2} \leq F_{ys}$$

$$F_{crs} = \frac{0.31(29,000)}{\left(\frac{5.5}{0.5}\right)^2} = 74.3 \text{ ksi} > F_{ys} = 50 \text{ ksi}$$

Therefore, use:  $F_{crs} = 50 \text{ ksi}$

The term,  $\rho_t$ , is equal to the larger of  $F_{yw}/F_{crs}$  (that is  $50 \text{ ksi}/50 \text{ ksi} = 1.0$ ) and 1.0. Therefore, in this case,  $\rho_t$  is equal to 1.0.

$$I_{t2} = \frac{(54)^4 (1.0)^{1.3}}{40} \left(\frac{50}{29,000}\right)^{1.5} = 15.21 \text{ in}^4$$

Since  $I_{t2} > I_{t1}$ , then:

$$I_t \geq I_{t1} + (I_{t2} - I_{t1})\rho_w$$

Since only one panel adjacent to this stiffener (i.e., the right panel) is subject to postbuckling tension-field action (the left panel is an end panel), then  $\rho_w$  is equal to the ratio of  $\left(\frac{V_u - \phi_v V_{cr}}{\phi_v V_n - \phi_v V_{cr}}\right)$  within the one panel subject to postbuckling tension-field action.

Therefore:

$$I_t \geq I_{t1} + (I_{t2} - I_{t1})\rho_w$$

$$I_t \geq 3.38 + (15.21 - 3.38)\left(\frac{423 - 367}{516 - 367}\right) = 7.83 \text{ in}^4$$

For single-sided stiffeners, the moment of inertia of the stiffener is to be taken about the edge in contact with the web. Therefore:

$$I_t = \frac{1}{3}(0.5)(5.5)^3 = 27.73 \text{ in}^4 > 7.83 \text{ in}^4 \quad \text{ok}$$

Use 1/2" x 5 1/2" stiffeners.

### Design Step 3.14 - Design for Flexure - Fatigue and Fracture Limit State - Negative Moment Region

S6.6.1

For this design example, sample nominal fatigue resistance computations were presented previously for the girder section at the location of maximum positive moment. Detail categories are explained and illustrated in *S*Table 6.6.1.2.3-1.

S6.10.5.3

In addition to the nominal fatigue resistance computations, a special fatigue requirement for webs must also be checked. This check is required to control out-of-plane flexing of the web due to shear under repeated live loading.

For this check, the live load shear stress from the fatigue load must be taken as that calculated using the Fatigue I load combination in Table 3-1. The Fatigue I load combination represents the heaviest truck expected to cross the bridge over its 75 year fatigue design life. The check is made using this live load shear in combination with the shear due to the unfactored permanent load (S6.6.1.2.1). This total shear is limited to the shear buckling resistance ( $V_{cr} = CV_p$ ), as follows:

$$V_u \leq V_{cr} \quad \text{S6.10.5.3}$$

Based on the unfactored shear values in Table 3-13:

$$V_u = 114.7K + 16.4K + 19.8K + (1.5 \cdot 46.5K) = 220.7K$$

$$C = 0.469 \quad \text{Design Step 3.11}$$

$$V_p = 783.0K \quad \text{Design Step 3.11}$$

$$V_{cr} = C \cdot V_p = 367.2K$$

$$V_u \leq V_{cr} \quad \text{OK}$$

Therefore, the special fatigue requirement for webs for shear is satisfied.

Other fatigue resistance calculations in the negative moment region are not shown here, but would be similar to the sample check illustrated previously for the positive moment region. Refer to Design Step 3.14 for the positive moment region for the sample checks.

### Design Step 3.15 - Design for Flexure - Service Limit State - Negative Moment Region

S6.10.4

The girder must be checked for service limit state control of permanent deformation. This check is intended to prevent objectionable permanent deflections due to expected severe traffic loadings that would impair rideability. The Service II load combination is used for this check.

C6.10.4.2.2

The flange stress checks of S6.10.4.2.2 will not control for composite sections in negative flexure for which the nominal flexural resistance under the strength load combinations given in *S*Table 3.4.1-1 is determined according to the slender-web provisions of S6.10.8, which is the case in this example.

S6.10.4.2.2

However, for sections in negative flexure, the web must satisfy the web bend buckling check given by *S*Equation 6.10.4.2.2-4 at the service limit state, using the appropriate value of the depth of the web in compression in the elastic range,  $D_c$ .

$$f_c \leq F_{crw}$$

$$F_{crw} = \frac{0.9 \cdot E \cdot k}{\left(\frac{D}{t_w}\right)^2} \quad \text{S6.10.1.9.1}$$

For which:

$$E = 29000ksi \quad D = 54.0in \quad t_w = 0.50in$$

The factored Service II flexural stresses were previously computed in Table 3-12 as follows:

$$f_{botgdr} = -34.87 ksi$$

$$f_{topgdr} = 23.51 ksi$$

S6.10.4.2.1

As previously explained, for this design example, since the maximum longitudinal tensile deck stress at the pier under the Service II loads is smaller than  $2f_r$  in this case, the deck may be considered fully effective for both positive and negative flexure at this section for the service limit state. Therefore, when this is the case,  $D_c$  for composite sections in negative flexure must be computed using the following equation given in *SAppendix D6.3.1*:

$$D_c = \left( \frac{-f_c}{|f_c| + f_t} \right) \cdot d - t_{fc} \geq 0$$

$$d = 59.25in \quad (\text{see Figure 3-4})$$

$$d_{comp} = \left( \frac{-f_c}{|f_c| + f_t} \right) \cdot d = 35.39in$$

$$t_{fc} = 2.75in$$

$$D_c = d_{comp} - t_{fc} = 32.64in > 0in \quad \text{OK}$$

$$k = \frac{9.0}{\left(\frac{D_c}{D}\right)^2} = 24.6 \quad (\text{for webs without longitudinal stiffeners}) \quad \text{S6.10.1.9.1}$$

$$t_w = 0.50in \quad (\text{see Figure 3-4})$$

$$\frac{0.9 \cdot E \cdot k}{\left(\frac{D}{t_w}\right)^2} = 55.12 \text{ksi}$$

$$F_{yw} = 50.0 \text{ksi}$$

$$F_{crw} = \min \left( \frac{0.9 \cdot E \cdot k}{\left(\frac{D}{t_w}\right)^2}, R_h \cdot F_{yc}, \frac{F_{yw}}{0.7} \right) = 50.0 \text{ksi}$$

$$f_c = -34.87 \text{ksi}$$

$$|-34.87 \text{ksi}| < 50.0 \text{ksi} \quad \text{OK}$$

### Design Step 3.16 - Design for Flexure - Constructibility Check - Negative Moment Region

S6.10.3.2

The girder must also be checked for flexure during construction. The girder has already been checked in its final condition when it behaves as a composite section. The constructibility must also be checked for the girder prior to the hardening of the concrete deck when the girder behaves as a noncomposite section.

For the interior girder in this case (where  $f_1 = 0$ ), the sizes of the flanges at the pier section are controlled by the strength limit state flexural resistance checks illustrated previously. Therefore, separate constructibility checks on the flanges need not be made. However, the web bend buckling resistance of the noncomposite pier section during construction must be checked according to *SEquation 6.10.3.2.1-3*, as follows:

$$f_{bu} \leq \phi_f \cdot F_{crw} \quad \text{S6.10.3.2.1}$$

Check first if the noncomposite section at the pier is a nonslender web section. From Table 3.5:

$$D_c = 28.718 \text{in} - 2.75 \text{in} = 25.968 \text{in}$$

$$\frac{2D_c}{t_w} = \frac{2(25.968)}{0.5} = 103.9 < \lambda_{rw} = 5.7 \sqrt{\frac{E}{F_{yc}}} = 137.3 \quad \text{S6.10.6.2.3}$$

The section is therefore a nonslender web section (i.e. a noncompact web section), web bend buckling need not be checked in this case according to S6.10.3.2.1.

### S6.10.3.3

In addition to checking the flexural resistance during construction, the shear resistance in the web must also be checked as follows:

$$V_{cr} = C \cdot V_p$$

to prevent shear buckling of the web during construction.

$$C = 0.469 \quad (\text{Design Step 3.11})$$

$$V_p = 783.0K \quad (\text{Design Step 3.11})$$

$$V_{cr} = 367.2K \quad (\text{Design Step 3.14})$$

$$\phi_v = 1.0$$

S6.5.4.2

$$V_r = \phi_v \cdot V_{cr} = 367.2K$$

$$V_u = (1.4 \cdot 114.7K) = 160.6K < 367.2K \quad \text{OK}$$

Therefore, the design section at the pier satisfies the constructibility specification checks.

### Design Step 3.17 - Check Wind Effects on Girder Flanges - Negative Moment Region

#### Strength Load Combination for Wind on Structure



For the strength limit state, wind on the structure is considered for the Strength III and Strength V load combinations. For Strength III, the load factor for wind on structure is 1.40 but live load is not considered. Due to the magnitude of the live load stresses, Strength III will clearly not control for this design example (and for most designs at the strength limit state)(S3.8.1.2). Therefore, for this design example, only the Strength V load combination will be investigated. The Strength III load combination is likely to be more critical when checking wind load effects during construction.

S6.10.1.6

As stated in Design Step 3.3, for this design example, the interior girder controls and is being designed.

C6.10.1.6 & C4.6.2.7.1

Wind effects generally do not control a steel girder design, and they are generally considered for the exterior girders only. However, for illustrative purposes, wind effects are presented below for the girder design section at the pier. A bridge height of greater than 30 feet is used in this design step to illustrate the required computations (S3.8.1.1). S3.8.1.1 states that the bridge height to be used is the height of structure at which wind loads are being calculated as measured from low ground, or from water level.

S6.10.8.1

The stresses in the bottom flange are combined as follows:

$$\left( f_{bu} + \frac{1}{3} f_l \right) \leq \phi_f \cdot F_{nc}$$

$$f_l = \frac{6 \cdot M_w}{t_{fb} \cdot b_{fb}^2}$$

C4.6.2.7.1

Since the deck provides horizontal diaphragm action and since there are cross frames assumed to be transmitting the wind force into the deck through frame action in the superstructure, the maximum wind moment on the loaded flange is determined as follows:

$$M_w = \frac{W \cdot L_b^2}{10}$$

$$L_b = 20.0 \text{ ft}$$

$$W = \frac{\eta \cdot \gamma \cdot P_D \cdot d}{2}$$

Assume that the bridge is to be constructed in Pittsburgh, Pennsylvania. The design horizontal wind pressure is computed as follows:

$$P_D = P_B \cdot \left( \frac{V_{DZ}}{V_B} \right)^2$$

$$P_B = 0.050 \text{ ksf}$$

STable 3.8.1.2.1-1



$$V_{DZ} = 2.5 \cdot V_0 \cdot \left( \frac{V_{30}}{V_B} \right) \cdot \ln \left( \frac{Z}{Z_0} \right)$$

$$V_0 = 12.0 \text{ MPH} \quad \text{for a bridge located in a city} \quad \text{S3.8.1.1}$$

$$V_{30} = 60 \text{ MPH} \quad \text{assumed wind velocity at 30 feet above low ground or above design water level at bridge site} \\ \text{STable 3.8.1.1-1}$$

$$V_B = 100 \text{ MPH} \quad \text{S3.8.1.1}$$

$$Z = 35 \text{ ft} \quad \text{assumed height of structure at which wind loads are being calculated as measured from low ground or from water level}$$

$$Z_0 = 8.20 \text{ ft} \quad \text{for a bridge located in a city} \quad \text{STable 3.8.1.1-1}$$

$$V_{DZ} = 2.5 \cdot V_0 \cdot \left( \frac{V_{30}}{V_B} \right) \cdot \ln \left( \frac{Z}{Z_0} \right) = 26.1 \text{ MPH} \quad \text{S3.8.1.1}$$

$$P_D = P_B \cdot \left( \frac{V_{DZ}}{V_B} \right)^2 = 0.00341 \text{ ksf} \quad \text{S3.8.1.2.1}$$

$$W = P_d \cdot d = 0.00341(9.23) = 0.0315 \text{ klf} < 0.3 \text{ klf} \quad \text{for beams, use (C4.6.2.7.1):}$$

$$P_d = \frac{W}{d} = \frac{0.3}{9.23} = 0.0325 \text{ ksf}$$

After the design horizontal wind pressure has been computed, the factored wind force per unit length applied to the flange is computed as follows:

$$W = \frac{\eta \cdot \gamma \cdot P_D \cdot d}{2}$$

$$\eta = 1.0 \quad \text{S1.3}$$

$$\gamma = 0.40 \quad \text{for Strength V load combination} \quad \text{STable 3.4.1-1}$$

$$d = 9.23 \text{ ft} \quad \text{from bottom of girder to top of barrier}$$

$$W = \frac{\eta \cdot \gamma \cdot P_D \cdot d}{2} = 0.060 \frac{\text{K}}{\text{ft}}$$

## C4.6.2.7.1

Next, the maximum lateral moment in the flange due to the factored wind loading is computed as follows:

$$M_w = \frac{W \cdot L_b^2}{10} = 2.40K \cdot ft$$

## S6.10.81.1

Finally, the flexural stress at the edges of the bottom flange due to factored wind loading is computed as follows:

$$t_{fb} = 2.75in \quad b_{fb} = 14.0in$$

$$f_l = \frac{6 \cdot M_w}{t_{fb} \cdot b_{fb}^2} = 0.32ksi$$

The load factor for live load is 1.35 for the Strength V load combination. However, it is 1.75 for the Strength I load combination, which has already been investigated. Therefore, it is clear that wind effects will not control the design of this steel girder. Nevertheless, the following computations are presented simply to demonstrate that wind effects do not control this design:

$$f_{bu} = (1.25 \cdot -16.84ksi) + (1.25 \cdot -2.15ksi) \\ + (1.50 \cdot -2.61ksi) + (1.35 \cdot -12.11ksi)$$

$$f_{bu} = -44.00ksi$$

$$f_l = -0.32ksi$$

$$f_{bu} + \frac{1}{3} \cdot f_l = -44.11ksi$$

$$F_{nc} = 50.0ksi \quad (\text{for both local buckling and lateral torsional buckling in this case – see Design Step 10})$$

Therefore:

$$\left( f_{bu} + \frac{1}{3} f_l \right) \leq \phi_f \cdot F_{nc} \quad \text{OK}$$

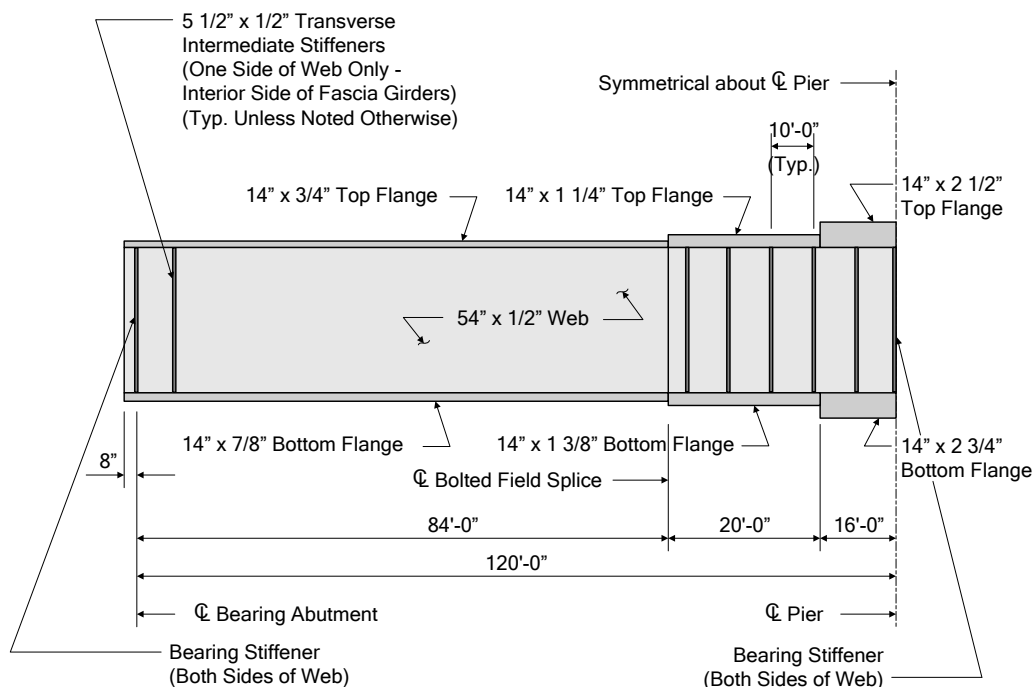
Wind effects do not control the design of this steel girder. For a discussion on checking for wind effects acting on the fully erected steel work prior to placing the concrete deck,

see RM Section 6.5.3.6. It should be noted that AASHTO may revise the computations for wind effects at some point in the future.

### Design Step 3.18 - Draw Schematic of Final Steel Girder Design

Since all of the specification checks were satisfied, the trial girder section presented in Design Step 3.2 is acceptable. If any of the specification checks were not satisfied or if the design were found to be overly conservative, then the trial girder section would need to be revised appropriately, and the specification checks would need to be repeated for the new trial girder section.

The following is a schematic of the final steel girder configuration:



**Figure 3-16 Final Plate Girder Elevation**

For this design example, only the location of maximum positive moment, the location of maximum negative moment, and the location of maximum shear were investigated. However, the above schematic shows the plate sizes and stiffener spacing throughout the entire length of the girder. Some of the design principles for this design example are presented in "tip boxes."

Design computations for a bolted field splice are presented in Design Step 4. Design computations and principles for shear connectors, bearing stiffeners, welded connections, and cross-frames are presented in Design Step 5. Design computations for an elastomeric bearing pad are presented in Design Step 6.

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## Bolted Field Splice Design Example

### Design Step 4

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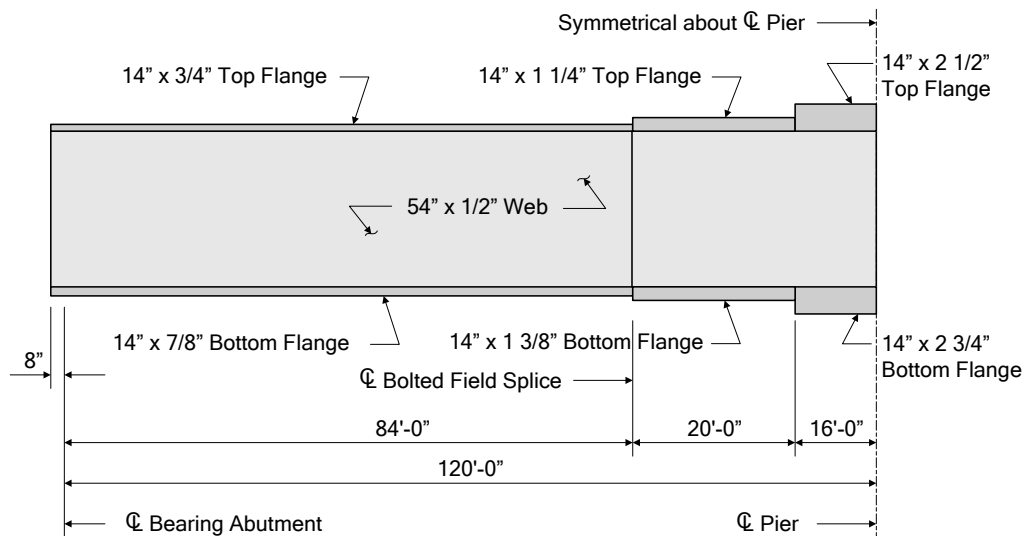
### Design Step 4.1 - Obtain Design Criteria

This splice design example is based on *AASHTO LRFD Bridge Design Specifications, Seventh Edition (2014)*, including the 2015 Interim Specifications. The design methods presented throughout the example are meant to be the most widely used in general bridge engineering practice.

The first design step is to identify the appropriate design criteria. This includes, but is not limited to, defining material properties, identifying relevant superstructure information, and determining the splice location.

Refer to Design Step 1 for introductory information about this design example. Additional information is presented about the design assumptions, methodology, and criteria for the entire bridge, including the splice.

Presented in Figure 4-1 is the final steel girder configuration as designed in Design Step 3. Included in Figure 4-1 is the bolted field splice location. This location was determined using the criteria presented in the narrative below.



**Figure 4-1 Plate Girder Elevation**

The following units are defined for use in this design example:

$$K = 1000lb \quad kcf = \frac{K}{ft^3} \quad ksi = \frac{K}{in^2}$$

For relatively long girders, field splices are generally required to reduce the girder shipping length. The location of the field splice is generally based on economy and includes the following considerations:

1. Field splices are generally located to provide girder segment lengths that do not exceed the allowable girder shipping length or weight. The allowable girder shipping length is often a function of the shipping route from the fabrication site to the construction site.
2. The Specification recommends locating splices near points of dead load contraflexure (*S6.13.6.1.4a*).
3. Field splices are generally located where total moment in the girder is relatively small. This minimizes the required splice plate thicknesses and the required number of bolts.

In Design Step 1.1, the steel properties of the girder were defined. These properties will be used for the splice plates as well.

Yield Strength:

*S*Table 6.4.1-1

$$F_y = 50ksi$$

Tensile Strength:

$$F_u = 65ksi$$

For Specifications equations requiring the flange yield strength:

Flange Yield Strength:

$$F_{yf} = 50ksi$$

Plate Dimensions of the Girder Section to the Left of the Bolted Splice as shown in Figure 4-1 (reference Design Step 3.18):

Web Thickness:

$$t_w = 0.50in$$

Web Depth:

$$D = 54in$$

Top Flange Width:

$$b_{flL} = 14in$$

Top Flange Thickness:

$$t_{flL} = 0.75in$$

Bottom Flange Width:

$$b_{fbL} = 14in$$

Bottom Flange Thickness:

$$t_{fbL} = 0.875in$$

Plate Dimensions of the Girder Section to the Right of the Bolted Splice as shown in Figure 4-1 (reference Design Step 3.18):

Web Thickness:

$$t_w = 0.50in$$

Web Depth:

$$D = 54in$$

Top Flange Width:

$$b_{ftR} = 14in$$

Top Flange Thickness:

$$t_{ftR} = 1.25in$$

Bottom Flange Width:

$$b_{fbR} = 14in$$

Bottom Flange Thickness:

$$t_{fbR} = 1.375in$$

Splice Bolt Properties:

S6.13.6.1.4a



**Bolt Holes**

Oversize or slotted holes are not permitted in either the member or the splice plates at bolted splices of flexural members for improved geometry control during erection and because a strength reduction may occur when oversize or slotted holes are used in eccentrically loaded bolted web connections.

Bolt Diameter:

$$d_{bolt} = 0.875in \quad S6.13.2.5$$

Bolt Hole Diameter (for design purposes):

$$d_{hole} = 0.9375in \quad S6.8.3$$

Bolt Tensile Strength (ASTM A 325):

$$F_{ubolt} = 120ksi \quad S6.4.3.1$$

Concrete Deck Properties (reference Design Step 3.3):

Effective Slab Thickness:

$$t_{seff} = 8in$$

Modular Ratio:

$$n = 8$$

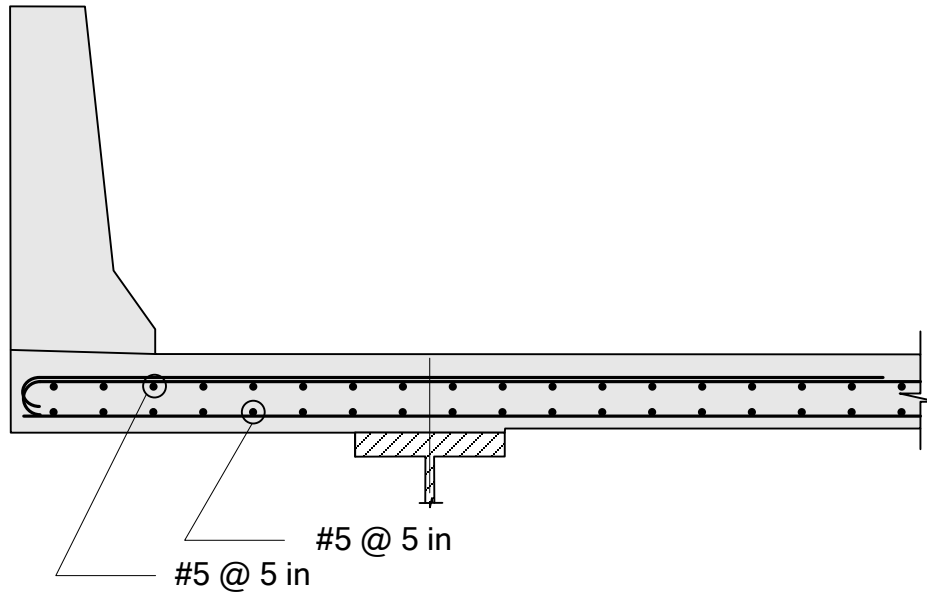
Haunch Depth (measured from top of web):

$$d_{haunch} = 3.5in$$

Effective Flange Width:

$$W_{eff} = 117in$$

Based on the concrete deck design example (FHWA Publication NHI-04-041, December 2003, Design Step 2) and as illustrated in Figure 2-18 (reproduced here as Figure 4-2), the area of longitudinal deck reinforcing steel in the negative moment region is computed as follows:



**Figure 4-2 Superstructure Negative Moment Deck Reinforcement**

For the top steel:

$$A_{deckreinftop} = (0.31in^2) \cdot \frac{W_{eff}}{5in}$$

$$A_{deckreinftop} = 7.254in^2$$

For the bottom steel:

$$A_{deckreinfbot} = (0.31in^2) \cdot \frac{W_{eff}}{5in}$$

$$A_{deckreinfbot} = 7.254in^2$$

S6.5.4.2

Resistance Factors:

Flexure:

$$\phi_f = 1.00$$

Shear:

$$\phi_v = 1.00$$

Axial Compression:

$$\phi_c = 0.95$$

Tension, fracture in net section:

$$\phi_u = 0.80$$

Tension, yielding in gross section:

$$\phi_y = 0.95$$

Bolts bearing on material:

$$\phi_{bb} = 0.80$$

A325 and A490 bolts in shear:

$$\phi_s = 0.80$$

Block shear:

$$\phi_{bs} = 0.80$$

## Design Step 4.2 - Select Girder Section as Basis for Field Splice Design

*S6.13.6.1.4a*

Where a section changes at a splice, the smaller of the two connected sections is to be used in the design. To determine the smaller section at the point of splice in a flexural member, it is suggested that the smaller section be taken as the side of the splice that has the smaller product of the calculated moment of inertia for the noncomposite steel section and the smallest specified minimum flange yield strength on the side of the splice under consideration. The smaller section is used to ensure that the splice is at least as strong as its smallest connected member. Therefore, the bolted field splice will be designed based on the girder section to the left of the bolted splice location. This will be referred to as the Left Girder throughout the calculations. The girder section located to the right of the bolted field splice will be designated the Right Girder. See Figure 4-1 for the girder sections described above.

### Design Step 4.3 - Compute Flange Splice Design Loads

#### Girder Moments at the Splice Location:

Based on the properties defined in Design Step 3 (Steel Girder Design), any number of commercially available software programs can be used to obtain the design dead and live loads at the splice. For this design example, the AASHTO BrD software was used. A summary of the unfactored moments at the splice from the initial trial of the girder design are listed below. The live load effects include the dynamic load allowance and distribution factors.

#### Dead Load Moments:

Noncomposite:

$$M_{NDL} = -51.8K \cdot ft$$

Composite:

$$M_{CDL} = 15.5K \cdot ft$$

Future Wearing Surface:

$$M_{FWS} = 18.8K \cdot ft$$

#### Live Load Moments:

HL-93 Positive:

$$M_{PLL} = 1307.8K \cdot ft$$

HL-93 Negative:

$$M_{NLL} = -953.3K \cdot ft$$

Fatigue Positive:

$$M_{PFL} = 394.3K \cdot ft$$

Fatigue Negative:

$$M_{NFL} = -284.0K \cdot ft$$

S6.13.6

Typically, splices are designed for the strength, service, and fatigue limit states. The load factors for the load combinations associated with these limit states are shown in Table 4-1:

*S*Table 3.4.1-1 and *S*Table 3.4.1-2

Load	Load Factors				
	Strength I		Service II	Fatigue I	Fatigue II
	$\gamma_{\max}$	$\gamma_{\min}$	$\gamma$	$\gamma$	$\gamma$
DC	1.25	0.90	1.00	-	-
DW	1.50	0.65	1.00	-	-
LL	1.75	1.75	1.30	1.50	0.75

**Table 4-1 Load Factors**

Flange Stress Computation Procedure:

S6.13.6

The stresses corresponding to each limit state will be computed at the midthickness of the top and bottom flanges. The appropriate section properties and load factors for use in computing stresses are described below. Where necessary, refer to the signs of the previously documented design moments.

*Strength Limit State:*

S6.13.6.1.4a

Case 1: Dead Load + Positive Live Load

*S*Table 3.4.1-2 and *S*6.10.1.1.1b

For this case, stresses will be computed using the gross section properties. In order to maximize the stresses, the appropriate load factors for this case for this particular example are as follows: the minimum load factor is used for the DC<sub>1</sub> dead load effects (noncomposite) since the DC<sub>1</sub> load effects are opposite in sign to the other load effects, and the maximum load factor is used for the future wearing surface composite dead load and live load effects. The composite dead load and future wearing surface act on the 3n-composite transformed section, and the live load acts on the n-composite transformed section.

Case 2: Dead Load + Negative Live Load

For this case, stresses will be computed using the gross section properties. The future wearing surface is conservatively excluded and the maximum load factor is

used for the DC (noncomposite) dead load effects and the live load effects. The minimum load factor is used for the composite dead load effects because they are opposite in sign to the other load effects. The live load acts on the section consisting of the steel girder plus the longitudinal reinforcement. The composite dead load is applied to this section as well, as a conservative assumption for simplicity and convenience, since the net effect of the live load is to induce tension in the slab. The reinforcing steel in the deck that is used corresponds to the negative moment deck reinforcement shown in Figure 4-2.

*S6.13.6.1.4a*

*Service Limit State:*

Case 1: Dead Load + Positive Live Load

For this case, stresses will be computed using the gross section properties. The future wearing surface is included and acts, along with the composite dead load, on the 3n-composite transformed section. The live load acts on the n-composite transformed section.

*S6.10.1.1.1c*

Case 2: Dead Load + Negative Live Load

For this case, stresses will be computed using the gross section properties. The future wearing surface is excluded. The composite dead load acts on the 3n-composite transformed section, and the live load acts on the n-composite transformed section since separate calculations (not shown) indicate that the longitudinal tensile deck stress due to the Service II loads at the splice is smaller than  $2f_r$  (*S6.13.6.1.4a* and *S6.10.4.2.1*).

*Fatigue Limit State:*

Case 1: Positive Live Load

For this case, stresses will be computed using the gross section properties. The live load acts on the n-composite transformed section.

*S6.13.6.1.4c*

Case 2: Negative Live Load

*S6.6.1.2.1*

For this case, stresses will be computed using the gross section properties. The live load acts on the n-composite transformed section.

Section Properties:

The transformed effective area of the concrete flange of the steel girder is now determined. This requires the modular ratio as follows:

$$A_c = \frac{\text{Effective Slab Width}}{\text{Modular Ratio}} \cdot t_{seff}$$

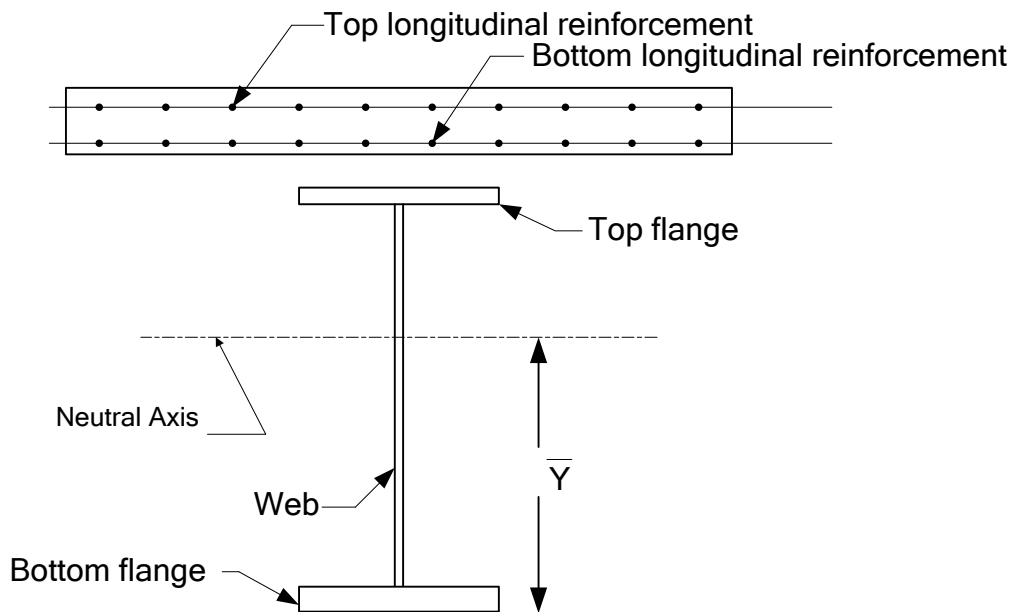
For the n-composite beam:

$$A_c = \frac{W_{eff}}{n} \cdot t_{seff} = 117.00in^2$$

For the 3n-composite beam:

$$A_{c3n} = \frac{W_{eff}}{3n} \cdot t_{seff} = 39.00in^2$$

The section properties for the Left Girder are calculated with the aid of Figure 4-3 shown below:



**Figure 4-3 Girder, Slab and Longitudinal Reinforcement**

The following tables contain the section properties for the left (i.e., smaller) girder section at the splice location. The properties in Table 4-2 are based on the gross area of the steel girder, and these properties are used for computation of stresses for the Strength I, Service II and Fatigue load combinations.

Gross Section Properties						
Section	Area, A (Inches <sup>2</sup> )	Centroid, d (Inches)	A*d (Inches <sup>3</sup> )	I <sub>o</sub> (Inches <sup>4</sup> )	A*y <sup>2</sup> (Inches <sup>4</sup> )	I <sub>total</sub> (Inches <sup>4</sup> )
Girder only:						
Top flange	10.500	55.250	580.1	0.5	8441.1	8441.6
Web	27.000	27.875	752.6	6561.0	25.8	6586.8
Bottom flange	12.250	0.438	5.4	0.8	8576.1	8576.9
Total	49.750	26.897	1338.1	6562.3	17043.0	23605.3
Deck Steel:						
Girder	49.750	26.897	1338.1	23605.3	3102.8	26708.0
Top Steel	7.254	63.438	460.2	0.0	5951.6	5951.6
Bottom Steel	7.254	60.313	437.5	0.0	4723.8	4723.8
Total	64.258	34.794	2235.8	23605.3	13778.1	37383.4
Composite (3n):						
Girder	49.750	26.897	1338.1	23605.3	12092.4	35697.6
Slab	39.000	62.375	2432.6	208.0	15425.5	15633.5
Total	88.750	42.487	3770.7	23813.3	27517.9	51331.2
Composite (n):						
Girder	49.750	26.897	1338.1	23605.3	30829.0	54434.3
Slab	117.000	62.375	7297.9	624.0	13108.9	13732.9
Total	166.750	51.790	8636.0	24229.3	43937.9	68167.2
Section	y <sub>botmid</sub> (Inches)	y <sub>topmid</sub> (Inches)	S <sub>botweb</sub> (Inches <sup>3</sup> )	S <sub>botmid</sub> (Inches <sup>3</sup> )	S <sub>stopmid</sub> (Inches <sup>3</sup> )	S <sub>stopweb</sub> (Inches <sup>3</sup> )
Girder only	26.459	28.353	907.1	892.1	832.5	843.7
Deck Steel	34.356	20.456	1102.1	1088.1	1827.5	1861.6
Composite (3n)	42.050	12.763	1233.6	1220.7	4021.9	4143.7
Composite (n)	51.353	3.460	1338.8	1327.4	19701.6	22096.4

**Table 4-2 Section Properties****Strength I Stresses - Dead Load + Positive Live Load:**

The section properties for this case have been calculated in Table 4-2. The stresses at the midthickness of the flanges are shown in Table 4-3, which immediately follows the sample calculation presented below.

A typical computation for the stresses occurring at the midthickness of the flanges is presented in the example below. The stress in the bottom flange of the girder is computed using the 3n-composite section for the composite dead load and future wearing surface, and the n-composite section for the live load:

$$f = \frac{M}{S}$$



Noncomposite DL:

Stress at the midthickness:

$$f = f_{botgdr_1}$$

Noncomposite DL moment:

$$M_{NDL} = -51.8K \cdot ft$$

Section modulus (girder only), from Table 4-2:

$$S_{botgdr_1} = 892.1in^3$$

Stress due to the noncomposite dead load:

$$f_{botgdr_1} = \frac{M_{NDL}}{S_{botgdr_1}} = -0.70ksi$$

Composite DL:

Stress at the midthickness:

$$f = f_{botgdr_2}$$

Composite DL moment:

$$M_{CDL} = 15.5K \cdot ft$$

Section modulus (3n-composite), from Table 4-2:

$$S_{botgdr_2} = 1220.7in^3$$

Stress due to the composite dead load:

$$f_{botgdr_2} = \frac{M_{CDL}}{S_{botgdr_2}} = 0.15ksi$$

Future Wearing Surface:

Stress at the midthickness:

$$f = f_{botgdr_3}$$

FWS moment:

$$M_{FWS} = 18.8K \cdot ft$$

Section modulus (3n-composite), from Table 4-2:

$$S_{botgdr_2} = 1220.7in^3$$

Stress due to the composite dead load:

$$f_{botgdr_3} = \frac{M_{FWS}}{S_{botgdr_3}} = 0.18ksi$$

Positive Live Load:

Stress at the midthickness:

$$f = f_{botgdr_4}$$

Live load moment:

$$M_{PLL} = 1307.8K \cdot ft$$

Section modulus (n-composite), from Table 4-2:

$$S_{botgdr_4} = 1327.4in^3$$

Stress due to the positive live load:

$$f_{botgdr_4} = \frac{M_{PLL}}{S_{botgdr_4}} = 11.82ksi$$

The preceding stresses are now factored by their respective load factors for the Strength I load combination to obtain the final factored stress at the midthickness of the bottom flange for this load case. The applicable load factors for this case were discussed previously. (*Table 3.4.1-1* and *Table 3.4.1-2*)

$$f_{botgdr} = (0.90 \cdot f_{botgdr_1} + 1.25 \cdot f_{botgdr_2} + 1.50 \cdot f_{botgdr_3} + 1.75 \cdot f_{botgdr_4}) f_{botgdr} = 20.53ksi$$

The stresses at the midthickness of the top flange for this load case are computed in a similar manner. The section properties used to obtain the stresses in the top flange are also from Table 4-2.

The top and bottom flange midthickness stresses are summarized in Table 4-3, shown below.

Strength I - Dead Load + Positive Live Load			
Summary of Unfactored Values			
Loading	Moment (K-ft)	$f_{botmid}$ (ksi)	$f_{topmid}$ (ksi)
Noncomposite DL	-51.80	-0.70	0.75
Composite DL	15.50	0.15	-0.05
FWS DL	18.80	0.18	-0.06
Live Load - HL-93	1307.80	11.82	-0.80
Summary of Factored Values			
Limit State			
Strength I	2289.61	20.53	-0.86

**Table 4-3 Strength I Flange Moments & Stresses for Dead + Pos. LL**

The computation of the midthickness flange stresses for the remaining load cases are computed in a manner similar to what was shown in the sample calculation that preceded Table 4-3.

Strength I - Dead Load + Negative Live Load:

The computed stresses in the following table require the use of section properties from Table 4-2.

Strength I - Dead Load + Negative Live Load			
Summary of Unfactored Values			
Loading	Moment (K-ft)	$f_{botmid}$ (ksi)	$f_{topmid}$ (ksi)
Noncomposite DL	-51.80	-0.70	0.75
Composite DL	15.50	0.17	-0.10
Live Load - HL-93	-953.30	-10.51	6.26
Summary of Factored Values			
Limit State			
Strength I	-1719.08	-19.12	11.80

**Table 4-4 Strength I Flange Moments & Stresses for Dead + Neg. LL**

Service II - Dead Load + Positive Live Load:

The computed stresses in the following table require the use of section properties from Table 4-2.

Service II - Dead Load + Positive Live Load			
Summary of Unfactored Values			
Loading	Moment (K-ft)	f <sub>botmid</sub> (ksi)	f <sub>topmid</sub> (ksi)
Noncomposite DL	-51.80	-0.70	0.75
Composite DL	15.50	0.15	-0.05
FWS	18.80	0.18	-0.06
Live Load - HL-93	1307.80	11.82	-0.80
Summary of Factored Values			
Limit State			
Service II	1682.64	15.01	-0.39

**Table 4-5 Service II Flange Moments & Stresses for Dead + Pos. LL**

Service II - Dead Load + Negative Live Load:

The computed stresses in the following table require the use of section properties from Table 4-2.

Service II - Dead Load + Negative Live Load			
Summary of Unfactored Values			
Loading	Moment (K-ft)	f <sub>botmid</sub> (ksi)	f <sub>topmid</sub> (ksi)
Noncomposite DL	-51.80	-0.70	0.75
Composite DL	15.50	0.15	-0.05
Live Load - HL-93	-953.30	-8.62	0.58
Summary of Factored Values			
Limit State			
Service II	-1275.59	-11.75	1.46

**Table 4-6 Service II Flange Moments & Stresses for Dead + Neg. LL**

Fatigue - Positive Live Load:

The computed stresses in the following table require the use of section properties from Table 4-2.

Fatigue - Positive Live Load			
Summary of Unfactored Values			
Loading	Moment (K-ft)	f <sub>botmid</sub> (ksi)	f <sub>topmid</sub> (ksi)
Live Load-Fatigue	394.30	3.56	-0.24
Summary of Factored Values			
Limit State			
Fatigue I	591.45	5.35	-0.36
Fatigue II	295.73	2.67	-0.18

**Table 4-7 Fatigue Flange Moments & Stresses for Positive LL**

Fatigue - Negative Live Load:

The computed stresses in the following table require the use of section properties from Table 4-2.

Fatigue - Negative Live Load			
Summary of Unfactored Values			
Loading	Moment (K-ft)	f <sub>botmid</sub> (ksi)	f <sub>topmid</sub> (ksi)
Live Load-Fatigue	-284.00	-2.57	0.17
Summary of Factored Values			
Limit State			
Fatigue I	-426.00	-3.85	0.26
Fatigue II	-213.00	-1.93	0.13

**Table 4-8 Fatigue Flange Moments & Stresses for Negative LL**

Fatigue:

The computed stresses in the following table require the use of section properties from Table 4-2.

Fatigue - Live Load			
Summary of Unfactored Values			
Loading	Moment (K-ft)	f <sub>botweb</sub> (ksi)	f <sub>topweb</sub> (ksi)
Live Load-Pos	394.3	3.53	-0.21
Live Load-Neg	-284.00	-2.55	0.15
Summary of Factored Values			
Limit State			
Pos Fatigue I	591.45	5.30	-0.32
Neg Fatigue I	-426.00	-3.82	0.23
Pos Fatigue II	295.73	2.65	-0.16
Neg Fatigue II	-213.00	-1.91	0.12

**Table 4-9 Fatigue Web Moments & Stresses for Positive and Negative Live Load**

A summary of the factored stresses at the midthickness of the top and bottom flanges for the Strength I, Service II, and Fatigue load combinations are presented below in Tables 4-10 through 4-12. Table 4-12 also contains the top and bottom web fatigue stresses.

Limit State	Location	Stress (ksi)	
		Dead + Pos. LL	Dead + Neg. LL
Strength I	Bottom Flange	20.53	-19.12
	Top Flange	-0.86	11.80

**Table 4-10 Strength I Flange Stresses**

Limit State	Location	Stress (ksi)	
		Dead + Pos. LL	Dead + Neg. LL
Service II	Bottom Flange	15.01	-11.75
	Top Flange	-0.39	1.46

**Table 4-11 Service II Flange Stresses**

Limit State	Location	Stress (ksi)	
		Positive LL	Negative LL
Fatigue I	Bottom Flange	5.35	-3.85
	Top Flange	-0.36	0.26
	Bottom of Web	5.30	-3.82
	Top of Web	-0.32	0.23
Fatigue II	Bottom Flange	2.67	-1.93
	Top Flange	-0.18	0.13
	Bottom of Web	2.65	-1.91
	Top of Web	-0.16	0.12

**Table 4-12 Fatigue Flange and Web Stresses**

Strength Limit State Minimum Design Force - Controlling Flange:

S6.13.6.1.4c

The next step is to determine the controlling flange of each load case (i.e., positive and negative live load). By inspection of Table 4-10, the bottom flange is the controlling flange for both positive and negative live load for the strength limit state.

S6.13.6.1.4c

The minimum design force for the controlling flange,  $P_{cu}$ , is taken equal to the design stress,  $F_{cf}$ , times the effective flange area,  $A_e$ , of the controlling flange. When the flange is in compression, the effective area of the flange is to be taken as  $A_e = A_g$ . When the flange is in tension, the effective area of the flange is to be taken as:

$$A_e = \left( \frac{\phi_u \cdot F_u}{\phi_y \cdot F_{yt}} \right) \cdot A_n \leq A_g \quad \text{SEquation 6.13.6.1.4c-2}$$

The gross area of the bottom flange of the steel girder is computed as follows:

$$A_g = t_{fbL} \cdot b_{fbL} = 12.25 \text{in}^2$$

S6.8.3

The net area of the bottom flange of the steel girder is defined as the product of the thickness of the flange and the smallest net width. The net width is determined by subtracting from the width of the flange the sum of the widths of all holes in the assumed failure chain, and then adding the quantity  $s^2/4g$  for each space between consecutive holes in the chain. Since the bolt holes in the flanges are lined up transverse to the loading direction (i.e. are not staggered), the governing failure chain is straight across the flange (i.e.,  $s^2/4g$  is equal to zero).

The net area of the bottom flange of the steel girder now follows:

$$A_n = (b_{fbL} - 4 \cdot d_{hole}) \cdot t_{fbL} = 8.97 \text{ in}^2$$

With the gross and net areas identified, the effective tension area of the bottom flange can now be computed as follows:

$$F_u = 65 \text{ ksi}$$

$$F_{yt} = 50 \text{ ksi}$$

$$A_e = \left( \frac{\phi_u \cdot F_u}{\phi_y \cdot F_{yt}} \right) \cdot A_n = 9.82 \text{ in}^2 \quad \text{SEquation 6.13.6.1.4c-2}$$

Check:

$$A_e = 9.82 \text{ in}^2 < A_g = 12.25 \text{ in}^2 \quad \text{OK}$$

Effective bottom flange area:

$$A_{e\text{bot}} = 9.82 \text{ in}^2$$

The calculation of the strength limit state design force is presented below for the load case of dead load with positive live load.

The strength limit state design stress for the controlling (bottom) flange is computed as follows:

$$F_{cf} = \frac{\left| \frac{f_{cf}}{R_h} \right| + \alpha \cdot \phi_f \cdot F_{yf} \cdot R_g}{2} \geq 0.75 \cdot \alpha \cdot \phi_f \cdot F_{yf} \cdot R_g \quad \text{SEquation 6.13.6.1.4c-1}$$

where:

Maximum flexural stress due to the factored loads at the midthickness of the controlling flange at the point of splice (from Table 4-10):

$$f_{cf} = 20.53 \text{ ksi}$$

Hybrid girder reduction factor. For homogeneous girders:

$$R_h = 1.0 \quad \text{S6.10.1.10.1}$$



Flange stress reduction factor:

$$\alpha = 1.0 \quad (\text{for I-girders})$$

Flange resistance modification factor:

$$R_g = \frac{|\alpha \cdot A_e \cdot F_{yf}|_{LS}}{|\alpha \cdot A_e \cdot F_{yf}|_{SS}} \leq 1.0$$

where:

$|\alpha \cdot A_e \cdot F_{yf}|_{LS}$  is the product of the effective area,  $A_e$ , times  $\alpha \cdot F_{yf}$  for the flange under consideration in the larger section at the point of splice.

$$|A_n|_{LS} = (b_{flbL} - 4 \cdot d_{hole}) \cdot t_{flbL} = 14.09 \text{ in}^2$$

$$|A_e|_{LS} = \left( \frac{\phi_u \cdot F_u}{\phi_y \cdot F_{yt}} \right) \cdot A_n = 15.42 \text{ in}^2$$

$$|A_e|_{LS} = 15.42 \text{ in}^2 < |A_g|_{LS} = 19.25 \text{ in}^2 \quad \text{OK}$$

$$|A_{ebot}|_{LS} = 15.42 \text{ in}^2$$

$$|\alpha \cdot A_e \cdot F_{yf}|_{LS} = 1.0 \cdot 15.42 \cdot 50 = 771$$

$|\alpha \cdot A_e \cdot F_{yf}|_{SS}$  is the product of the effective area,  $A_e$ , times  $\alpha \cdot F_{yf}$  for the flange under consideration in the smaller section at the point of splice.

$$|\alpha \cdot A_e \cdot F_{yf}|_{SS} = 1.0 \cdot 9.87 \cdot 50 = 493.5$$

$$R_g = \frac{|\alpha \cdot A_e \cdot F_{yf}|_{LS}}{|\alpha \cdot A_e \cdot F_{yf}|_{SS}} = \frac{771}{493.5} = 1.56 > 1.0 \quad \therefore R_g = 1.0$$

Resistance factor for flexure (Design Step 4.1):

$$\phi_f = 1.0$$

Minimum yield strength of the flange:

$$F_{yf} = 50 \text{ ksi}$$

$$F_{cf_1} = \frac{\left| \frac{f_{cf}}{R_h} \right| + \alpha \cdot \phi_f \cdot F_{yf} \cdot R_g}{2} = 35.27 \text{ksi}$$

Compute the minimum required design stress:

$$F_{cf_2} = 0.75 \cdot \alpha \cdot \phi_f \cdot F_{yf} \cdot R_g = 37.50 \text{ksi}$$

The controlling design stress for the bottom flange for this load case is:

$$F_{cf} = \max(F_{cf_1}, F_{cf_2}) = 37.50 \text{ksi}$$

The design force now follows:

$$P_{cu} = F_{cf} \cdot A_e$$

Since the bottom flange force for this load case is a tensile force, the effective area of the controlling bottom flange of the Left Girder will be used. This value was computed previously to be:

$$A_{ebot} = 9.82 \text{in}^2$$

Therefore:

$$P_{cu} = F_{cf} \cdot A_{ebot} = 368.25 \text{K}$$

Table 4-13 presents the minimum strength limit state design forces for the controlling flange for both the positive and negative live load cases.

		Strength Limit State			
		Controlling Flange			
Load Case	Location	$f_{cf}$ (ksi)	$F_{cf}$ (ksi)	Area (in <sup>2</sup> )	$P_{cu}$ (kips)
Dead + Pos. LL	Bot. Flange	20.53	37.5	9.82	368.25
Dead + Neg. LL	Bot. Flange	-19.12	-37.5	12.25	-459.38

**Table 4-13 Strength Limit State Controlling Flange Stresses and Forces**

In the above table, the design controlling flange force ( $P_{cu}$ ) is a compressive force for negative live load.

Strength Limit State Minimum Design Force - Noncontrolling Flange:

S6.13.6.1.4c

The next step is to determine the minimum design forces for the noncontrolling flange of each load case (i.e., positive and negative live load). By inspection of Table 4-10, the top flange is the noncontrolling flange for both positive and negative live load for the strength limit state.

The minimum design force for the noncontrolling flange,  $P_{ncu}$ , is taken equal to the design stress,  $F_{ncf}$ , times the effective flange area,  $A_e$ , of the noncontrolling flange. When the flange is in compression, the effective area of the flange is to be taken as  $A_e = A_g$ . When the flange is in tension, the effective flange area is calculated as illustrated previously.

The calculation of the strength limit state minimum design force is presented below for the load case of dead load with positive live load.

The strength limit state minimum design stress for the noncontrolling (top) flange is computed as follows:

$$F_{ncf} = R_{cf} \cdot \left| \frac{f_{ncf}}{R_h} \right| \geq 0.75 \cdot \alpha \cdot \phi_f \cdot F_{yf} \cdot R_g \quad \text{SEquation 6.13.6.1.4c-3}$$

where:

Maximum flexural stress due to the factored loads at the midthickness of the noncontrolling flange at the point of splice concurrent with  $f_{cf}$  (see Table 4-10):

$$f_{ncf} = -0.86 \text{ksi}$$

Controlling flange design stress:

$$F_{cf} = 37.50 \text{ksi}$$

Controlling flange actual stress:

$$f_{cf} = 20.53 \text{ksi}$$

Controlling flange stress ratio:

$$R_{cf} = \left| \frac{F_{cf}}{f_{cf}} \right| = 1.83$$

Hybrid girder reduction factor:

$$R_h = 1.00$$

Flange Resistance Modification Factor:

$$R_g = \frac{|\alpha \cdot A_e \cdot F_{yf}|_{LS}}{|\alpha \cdot A_e \cdot F_{yf}|_{SS}} \leq 1.0$$

For the positive live load case, the top flange is in compression. The effective compression flange area for the larger flange at the point of the splice shall be taken as:

$$(A_e)_{LS} = (A_g)_{LS} = (b_{flR} \cdot t_{flR})_{LS} = 17.50 \text{ in}^2$$

The effective compression flange area for the smaller flange at the point of the splice is taken as:

$$(A_e)_{SS} = (A_g)_{SS} = (b_{flR} \cdot t_{flR})_{SS} = 10.50 \text{ in}^2$$

$$R_g = \frac{|\alpha \cdot A_e \cdot F_{yf}|_{LS}}{|\alpha \cdot A_e \cdot F_{yf}|_{SS}} = \frac{|1.0 \cdot 17.50 \cdot 50|}{|1.0 \cdot 10.50 \cdot 50|} = 1.67 > 1.0 \quad \therefore R_g = 1.0$$

Therefore:

$$F_{ncf_1} = R_{cf} \cdot \left| \frac{f_{ncf}}{R_h} \right| = 1.57 \text{ ksi}$$

Compute the minimum required design stress:

$$F_{ncf_2} = 0.75 \cdot \alpha \cdot \phi_f \cdot F_{yf} \cdot R_g = 37.50 \text{ ksi}$$

The minimum design stress in the top flange is:

$$F_{ncf} = \max(F_{ncf_1}, F_{ncf_2}) = 37.50 \text{ ksi}$$

The minimum design force now follows:

$$P_{ncu} = F_{ncf} \cdot A_e$$

For the positive live load case, the top flange is in compression. The effective compression flange area is taken as:

$$A_e = A_g = b_{flL} \cdot t_{flL} = 10.50 \text{ in}^2$$

Therefore:

$$P_{ncu} = F_{ncf} \cdot A_g = 393.75K \text{ (compression)}$$

Table 4-14 presents the strength limit state minimum design forces for the noncontrolling flange for both the positive and negative live load cases.

		Strength Limit State Noncontrolling Flange			
Load Case	Location	$f_{ncf}$ (ksi)	$F_{ncf}$ (ksi)	Area (in <sup>2</sup> )	$P_{ncu}$ (kips)
Dead + Pos. LL	Top Flange	-0.86	37.5	10.5	-393.75
Dead + Neg. LL	Top Flange	11.80	37.5	7.69	288.38

**Table 4-14 Strength Limit State Noncontrolling Flange Stresses & Forces**

In the above table, the design noncontrolling flange force ( $P_{ncu}$ ) is a tensile force for negative live load.

Service II Limit State Flange Forces:

S6.13.6.1.4c

Per the Specifications, bolted connections for flange splices are to be designed as slip-critical connections for the service level flange design force. This design force for the flange under consideration is to be taken as the Service II design stress,  $F_s$ , multiplied by the gross area of that flange in the smaller section at the point of the splice.

$F_s$  is defined as follows:

$$F_s = \frac{f_s}{R_h} \quad \text{SEquation 6.13.6.1.4c-5}$$

$f_s$  = maximum flexural Service II stress at the midthickness of the flange under consideration.

The factored Service II design stresses and forces are shown in Table 4-15 below.

		Service II Limit State		
Load Case	Location	$F_s$ (ksi)	$A_{gross}$ (in <sup>2</sup> )	$P_s$ (kips)
Dead + Pos. LL	Bot. Flange	15.01	12.25	183.87
	Top Flange	-0.39	10.50	-4.10
Dead + Neg. LL	Bot. Flange	-11.75	12.25	-143.94
	Top Flange	1.46	10.50	15.33

**Table 4-15 Service II Flange Stresses & Forces**

## S6.13.6.1.4a

It is important to note here that the flange slip resistance must exceed the larger of: (1) the Service II flange forces or (2) the factored flange forces from the moments at the splice due to constructibility (erection and/or deck pouring sequence). However, in this design example, no special erection procedure is prescribed and, per the Introduction in Design Step 1, the deck is assumed placed in a single pour in this example. Therefore, the constructibility moment is equal to the noncomposite dead load moment shown at the beginning of this design step. By inspection, the Service II load combination will control for checking of slip-critical connections for the flanges and the web in this example.

Fatigue Stresses:

## C6.13.6.1.4a

The final portion of this design step is to determine the range of the stresses at the midthickness of both flanges, and at the top and bottom of the web for the Fatigue load combinations. The ranges are calculated below and presented in Table 4-16.

A typical calculation of the stress range for the bottom flange for the Fatigue II load combination is shown below.

From Tables 4-7 and 4-8, the factored stresses at the midthickness of the bottom flange are:

*Case 1 - Positive Live Load:*

$$f_{spos} = 2.67ksi$$

*Case 2 - Negative Live Load:*

$$f_{sneg} = -1.93ksi$$

The stress range is determined as:

$$\Delta f = |f_{spos}| + |f_{sneg}| = 4.60ksi$$

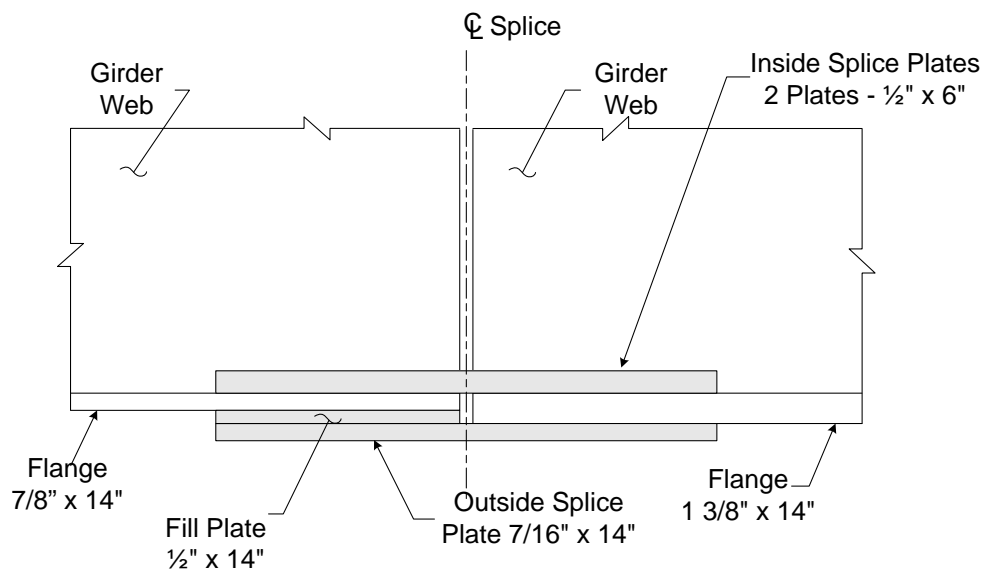
Load Case	Location	Fatigue I	Fatigue II
		Stress Range (ksi)	Stress Range (ksi)
Dead + Pos. LL	Bottom Flange	9.20	4.60
	Top Flange	0.62	0.31
Dead + Neg. LL	Bottom of Web	9.12	4.56
	Top of Web	0.55	0.28

**Table 4-16 Fatigue Stress Ranges**

### Design Step 4.4 - Design Bottom Flange Splice

#### Splice Plate Dimensions:

The width of the outside plate should be at least as wide as the width of the narrowest flange at the splice. Therefore, try a 7/16" x 14" outside splice plate with two 1/2" x 6" inside splice plates. Include a 1/2" x 14" fill plate on the outside. Figure 4-4 illustrates the initial bottom flange splice configuration.



**Figure 4-4 Bottom Flange Splice**

The dimensions of the elements involved in the bottom flange splice from Figure 4-4 are:

Thickness of the inside splice plate:

$$t_{in} = 0.50in$$

Width of the inside splice plate:

$$b_{in} = 6in$$

Thickness of the outside splice plate:

$$t_{out} = 0.4375in$$

Width of the outside splice plate:

$$b_{out} = 14in$$

Thickness of the fill plate:

$$t_{fill} = 0.50in$$

Width of the fill plate:

$$b_{fill} = 14in$$

C6.13.6.1.4c

If the combined area of the inside splice plates is within ten percent of the area of the outside splice plate, then both the inside and outside splice plates may be designed for one-half the flange design force.

Gross area of the inside and outside splice plates:

Inside:

$$A_{gross\_in} = 2 \cdot t_{in} \cdot b_{in} = 6.00in^2$$

Outside:

$$A_{gross\_out} = t_{out} \cdot b_{out} = 6.12in^2$$

Check:

$$\left( 1 - \frac{A_{gross\_in}}{A_{gross\_out}} \right) \cdot 100\% = 2.04\%$$

The combined areas are within ten percent.

S6.13.6.1.4c



If the areas of the inside and outside splice plates had differed by more than ten percent, the flange design force would be proportioned to the inside and outside splice plates. This is calculated by multiplying the flange design force by the ratio of the area of the splice plate under consideration to the total area of the inner and outer splice plates.

### Yielding and Fracture of Splice Plates:

For a detailed discussion on the resistance of connected elements, see RM Section 6.6.4.2.5.6.

S6.13.5.2

#### *Case 1 - Tension:*

At the strength limit state, the design force in the splice plates subjected to tension is not to exceed the factored resistances for yielding, fracture, and block shear.

From Table 4-13, the critical strength limit state bottom flange tension design force is:

$$P_{cu} = 368.25K$$

The factored tensile resistance for yielding on the gross section is:

$$P_r = \phi_y \cdot P_{ny} = \phi_y \cdot F_y \cdot A_g \quad \text{SEquation 6.8.2.1-1}$$

$$F_y = 50ksi \quad (\text{Design Step 4.1})$$

$$\phi_y = 0.95 \quad (\text{Design Step 4.1})$$

For yielding of the outside splice plate:

$$A_g = A_{gross\_out}$$

$$P_r = \phi_y \cdot F_y \cdot A_g = 290.94K$$

The outside splice plate takes half of the design load:

$$\frac{P_{cu}}{2} = 184.13K$$

$$P_r = 290.94K > \frac{P_{cu}}{2} = 184.13K \quad \text{OK}$$

For yielding of the inside splice plates:

$$A_g = A_{gross\_in}$$

$$P_r = \phi_y \cdot F_y \cdot A_g = 285.00K$$

The inside splice plate takes half of the design load:

$$\frac{P_{cu}}{2} = 184.13K$$

$$P_r = 285.00K > \frac{P_{cu}}{2} = 184.13K \quad \text{OK}$$

The factored tensile resistance for fracture on the net section is:

$$P_r = \phi_u \cdot P_{nu} = \phi_u \cdot F_u \cdot A_n \cdot R_p \cdot U \quad \text{SEquation 6.5.2.1-2}$$

$$F_u = 65ksi \quad (\text{Design Step 4.1})$$

$$\phi_u = 0.80 \quad (\text{Design Step 4.1})$$

$$U = 1.0 \quad \text{S6.13.5.2}$$

$$R_p = 1.0 \quad (\text{for drilled holes})$$

To compute the net area of the splice plates, assume four 7/8" bolts across the width of the splice plate.

S6.8.3

The net width is to be determined for each chain of holes extending across the member along any transverse, diagonal or zigzag line. This is determined by subtracting from the width of the element the sum of the width of all holes in the chain and adding the quantity  $s^2/4g$  for each space between consecutive holes in the chain. For non-staggered holes, such as in this design example, the minimum net width is the width of the element minus the number of bolt holes in a line straight across the width. For a detailed discussion on the spacing and layout of bolt holes, refer to RM Section 6.6.4.2.2.2.

For fracture of the outside splice plate:

The net width is:

$$b_{n\_out} = b_{out} - 4 \cdot d_{hole}$$

$$d_{hole} = 0.9375in \quad (\text{Design Step 4.1})$$

$$b_{n\_out} = 10.25in$$

The nominal area is determined to be:

$$A_{n\_out} = b_{n\_out} \cdot t_{out} = 4.48in^2$$

S6.13.5.2

The net area of the connecting element is limited to 0.85 A<sub>g</sub>:

$$A_n \leq 0.85A_g$$

$$A_{gross\_out} = 6.12in^2$$

$$A_{n\_out} = 4.48in^2 < 0.85 \cdot A_{gross\_out} = 5.21in^2 \quad \text{OK}$$

$$P_r = \phi_u \cdot F_u \cdot A_{n\_out} \cdot R_p \cdot U = 232.96K$$

The outside splice plate takes half of the design flange force:

$$P_r = 232.96K > \frac{P_{cu}}{2} = 184.13K \quad \text{OK}$$

For fracture of the inside splice plates:

The net width is:

$$b_{n\_in} = b_{in} - 2 \cdot d_{hole} = 4.125in$$

The nominal area is determined to be:

$$A_{n\_in} = 2(b_{n\_in} \cdot t_{in}) = 4.125in^2$$

S6.13.5.2

The net area of the connecting element is limited to 0.85 A<sub>g</sub>:

$$A_n \leq 0.85 \cdot A_g$$

$$A_{gross\_in} = 6.00in^2$$

$$A_{n\_in} = 4.125in^2 < 0.85 \cdot A_{gross\_in} = 5.10in^2 \quad \text{OK}$$

$$P_r = \phi_u \cdot F_u \cdot A_{n\_in} \cdot R_p \cdot U = 214.50K$$

The inside splice plate takes half of the design flange force:

$$P_r = 214.50K > \frac{P_{cu}}{2} = 184.13K \quad \text{OK}$$

S6.13.6.1.4c

*Case 2 - Compression:*

From Table 4-13, the strength limit state bottom flange compression design force is:

$$P_{cu} = 459.38K$$

This force is distributed equally to the inside and outside splice plates.

The factored resistance of the splice plate is:

$$R_r = \phi_c \cdot F_y \cdot A_s \quad \text{SEquation 6.13.6.1.4c-4}$$

$$\phi_c = 0.95 \quad (\text{Design Step 4.1})$$

For yielding of the outside splice plate:

$$A_s = A_{gross\_out}$$

$$R_{r\_out} = \phi_c \cdot F_y \cdot A_s = 290.94K$$

$$R_{r\_out} = 290.94K > \frac{P_{cu}}{2} = 229.69K \quad \text{OK}$$

For yielding of the inside splice plates:

$$A_s = A_{gross\_in}$$

$$R_{r\_in} = \phi_c \cdot F_y \cdot A_s = 285.00K$$

$$R_{r\_out} = 285.00K > \frac{P_{cu}}{2} = 229.69K \quad \text{OK}$$

Block Shear:

S6.13.6.1.4c, S6.13.5.2, S6.13.4

All tension connections, including connection plates, splice plates and gusset plates, are to be investigated to ensure that adequate connection material is provided to develop

the factored resistance of the connection. Block shear rupture will usually not govern the design of splice plates of typical proportion. However, the block shear checks are carried out here for completeness. For a detailed discussion of block shear, refer to RM Section 6.6.4.2.5.6.

From Table 4-13, the strength limit state bottom flange tension design force is:

$$P_{cu} = 368.25K$$

The block shear resistance is given by:

$$R_r = \phi_{bs} \cdot (0.58 \cdot F_u \cdot A_{vn} + U_{bs} \cdot F_u \cdot A_m) \quad \text{SEquation 6.13.4-1}$$

$$< \phi_{bs} \cdot (0.58 \cdot F_y \cdot A_{vg} + U_{bs} \cdot F_u \cdot A_m)$$

where from Design Step 4.1:

Minimum yield strength of the connected material:

$$F_y = 50ksi$$

Minimum tensile strength of the connected material:

$$F_u = 65ksi$$

Resistance factor for block shear:

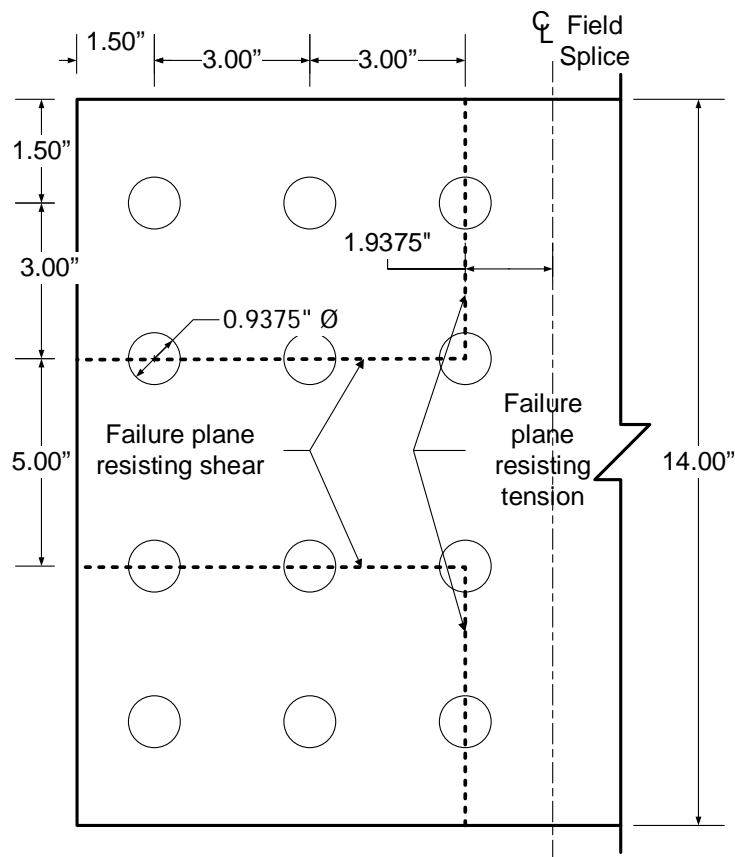
$$\phi_{bs} = 0.80$$

Reduction factor for block shear (for uniform tension stress):

$$U_{bs} = 1.0$$

*Outside Splice Plate:*

A bolt pattern must be assumed prior to checking an assumed block shear failure mode. An initial bolt pattern for the bottom flange splice, along with an assumed failure mode for the outside splice plate, is shown in Figure 4-5. The outside splice plate will now be checked for block shear.



**Figure 4-5 Outside Splice Plate – Block Shear Check**

Applying the factored resistance equations presented previously to the outside splice plate:

Gross area along the planes resisting shear stress:

$$A_{vg} = 2[2 \cdot (3.00in) + 1.50in] \cdot t_{out} = 6.56in^2$$

Net area along the planes resisting shear stress:

$$A_{vn} = 2[2 \cdot (3.00in) + 1.50in - 2.5 \cdot d_{hole}] \cdot t_{out} = 4.51in^2$$

Net area along the planes resisting tension stress:

$$A_m = 2[(3.00in + 1.50in) - 1.5 \cdot d_{hole}] \cdot t_{out} = 2.71in^2$$

Factored resistance:

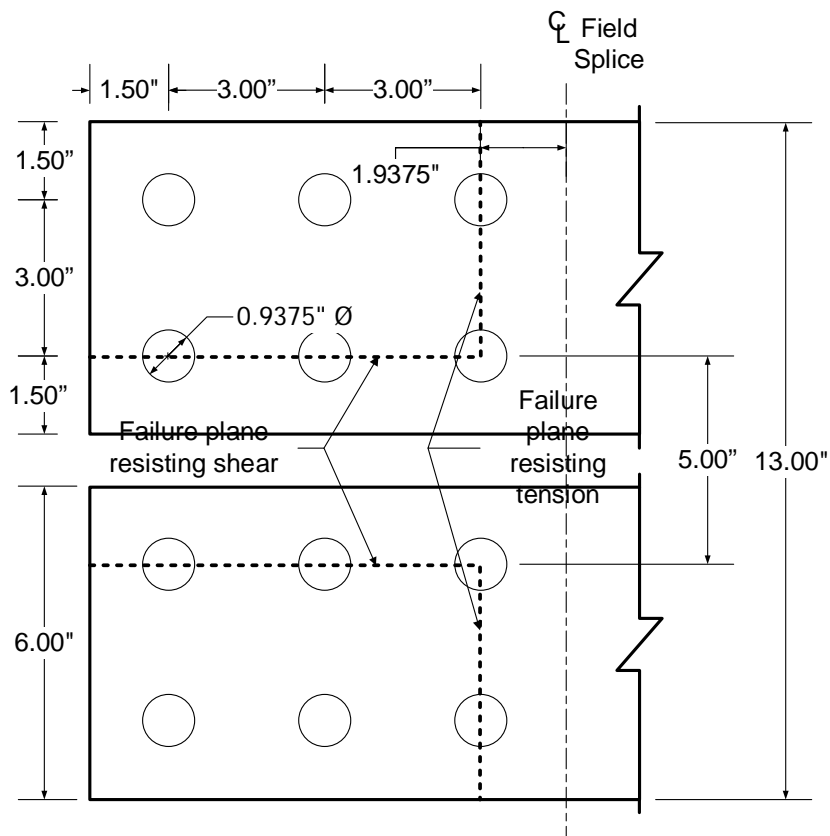
$$R_r = 0.80 \cdot (0.58 \cdot 65 \cdot 4.51 + 1.0 \cdot 65 \cdot 2.71) = 276.94K$$

$$< 0.80 \cdot (0.58 \cdot 50 \cdot 6.56 + 1.0 \cdot 65 \cdot 2.71) = 293.11K$$

$$\therefore R_r = 276.94K > \frac{368.25}{2} = 184.13K \quad \text{OK}$$

### Inside Splice Plates:

The inside splice plates will now be checked for block shear. See Figure 4-6 for the assumed failure mode:



**Figure 4-6 Inside Splice Plates - Block Shear Check**

Applying the factored resistance equations presented previously to the inside splice plates for the assumed failure mode:

Gross area along the planes resisting shear stress:

$$A_{vg} = 2[2 \cdot (3.00in) + 1.50in] \cdot t_{in} = 7.50in^2$$

Net area along the planes resisting shear stress:

$$A_{vn} = 2[2 \cdot (3.00in) + 1.50in - 2.5 \cdot d_{hole}] \cdot t_{in} = 5.16in^2$$

Net area along the planes resisting tension stress:

$$A_m = 2[(3.00in + 1.50in) - 1.5 \cdot d_{hole}] \cdot t_{in} = 3.09in^2$$

Factored resistance:

$$R_r = 0.80 \cdot (0.58 \cdot 65 \cdot 5.16 + 1.0 \cdot 65 \cdot 3.09) = 316.31K$$

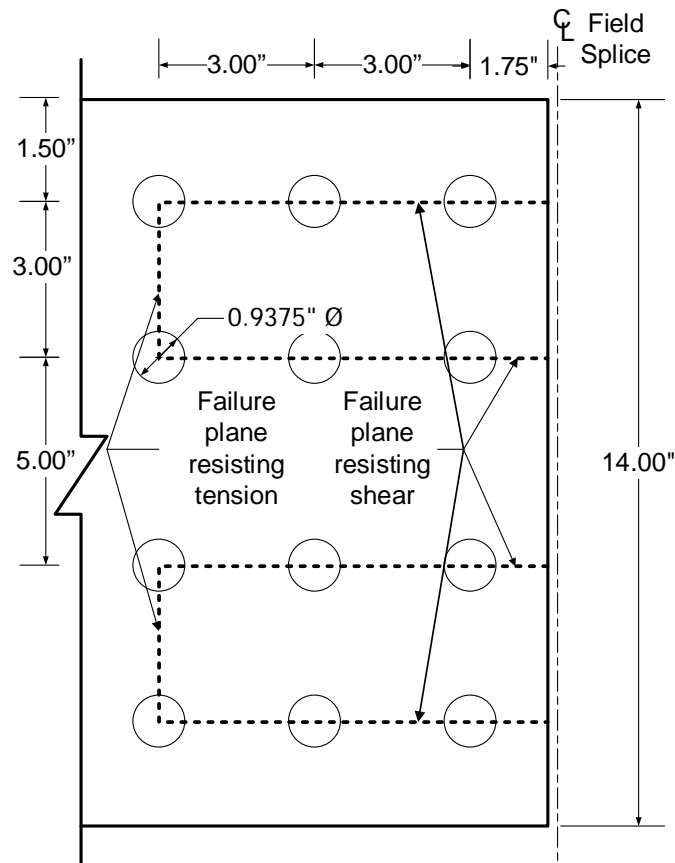
$$< 0.80 \cdot (0.58 \cdot 50 \cdot 7.50 + 1.0 \cdot 65 \cdot 3.09) = 334.68K$$

$$\therefore R_r = 316.31K > \frac{368.25}{2} = 184.13K \quad \text{OK}$$

*Girder Bottom Flange:*

The girder bottom flange will now be checked for block shear. For the flange, two different failure modes are possible. One failure mode is shown in Figure 4-7 and the accompanying computations are shown immediately following Figure 4-7. The second failure mode is similar to the failure mode investigated previously in this design example for the inside splice plates. Since these computations were previously shown for the inside splice plates, they will not be repeated here.





**Figure 4-7 Bottom Flange - Block Shear Check (Failure Mode 1)**

Applying the factored resistance equations presented previously to the bottom flange for the assumed failure mode:

Gross area along the planes resisting shear stress:

$$A_{vg} = 4[2 \cdot (3.00in) + 1.75in] \cdot t_{flbL} = 27.12in^2$$

Net area along the planes resisting shear stress:

$$A_{vn} = 4[2 \cdot (3.00in) + 1.75in - 2.5 \cdot d_{hole}] \cdot t_{flbL} = 18.92in^2$$

Net area along the planes resisting tension stress:

$$A_{tn} = 2[(3.00in) - 1.0 \cdot d_{hole}] \cdot t_{flbL} = 3.61in^2$$

Factored resistance:

$$R_r = 0.80 \cdot (0.58 \cdot 65 \cdot 18.92 + 1.0 \cdot 65 \cdot 3.61) = 758.35K$$

$$< 0.80 \cdot (0.58 \cdot 50 \cdot 27.12 + 1.0 \cdot 65 \cdot 3.61) = 816.90K$$

$$\therefore R_r = 758.35K > 368.25K \quad \text{OK}$$

It should be noted that although the block shear checks performed in this design example indicate an overdesign, the number of bolts cannot be reduced prior to checking shear on the bolts and bearing at the bolt holes. These checks are performed in what follows.

### Net Section Fracture

#### S6.10.1.8

When checking flexural members at the strength limit state or for constructibility, the following additional requirement is to be satisfied at all cross sections of the girder containing holes in the tension flange:

$$f_t \leq 0.84 \cdot \left( \frac{A_n}{A_g} \right) \cdot F_u \leq F_{yt} \quad \text{SEquation 6.10.1.8-1}$$

$$A_n = (b_{flbL} - 4 \cdot d_{hole}) \cdot t_{flbL} = 8.97in^2$$

$$A_g = t_{flbL} \cdot b_{flbL} = 12.25in^2$$

$$F_u = 65ksi$$

$$0.84 \cdot \left( \frac{A_n}{A_g} \right) \cdot F_u = 39.98ksi \quad F_{yt} = 50ksi$$

$$f_t = 20.70ksi$$

$$f_t \leq 0.84 \cdot \left( \frac{A_n}{A_g} \right) \cdot F_u \leq f_{yt} \quad \text{OK}$$

### Flange Bolts - Shear:

Determine the number of bolts for the bottom flange splice plates that are required to develop the strength limit state minimum design force in the flange in shear assuming the bolts in the connection have slipped and gone into bearing. A minimum of two rows of bolts should be provided to ensure proper alignment and stability of the girder during construction.

The critical strength limit state flange design force to be used in this check was previously computed (reference Table 4-13):

$$P_{uc} = 459.38K$$

The factored resistance of an ASTM A325 7/8" diameter high-strength bolt in shear must be determined, assuming the threads are excluded from the shear planes. For this case, the number of bolts required to provide adequate shear strength is determined by assuming the design force acts on two shear planes, known as double shear.

### Threads in the Shear Plane



If the ply thickness (i.e. splice plate thickness) closest to the nut is greater than or equal to 1/2-in. thick, the nominal shear resistance of the bolts should be determined assuming the threads are excluded from the shear planes for bolts less than 1 in. in diameter. For bolts greater than or equal to 1 in. in diameter, the nominal shear resistance of the bolts should be determined assuming the threads are excluded from the shear planes if the ply thickness closest to the nut is greater than 3/4-in. in thickness. Otherwise, the threads should be assumed included in the shear planes. The preceding assumes there is one washer under the turned element (i.e. the nut), and that there is no stick-out beyond the nut (which represents the worst case for this determination).

The nominal shear resistance is computed first as follows:

$$R_n = (0.48 \cdot A_b \cdot F_{ub} \cdot N_s) \quad \text{SEquation 6.13.2.7-1}$$

where:

Area of the bolt corresponding to the nominal diameter:

$$A_b = \frac{\pi}{4} \cdot d_{bolt}^2 = 0.60in^2$$

Specified minimum tensile strength of the bolt from Design Step 4.1:

$$F_{ub} = F_{ubolt} = 120ksi$$

Number of shear planes per bolt:

$$N_s = 2$$

$$R_n = 2(0.48 \cdot A_b \cdot F_{ub}) = 69.27K$$

The factored shear resistance now follows:

$$\phi_s = 0.80 \quad (\text{Design Step 4.1})$$

$$R_u = \phi_s \cdot R_n = 55.42K$$

### S6.13.6.1.5

When bolts carrying loads pass through fillers 0.25 inches or more in thickness in axially loaded connections, including girder flange splices, either:

The fillers are to be extended beyond the gusset or splice material and are to be secured by enough additional bolts to distribute the total stress in the member uniformly over the combined section of the member and the filler.

or

The fillers need not be extended and developed provided that the factored resistance of the bolts in shear at the strength limit state, specified in Article 6.13.2.2, is reduced by an appropriate factor:

In this design example, the reduction factor approach will be used. The reduction factor per the Specifications is:

$$R = \frac{(1 + \gamma)}{(1 + 2 \cdot \gamma)} \quad \text{SEquation 6.13.6.1.5-1}$$

where:

$$\gamma = \frac{A_f}{A_p}$$

Sum of the area of the fillers on the top and bottom of the connected plate:

$$A_f = b_{fill} \cdot t_{fill} = 7.00in^2$$

The smaller of either the connected plate area (i.e., girder flange) or the sum of the splice plate areas on the top and bottom of the connected plate determines  $A_p$ .

Bottom flange area:

$$b_{flbL} = 14in$$

$$t_{flbL} = 0.875in$$

$$A_{p1} = (b_{flbL})(t_{flbL}) = 12.25in^2$$

Sum of splice plate areas is equal to the sum of the gross areas of the inside and outside splice plates:

$$A_{gross\_in} = 6.00in^2$$

$$A_{gross\_out} = 6.12in^2$$

$$A_{p2} = A_{gross\_in} + A_{gross\_out} = 12.12in^2$$

The minimum of the areas is:

$$A_p = \min(A_{p1}, A_{p2}) = 12.12in^2$$

Therefore:

$$\gamma = \frac{A_f}{A_p} = 0.58$$

The reduction factor is determined to be:

$$R_{fill} = \frac{(1 + \gamma)}{(1 + 2 \cdot \gamma)} = 0.73$$

To determine the total number of bolts required for the bottom flange splice, divide the strength limit state minimum flange design force by the reduced allowable bolt shear strength:

$$R = R_u \cdot R_{fill} = 40.46K$$

The number of bolts required per side is:

$$N = \frac{P_{cu}}{R} = 11.3$$

The minimum number of bolts required on the side of the splice with the filler to resist the critical strength limit state minimum flange design force in shear is twelve. For practical reasons, use the same number of bolts on each side of the splice.

Flange Bolts - Slip Resistance:

S6.13.6.1.4c

Bolted connections for flange splices are to be designed as slip-critical connections for the Service II flange design force, or the flange design force from constructibility, whichever governs. In this design example, the Service II flange force controls (see previous discussion in Design Step 4.3).

C6.13.6.1.4c

When checking for slip of the bolted connection for a flange splice with inner and outer splice plates, the slip resistance should always be determined by dividing the flange design force equally to the two slip planes regardless of the ratio of the splice plate areas. Slip of the connection cannot occur unless slip occurs on both planes.

From Table 4-15, the critical Service II bottom flange design force is:

$$P_s = 183.87K \quad \text{SEquation 6.13.2.2-1}$$

The factored resistance for slip-critical connections is:

$$R_r = R_n = K_h \cdot K_s \cdot N_s \cdot P_t \quad \text{SEquation 6.13.2.8-1}$$

Determine the factored resistance per bolt assuming a Class B surface condition for the faying surface, standard holes (which are required per S6.13.6.1.4a) and two slip planes per bolt:

S6.13.2.8

Class B surfaces are unpainted blast-cleaned surfaces and blast-cleaned surfaces with Class B coatings.

Additionally:

Number of slip planes per bolt:

$$N_s = 2$$

Minimum required bolt tension:

$$P_t = 39.0K \quad \text{STable 6.13.2.8-1}$$

Hole size factor:

$$K_h = 1.0 \quad \text{STable 6.13.2.8-2}$$

Surface condition factor for Class B surface conditions:

$$K_s = 0.50$$

*S*Table 6.13.2.8-3

$$R_r = R_n = K_h \cdot K_s \cdot N_s \cdot P_t = 39.00K$$

The minimum number of bolts required to prevent slip is:

$$N = \frac{P_s}{R_r} = 4.7$$

Use:

$N = 5 < N = 12$  bolts determined previously to satisfy the bolt shear requirements.

Therefore, the number of bolts required for the bottom-flange splice is controlled by the bolt shear requirements. Arrange the bolts in three rows of four bolts per line with no stagger.

#### Friction Coefficient Selection



Weathering steel can be blasted for a Class B surface. Also, for painted steel, most inorganic zinc (IOZ) primers provide a Class B surface.

#### Flange Bolts - Minimum Spacing:

S6.13.2.6.1

The minimum spacing between centers of bolts in standard holes is to be no less than three times the diameter of the bolt.

$$d_{bolt} = 0.875in \quad (\text{Design Step 4.1})$$

$$s_{min} = 3 \cdot d_{bolt} = 2.63in$$

For this example:

$$s = 3.00in \quad (\text{see Figures 4-4 through 4-7})$$

The minimum spacing requirement is satisfied.

Flange Bolts - Maximum Spacing for Sealing:

S6.13.2.6.2

The maximum spacing of the bolts is limited to seal against an excessive penetration of moisture in the joints.

For a single line adjacent to a free edge of an outside plate or shape (for example, the bolts along the edges of the plate parallel to the direction of the applied force):

$$s \leq (4.00 + 4.00 \cdot t) \leq 7.00 \quad \text{SEquation 6.13.2.6.2-1}$$

where:

Thickness of the thinner outside plate or shape:

$$t \cdot out = 0.4375in$$

Maximum spacing for sealing:

$$4.0in + 4.0 \cdot t_{out} = 5.75in \leq 7.00in$$

$$s \leq 5.75in \quad \text{OK}$$

Next, check for sealing along the free edge at the end of the splice plate. The bolts are not staggered, therefore the applicable equation is:

$$s \leq (4.00 + 4.00 \cdot t) \leq 7.00$$

Maximum spacing along the free edge at the end of the splice plate (see Figures 4-4 through 4-7):

$$s_{end} = 5.00in$$

Maximum spacing for sealing:

$$4.0in + 4.0 \cdot t_{out} = 5.75in \leq 7.00in$$

$$s_{end} \leq 5.75in \quad \text{OK}$$

Therefore the requirement is satisfied.

Flange Bolts - Maximum Pitch for Stitch Bolts:

S6.13.2.6.3



The maximum pitch requirements are applicable only for mechanically fastened built-up members and do not apply to this example.

Flange Bolts – Edge and End Distance:

S6.13.2.6.6

*Minimum:*

The minimum required edge distance is measured as the distance from the center of any bolt in a standard hole to an edge of the plate perpendicular to the line of the force. The end distance is measured parallel to the line of force. The requirements are the same for both.

S*Table* 6.13.2.6.6-1

For a 7/8" diameter bolt measured to a gas-cut edge (assumed for this example), the minimum edge and end distance is 1 1/8".

Referring to Figures 4-4 through 4-7, it is clear that the minimum edge and end distances specified for this example are 1 1/2" and thus satisfy the minimum requirement.

*Maximum:*

The maximum edge and end distances are not to be more than eight times the thickness of the thinnest outside plate or five inches.

$$8 \cdot t \leq 5.00in$$

where:

$$t = t_{out} = 0.4375in$$

The maximum permitted edge and end distance is:

$$8 \cdot t_{out} = 3.50in$$

The maximum distance from the corner bolts to the corner of the splice plate or girder flange is equal to (reference Figure 4-7):

$$\sqrt{(1.50in)^2 + (1.75in)^2} = 2.30in$$

and satisfies the maximum edge and end distance requirement.

$$2.30in \leq 3.50in \quad \text{OK}$$

Flange Bolts - Bearing at Bolt Holes:

S6.13.2.9

Check bearing of the bolts on the connected material under the critical strength limit state minimum design force. The critical strength limit state minimum bottom flange design force from Table 4-13 is:

$$P_{cu} = 459.38K$$

The design bearing strength of the connected material is calculated as the sum of the bearing strengths of the individual bolt holes parallel to the line of the applied force.

The element of the bottom flange splice that controls the bearing check in this design example is the outer splice plate.

To determine the applicable equation for the calculation of the nominal resistance, the clear distance between holes and the clear end distance must be calculated and compared to the value of two times the nominal diameter of the bolt. This check yields:

$$d_{bolt} = 0.875in \quad (\text{Design Step 4.1})$$

$$2 \cdot d_{bolt} = 1.75in$$

For the bolts adjacent to the end of the splice plate, the edge distance is 1 1/2". Therefore, the clear end distance between the edge of the hole and the end of the splice plate:

$$d_{hole} = 0.9375in \quad (\text{Design Step 4.1})$$

$$L_{c_1} = 1.50in - \frac{d_{hole}}{2} = 1.03in$$

The center-to-center distance between bolts in the direction of the force is three inches. Therefore, the clear distance between edges of adjacent holes is computed as:

$$L_{c_2} = 3.00in - d_{hole} = 2.0625in$$

For standard holes, where either the clear distance between holes or the clear end distance is less than twice the bolt diameter:

$$R_n = 1.2 \cdot L_c \cdot t \cdot F_u \quad \text{SEquation 6.13.2.9-2}$$

For the outside splice plate:

Thickness of the connected material:

$$t_{out} = 0.4375in$$

Tensile strength of the connected material (Design Step 4.1):

$$F_u = 65 \text{ksi}$$

The nominal resistance for the end row of bolt holes is computed as follows:

$$R_{n_1} = 4 \cdot (1.2 \cdot L_{c_1} \cdot t_{out} \cdot F_u) = 140.60K$$

The nominal resistance for the remaining bolt holes is computed as follows:

$$R_{n_2} = 8 \cdot (1.2 \cdot L_{c_2} \cdot t_{out} \cdot F_u) = 563.06K$$

The total nominal resistance of the bolt holes is:

$$R_n = R_{n_1} + R_{n_2} = 703.66K$$

$$\phi_{bb} = 0.80 \quad (\text{Design Step 4.1})$$

$$R_r = \phi_{bb} \cdot R_n = 562.93K$$

Check:

$$\frac{P_{cu}}{2} = 229.69K < R_r = 562.93K \quad \text{OK}$$

### Fatigue of Splice Plates:

S6.6.1

SC6.13.6.1.4a

Check the fatigue stresses in the base metal of the bottom flange splice plates adjacent to the high-strength bolted slip-critical connections. Fatigue normally does not govern the design of the splice plates, and therefore, an explicit check is not specified. However, a fatigue check of the splice plates is recommended whenever the combined area of the inside and outside flange splice plates is less than the area of the smaller flange at the splice.

S*Table* 6.6.1.2.3-1

The fatigue detail category under the condition of Mechanically Fastened Connections for checking the base metal at the gross section of high-strength bolted slip-resistant connections with holes drilled full size is Category B (Condition 2.1). From Table S6.6.1.2.3-2, since (ADTT)<sub>SL</sub> exceeds 860 trucks per day for a Category B detail, the nominal fatigue resistance is computed for the Fatigue I load combination and infinite life. From Table 4-16, the factored fatigue stress range due to the Fatigue I load combination at the midthickness of the bottom flange is:

$$\gamma(\Delta f) = 9.20 \text{ ksi}$$

For load-induced fatigue considerations, each detail shall satisfy:

$$\gamma \cdot (\Delta f) \leq \Delta F_n \quad \text{SEquation 6.6.1.2.2-1}$$

Nominal fatigue resistance:

$$\Delta F_n = \Delta F_{TH} \quad \text{SEquation 6.6.1.2.5-1}$$

Constant-amplitude fatigue threshold:

$$\Delta F_{TH} = 16 \text{ ksi} \quad \text{STable 6.6.1.2.5-3}$$

S6.10.5.2

The range of flange force in the bottom flange is computed from the stress range as follows:

$$\Delta P = 9.20 \cdot 0.875 \cdot 14 = 112.7 \text{ K}$$

The flange force is equally distributed to the inner and outer splice plates due to the areas of the flanges being within 10 percent of each other:

$$P = \frac{112.7}{2} = 56.35 \text{ K}$$

The range of fatigue stress in the outside splice plate is computed as:

$$\Delta f = \frac{56.35}{(0.4375)(14)} = 9.2 \text{ ksi} < 16.0 \text{ ksi} \quad \text{OK}$$

The range of fatigue stress in the inside splice plates is computed as:

$$\Delta f = \frac{56.35}{2 \cdot (0.5)(6)} = 9.4 \text{ ksi} < 16.0 \text{ ksi} \quad \text{OK}$$

#### Control of Permanent Deformation - Splice Plates:

A check of the flexural stresses in the splice plates at the service limit state is not explicitly specified in the specifications. However, whenever the combined area of the inside and outside flange splice plates is less than the area of the smaller flange at the splice (which is the case for the bottom flange splice in this example), such a check is recommended.

The critical Service II flange force in the bottom flange is taken from Table 4-15:

$$P_s = 183.87K$$

The following criteria will be used to make this check. The equation presented is for both steel flanges of a composite section assuming no lateral flange bending:

$$f_f \leq 0.95 \cdot R_h \cdot F_{yf} \quad \text{SEquation 6.10.4.2.2-1}$$

where:

Elastic flange stress caused by the factored loading:

$$f_f$$

Specified minimum yield strength of the flange (Design Step 4.1):

$$F_{yf} = 50ksi$$

The flange force is equally distributed to the inner and outer splice plates due to the areas of the flanges being within 10 percent of each other:

$$P = \frac{P_s}{2} = 91.94K$$

The resulting stress in the outside splice plate is:

$$A_{gross\_out} = 6.12in^2$$

$$f_{\_out} = \frac{P}{A_{gross\_out}} = 15.02ksi$$

$$f_{\_out} = 15.02ksi < 0.95 \cdot R_h \cdot F_{yf} = 47.50ksi \quad \text{OK}$$

The resulting stress in the inside splice plates is:

$$A_{gross\_in} = 6.00in^2$$

$$f_{\_in} = \frac{P}{A_{gross\_in}} = 15.32ksi$$

$$f_{\_in} = 15.32ksi < 0.95 \cdot R_h \cdot F_{yf} = 47.50ksi \quad \text{OK}$$

### Design Step 4.5 - Design Top Flange Splice

The design of the top flange splice is not included in this design example (for the sake of simplicity and brevity). However, the top flange splice is designed using the same procedures and methods presented in this design example for the bottom flange splice.

### Design Step 4.6 - Compute Web Splice Design Loads

*S6.13.6.1.4b*

Web splice plates and their connections are to be designed for shear, the moment due to the eccentricity of the shear at the point of splice, and the portion of the flexural moment assumed to be resisted by the web at the point of the splice.

#### Girder Shear Forces at the Splice Location:

Based on the girder properties defined in Design Step 3 (Steel Girder Design), any number of commercially available software programs can be used to obtain the design dead and live loads at the splice. For this design example, the AASHTO BrD software was used. A summary of the unfactored shears at the splice from the initial trial of the girder design are listed below. The live load shears include the dynamic load allowance and distribution factors.

Dead Loads:

Noncomposite:

$$V_{NDL} = -60.8K$$

Composite:

$$V_{CDL} = -8.7K$$

Future Wearing Surface:

$$V_{FWS} = -10.6K$$

Live Loads:

HL-93 Positive:

$$V_{PLL} = 14.5K$$

HL-93 Negative:

$$V_{NLL} = -91.1K$$

Fatigue Positive:

$$V_{PFL} = 5.0K$$

Fatigue Negative:

$$V_{NFL} = -33.4K$$

C6.13.6.1.4b

#### Web Moments and Horizontal Force Resultant:

Because the portion of the flexural moment assumed to be resisted by the web is to be applied at the mid-depth of the web, a horizontal design force resultant,  $H_{uw}$ , must also be applied at the mid-depth of the web to maintain equilibrium. The web moment and horizontal force resultant are applied together to yield a combined stress distribution equivalent to the unsymmetrical stress distribution in the web. For sections with equal compressive and tensile stresses at the top and bottom of the web (i.e., with the neutral axis located at the mid-depth of the web),  $H_{uw}$  will equal zero.

In the computation of the portion of the flexural moment assumed to be resisted by the web,  $M_{uw}$ , and the horizontal design force resultant,  $H_{uw}$ , in the web, the flange stresses at the midthickness of the flanges are conservatively used (C6.13.6.2.4b). This allows use of the same stress values for both the flange and web splices, which simplifies the calculations. It is important to note that all flange stresses are taken as signed quantities in determining  $M_{uw}$  and  $H_{uw}$  (positive for tension; negative for compression).

The moment,  $M_{uw}$ , due to the eccentricity of the design shear,  $V_{uw}$ , is resisted solely by the web and always acts about the mid-depth of the web (i.e., horizontal force resultant is zero). This moment is computed as:

$$M_{uw} = V_{uw} \cdot e$$

S6.13.6.1.4b

where  $e$  is defined as the distance from the centerline of the splice to the centroid of the connection on the side of the joint under consideration. For this design example:

$$e = 1.9375in + \frac{3.00in}{2} = 3.44in \quad (\text{reference Figure 4-8})$$

The total web moment for each load case is computed as follows:

$$M_{total} = M_{uw} + M_{uv}$$

In general, and in this example, the web splice is designed under the conservative assumption that the maximum moment and shear at the splice will occur under the same loading condition.

Strength Limit State:

*Design Shear:*

S6.13.6.1.4b

For the strength limit state, the girder web nominal shear resistance is required when determining the design shear. Assume an unstiffened web at the splice location (S6.10.9.2).

$$\phi_v = 1.00 \quad (\text{Design Step 4.1})$$

$$V_u \leq \phi_v \cdot V_n \quad \text{SEquation 6.10.9.1-1}$$

$$V_n = C \cdot V_p \quad \text{SEquation 6.10.9.2-1}$$

$$V_p = 0.58 \cdot F_{yw} \cdot D \cdot t_w \quad \text{SEquation 6.10.9.2-2}$$

where:

S6.10.9.3.2

The ratio of the shear buckling stress to the shear yield strength, C, is dependent upon the ratio of D/t<sub>w</sub> in comparison to:

$$1.12 \cdot \sqrt{\frac{E \cdot k}{F_{yw}}} \quad \text{and} \quad 1.40 \cdot \sqrt{\frac{E \cdot k}{F_{yw}}}$$

$$k = 5.0 \quad \text{S6.10.9.2}$$

Modulus of elasticity:

$$E = 29000 \text{ksi}$$

Specified minimum yield strength of the web (Design Step 4.1):

$$F_{yw} = 50 \text{ksi}$$

Web Depth:

$$D = 54 \text{in} \quad (\text{Figure 4-1})$$



Thickness of the web:

$$t_w = 0.50in \quad (\text{Figure 4-1})$$

Compare:

$$\frac{D}{t_w} = 108.00$$

to the values for:

$$1.12 \cdot \sqrt{\frac{E \cdot k}{F_{yw}}} = 60.31 \quad \text{and} \quad 1.40 \cdot \sqrt{\frac{E \cdot k}{F_{yw}}} = 75.39$$

Since  $D/t_w$  is greater than the above two values, use the following equation to determine  $C$ :

$$C = \frac{1.57}{\left(\frac{D}{t_w}\right)^2} \cdot \left(\frac{E \cdot k}{F_{yw}}\right) = 0.39 \quad \text{SEquation 6.10.9.3.2-6}$$

The nominal shear resistance is computed as follows:

$$V_p = 0.58 \cdot F_{yw} \cdot D \cdot t_w = 783.00K$$

$$V_n = C \cdot V_p = 305.64K$$

The factored shear resistance now follows:

$$V_r = \phi_v \cdot V_n = 305.64K$$

At the strength limit state, the design shear,  $V_{uw}$ , is to be taken as:

If  $V_u < 0.5 \phi_v V_n$ , then:

$$V_{uw} = 1.5 \cdot V_u \quad \text{SEquation 6.13.6.1.4b-1}$$

Otherwise:

$$V_{uw} = \frac{V_u + (\phi_v \cdot V_n)}{2} \quad \text{SEquation 6.13.6.1.4b-2}$$

The shear at the strength limit state at the point of splice,  $V_u$ , is computed from the girder shear forces at the splice location listed at the beginning of this design step.

For the strength limit state, the factored shear for the positive live load is (the shear due to the future wearing surface is conservatively ignored):

$$V_{upos} = 0.90 \cdot (V_{NDL} + V_{CDL}) + 1.75 \cdot V_{PLL} = -37.18K$$

For the strength limit state, the factored shear for the negative live load is:

$$V_{uneg} = 1.25 \cdot (V_{NDL} + V_{CDL}) + 1.50 \cdot V_{FWS} + 1.75 \cdot V_{NLL} = -262.20K$$

(controls)

Therefore:

$$V_u = |V_{uneg}|$$

Since  $V_u$  exceeds one-half of  $\phi_v V_n$ :

$$V_{uw} = \frac{V_u + (\phi_v \cdot V_n)}{2} = 283.92K$$

*Web Moments and Horizontal Force Resultants:*

Case 1 - Dead Load + Positive Live Load:

For the loading condition with positive live load, the controlling flange was previously determined to be the bottom flange. The maximum elastic flexural stress due to the factored loads at the midthickness of the controlling flange,  $f_{cf}$ , and the design stress for the controlling flange,  $F_{cf}$ , were previously computed for this loading condition. From Design Step 4.3, Table 4-13:

$$f_{cf} = 20.53ksi$$

$$F_{cf} = 37.50ksi$$

For the same loading condition, the concurrent flexural stress at the midthickness of the noncontrolling (top) flange,  $f_{ncf}$ , was previously computed. From Design Step 4.3, Table 4-14:

$$f_{ncf} = -0.86ksi$$

Therefore, the portion of the flexural moment assumed to be resisted by the web is computed as:

$$M_{uw} = \frac{t_w \cdot D^2}{12} \cdot |R_h \cdot F_{cf} - R_{cf} \cdot f_{ncf}|$$

*CEquation 6.13.6.1.4b-1*

where:

The hybrid girder reduction factor:

$$R_h = 1.00$$

The ratio  $R_{cf}$  is computed as follows:

$$R_{cf} = \left| \frac{F_{cf}}{f_{cf}} \right| = 1.83$$

Web Depth:

$$D = 54in \quad (\text{Figure 4-1})$$

Thickness of the web:

$$t_w = 0.50in \quad (\text{Figure 4-1})$$

Compute the portion of the flexural moment to be resisted by the web:

$$M_{uw\_str\_pos} = \frac{t_w \cdot D^2}{12} \cdot |R_h \cdot F_{cf} - R_{cf} \cdot f_{ncf}| \cdot \left( \frac{1}{12 \cdot \frac{in}{ft}} \right) = 395.62K \cdot ft$$

The total web moment is:

$$V_{uw} = 283.92K$$

$$e = 3.44in$$

$$M_{tot\_str\_pos} = M_{uw\_str\_pos} + (V_{uw} \cdot e) \cdot \left( \frac{1}{12 \cdot \frac{in}{ft}} \right) = 477.01K \cdot ft$$

Compute the horizontal force resultant (the variables included in this equation are as defined for  $M_{uw\_str\_pos}$ ):

$$H_{uw\_str\_pos} = \frac{t_w \cdot D}{2} \cdot (R_h \cdot F_{cf} + R_{cf} \cdot f_{ncf}) = 485.12K \quad \text{CEquation 6.13.6.1.4b-2}$$

The above value is a signed quantity, positive for tension and negative for compression.

## Case 2 - Dead Load + Negative Live Load:

Similarly, for the loading condition with negative live load, the controlling flange was determined to be the bottom flange. For this case the stresses were previously computed. From Design Step 4.3, Table 4-13:

$$f_{cf} = -19.12ksi$$

$$F_{cf} = -37.50ksi$$

For the noncontrolling (top) flange, the flexural stress at the midthickness of the flange, from Design Step 4.3, Table 4-14:

$$f_{ncf} = 11.80ksi$$

The ratio,  $R_{cf}$ , is computed as follows:

$$R_{cf} = \left| \frac{F_{cf}}{f_{cf}} \right| = 1.96$$

Therefore:

$$M_{uw\_st\_neg} = \frac{t_w \cdot D^2}{12} \cdot \left| R_h \cdot F_{cf} - R_{cf} \cdot f_{ncf} \right| \cdot \left( \frac{1}{12 \cdot \frac{in}{ft}} \right) = 613.86K \cdot ft$$

The total web moment is:

$$V_{uw} = 283.92K$$

$$e = 3.44in$$

$$M_{tot\_str\_neg} = M_{uw\_str\_neg} + (V_{uw} \cdot e) \cdot \left( \frac{1}{12 \cdot \frac{in}{ft}} \right) = 695.25K \cdot ft$$

Compute the horizontal force resultant:

$$H_{uw\_str\_neg} = \frac{t_w \cdot D}{2} \cdot (R_h \cdot F_{cf} + R_{cf} \cdot f_{ncf}) = -194.02K$$

The above value is a signed quantity, positive for tension, and negative for compression.

Service Limit State:

S6.13.6.1.4b

*Design Shear:*

As a minimum, for checking slip of the web splice bolts, the design shear is to be taken as the shear at the point of splice under the Service II Load Combination, or the shear from constructibility, whichever governs. In this design example, the Service II shear controls (see previous discussion in Design Step 4.3).

The elastic shears due to the unfactored loads at the point of the splice are listed at the beginning of this design step.

For Service II, the factored shear for the positive live load is computed below. The future wearing surface is conservatively not included since it doesn't maximize the shear for the positive live load.

$$V_{ser\_pos} = 1.00 \cdot V_{NDL} + 1.00 \cdot V_{CDL} + 1.30 \cdot V_{PLL} = -50.65K$$

For Service II, the factored shear for the negative live load is computed below. The future wearing surface is included since it helps to maximize the shear for the negative live load.

$$V_{ser\_neg} = 1.00 \cdot V_{NDL} + 1.00 \cdot V_{CDL} + 1.00 \cdot V_{FWS} + 1.30 \cdot V_{PLL} = -198.53K$$

(governs)

Therefore:

$$V_{w\_ser} = |V_{ser\_neg}|$$

*Web Moments and Horizontal Force Resultants:*

C6.13.6.1.4b

The web design moment and horizontal force resultant are computed using *CEquation 6.13.6.1.4b-1* and *CEquation 6.13.6.1.4b-2*, modified for the service limit state as follows:

$$M_{uw\_ser} = \frac{t_w \cdot D^2}{12} \cdot |f_s - f_{os}|$$

$$H_{uw\_ser} = \frac{t_w \cdot D}{2} \cdot (f_s + f_{os})$$

In the above equations,  $f_s$  is the maximum Service II midthickness flange stress for the load case considered (i.e., positive or negative live load). The Service II midthickness flange stress in the other flange, concurrent with  $f_s$ , is termed  $f_{os}$ .

Case 1 - Dead Load + Positive Live Load:

The maximum midthickness flange flexural stress for the load case with positive live load moment for the service limit state occurs in the bottom flange. From Table 4-11:

$$f_{s\_bot\_pos} = 15.01ksi$$

$$f_{os\_top\_pos} = -0.39ksi$$

Therefore, for the load case of positive live load:

$$M_{uw\_ser\_pos} = \frac{t_w \cdot D^2}{12} \cdot |f_{s\_bot\_pos} - f_{os\_top\_pos}| \cdot \left( \frac{1}{12 \cdot \frac{in}{ft}} \right) = 155.93K \cdot ft$$

The total web moment is:

$$V_{w\_ser} = 198.53K$$

$$e = 3.44in$$

$$M_{tot\_ser\_pos} = M_{uw\_ser\_pos} + (V_{w\_ser} \cdot e) \cdot \left( \frac{1}{12 \cdot \frac{in}{ft}} \right) = 212.84K \cdot ft$$

Compute the horizontal force resultant:

$$H_{uw\_ser\_pos} = \frac{t_w \cdot D}{2} \cdot (f_{s\_bot\_pos} + f_{os\_top\_pos}) = 197.37K$$

The above value is a signed quantity, positive for tension, and negative for compression.

Case 2 - Dead Load + Negative Live Load:

The maximum midthickness flange flexural stress for the load case with negative live load moment for the service limit state occurs in the bottom flange. From Table 4-11:

$$f_{s\_bot\_neg} = -11.75ksi$$

$$f_{os\_top\_neg} = 1.46ksi$$

Therefore:

$$M_{uw\_ser\_neg} = \frac{t_w \cdot D^2}{12} \cdot |f_{s\_bot\_neg} - f_{os\_top\_neg}| \cdot \left( \frac{1}{12 \cdot \frac{in}{ft}} \right) = 133.75K \cdot ft$$

The total web moment is:

$$V_{w\_ser} = 198.53K$$

$$e = 3.44in$$

$$M_{tot\_ser\_neg} = M_{uw\_ser\_neg} + (V_{w\_ser} \cdot e) \cdot \left( \frac{1}{12 \cdot \frac{in}{ft}} \right) = 190.66K \cdot ft$$

Compute the horizontal force resultant:

$$H_{uw\_ser\_neg} = \frac{t_w \cdot D}{2} \cdot (f_{s\_bot\_neg} + f_{os\_top\_neg}) = -138.92K$$

The above value is a signed quantity, positive for tension, and negative for compression.

Fatigue Limit State:

C6.13.6.1.4a

Fatigue of the base metal adjacent to the slip-critical connections in the splice plates may be checked as specified in *S*Table 6.6.1.2.3-1 using the gross section of the splice plates and member. However, the areas of the web splice plates will often equal or exceed the area of the web to which they are attached (the case in this design example). Therefore, fatigue will generally not govern the design of the splice plates, but is carried out in this example for completeness.

*Design Shear:*

For the fatigue limit state, the factored Fatigue I shear for the positive live load is (Design Step 4.6):

$$V_{fat\_pos} = 1.5 \cdot V_{PFL} = 7.5K$$

For the fatigue limit state, the factored Fatigue I shear for the negative live load is (Design Step 4.6):

$$V_{fat\_neg} = 1.5 \cdot V_{NFLL} = -50.1K$$

*Web Moments and Horizontal Force Resultants:*

The portion of the flexural moment to be resisted by the web and the horizontal force resultant are computed from equations similar to *CEquations 6.13.6.1.4b-1 and 6.13.6.1.4b-2*, respectively, with appropriate substitutions of the stresses in the web caused by the fatigue-load moment for the flange stresses in the equations. Also, the absolute value signs are removed to keep track of the signs. This yields the following equations:

$$M_{uw} = \frac{t_w \cdot D^2}{12} \cdot (f_{botweb} - f_{topweb})$$

$$H_{uw} = \frac{t_w \cdot D}{2} \cdot (f_{botweb} + f_{topweb})$$

Case 1 - Positive Live Load:

The factored Fatigue I stresses due to the positive live load moment for the Fatigue Limit State at the top and bottom of the web, from Table 4-12, are:

$$f_{topweb\_pos} = -0.32ksi$$

$$f_{botweb\_pos} = 5.30ksi$$

Therefore:

$$M_{uw\_fat\_pos} = \frac{t_w \cdot D^2}{12} \cdot (f_{botweb\_pos} - f_{topweb\_pos}) \cdot \left( \frac{1}{12 \cdot \frac{in}{ft}} \right) = 56.90K \cdot ft$$

The total web moment is:

$$V_{fat\_pos} = 7.5K$$

$$e = 3.44in$$



$$M_{tot\_fat\_pos} = M_{uw\_fat\_pos} + (V_{fat\_pos} \cdot e) \cdot \left( \frac{1}{12 \cdot \frac{in}{ft}} \right) = 59.05K \cdot ft$$

Compute the horizontal force resultant:

$$H_{uw\_fat\_pos} = \frac{t_w \cdot D}{2} \cdot (f_{botweb\_pos} + f_{topweb\_pos}) = 67.23K$$

The above value is a signed quantity, positive for tension, and negative for compression.

Case 2 - Negative Live Load:

The factored Fatigue I stresses due to the negative live load moment for the Fatigue Limit State at the top and bottom of the web, from Table 4-12, are:

$$f_{botweb\_neg} = -3.82ksi$$

$$f_{topweb\_neg} = 0.23ksi$$

Therefore:

$$M_{uw\_fat\_neg} = \frac{t_w \cdot D^2}{12} \cdot (f_{botweb\_neg} - f_{topweb\_neg}) \cdot \left( \frac{1}{12 \cdot \frac{in}{ft}} \right) = -41.01K \cdot ft$$

The total web moment is:

$$V_{fat\_neg} = -50.1K$$

$$e = 3.44in$$

$$M_{tot\_fat\_neg} = M_{uw\_fat\_neg} + (V_{fat\_neg} \cdot e) \cdot \left( \frac{1}{12 \cdot \frac{in}{ft}} \right) = -55.37K \cdot ft$$

Compute the horizontal force resultant:

$$H_{uw\_fat\_neg} = \frac{t_w \cdot D}{2} \cdot (f_{botweb\_neg} + f_{topweb\_neg}) = -48.47K$$

The above value is a signed quantity, positive for tension, and negative for compression.

### Design Step 4.7 - Design Web Splice

#### Web Splice Configuration:

Two vertical rows of bolts with sixteen bolts per row will be investigated. The typical bolt spacing, both horizontally and vertically, are as shown in Figure 4-8. The outermost rows of bolts are located 4 1/2" from the flanges to provide clearance for assembly (see the *AISC Manual of Steel Construction* for required bolt assembly clearances). The web is spliced symmetrically by plates on each side with a thickness not less than one-half the thickness of the web. Assume 5/16" x 48" splice plates on each side of the web. No web fill plate is necessary for this example.

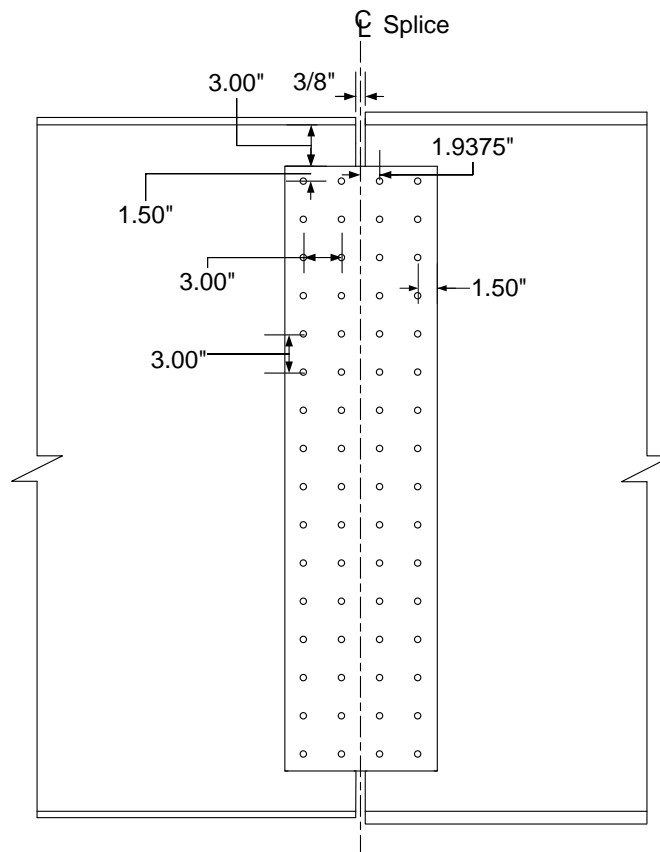


Figure 4-8 Web Splice

**Web Splice Design**

It is recommended to extend the web splice plates as near as practical to the full depth of the web between flanges without impinging on bolt assembly clearances (Refer to the AISC Manual for bolt assembly clearances). Also, S6.13.6.1.4a specifies that there shall not be less than two rows of bolts on each side of the joint.

**Web Bolts - Minimum Spacing:**

S6.13.2.6.1

This check is only dependent upon the bolt diameter, and is therefore satisfied for a three-inch spacing per the check for the flange bolts from Design Step 4.4.

**Web Bolts - Maximum Spacing for Sealing:**

S6.13.2.6.2

The maximum spacing of the bolts is limited to seal against excessive penetration of moisture in the joints.

For a single line adjacent to a free edge of an outside plate or shape (for example, the bolts along the edges of the plate parallel to the direction of the applied force):

$$s \leq (4.00 + 4.00 \cdot t) \leq 7.00$$

where:

Thickness of the thinner outside plate or shape, in this case the web plate:

$$t_{wp} = 0.3125in$$

Maximum spacing for sealing:

$$4.0in + 4.0 \cdot t_{wp} = 5.25in \leq 7.00in$$

$$s = 3.00in \leq 5.25in \quad \text{OK}$$

**Web Bolts - Maximum Pitch for Stitch Bolts:**

S6.13.2.6.3

The maximum pitch requirements are applicable only for mechanically fastened built-up members and do not apply to this example.

Web Bolts - Edge and End Distance:

S6.13.2.6.6

*Minimum:*

The minimum required edge and end distance is measured as the distance from the center of any bolt in a standard hole to an edge of the plate.

STable 6.13.2.6.6-1

For a 7/8" diameter bolt measured to a gas-cut edge, the minimum edge and end distance is 1 1/8".

Referring to Figure 4-8, it is clear that the minimum edge and end distance specified for this example is 1 1/2" and thus satisfies the minimum requirement.

*Maximum:*

The maximum edge and end distance is not to be more than eight times the thickness of the thinnest outside plate or five inches.

$$8 \cdot t \leq 5.00in$$

where:

$$t = t_{wp} = 0.3125in$$

The maximum permitted edge and end distance is:

$$8t_{wp} = 2.50in$$

The maximum distance from the corner bolts to the corner of the splice plate is equal to (reference Figure 4-8):

$$\sqrt{(1.50in)^2 + (1.50in)^2} = 2.12in$$

and satisfies the maximum edge and end distance requirement.

$$2.12in \leq 2.50in \quad \text{OK}$$

Web Bolts - Shear:

Calculate the polar moment of inertia,  $I_p$ , of the bolt group on each side of the centerline with respect to the centroid of the connection. This is required for determination of the shear force in a given bolt due to the applied web moments.

$$I_p = \frac{n \cdot m}{12} \cdot [s^2 \cdot (n^2 - 1) + g^2 \cdot (m^2 - 1)] \quad \text{CEquation 6.13.6.1.4b-3}$$

where:

Number of vertical rows of bolts:

$$m = 2$$

Number of bolts in one vertical row:

$$n = 16$$

Vertical pitch:

$$s = 3.00in$$

Horizontal pitch:

$$g = 3.00in$$

The polar moment of inertia is:

$$I_p = \frac{n \cdot m}{12} \cdot [s^2 \cdot (n^2 - 1) + g^2 \cdot (m^2 - 1)] = 6192.00in^2$$

The total number of web bolts on each side of the splice, assuming two vertical rows per side with sixteen bolts per row, is:

$$N_b = 32$$

#### *Strength Limit State:*

Under the most critical combination of the minimum design shear, moment and horizontal force, it is assumed that the bolts in the web splice have slipped and gone into bearing. Since 7/8 in. diameter bolts are used and the web splice plates are less than 1/2 in. thick, the bolts are included in the shear plane.

The resistance of a bolt when threads are included in the shear plane is given as:

$$R_u = 0.38 \cdot A_b \cdot F_{ub} \cdot N_s$$

where:

Area of the bolt corresponding to the nominal diameter:

$$A_b = \frac{\pi}{4} \cdot d_{bolt}^2 = 0.60in^2$$

Specified minimum tensile strength of the bolt from Design Step 4.1:

$$F_{ub} = F_{ubolt} = 120ksi$$

Number of shear planes per bolt:

$$N_s = 2$$

$$R_n = 2(0.38 \cdot A_b \cdot F_{ub}) = 54.72K$$

The factored shear resistance now follows:

$$\phi_s = 0.80 \quad (\text{Design Step 4.1})$$

$$R_u = \phi_s \cdot R_n = 43.78K$$

$$R_u = 43.78K$$

Case 1 - Dead Load + Positive Live Load:

The following forces were computed in Design Step 4.6:

$$V_{uw} = 283.92K$$

$$M_{tot\_str\_pos} = 477.01K \cdot ft$$

$$H_{uw\_str\_pos} = 485.12K$$

The vertical shear force in the bolts due to the applied shear force:

$$P_{v\_str} = \frac{V_{uw}}{N_b} = 8.87K$$

The horizontal shear force in the bolts due to the horizontal force resultant:

$$P_{H\_str\_pos} = \frac{H_{uw\_str\_pos}}{N_b} = 15.16K$$

Determine the horizontal and vertical components of the bolt shear force on the extreme bolt due to the total moment in the web:

$$P_{Mv} = \frac{M_{total} \cdot x}{I_p}$$

$$P_{Mh} = \frac{M_{total} \cdot y}{I_p}$$

For the vertical component:

$$x = \frac{g}{2} = 1.50in$$

For the horizontal component:

$$y = \frac{15 \cdot s}{2} = 22.50in$$

Calculating the components:

$$P_{Mv\_str\_pos} = \frac{M_{tot\_str\_pos} \cdot x}{I_p} \cdot \left(12 \cdot \frac{in}{ft}\right) = 1.39K$$

$$P_{Mh\_str\_pos} = \frac{M_{tot\_str\_pos} \cdot y}{I_p} \cdot \left(12 \cdot \frac{in}{ft}\right) = 20.80K$$

The resultant bolt force for the extreme bolt is:

$$P_{r\_str\_pos} = \sqrt{\left(P_{v\_str} + P_{Mv\_str\_pos}\right)^2 + \left(P_{H\_str\_pos} + P_{Mh\_str\_pos}\right)^2} = 37.40K$$

Case 2 - Dead Load + Negative Live Load:

The following forces were computed in Design Step 4.6:

$$V_{uw} = 283.92K$$

$$M_{tot\_str\_neg} = 695.25K \cdot ft$$

$$H_{uw\_str\_neg} = -194.02K$$

The vertical shear force in the bolts due to the applied shear force:

$$P_{v\_str} = \frac{V_{uw}}{N_b} = 8.87K$$

The horizontal shear force in the bolts due to the horizontal force resultant:

$$P_{H\_str\_neg} = \frac{|H_{uw\_str\_neg}|}{N_b} = 6.06K$$

Determine the horizontal and vertical components of the bolt shear force on the extreme bolt due to the total moment in the web:

Calculating the components:

$$P_{Mv\_str\_neg} = \frac{M_{tot\_str\_neg} \cdot x}{I_p} \cdot \left(12 \cdot \frac{in}{ft}\right) = 2.02K$$

$$P_{Mh\_str\_neg} = \frac{M_{tot\_str\_neg} \cdot y}{I_p} \cdot \left(12 \cdot \frac{in}{ft}\right) = 30.32K$$

The resultant bolt force is:

$$P_{r\_str\_neg} = \sqrt{(P_{v\_str} + P_{Mv\_str\_neg})^2 + (P_{H\_str\_neg} + P_{Mh\_str\_neg})^2} = 37.97K$$

The governing resultant bolt force is:

$$P_{r\_str} = \max(P_{r\_str\_pos}, P_{r\_str\_neg}) = 37.97K$$

Check:

$$P_{r\_str} = 37.97K < R_u = 43.78K \quad \text{OK}$$

*Service Limit State:*

The factored slip resistance,  $R_r$ , for a 7/8" diameter high-strength bolt in double shear for a Class B surface and standard holes was determined from Design Step 4.4 to be:

$$R_r = 39.00K$$

Case 1 - Dead Load + Positive Live Load:

The following forces were computed in Design Step 4.6:

$$V_{uw\_ser} = 198.53K$$

$$M_{tot\_ser\_pos} = 212.84K \cdot ft$$

$$H_{uw\_ser\_pos} = 197.37K$$



The vertical shear force in the bolts due to the applied shear force:

$$P_{s\_ser} = \frac{V_{w\_ser}}{N_b} = 6.20K$$

The horizontal shear force in the bolts due to the horizontal force resultant:

$$P_{H\_ser\_pos} = \frac{H_{uw\_ser\_pos}}{N_b} = 6.16K$$

Determine the horizontal and vertical components of the bolt shear force on the extreme bolt due to the total moment in the web:

For the vertical component:

$$x = 1.50in$$

$$P_{Mv\_ser\_pos} = \frac{M_{tot\_ser\_pos} \cdot x}{I_p} \cdot \left(12 \frac{in}{ft}\right) = 0.62K$$

For the horizontal component:

$$y = 22.50in$$

$$P_{Mh\_ser\_pos} = \frac{M_{tot\_ser\_pos} \cdot y}{I_p} \cdot \left(12 \frac{in}{ft}\right) = 9.28K$$

The resultant bolt force is:

$$P_{r\_ser\_pos} = \sqrt{(P_{s\_ser} + P_{Mv\_ser\_pos})^2 + (P_{H\_ser\_pos} + P_{Mh\_ser\_pos})^2} = 16.88K$$

Case 2 - Dead Load + Negative Live Load:

The following forces were computed in Design Step 4.6:

$$V_{uw\_ser} = 198.53K$$

$$M_{tot\_ser\_neg} = 190.66K \cdot ft$$

$$H_{uw\_ser\_neg} = -138.92K$$

The vertical shear force in the bolts due to the applied shear force:

$$P_{s\_ser} = \frac{V_{w\_ser}}{N_b} = 6.20K$$

The horizontal shear force in the bolts due to the horizontal force resultant:

$$P_{H\_ser\_neg} = \frac{|H_{uw\_ser\_neg}|}{N_b} = 4.34K$$

Determine the horizontal and vertical components of the bolt shear force on the extreme bolt due to the total moment in the web:

For the vertical component:

$$P_{Mv\_ser\_neg} = \frac{M_{tot\_ser\_neg} \cdot x}{I_p} \cdot \left(12 \frac{in}{ft}\right) = 0.55K$$

For the horizontal component:

$$P_{Mh\_ser\_neg} = \frac{M_{tot\_ser\_neg} \cdot y}{I_p} \cdot \left(12 \frac{in}{ft}\right) = 8.31K$$

The resultant bolt force is:

$$P_{r\_ser\_neg} = \sqrt{(P_{s\_ser} + P_{Mv\_ser\_neg})^2 + (P_{H\_ser\_neg} + P_{Mh\_ser\_neg})^2} = 14.34K$$

The governing resultant bolt force is:

$$P_{r\_ser} = \max(P_{r\_ser\_pos}, P_{r\_ser\_neg}) = 16.88K$$

Check:

$$P_{r\_ser} = 16.88K < R_r = 39.00K \quad \text{OK}$$

Thirty-two 7/8" diameter high-strength bolts in two vertical rows on each side of the splice provides sufficient resistance against bolt shear and slip.

Shear Yielding and Shear Rupture of Splice Plates:

S6.13.6.1.4b

Check for shear yielding on the gross section of the web splice plates under the strength limit state minimum design shear force,  $V_{uw}$ :

$$V_{uw} = 283.92K$$

The factored resistance of the splice plates is taken as:

$$R_r = \phi_v \cdot R_n \quad \text{SEquation 6.13.5.3-1}$$

$$R_n = 0.58 \cdot A_g \cdot F_y \quad \text{SEquation 6.13.5.3-2}$$

The gross area of the web splice is calculated as follows:

Number of splice plates:

$$N_{wp} = 2$$

Thickness of plate:

$$t_{wp} = 0.3125in$$

Depth of splice plate:

$$d_{wp} = 48in$$

$$A_{gross\_wp} = N_{wp} \cdot t_{wp} \cdot d_{wp} = 30.00in^2$$

From Design Step 4.1:

Specified minimum yield strength of the connection element:

$$F_y = 50ksi$$

Resistance factor for shear:

$$\phi_v = 1.0$$

The factored shear resistance is then:

$$R_r = \phi_v \cdot (0.58) \cdot (A_{gross\_wp}) \cdot (F_y) = 870.00K$$

Check:

$$V_{uw} = 283.92K < R_r = 870.00K \quad \text{OK}$$

Check for shear rupture on the gross section of the web splice plates under the strength limit state minimum design shear force,  $V_{uw}$ :

$$R_r = \phi_{vu} \cdot 0.58 \cdot R_p \cdot F_u A_{vn}$$

where:

Net Area Resisting Shear:

$$A_{vn} = 2 \cdot [48 - 16 \cdot (0.9375)] \cdot 0.3125 = 20.625 \text{ in}^2$$

The factored shear rupture resistance is then:

$$R_r = 0.80 \cdot 0.58 \cdot 1.0 \cdot 65 \cdot 20.625 = 622.05 \text{ K}$$

$$V_{uw} = 283.92 \text{ K} < R_r = 622.05 \text{ K} \quad \text{OK}$$

S6.13.6.1.4b

#### Block Shear Rupture of the Web Splice Plates:

Strength limit state checks for block shear rupture normally do not govern for many designs but are provided in this example for completeness.

From Design Step 4.6, the strength limit state minimum design shear force was determined to be:

$$V_{uw} = 283.92 \text{ K}$$

C6.13.4

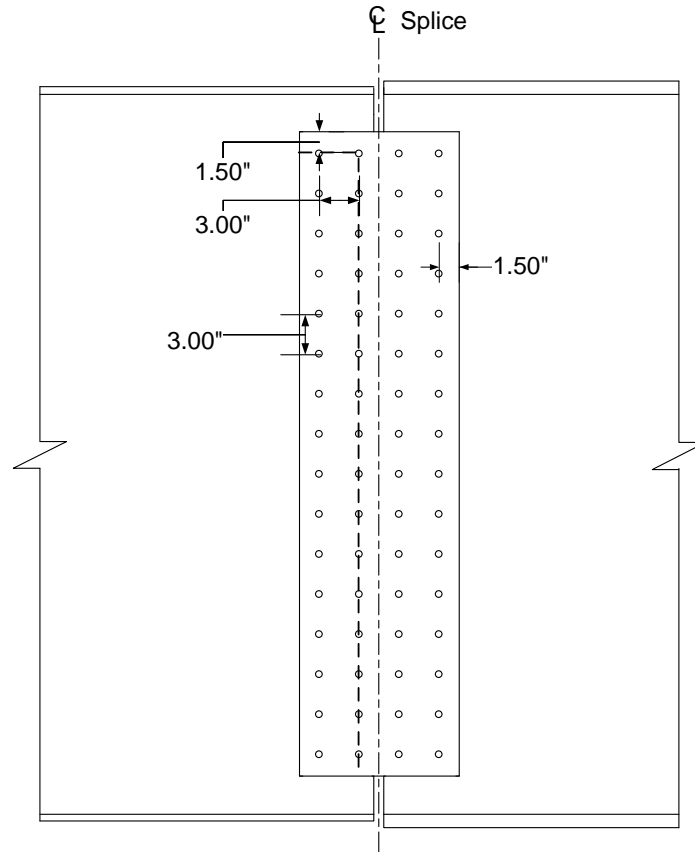
#### *Block Shear Rupture Resistance:*

Connection plates, splice plates and gusset plates are to be investigated to ensure that adequate connection material is provided to develop the factored resistance of the connection.

Assume the block shear failure mode in the web splice plates shown in Figure 4-9. The block shear resistance is computed as:

$$R_r = \phi_{bs} \cdot R_p (0.58 \cdot F_u \cdot A_{vn} + U_{bs} \cdot F_u \cdot A_m) \quad \text{SEquation 6.13.4-1}$$

$$< \phi_{bs} \cdot (0.58 \cdot F_y \cdot A_{vg} + U_{bs} \cdot F_u \cdot A_m)$$



**Figure 4-9 Block Shear Failure Mode - Web Splice Plate**

Gross area along the planes resisting shear stress:

$$A_{vg} = N_{wp} \cdot (d_{wp} - 1.50in) \cdot t_{wp} = 29.06in^2$$

Net area along the planes resisting shear stress:

$$A_{vn} = N_{wp} \cdot (d_{wp} - 1.50in - 15.50 \cdot d_{hole}) \cdot t_{wp} = 19.98in^2$$

Net area along the planes resisting tension stress:

$$A_{tn} = N_{wp} \cdot (1.50in + 3.0in - 1.5 \cdot d_{hole}) \cdot t_{wp} = 1.93in^2$$

Check:

$$R_r = 0.80 \cdot 1.0(0.58 \cdot 65 \cdot 19.98 + 1.0 \cdot 65 \cdot 1.93) = 702.96K$$

$$< 0.80 \cdot 1.0 \cdot (0.58 \cdot 50 \cdot 29.06 + 1.0 \cdot 65 \cdot 1.93) = 774.55K$$

$$V_{uw} = 283.92K < R_r = 702.96K \quad \text{OK}$$

Flexural Yielding of Splice Plates:

S6.13.6.1.4b

Check for flexural yielding on the gross section of the web splice plates at the strength limit state due to the total web moment and the horizontal force resultant:

$$f = \frac{M_{Total}}{S_{pl}} + \frac{H_{uw}}{A_{gross\_wp}} \leq \phi_f \cdot F_y$$

where:

Resistance factor for flexure (Design Step 4.1):

$$\phi_f = 1.0$$

Section modulus of the web splice plates:

$$S_{pl} = \frac{1}{6} \cdot A_{gross\_wp} \cdot d_{wp} = 240.00 \text{ in}^3$$

Case 1 - Dead Load + Positive Live Load:

$$M_{tot\_str\_pos} = 477.01 \text{ K} \cdot \text{ft}$$

$$H_{uw\_str\_pos} = 485.12 \text{ K}$$

$$f_{str\_pos} = \frac{M_{tot\_str\_pos}}{S_{pl}} \cdot \left( 12 \cdot \frac{\text{in}}{\text{ft}} \right) + \frac{H_{uw\_str\_pos}}{A_{gross\_wp}} = 40.02 \text{ ksi}$$

$$f_{str\_pos} = 40.02 \text{ ksi} < \phi_f \cdot F_y = 50 \text{ ksi} \quad \text{OK}$$

Case 2 - Dead Load + Negative Live Load:

$$M_{tot\_str\_neg} = 695.25 \text{ K} \cdot \text{ft}$$

$$H_{uw\_str\_neg} = -194.02 \text{ K}$$

$$f_{str\_neg} = \frac{M_{tot\_str\_neg}}{S_{pl}} \cdot \left( 12 \cdot \frac{\text{in}}{\text{ft}} \right) + \frac{|H_{uw\_str\_neg}|}{A_{gross\_wp}} = 41.23 \text{ ksi}$$

$$f_{str\_neg} = 41.23 \text{ ksi} < \phi_f \cdot F_y = 50 \text{ ksi} \quad \text{OK}$$

Control of Permanent Deflection - Splice Plates:

S6.10.4.2

Check the maximum normal stress on the gross section of the web splice plates at the service limit state due to the total web moment and horizontal force resultant:

$$f = \frac{M_{Total}}{S_{pl}} + \frac{H_w}{A_{gross\_wp}} \leq 0.95F_y$$

where:

$$S_{pl} = 240.00in^3$$

$$A_{gross\_wp} = 30.00in^2$$

Case 1 - Dead Load + Positive Live Load:

$$M_{tot\_ser\_pos} = 212.84K - ft$$

$$H_{uw\_ser\_pos} = 197.37K$$

$$f_{ser\_pos} = \frac{M_{tot\_ser\_pos}}{S_{pl}} \left( 12 \frac{in}{ft} \right) + \frac{H_{uw\_ser\_pos}}{A_{gross\_wp}} = 17.22ksi$$

$$f_{ser\_pos} = 17.22ksi < 0.95F_y = 47.50ksi \quad \text{OK}$$

Case 2 - Dead Load + Negative Live Load:

$$M_{tot\_ser\_neg} = 190.66K - ft$$

$$H_{uw\_ser\_neg} = -138.92K$$

$$f_{ser\_neg} = \frac{M_{tot\_ser\_neg}}{S_{pl}} \left( 12 \frac{in}{ft} \right) + \frac{|H_{uw\_ser\_neg}|}{A_{gross\_wp}} = 14.16ksi$$

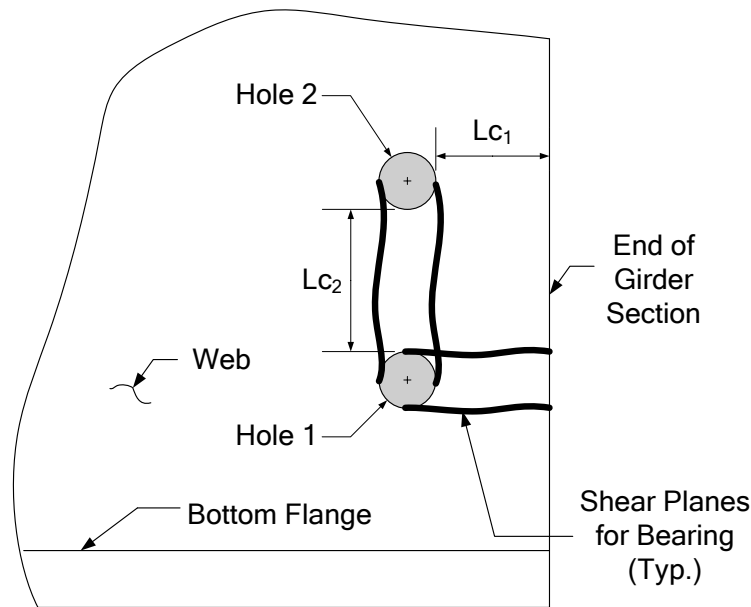
$$f_{ser\_neg} = 14.16ksi < 0.95F_y = 47.50ksi \quad \text{OK}$$

Web Bolts - Bearing Resistance at Bolt Holes:

S6.13.2.9

Since the girder web thickness is less than the sum of the thicknesses of the web splice plates, the girder web will control for the bearing check.

Check the bearing of the bolts on the connected material at the strength limit state assuming the bolts have slipped and gone into bearing. The design bearing strength of the girder web at the location of the extreme bolt in the splice is computed as the minimum resistance along the two orthogonal shear failure planes shown in Figure 4-10. The maximum force (vector resultant) acting on the extreme bolt is compared to this calculated strength, which is conservative since the components of this force parallel to the failure surfaces are smaller than the maximum force.



**Figure 4-10 Bearing Resistance - Girder Web**

S6.13.2.9

To determine the applicable equation for the calculation of the nominal bearing resistance, the clear distance between holes and the clear end distance must be calculated and compared to the value of two times the nominal diameter of the bolt. This check yields:

$$d_{bolt} = 0.875in \quad (\text{Design Step 4.1})$$

$$2 \cdot d_{bolt} = 1.75in$$

S6.13.2.6.6

The edge distance from the center of the hole to the edge of the girder is taken as 1.75". Therefore, the clear distance between the edge of the hole and the edge of the girder is computed as follows:



$$L_{c_1} = 1.75in - \frac{d_{hole}}{2}$$

$$d_{hole} = 0.9375in \quad (\text{Design Step 4.1})$$

$$L_{c_1} = 1.28in$$

The center-to-center distance between adjacent holes is 3". Therefore, the clear distance between holes is:

$$L_{c_2} = 3.00in - d_{hole} = 2.0625in$$

For standard holes, where either the clear distance between holes is less than 2.0d, or the clear end distance is less than 2.0d:

$$R_n = 1.2 \cdot L_c \cdot t \cdot F_u \quad \text{SEquation 6.13.2.9-2}$$

Thickness of the connected material:

$$t_w = 0.50in \quad (\text{Design Step 4.1})$$

Tensile strength of the connected material:

$$F_u = 65ksi \quad (\text{Design Step 4.1})$$

The nominal bearing resistance at the extreme bolt hole is as follows:

$$R_n = 1.2 \cdot L_{c_1} \cdot t_w \cdot F_u = 49.92K$$

The factored bearing resistance is:

$$R_r = \phi_{bb} \cdot R_n$$

$$\phi_{bb} = 0.80 \quad (\text{Design Step 4.1})$$

$$R_r = 39.94K$$

The controlling strength limit state resultant bolt force was previously computed to be:

$$P_{r\_str} = 37.97K < R_r = 39.94K \quad \text{OK}$$

### Bearing Resistance at Web Bolt Holes



Should the bearing resistance be exceeded, it is recommended that the edge distance be increased slightly in lieu of increasing the number of bolts or thickening the web.

#### Fatigue of Splice Plates:

For load-induced fatigue considerations, each detail is to satisfy:

$$\gamma \cdot (\Delta f) \leq \Delta F_n \quad \text{SEquation 6.6.1.2.2-1}$$

Fatigue is checked at the bottom edge of the splice plates, which by inspection are subject to a net tensile stress.

The normal stresses at the bottom edge of the splice plates due to the total positive and negative fatigue-load web moments and the corresponding horizontal force resultants are as follows:

$$f = \frac{M_{Total}}{S_{pl}} + \frac{H_{uw}}{A_{gross\_wp}}$$

From previous calculations in this design step:

$$S_{pl} = 240.00 \text{ in}^3$$

$$A_{gross\_wp} = 30.00 \text{ in}^2$$

#### Case 1 - Positive Live Load:

From Design Step 4.6:

$$M_{tot\_fat\_pos} = 59.05 \text{ K} \cdot \text{ft}$$

$$H_{uw\_fat\_pos} = 67.23 \text{ K}$$

$$f_{fat\_pos} = \frac{M_{tot\_fat\_pos}}{S_{pl}} \cdot \left( 12 \cdot \frac{\text{in}}{\text{ft}} \right) + \frac{H_{uw\_fat\_pos}}{A_{gross\_wp}} = 5.19 \text{ ksi}$$

#### Case 2 - Negative Live Load:

$$M_{tot\_fat\_neg} = -55.37 \text{ K} \cdot \text{ft}$$

$$H_{uw\_fat\_neg} = -48.47 K$$

$$f_{fat\_neg} = \frac{M_{tot\_fat\_neg}}{S_{pl}} \cdot \left( 12 \cdot \frac{in}{ft} \right) + \frac{H_{uw\_fat\_neg}}{A_{gross\_wp}} = -4.38 ksi$$

The total fatigue-load stress range at the bottom edge of the web splice plates is therefore:

$$\gamma \Delta f = |f_{fat\_pos}| + |f_{fat\_neg}| = 9.57 ksi$$

From Design Step 4.4, the fatigue resistance was determined as:

$$\Delta F_n = 16.0 ksi$$

The fatigue check is now completed as follows:

$$\gamma \Delta f = 9.57 ksi < \Delta F_n = 16.0 ksi \quad \text{OK}$$

#### Design Step 4.8 - Draw Schematic of Final Bolted Field Splice Design

Figure 4-11 shows the final bolted field splice as determined in this design example.

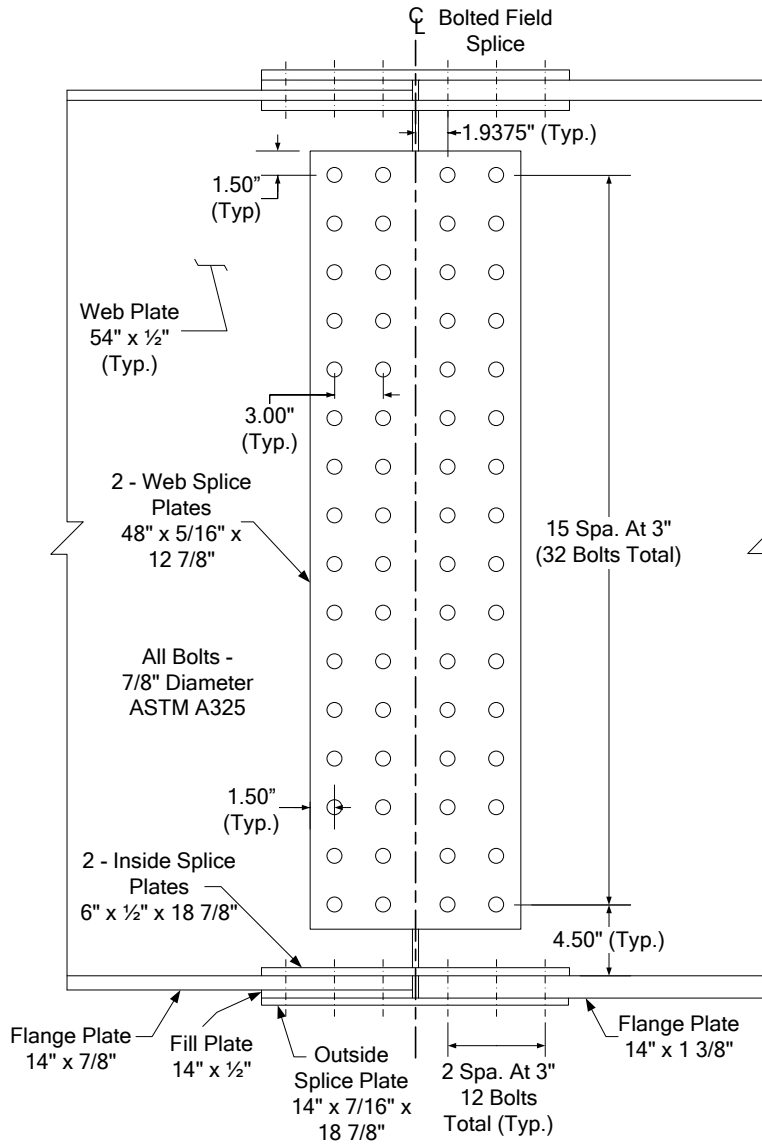


Figure 4-11 Final Bolted Field Splice Design

**Miscellaneous Steel Design Example**

**Design Step 5**

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(It should be noted that Design Step 5.4 presents a narrative description rather than design computations.)

Design Step 5 consists of various design computations associated with the steel girder but not necessarily required to design the actual plates of the steel girder. Such miscellaneous steel design computations include the following:

1. Shear connectors
2. Bearing stiffeners
3. Welded connections
4. Diaphragms and cross-frames
5. Lateral bracing
6. Girder camber

For this design example, computations for the shear connectors, a bearing stiffener, a welded connection, and a cross-frame will be presented. The other features must also be designed, but their design computations are not included in this design example.

Refer to Design Step 1 for introductory information about this design example. Additional information is presented about the design assumptions, methodology, and criteria for the entire bridge, including the design features included in this design step.

### **Design Step 5.1 - Design Shear Connectors**

*S6.10.10.1*

Since the steel girder has been designed as a composite section, shear connectors must be provided at the interface between the concrete deck slab and the steel section to resist the interface shear. For continuous composite bridges, shear connectors are normally provided throughout the length of the bridge. In the negative flexure region, since the longitudinal reinforcement is considered to be a part of the composite section, shear connectors must be provided.

*S6.10.10.1.1*

Studs or channels may be used as shear connectors. For this design example, stud shear connectors are being used throughout the length of the bridge. The shear connectors must permit a thorough compaction of the concrete to ensure that their entire surfaces are in contact with the concrete. In addition, the shear connectors must be capable of resisting both horizontal and vertical movement between the concrete and the steel.

The following figure shows the stud shear connector proportions, as well as the location of the stud head within the concrete deck.

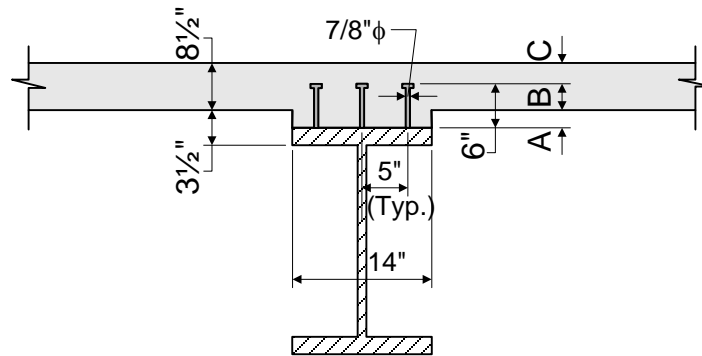


Figure 5-1 Stud Shear Connectors

Shear Connector Embedment			
Flexure Region	A	B	C
Positive	2.875"	3.125"	5.375"
Intermediate	2.25"	3.75"	4.75"
Negative	1.00"	5.00"	3.50"

Table 5-1 Shear Connector Embedment

**Shear Connector Layout**



It is common to use several stud shear connectors per transverse row along the top flange of the girder. The number of shear connectors per transverse row will depend on the top flange width. Refer to S6.10.10.1.3 for transverse spacing requirements.

**Shear Connector Length**



The stud shear connector length is commonly set such that its head is located near the middle of the deck slab. Refer to S6.10.10.1.4 for shear connector embedment requirements.

S6.10.10.1.1

The ratio of the height to the diameter of a stud shear connector must not be less than 4.0. For this design example, the ratio is computed based on the dimensions presented in Figure 5-1, as follows:

$$Height_{stud} = 6.0in$$

$$Diameter_{stud} = 0.875in$$

$$\frac{Height_{stud}}{Diameter_{stud}} = 6.86 \quad \text{OK}$$

S6.10.10.1.2

The pitch of the shear connectors must be determined to satisfy the fatigue limit state as specified in S6.10.10.2 and S6.10.10.3, as applicable. The resulting number of shear connectors must not be less than the number required to satisfy the strength limit state as specified in S6.10.10.4.

The pitch,  $p$ , of the shear connectors must satisfy the following equation:

$$p \leq \frac{n \cdot Z_r}{V_{sr}} \quad \text{SEquation 6.10.10.1.2-1}$$

The horizontal fatigue shear range per unit length,  $V_{sr}$ , is taken as:

$$V_{sr} = \sqrt{V_{fat}^2 + F_{fat}^2} \quad \text{SEquation 6.10.10.1.2-2}$$

$$V_{fat} = \frac{V_f \cdot Q}{I} \quad \text{SEquation 6.10.10.1.2-3}$$

$$F_{fat} = \max(F_{fat1}, F_{fat2}) \quad \text{(Radial fatigue shear range)}$$

$$F_{fat1} = \frac{A_{bot} \cdot \sigma_{flg} \cdot l}{w \cdot R} \quad \text{SEquation 6.40.40.1.2-4}$$

$$F_{fat2} = \frac{F_{rc}}{w} \quad \text{SEquation 6.10.10.1.2-5}$$

Since this bridge utilizes straight spans and has no skew, the radial fatigue shear range,  $F_{fat}$  is taken as zero.

$$V_{sr} = V_{fat}$$

According to S6.10.10.2, since the  $(ADTT)_{SL}$  exceeds 960 trucks per day, the Fatigue I load combination is to be used to design the stud shear connectors in combination with the fatigue shear resistance for infinite life.

In the positive flexure region, the maximum fatigue live load shear range is located at the abutment. The factored value for Fatigue I is computed as follows:



$$V_f = 1.50 \cdot (41.45K + 5.18K) = 69.95K \quad (\text{from live load analysis computer run})$$

The parameters  $I$  and  $Q$  are based on the short-term composite section and are determined using the deck within the effective flange width.

In the positive flexure region:

$$n = 3 \quad (\text{see Figure 5-1})$$

$$I = 68167.2in^4 \quad (\text{see Table 3-4}) \quad S6.10.1.1.1b$$

$$Q = \left[ \frac{(8.0in) \cdot (117.0in)}{8} \right] \cdot (62.375in - 51.790in) = 1238.4in^3$$

$$V_{fat} = \frac{V_f \cdot Q}{I} = 1.27 \frac{K}{in}$$

$$V_{sr} = V_{fat} = 1.27 \frac{K}{in}$$

$$Z_r = 5.5 \cdot d^2 \quad S6.10.10.2$$

$$d = 0.875in$$

$$5.5 \cdot d^2 = 4.21$$

Therefore:

$$Z_r = 4.21K$$

$$p = \frac{n \cdot Z_r}{V_{sr}} = 9.9in$$

In the negative flexure region:

$$n = 3 \quad (\text{see Figure 5-1})$$

C6.10.10.1.2

In the negative flexure region, the parameters  $I$  and  $Q$  may be determined using the reinforcement within the effective flange width for negative moment, unless the concrete slab is considered to be fully effective for negative moment in computing the longitudinal range of stress, as permitted in S6.6.1.2.1. For this design example,  $I$  and  $Q$  are assumed to be computed considering the concrete slab to be fully effective.

$$I = 134304.1in^4 \quad (\text{see Table 3-5})$$

$$Q = \left[ \frac{(8.0in) \cdot (117.0in)}{8} \right] \cdot (64.250in - 47.832in) = 1920.9in^3$$

$$V_f = 1.50 \cdot (0.00K + 46.53K) = 69.80K$$

(from live load analysis computer run)

$$V_{fat} = \frac{V_f \cdot Q}{I} = 1.00 \frac{K}{in}$$

$$V_{sr} = V_{fat} = 1.00 \frac{K}{in}$$

$$Z_r = 4.21K \quad (\text{see previous computation}) \quad S6.10.10.2$$

$$p = \frac{n \cdot Z_r}{V_{sr}} = 12.7in$$

Therefore, based on the above pitch computations to satisfy the fatigue limit state, use the following pitch throughout the entire girder length:

$$p = 9in$$

### Shear Connector Pitch



The shear connector pitch does not necessarily have to be the same throughout the entire length of the girder. Many girder designs use a variable pitch, and this can be economically beneficial.

However, for this design example, the required pitch for fatigue does not vary significantly over the length of the bridge. Therefore, a constant shear connector pitch of 9 inches will be used.

S6.10.10.1.2

In addition, the shear connectors must satisfy the following pitch requirements:

$$p \leq 24in \quad \text{OK}$$

$$p \geq 6 \cdot d$$

$$d = 0.875in$$

$$6 \cdot d = 5.25in \quad \text{OK}$$

## S6.10.10.1.3

For transverse spacing, the shear connectors must be placed transversely across the top flange of the steel section and may be spaced at regular or variable intervals.

Stud shear connectors must not be closer than 4.0 stud diameters center-to-center transverse to the longitudinal axis of the supporting member.

$$4 \cdot d = 3.5in$$

$$Spacing_{transverse} = 5.0in \quad (\text{see Figure 5-1}) \quad \text{OK}$$

In addition, the clear distance between the edge of the top flange and the edge of the nearest shear connector must not be less than 1.0 inch.

$$Distance_{clear} = \frac{14in}{2} - 5in - \frac{d}{2} = 1.56in \quad (\text{see Figure 5-1}) \quad \text{OK}$$

## S6.10.10.1.4

The clear depth of concrete cover over the tops of the shear connectors should not be less than 2.0 inches, and shear connectors should penetrate at least 2.0 inches into the deck. Based on the shear connector penetration information presented in Table 5-1, both of these requirements are satisfied.

The designer must detail any splice plates or section transitions to allow for the shear connector layout.

## S6.10.10.4.1

For the strength limit state, the factored resistance of the shear connectors,  $Q_r$ , is computed as follows:

$$Q_r = \phi_{sc} \cdot Q_n$$

$$\phi_{sc} = 0.85$$

S6.5.4.2

## S6.10.10.4.3

The nominal shear resistance of one stud shear connector embedded in a concrete slab is computed as follows:

$$Q_n = 0.5 \cdot A_{sc} \cdot \sqrt{f'_c \cdot E_c} \leq A_{sc} \cdot F_u$$

$$A_{sc} = \pi \cdot \frac{d^2}{4} = 0.601in^2$$

$$f'_c = 4.0 \text{ksi} \quad (\text{see Design Step 3.1}) \quad \text{S5.4.2.1}$$

$$E_c = 3834 \text{ksi} \quad (\text{see Design Step 3.3}) \quad \text{S5.4.2.4}$$

$$F_u = 60.0 \text{ksi} \quad \text{S6.4.4}$$

$$0.5 \cdot A_{sc} \cdot \sqrt{f'_c \cdot E_c} = 37.21 \text{K}$$

$$A_{sc} \cdot F_u = 36.06 \text{K}$$

Therefore:

$$Q_n = 36.06 \text{K}$$

$$Q_r = \phi_{sc} \cdot Q_n = 30.65 \text{K}$$

S6.10.10.4.1

The number of shear connectors provided over the section being investigated must not be less than the following:

$$n = \frac{P}{Q_r}$$

S6.10.10.4.2

For continuous spans that are composite for negative flexure in their final condition, the nominal shear force, P, must be calculated for the following regions:

1. Between points of maximum positive design live load plus impact moments and adjacent ends of the member
2. Between points of maximum positive design live load plus impact moment and centerlines of adjacent interior supports

For region 1:

$$P = \sqrt{P_p^2 + F_p^2}$$

where  $P_p$  is taken as the lesser of:

$$P_{1p} = 0.85 \cdot f'_c \cdot b_s \cdot t_s$$

or

$$P_{2p} = F_{yw} \cdot D \cdot t_w + F_{yt} \cdot b_{ft} \cdot t_{ft} + F_{yc} \cdot b_{fc} \cdot t_{fc}$$

where:

$$f'_c = 4.0ksi \quad (\text{see Design Step 3.1}) \quad S5.4.2.1$$

$$b_s = 117.0in \quad (\text{see Design Step 3.3})$$

$$t_s = 8.0in \quad (\text{see Design Step 3.1})$$

$$F_{yw} = 50ksi \quad (\text{see Design Step 3.1}) \quad STable 6.4.1-1$$

$$D = 54in \quad (\text{see Design Step 3.18})$$

$$t_w = 0.50in \quad (\text{see Design Step 3.18})$$

$$F_{yt} = 50ksi \quad (\text{see Design Step 3.1}) \quad STable 6.4.1-1$$

$$b_{ft} = 14in \quad (\text{see Design Step 3.18})$$

$$t_{ft} = 0.875in \quad (\text{see Design Step 3.18})$$

$$F_{yc} = 50ksi \quad (\text{see Design Step 3.1}) \quad STable 6.4.1-1$$

$$b_{fc} = 14in \quad (\text{see Design Step 3.18})$$

$$t_{fc} = 0.75in \quad (\text{see Design Step 3.18})$$

$$0.85 \cdot f'_c \cdot b_s \cdot t_s = 3182K$$

$$F_{yw} \cdot D \cdot t_w + F_{yt} \cdot b_{ft} \cdot t_{ft} + F_{yc} \cdot b_{fc} \cdot t_{fc} = 2488K$$

Therefore:

$$P_p = 2488K$$

For straight spans or segments,  $F_p$  may be taken equal to zero which gives:

$$P = P_p = 2488K$$

S6.10.10.4.1

Therefore, the number of shear connectors provided between the section of maximum positive moment and each adjacent end of the member must not be less than the following:

$$n = \frac{P}{Q_r} = 81.2$$

For region 2:

$$P = \sqrt{P_T^2 + F_T^2}$$

where  $P_T$  is taken as:

$$P_T = P_p + P_n$$

and where  $P_n$  is taken as the lesser of:

$$P_{1n} = F_{yw} \cdot D \cdot t_w + F_{yt} \cdot b_{ft} \cdot t_{ft} + F_{yc} \cdot b_{fc} \cdot t_{fc}$$

or

$$P_{2n} = 0.45 \cdot f'_c \cdot b_s \cdot t_s$$

where:

$$f'_c = 4.0 \text{ksi} \quad (\text{see Design Step 3.1}) \quad S5.4.2.1$$

$$b_s = 117.0 \text{in} \quad (\text{see Design Step 3.3})$$

$$t_s = 8.0 \text{in} \quad (\text{see Design Step 3.1})$$

$$F_{yw} = 50 \text{ksi} \quad (\text{see Design Step 3.1}) \quad STable 6.4.1-1$$

$$D = 54 \text{in} \quad (\text{see Design Step 3.18})$$

$$t_w = 0.50 \text{in} \quad (\text{see Design Step 3.18})$$

$$F_{yt} = 50 \text{ksi} \quad (\text{see Design Step 3.1}) \quad STable 6.4.1-1$$

$$b_{ft} = 14 \text{in} \quad (\text{see Design Step 3.18})$$

$$t_{ft} = 2.5 \text{in} \quad (\text{see Design Step 3.18})$$

$$F_{yc} = 50ksi \quad (\text{see Design Step 3.1}) \quad \text{STable 6.4.1-1}$$

$$b_{fc} = 14in \quad (\text{see Design Step 3.18})$$

$$t_{fc} = 2.75in \quad (\text{see Design Step 3.18})$$

$$F_{yw} \cdot D \cdot t_w + F_{yt} \cdot b_{ft} \cdot t_{ft} + F_{yc} \cdot b_{fc} \cdot t_{fc} = 5025K$$

$$0.45 \cdot f'_c \cdot b_s \cdot t_s = 1685K$$

Therefore:

$$P_n = 1685K$$

$$P_T = P_p + P_n = 4173K$$

For straight spans or segments,  $F_T$  may be taken equal to zero which gives:

$$P = P_T = 4173K$$

S6.10.10.4.1

Therefore, the number of shear connectors provided between the section of maximum positive moment and the centerline of the adjacent interior pier must not be less than the following:

$$n = \frac{P}{Q_r} = 136$$

The distance between the end of the girder and the location of maximum positive moment is approximately equal to:

$$L = 48.0ft \quad (\text{see Table 3-7})$$

Using a pitch of 9 inches, as previously computed for the fatigue limit state, and using the above length, the number of shear connectors provided is as follows:

$$n = 3 \cdot \frac{L \cdot \left(12 \frac{in}{ft}\right)}{p} = 192 \quad \text{OK}$$

Similarly the distance between the section of the maximum positive moment and the interior support is equal to:

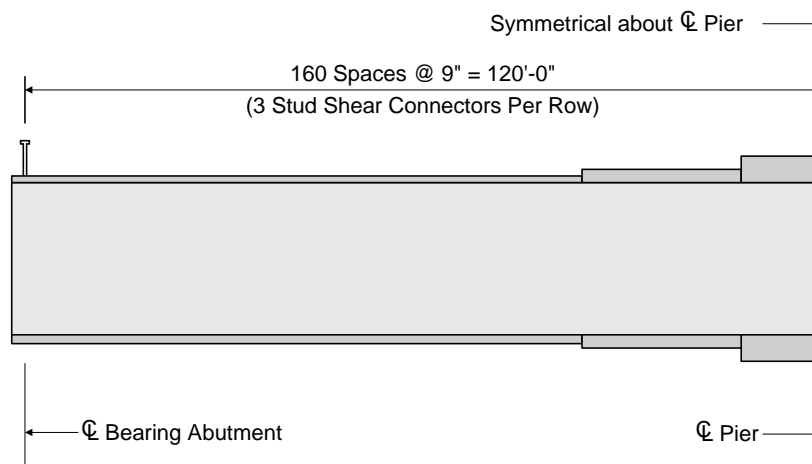
$$L = 120.0ft - 48.0ft = 72.0ft \quad (\text{see Table 3-7})$$

Using a pitch of 9 inches, as previously computed for the fatigue limit state, and using the above length, the number of shear connectors provided is as follows:

$$n = 3 \cdot \frac{L \cdot \left(12 \frac{\text{in}}{\text{ft}}\right)}{p} = 288 \quad \text{OK}$$

Therefore, using a pitch of 9 inches for each row, with three stud shear connectors per row, throughout the entire length of the girder satisfies both the fatigue limit state requirements of S6.10.10.1.2 and S6.10.10.2 and the strength limit state requirements of S6.10.10.4.

Therefore, use a shear stud spacing as illustrated in the following figure.



**Figure 5-2 Shear Connector Spacing**

### Design Step 5.2 - Design Bearing Stiffeners

S6.10.11.2.1

Bearing stiffeners are required to resist the bearing reactions and other concentrated loads, either in the final state or during construction.

For plate girders, bearing stiffeners are required to be placed on the webs at all bearing locations. At all locations supporting concentrated loads where the loads are not transmitted through a deck or deck system, either bearing stiffeners are to be provided or the web must satisfy the provisions of *SAppendix D6.5*.

Therefore, for this design example, bearing stiffeners are required at both abutments and at the pier. The following design of the abutment bearing stiffeners illustrates the bearing stiffener design procedure.

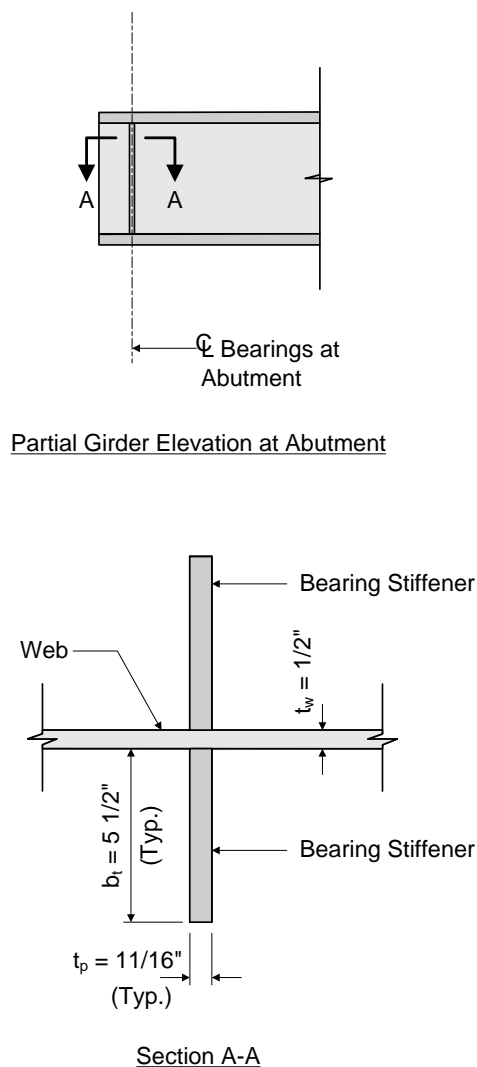


The bearing stiffeners in this design example consist of one plate welded to each side of the web. The connections to the web will be designed to transmit the full bearing force due to factored loads and is presented in Design Step 5.3.

The stiffeners extend the full depth of the web and, as closely as practical, to the outer edges of the flanges.

Each stiffener will be milled to fit against the flange through which it receives its reaction. Since the stiffeners are attached to cross-frames at this location, the stiffeners are positively attached by fillet welds to both flanges.

The following figure illustrates the bearing stiffener layout at the abutments.



**Figure 5-3 Bearing Stiffeners at Abutments**

**Bearing Stiffener Plates**

Bearing stiffeners usually consist of one plate connected to each side of the web. This is generally a good starting assumption for the bearing stiffener design. Then, if this configuration does not provide sufficient resistance, two plates can be used on each side of the web.

## S6.10.11.2.2

The projecting width,  $b_t$ , of each bearing stiffener element must satisfy the following equation. This provision is intended to prevent local buckling of the bearing stiffener plates.

$$b_t = 0.48 \cdot t_p \cdot \sqrt{\frac{E}{F_{ys}}}$$

$$t_p = \frac{11}{16} \text{ in} \quad (\text{see Figure 5-3})$$

$$E = 29000 \text{ ksi} \quad \text{S6.4.1}$$

$$F_{ys} = 50 \text{ ksi} \quad \text{STable 6.4.1-1}$$

$$0.48 \cdot t_p \cdot \sqrt{\frac{E}{F_{ys}}} = 7.95 \text{ in}$$

$$b_t = 5.5 \text{ in} < 7.95 \text{ in} \quad (\text{see Figure 5-3}) \quad \text{OK}$$

## S6.10.11.2.3

The bearing resistance must be sufficient to resist the factored reaction acting on the bearing stiffeners. The factored bearing resistance,  $R_{sbr}$ , is computed as follows:

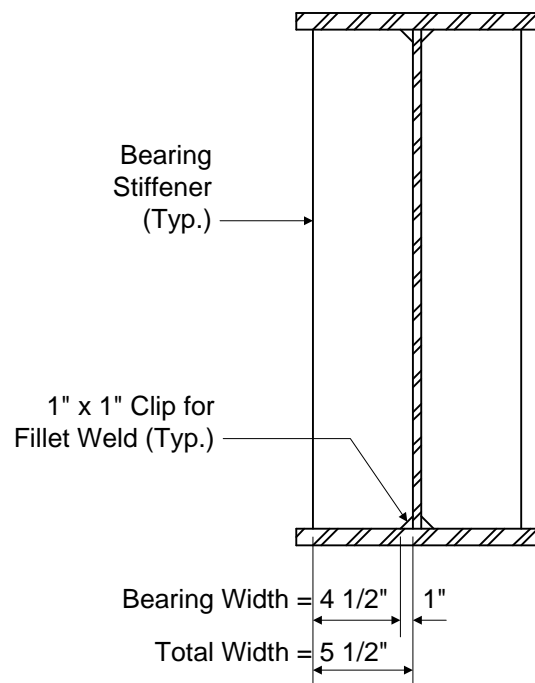
$$R_{sbr} = \phi_b \cdot R_{sbn}$$

$$R_{sbn} = 1.4 \cdot A_{pn} \cdot F_{ys}$$

$$\phi_b = 1.00 \quad \text{S6.5.4.2}$$

Part of the stiffener must be clipped to clear the web-to-flange weld. Thus the area of direct bearing is less than the gross area of the stiffener. The bearing area,  $A_{pn}$ , is taken as the area of the projecting elements of the stiffener outside of the web-to-flange

fillet welds but not beyond the edge of the flange. This is illustrated in the following figure:



**Figure 5-4 Bearing Width**

$$b_{brg} = b_t - 1.0in = 4.5in$$

$$A_{pn} = 2 \cdot b_{brg} \cdot t_p = 6.19in^2$$

$$F_{ys} = 50ksi$$

$$R_{sbr} = \phi_b \cdot 1.4 \cdot A_{pn} \cdot F_{ys} = 433.1K$$

The factored bearing reaction at the abutment is computed as follows, using load factors as presented in *S*Table 3.4.1-1 and *S*Table 3.4.1-2 and using reactions obtained from a computer analysis run:

$$React_{Factored} = (1.25 \cdot 68.7K) + (1.50 \cdot 11.0K) + (1.75 \cdot 110.5K) = 295.8K$$

$$< 433.1K \quad OK$$

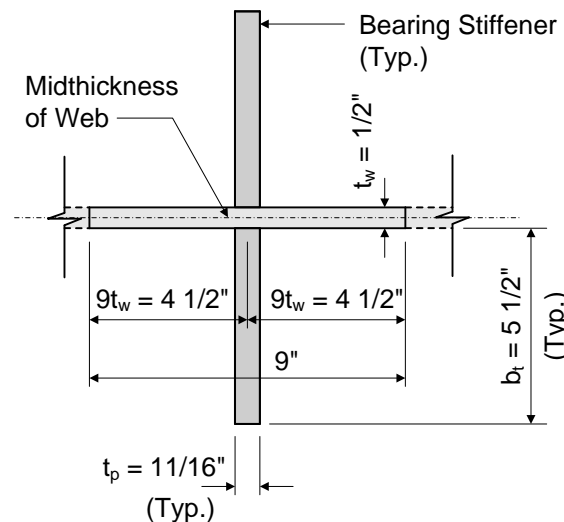
Therefore, the bearing stiffener at the abutment satisfies the bearing resistance requirements.

*S6.10.11.2.4 and S6.10.11.2.4a*

The final bearing stiffener check relates to the axial resistance of the bearing stiffeners. The factored axial resistance is determined as specified in S6.9.2.1. The radius of gyration is computed about the midthickness of the web, and the effective length is taken as  $0.75D$ , where  $D$  is the web depth.

#### S6.10.11.2.4b

For stiffeners consisting of two plates welded to the web, the effective column section consists of the two stiffener elements, plus a centrally located strip of web extending not more than  $9t_w$  on each side of the stiffeners. This is illustrated in the following figure:



**Figure 5-5 Bearing Stiffener Effective Section**

$$P_r = \phi_c \cdot P_n \quad \text{S6.9.2.1}$$

$$\phi_c = 0.95 \quad \text{S6.5.4.2}$$

The limiting slenderness ratio must be checked, as follows:

$$A_g = (0.6875in \cdot 11.5in) + (8.3125in \cdot 0.5in) = 12.06in^2$$

$$I_s = \frac{[0.6875in \cdot (11.5in)^3] + [8.3125in \cdot (0.5in)^3]}{12} = 87.22in^4$$

$$r_s = \sqrt{\frac{I_s}{A_g}} = 2.69in$$

$$K \cdot l = (0.75) \cdot (54in)$$

$$\frac{K \cdot l}{r_s} = \frac{(0.75) \cdot (54in)}{2.69in} = 15.1 < 120 \quad \text{OK}$$

Next, the elastic critical flexural buckling resistance,  $P_e$ , is computed, as follows:

$$P_e = \frac{\pi^2 E}{\left(\frac{kl}{r_s}\right)^2} \cdot A_g \quad \text{S6.9.4.1.2}$$

$$P_e = \frac{\pi^2 (29,000)}{(15.1)^2} \cdot (12.06) = 15,139 \text{ kips}$$

Next the equivalent nominal yield resistance,  $P_o$ , is computed. The slender element reduction factor,  $Q$ , is taken equal to 1.0 for bearing stiffeners.

$$P_o = Q F_{ys} A_g$$

$$P_o = (1.0)(50.0)(12.06) = 603 \text{ kips}$$

$$\frac{P_e}{P_o} = \frac{15,139}{603} = 25.1 > 0.44$$

Since  $P_e/P_o$  is greater than 0.44,  $P_n$  is computed using the following equation:

$$P_n = \left[ 0.658 \left( \frac{P_o}{P_e} \right) \right] P_o \quad \text{S6.9.4.1.1}$$

$$P_n = \left[ 0.658 \left( \frac{603}{15,139} \right) \right] 603 = 593 \text{ kips}$$

Finally, the factored resistance is computed, as follows:

$$P_r = \phi_c \cdot P_n \quad \text{S6.9.2.1}$$

$$P_r = 0.95 \cdot (593 \text{ kips}) = 563 \text{ kips} > R_u = 295.8 \text{ kips} \quad \text{OK}$$

Therefore, the bearing stiffener at the abutment satisfies the axial bearing resistance requirements.

The bearing stiffener at the abutment satisfies all bearing stiffener requirements. Therefore, use the bearing stiffener as presented in Figures 5-3 and 5-4.

## Design Step 5.3 - Design Welded Connections

S6.13.3

Welded connections are required at several locations on the steel superstructure. Base metal, weld metal, and welding design details must conform to the requirements of the *ANSI/AASHTO/AWS Bridge Welding Code D1.5*.

For this design example, two fillet welded connection designs will be presented using E70 electrodes:

1. Welded connection between the bearing stiffeners and the web.
2. Welded connection between the web and the flanges.

For the welded connection between the bearing stiffeners and the web, the fillet weld must resist the factored reaction computed in Design Step 5.2.

$$React_{Factored} = 295.8K$$

Assume a fillet weld thickness of 1/4 inches.

$$Thickness_{Weld} = 0.25in$$

### Fillet Weld Thickness



In most cases, the minimum weld thickness, as specified in Table 5-2, provides a welded connection that satisfies all design requirements. Therefore, the minimum weld thickness is generally a good starting point when designing a fillet weld.

S6.13.3.2.4b

The resistance of the fillet weld in shear is the product of the effective area and the factored resistance of the weld metal. The factored resistance of the weld metal is computed as follows:

$$R_r = 0.6 \cdot \phi_{e2} \cdot F_{exx}$$

$$\phi_{e2} = 0.80$$

S6.5.4.2

$$F_{e70} = 70ksi$$

SC6.13.3.2.1

$$R_r = 0.6 \cdot \phi_{e2} \cdot F_{e70} = 33.60ksi$$

S6.13.3.2.4b

## S6.13.3.3

The effective area equals the effective weld length multiplied by the effective throat. The effective throat is the shortest distance from the joint root to the weld face.

$$Length_{Eff} = 4 \cdot (54in - 2in) = 208.0in$$

$$Throat_{Eff} = \frac{Thickness_{Weld}}{\sqrt{2}} = 0.177in$$

$$Area_{Eff} = Length_{Eff} \cdot Throat_{Eff} = 36.77in^2$$

## S6.13.3.2.4b

The resistance of the fillet weld is then computed as follows:

$$Resistance = R_r \cdot Area_{Eff} = 1235K \quad \text{OK}$$

## S6.13.3.4

For material 0.25 inches or more in thickness, the maximum size of the fillet weld is 0.0625 inches less than the thickness of the material, unless the weld is designated on the contract documents to be built out to obtain full throat thickness.

For the fillet weld connecting the bearing stiffeners to the web, the bearing stiffener thickness is 11/16 inches and the web thickness is 1/2 inches. Therefore, the maximum fillet weld size requirement is satisfied.

## S6.13.3.4

The minimum size of fillet welds is as presented in Table 5-2. In addition, the weld size need not exceed the thickness of the thinner part joined.

STable 6.13.3.4-1

Minimum Size of Fillet Welds	
Base Metal Thickness of Thicker Part Joined (T) (Inches)	Minimum Size of Fillet Weld (Inches)
$T \leq 3/4$	1/4
$T > 3/4$	5/16

Table 5-2 Minimum Size of Fillet Welds

In this case, the thicker part joined is the bearing stiffener plate, which is 11/16 inches thick. Therefore, based on Table 5-2, the minimum size of fillet weld is 1/4 inch, and this requirement is satisfied.

## S6.13.3.5

The minimum effective length of a fillet weld is four times its size and in no case less than 1.5 inches. Since the length of each weld is 54 in – 2 in = 52 in, this requirement is also satisfied.

Since the stiffeners are located at an abutment, fatigue of the base metal adjacent to these welds need not be checked.

Since all weld design requirements are satisfied, use a 1/4 inch fillet weld for the connection of the bearing stiffeners to the web.

## S6.13.3

For the welded connection between the web and the flanges, the fillet weld must resist a factored horizontal shear per unit length based on the following equation:

$$v = \frac{VQ}{I}$$

This value is greatest at the pier, where the factored shear has its highest value.

The following computations are for the welded connection between the web and the top flange. The welded connection between the web and the bottom flange is designed in a similar manner.

The shear is computed based on the individual section properties and load factors for each loading, as presented in Design Steps 3.3 and 3.6:

For the noncomposite section, the factored horizontal shear is computed as follows:

$$V_{Noncomp} = (1.25 \cdot 114.7K) = 143.4K$$

$$Q_{Noncomp} = (14in \cdot 2.5in) \cdot (58.00in - 28.718in) = 1024.9in^3$$

$$I_{Noncomp} = 65426.6in^4$$

$$v_{Noncomp} = \frac{V_{Noncomp} \cdot Q_{Noncomp}}{I_{Noncomp}} = 2.25 \frac{K}{in}$$

For the composite section, the factored horizontal shear is computed as follows:



$$V_{Comp} = (1.25 \cdot 16.4K) + (1.50 \cdot 19.8K) + (1.75 \cdot 131.4K) = 280.2K$$

$$Q_{Comp} = (14in \cdot 2.5in) \cdot (58.00in - 33.137in) = 870.2in^3$$

$$I_{Comp} = 80985.1in^4$$

$$v_{Comp} = \frac{V_{Comp} \cdot Q_{Comp}}{I_{Comp}} = 3.01 \frac{K}{in}$$

Based on the above computations, the total factored horizontal shear is computed as follows:

$$v_{Total} = v_{Noncomp} + v_{Comp} = 5.26 \frac{K}{in}$$

Assume a fillet weld thickness of 5/16 inches.

$$Thickness_{Weld} = 0.3125in$$

S6.13.3.2.4b

The resistance of the fillet weld in shear is the product of the effective area and the factored resistance of the weld metal. The factored resistance of the weld metal was previously computed as follows:

$$R_r = 0.6 \cdot \phi_{e2} \cdot F_{e70} = 33.60ksi$$

S6.13.3.3

The effective area equals the effective weld length multiplied by the effective throat. The effective throat is the shortest distance from the joint root to the weld face. In this case, the effective area is computed per unit length, based on the use of one weld on each side of the web.

$$Throat_{Eff} = \frac{Thickness_{Weld}}{\sqrt{2}} = 0.221in$$

$$Area_{Eff} = 2 \cdot Throat_{Eff} = 0.442 \frac{in^2}{in}$$

The resistance of the fillet weld is then computed as follows:

S6.13.3.2.4b

$$Resistance = R_r \cdot Area_{Eff} = 14.85 \frac{K}{in} \quad \text{OK}$$

## S6.13.3.4

For material 0.25 inches or more in thickness, the maximum size of the fillet weld is 0.0625 inches less than the thickness of the material, unless the weld is designated on the contract documents to be built out to obtain full throat thickness.

For the fillet weld connecting the web to the flanges, the web thickness is 0.5 inches, the minimum flange thickness is 0.75 inches, and the maximum flange thickness is 2.75 inches. Therefore, the maximum fillet weld size requirement is satisfied.

## S6.13.3.4

The minimum size of fillet welds is as presented in Table 5-2. In addition, the weld size need not exceed the thickness of the thinner part joined.

In this case, the thicker part joined is the flange, which has a minimum thickness of 0.75 inches and a maximum thickness of 2.75 inches. Therefore, based on Table 5-2, the minimum size of fillet weld is 5/16 inch, and this requirement is satisfied.

## S6.13.3.5

The minimum effective length of a fillet weld is four times its size and in no case less than 1.5 inches. Therefore, this requirement is also satisfied.

Since all weld design requirements are satisfied, use a 5/16 inch fillet weld for the connection of the web and the top flange. The welded connection between the web and the bottom flange is designed in a similar manner.

## S6.6.1.2.5

Load-induced fatigue must be considered in the base metal at a welded connection.

The specific fatigue considerations depend on the unique characteristics of the girder design. Specific fatigue details and detail categories are explained and illustrated in *S*Table 6.6.1.2.3-1. For continuous fillet welded connections parallel to the direction of the applied stress (e.g. flange-to-web fillet welds), the fatigue category is Category B.

In Design Step 3.14 for the positive moment region, the fatigue check is illustrated for the fillet-welded connection of the transverse intermediate stiffeners to the girder. A similar procedure must be followed for checking the base metal at all welded connections subject to a net tensile stress according to the criteria given in *S*6.6.1.2.1.

Additional weld connection requirements are presented in *S*6.13.3 and in *ANSI/AASHTO/AWS Bridge Welding Code D1.5*.

## Design Step 5.4 - Design Cross-frames

S6.7.4.1

Diaphragms and cross-frames may be placed at the following locations along the bridge:

- At the end of the structure
- Across interior supports
- Intermittently along the span

### Diaphragm or Cross-frame Spacing



A common rule of thumb, based on previous editions of the AASHTO Specifications, is to use a maximum diaphragm or cross-frame spacing of around 25 feet. Based on C6.7.4.1, the arbitrary requirement for a strict 25 foot maximum spacing has been replaced by a requirement for a rational analysis to determine the necessary cross-frame spacing for straight I-girder bridges.

For this design example, cross-frames are used at a spacing of 20 feet.

The need for diaphragms or cross-frames must be investigated for:

- All stages of assumed construction procedures
- The final condition

### Difference Between Diaphragms and Cross-frames



The difference between diaphragms and cross-frames is that diaphragms consist of a transverse solid web member, while cross-frames consist of a transverse truss framework.

Both diaphragms and cross-frames connect adjacent longitudinal flexural components.

When investigating the need for diaphragms or cross-frames and when designing them, the following must be considered:

- Transfer of lateral wind loads from the bottom of the girder to the deck and from the deck to the bearings
- Stability of the bottom flange for all loads when it is in compression
- Stability of the top flange in compression prior to curing of the deck
- Consideration of any flange lateral bending effects
- Distribution of vertical dead and live loads applied to the structure
- Sufficient stiffness in the cross-frames to provide the necessary stability to the girder system

At a minimum, the Specifications require that diaphragms and cross-frames be designed for the following:

- Transfer of wind loads according to the provisions of S4.6.2.7
- Applicable slenderness requirements in S6.8.4 or S6.9.3

S6.7.4.1 & S6.13.1

When diaphragms or cross-frames are included in the structural model used to determine force effects, or are designed for explicitly calculated force effects from the results of a separate investigation, the members and end connections are to be designed for the calculated factored member force effects.

In addition, connection plates must satisfy the requirements of S6.6.1.3.1.

#### Cross-frame Types



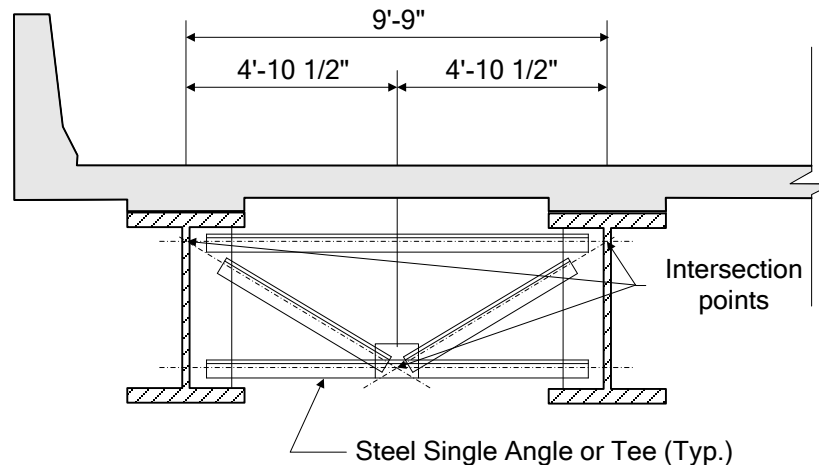
K-type cross-frames are as shown in Figure 5-6, while X-type cross-frames have an X-shape configuration of angles or structural tees rather than a K-shape configuration of angles or structural tees.

For guidelines on when to use each cross-frame type, see RM Section 6.3.2.9.5.

For this design example, K-type cross-frames will be used.

The geometry of a typical K-type cross-frame for an intermediate cross-frame is illustrated in Figure 5-6.

As illustrated in Figure 5-6, the intersection of the centroidal axes of the two diagonals coincides with the centroidal axis of the bottom strut. In addition, the intersection of the centroidal axis of each diagonal and the centroidal axis of the top strut coincides with the vertical centerlines of the girders.



**Figure 5-6 K-Type Cross-frame**

Based on previous computations in Design Step 3.17 for the negative moment region, the unfactored wind load is computed as follows:

$$W = \frac{\eta \cdot \gamma \cdot P_D \cdot d}{2} \quad C4.6.2.7.1$$

$$\eta = 1.0 \quad S1.3$$

$$\gamma = 1.40 \quad (\text{for Strength III Limit State - controls}) \quad STable 3.4.1-1$$

$$P_D = 0.0325ksf \quad (\text{see Design Step 3.17})$$

$$d = 4.9375ft \quad (\text{maximum value})$$

$$W = \frac{\eta \cdot \gamma \cdot P_D \cdot d}{2} = 0.112 \frac{K}{ft}$$

The horizontal wind force applied to the brace point may then be computed as specified in C4.6.2.7.1, as follows:

$$P_w = W \cdot L_b \quad C4.6.2.7.1$$

$$W = 0.112 \frac{K}{ft}$$

$$L_b = 20 \text{ ft}$$

$$P_w = W \cdot L_b = 2.25 \text{ K}$$

For the design of the cross-frame members and their connections, the reader is referred to RM Sections 6.6.3 and 6.6.4 for a more detailed discussion along with illustrative examples.

## Bearing Design

### Design Step 6

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### Design Step 6.1 - Obtain Design Criteria

For this bearing design example, an abutment bearing was chosen. It was decided that the abutment would have expansion bearings. Therefore, the bearing design will be for an expansion bearing.

Refer to Design Step 1 for introductory information about this design example. Additional information is presented about the design assumptions, methodology, and criteria for the entire bridge, including the bearing design.

The following units are defined for use in this design example:

$$K = 1000lb \quad ksi = \frac{K}{in^2}$$

For bearing design, the required design criteria includes:

1. Longitudinal and transverse movement
2. Longitudinal, transverse, and vertical rotation
3. Longitudinal, transverse, and vertical loads

Most of the above information is typically obtained from the superstructure design software output, which is the case for this bearing design (first trial of girder design):

$$DL_{serv} = 78.4K \quad \text{Service I load combination dead load}$$

$$LL_{serv} = 110.4K \quad \text{Service I load combination live load (including dynamic load allowance)}$$

$$\theta_{sx} = 0.0121rad \quad \text{Service I load combination total rotation about the transverse axis (see Figure 6-1)}$$

### Design Step 6.2 - Select Optimum Bearing Type

S14.6.2

Selecting the optimum bearing type depends on the load, movement capabilities, and economics. Refer to *S*Table 14.6.2-1 and *S*Figure 14.6.2-1 for guidance on selecting the most practical bearing type. For the abutment bearing design, a steel-reinforced elastomeric bearing was selected. If the loads were considerably larger, pot bearings, which are more expensive than elastomeric bearings, would be an option.



### Design Step 6.3 - Select Preliminary Bearing Properties

Once the most practical bearing type has been selected, the preliminary bearing properties must be defined. The bearing properties are obtained from the Specifications, as well as from past experience. The following preliminary bearing properties were selected:

#### Bearing Pad Configuration

Pad length (bridge longitudinal direction):

$$L_{pad} = 14in$$

Pad width (bridge transverse direction):

$$W_{pad} = 15in$$

Elastomer cover thickness:

$$h_{r\ cover} = 0.25in$$

Elastomer internal layer thickness:

$$h_{r\ internal} = 0.375in$$

Number of steel reinforcement layers:

$$N_{stayers} = 9$$

Steel reinforcement thickness:

$$h_{reinf} = 0.1196in$$

S14.7.6.2 & S14.7.5.2

#### Material Properties

Elastomer nominal hardness:

$$H_{shoreA} = 50$$

S14.7.6.2-1

Elastomer specified shear modulus (as described in S14.7.6.2, use the least favorable value from the range):

$$G = 0.095ksi - 0.130ksi$$

*S*Table 14.7.6.2-1

Elastomer creep deflection at 25 years divided by the initial deflection:

$$C_d = 0.25$$

Steel reinforcement yield strength:

$$F_y = 50ksi$$

### Design Step 6.4 - Select Design Method (A or B)

*C*14.7.5.1

For this design example, Method A will be used. Method A usually results in a bearing with a lower capacity than a bearing designed with Method B. However, Method B requires additional testing and quality control. Method A is described in *S*14.7.6, while Method B is described in *S*14.7.5.

### Design Step 6.5 - Compute Shape Factor

*S*14.7.6.1 & *S*14.7.5.1

The shape factor for individual elastomer layers is the plan area divided by the area of perimeter free to bulge.

*S*14.7.6.1 & *S*14.7.5.1

For steel-reinforced elastomeric bearings, the following requirements must be met prior to calculating the shape factor:

1. All internal layers of elastomer must be the same thickness.
2. The thickness of the cover layers cannot exceed 70 percent of the thickness of the internal layers.

From Design Step 6.3, all internal elastomer layers are the same thickness, which satisfies Requirement 1. The following calculation verifies that Requirement 2 is satisfied:

$$0.70 \cdot h_{r\text{internal}} = 0.26in$$

$$h_{r\text{cover}} = 0.25in \quad \text{OK}$$

For rectangular bearings without holes, the shape factor for the  $i^{\text{th}}$  layer is:

$$S_i = \frac{L \cdot W}{2 \cdot h_{r_i} \cdot (L + W)} \quad S14.7.5.1$$

The shape factor for the cover layers is then:

$$S_{cov} = \frac{L_{pad} \cdot W_{pad}}{2 \cdot h_{r_{cover}} \cdot (L_{pad} + W_{pad})}$$

$$S_{cov} = 14.48$$

The shape factor for the internal layers is then:

$$S_{int} = \frac{L_{pad} \cdot W_{pad}}{2 \cdot h_{r_{internal}} \cdot (L_{pad} + W_{pad})}$$

$$S_{int} = 9.66$$

Check if:

S14.7.6.1

$$\frac{S_i^2}{n} < 22$$

$$S_i = 9.66$$

$$n = 9$$

since each exterior layer of elastomer is thicker than one-half of the interior elastomer layer thickness, they each count as an additional one-half layer for this calculation.

$$\frac{S_i^2}{n} = \frac{(9.66)^2}{9} = 10.4 < 22 \quad \text{OK}$$

### Design Step 6.6 - Check Compressive Stress or Combined Compression, Rotation, and Shear

S14.7.6.3.2

The compressive stress check limits the compressive stress in the elastomer at the service limit state as follows:

$$\sigma_s \leq 1.25 \text{ksi} \quad \text{and} \quad \sigma_s \leq 1.25 \cdot G \cdot S_i$$

The compressive stress is taken as the total reaction at one of the abutment bearings for the service limit state divided by the elastomeric pad plan area. The service limit state dead and live load reactions are obtained from the BrD superstructure output. The shape factor used in the above equation should be for an interior elastomer layer.

Service I load combination dead load:

$$DL_{serv} = 78.4K$$

$$\sigma_d = \frac{DL_{serv}}{L_{pad} \cdot W_{pad}}$$

$$\sigma_d = 0.373ksi$$

Service I load combination live load (including dynamic load allowance):

$$LL_{serv} = 110.4K$$

$$\sigma_L = \frac{LL_{serv}}{L_{pad} \cdot W_{pad}}$$

$$\sigma_L = 0.526ksi$$

$$\sigma_s = \sigma_d + \sigma_L$$

$$\sigma_s = 0.899ksi$$

$$1.25 \cdot G \cdot S_{int} = 1.14ksi \quad \text{OK}$$

### Design Step 6.7 - Check Compressive Deflection

S14.7.5.3.6

The compressive deflections due to the live and dead loads at the service limit state are obtained from the following equations:

$$\delta_L = \sum \varepsilon_{Li} \cdot h_{ri}$$

$$\delta_d = \sum \varepsilon_{di} \cdot h_{ri}$$

CTable 14.7.6.3.3-1

For this design example, the compressive strains were approximated from CTable 14.7.6.3.3-1 for 50 durometer reinforced bearings using compressive stresses of 0.526 ksi (live load) and 0.373 ksi (dead load) and shape factors of 9.66 for the interior layers

and 14.48 for the cover layers. The compressive strain for the cover layer is approximated using a shape factor of 12 since  $S_{cov} > 12$ .

$$\varepsilon_{int} = 0.025 (live), 0.015 (dead)$$

$$\varepsilon_{cov} = 0.0225 (live), 0.015 (dead)$$

The deflections are then:

$$\delta_{L,d} = 2 \cdot \varepsilon_{cov} \cdot h_{r_{cover}} + 8 \cdot \varepsilon_{int} \cdot h_{r_{internal}}$$

$$\delta_L = 0.086 in$$

$$\delta_d = 0.052 in$$

The effects of creep should also be considered. For this design example, material-specific data is not available. Therefore, calculate the creep deflection value as follows:

$$\delta_{creep} = a_{cr} \cdot \delta_d \quad \text{SEquation 14.7.5.3.6-3}$$

$$a_{cr} = 0.25 \quad \text{STable 14.7.6.2-1}$$

$$\delta_{creep} = 0.013 in$$

The total deflection is then:

$$\delta_{total} = \delta_L + \delta_d + \delta_{creep}$$

$$\delta_{total} = 0.151 in$$

C14.7.5.3.6

SC14.7.5.3.6 suggests that the maximum relative deflection across a joint be limited to 0.125 in. For this example, if we performed the above deflection computations just for live load, the instantaneous deflection would be 0.086 in, much less than 0.125 in. This constraint is meant to limit the small step that occurs when the elastomeric bearing instantaneously deflects due to live load passage.

S14.7.6.3.3

The compressive deflection in an internal layer of a steel-reinforced elastomeric bearing at the service limit state without dynamic load allowance shall not exceed  $0.09h_{ri}$ .

In order to reduce design steps, the above requirement will be checked using the deflection calculated for the service limit state including dynamic load allowance. If the

compressive deflection is greater than  $0.09h_{ri}$ , then the deflection without dynamic load allowance would need to be calculated.

$$\delta_{layer} = \varepsilon \cdot h_r$$

$$\delta_{intlayer} = 0.015in$$

$$0.09h_{rinternal} = 0.034in \quad \text{OK}$$

$$\delta_{coverlayer} = 0.009in$$

$$0.09h_{rcover} = 0.022in \quad \text{OK}$$

### Design Step 6.8 - Check Shear Deformation

*S14.7.6.3.4 and C14.7.5.3.2*

The shear deformation is checked to ensure that the bearing is capable of allowing the anticipated horizontal bridge movement. Also, the shear deformation is limited in order to avoid rollover at the edges and delamination due to fatigue caused by cyclic expansion and contraction deformations. The horizontal movement for this bridge design example is based on thermal effects only in the longitudinal direction. The thermal movement is taken from Design Step 7.6 for the controlling movement, which is contraction. For this particular example, other criteria that could add to the shear deformation include construction tolerances, braking force, and longitudinal wind if applicable. For this particular example, one factor that can reduce the amount of shear deformation is the abutment deflection. Since the abutment height is relatively short and the shear deformation is relatively small, the abutment deflection will not be taken into account. Refer to C14.7.5.3.2 for a more detailed description of substructure deflections and how they affect the shear deformation.

The bearing must satisfy:

$$h_{rt} \geq 2 \cdot \Delta_s$$

$$h_{rt} = 2 \cdot h_{rcover} + 8 \cdot h_{rinternal}$$

$$h_{rt} = 3.50in$$

$$\Delta_{contr} = 0.636in \quad \text{from Design Step 7.6 for thermal contraction}$$

$$\gamma_{TU} = 1.20 \quad \text{for the service limit state} \quad \text{Table 3.4.1-1 \& S3.4.1}$$

$$\Delta_s = \gamma_{TU} \cdot \Delta_{contr}$$

$$\Delta_s = 0.76in$$

$$2 \cdot \Delta_s = 1.53in$$

$$3.50in \geq 1.53in \quad \text{OK}$$

### Design Step 6.9 - Check Rotation or Combined Compression, Rotation, and Shear

S14.7.6.3.5

From C14.7.6.3.5a, "design for rotation in Method A is implicit in the geometric and stress limits given." Because of this, Method A no longer has a separate rotation criteria for steel-reinforced elastomeric bearings.

### Design Step 6.10 - Check Stability

S14.7.6.3.6

The total thickness of the pad shall not exceed the least of L/3 or W/3.

$$\frac{L_{pad}}{3} = 4.67in \quad \frac{W_{pad}}{3} = 5.00in$$

The total thickness of the pad based on the preliminary dimensions is:

$$h_{total} = 2 \cdot h_{cover} + 8 \cdot h_{internal} + N_{layers} \cdot h_{reinf}$$

$$h_{total} = 4.5764in \quad \text{OK}$$

### Design Step 6.11 - Check Reinforcement

S14.7.6.3.7 and S14.7.5.3.5

The thickness of the steel reinforcement must be able to sustain the stresses induced by compression in the bearing. The reinforcement thickness must also satisfy the requirements of the *AASHTO LRFD Bridge Construction Specifications* and also must be greater than 0.0625 in, as per AASHTO M 251.

For the service limit state:

$$h_s \geq \frac{3h_{ri} \cdot \sigma_s}{F_y}$$

$$h_{ri} = 0.375in$$

$$\sigma_s = 0.899ksi \quad F_y = 50ksi$$

$$\frac{3 \cdot h_{ri} \cdot \sigma_s}{F_y} = 0.0202in$$

$$h_{reinf} = 0.1196in \quad \text{OK}$$

For the fatigue limit state:

$$h_s \geq \frac{2h_{ri} \cdot \sigma_L}{\Delta F_{TH}}$$

From Design Step 6.6, the service average compressive stress due to live load only is:

$$\sigma_L = 0.526ksi$$

$$\Delta F_{TH} = 24.0ksi \quad (\text{Detail Category A})$$

*S*Table 6.6.1.2.5-3

$$\frac{2 \cdot h_{ri} \cdot \sigma_L}{\Delta F_{TH}} = 0.0164in$$

$$h_{reinf} = 0.1196in$$

## Design Step 6.12 - Design for Extreme Event Provisions

*S*14.7.6.3.8

In accordance with *S*14.7.6.3.8, expansion bearings designed using Method A shall be provided with adequate seismic and other extreme event resistant anchorage to resist the horizontal forces in excess of those accommodated by shear in the pad, unless the bearing is intended to act as a fuse or unless irreplaceable damage is permitted.

In addition, the sole plate and the base plate must be wider than the bearing pad to accommodate the anchor bolts. Inserts through the elastomer are not permitted, unless approved by the Engineer.



For this design example, neither seismic loads nor other extreme event loads are provided. However, this step must be considered during bridge design.

### Design Step 6.13 - Design for Anchorage

Design Method A no longer requires an anchorage check for the deformed pad.

### Design Step 6.14 - Design Anchorage for Fixed Bearings

The abutment bearings are expansion in the longitudinal direction but fixed in the transverse direction. Therefore, the bearings must be restrained in the transverse direction. The factored resistance of the anchor bolts shall be greater than the factored force effects due to Strength I or II load combinations and to all applicable extreme event load combinations. However, based on S3.10.9.2, the horizontal connection force in the restrained direction cannot be less than 0.15 times the vertical reaction due to the tributary permanent load and the tributary live loads assumed to exist during an earthquake (which controls over Strength I and II). In addition, since all abutment bearings are restrained in the transverse direction, the tributary permanent load can be taken as the reaction at the bearing. Also,  $\gamma_{EQ}$  is assumed to be zero. Therefore, no tributary live loads will be considered. This transverse load will be used to design the bearing anchor bolts for this design example.

For the controlling girder (interior):

$$DL_{serv} = 78.4K$$

The maximum transverse horizontal earthquake load per bearing is then:

$$H_{EQ} = 0.15 \cdot DL_{serv}$$

$$H_{EQ} = 11.8K$$

C14.8.3.1

Based on SC14.8.3.1, "Elastomeric bearings may be left without anchorage if adequate friction is available. A design coefficient of friction of 0.2 may be assumed between elastomer and clean concrete or steel." This can be interpreted to mean that if the calculated horizontal force is less than 20% of the applied vertical force, the engineer can conclude that anchorage is not necessary. For the purposes of this example, it is assumed that anchorage is necessary and the anchor bolts will be designed.

The factored shear resistance of the anchor bolts per bearing is then:

S6.4.3.1

Assume two 1.0" diameter ASTM F1554 or ASTM A307 Grade C anchor bolts with a minimum tensile strength of 60 ksi:

$$R_n = 0.48 \cdot A_b \cdot F_{ub} \cdot N_s \quad \text{for threads included in shear plane} \quad S6.13.2.12$$

$$\phi_s = 0.75 \quad \text{resistance factor for ASTM A307 bolts in shear} \quad S6.5.4.2$$

$$A_b = \frac{\pi \cdot (1.00in)^2}{4}$$

$$A_b = 0.785in^2$$

$$F_{ub} = 60ksi$$

$$N_s = 2 \quad (\text{number of bolts})$$

$$R_n = 0.48 \cdot A_b \cdot F_{ub} \cdot N_s \quad R_n = 45.2K$$

$$R_r = \phi_s \cdot R_n \quad R_r = 33.9K$$

$$R_r \geq H_{EQ} \quad \text{OK}$$

Although not checked in this example, the bearing resistance at bolt holes must also be checked in accordance with S6.13.2.9. The induced bending stress in the anchor bolt should also be checked.

*S14.8.3.1 and C14.8.3.1*

Once the anchor bolt quantity and size are determined, the anchor bolt length must be computed. As an approximation, the bearing stress may be assumed to vary linearly from zero at the end of the embedded length to its maximum value at the top surface of the concrete. The bearing resistance of the concrete is based on S5.7.5.

$$\phi_b \cdot P_n = \phi_b \cdot 0.85 \cdot f_c' \cdot A_1 \cdot m \quad S5.7.5$$

$$Stress_{brg} = \frac{\phi_b \cdot P_n}{A_1}$$

$$Stress_{brg} = \phi_b \cdot 0.85 \cdot f_c' \cdot m$$

Assume

$$m = 0.75 \quad (\text{conservative assumption})$$

$$\phi_b = 0.70 \quad \text{for bearing on concrete} \quad \text{S5.5.4.2.1}$$

$$Stress_{brg} = \phi_b \cdot 0.85 \cdot (4ksi) \cdot m$$

$$Stress_{brg} = 1.78ksi$$

The total transverse horizontal load is:

$$H_{EQ} = 11.8K$$

The transverse load per anchor bolt is then:

$$P_{1bolt} = \frac{H_{EQ}}{2}$$

$$P_{1bolt} = 5.9K$$

Using the bearing stress approximation from above, the required anchor bolt area resisting the transverse horizontal load can be calculated.

$$A_1 = \frac{P_{1bolt}}{\frac{Stress_{brg} + 0}{2}}$$

$$A_1 = 6.6in^2$$

$A_1$  is the product of the anchor bolt diameter and the length the anchor bolt is embedded into the concrete pedestal/beam seat. Since we know the anchor bolt diameter, we can now solve for the required embedment length.

$$L_{embed} = \frac{A_1}{1.00in}$$

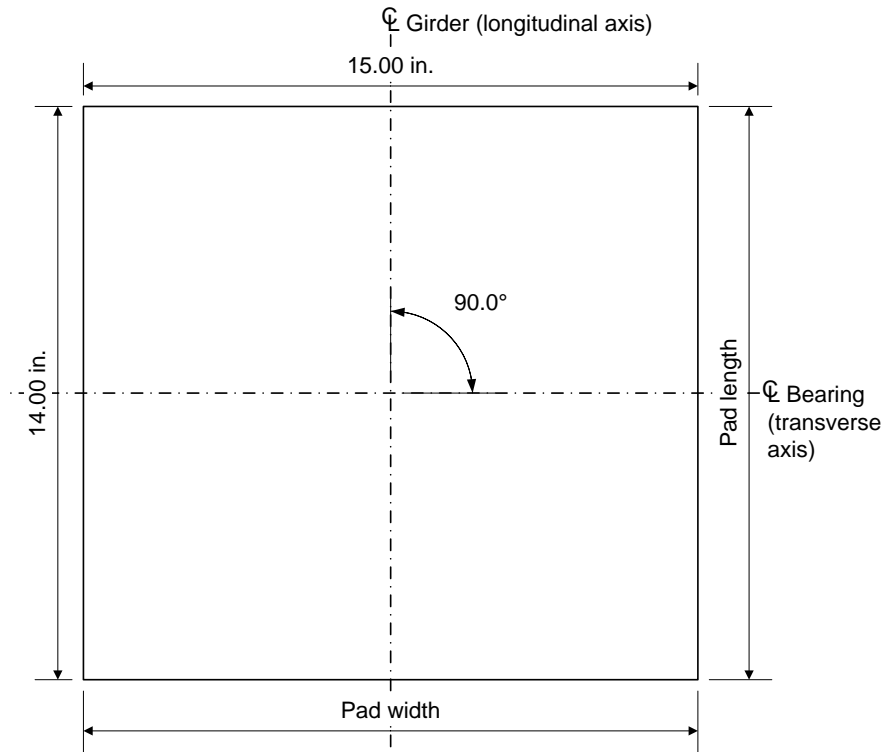
$$L_{embed} = 6.6in$$

Individual states and agencies have their own minimum anchor bolt embedment lengths. For this design example, a minimum of 12 inches will be used.

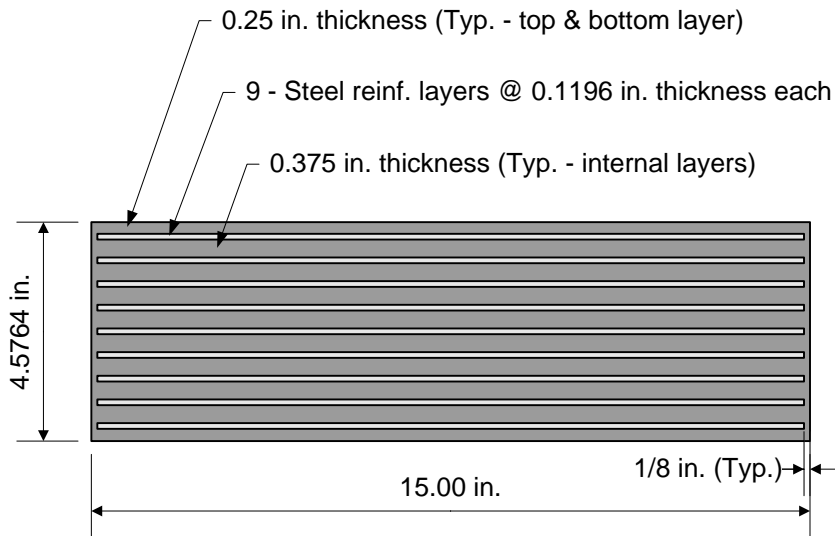
Use:

$$L_{embed} = 12.0in$$

**Design Step 6.15 - Draw Schematic of Final Bearing Design**



**Figure 6-1 Bearing Pad Plan View**



**Figure 6-2 Bearing Pad Elevation View**

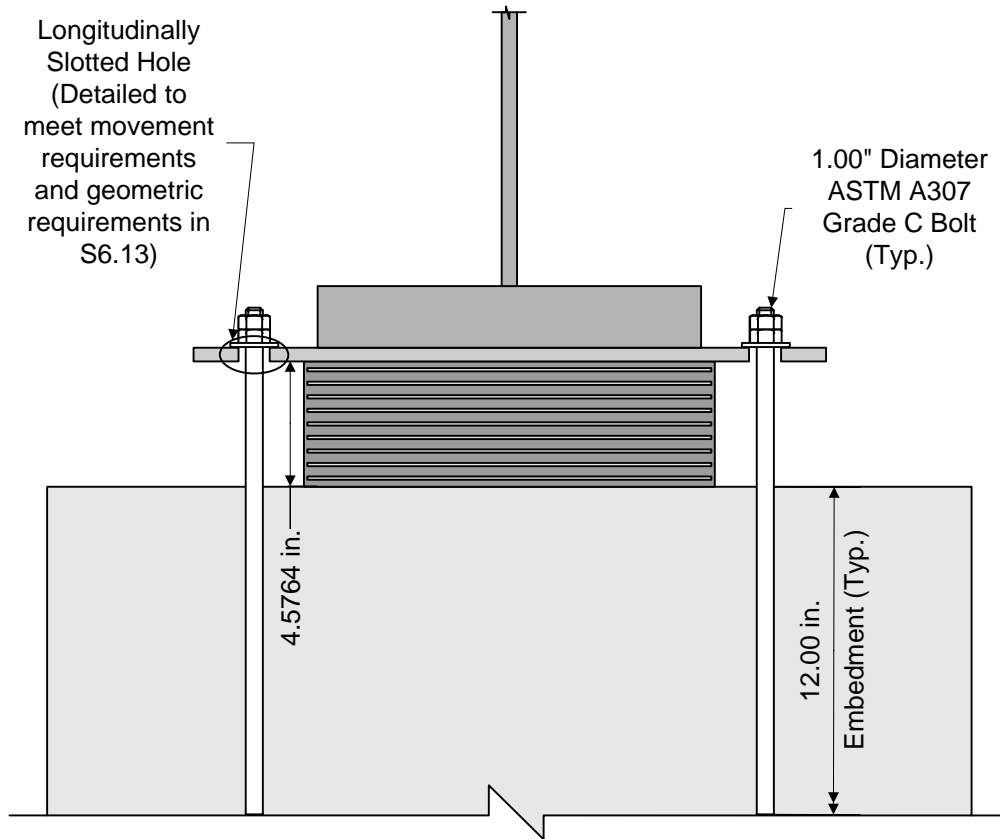


Figure 6-3 Anchor Bolt Embedment

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**COMPREHENSIVE DESIGN EXAMPLE FOR PRESTRESSED  
CONCRETE (PSC) GIRDER SUPERSTRUCTURE BRIDGE  
WITH COMMENTARY**

(Task Order DTFH61-11-D-00046-T-13007)

**US CUSTOMARY UNITS**

Submitted to  
**THE FEDERAL HIGHWAY ADMINISTRATION**

Prepared By  
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## **1.0 INTRODUCTION**

This example is part of a series of design examples sponsored by the Federal Highway Administration. The design specifications used in these examples is the AASHTO LRFD Bridge design Specifications. The intent of these examples is to assist bridge designers in interpreting the specifications and limit differences in interpretation between designers. For this example, the Seventh Edition with 2015 Interims of the AASHTO-LRFD Specifications is used.

This design example is intended to provide guidance on the application of the AASHTO-LRFD Bridge Design Specifications when applied to prestressed concrete superstructure bridges supported on intermediate multicolumn bents and integral end abutments. The example and commentary are intended for use by designers who have knowledge of the requirements of the AASHTO-LRFD Bridge Design Specifications and have designed at least one prestressed concrete girder bridge, including the bridge substructure. Designers who have not designed prestressed concrete bridges, but have used the AASHTO Specification to design other types of bridges may be able to follow the design example, however, they will first need to familiarize themselves with the basic concepts of prestressed concrete design.

This design example was not intended to follow the design and detailing practices of any particular agency. Rather, it is intended to follow common practices widely used and to adhere to the requirements of the specifications. It is expected that some users may find differences between the procedures used in the design compared to the procedures followed in the jurisdiction they practice in due to Agency-specific requirements that may deviate from the requirements of the specifications. This difference should not create the assumption that one procedure is superior to the other.

Reference is made to AASHTO-LRFD specifications article numbers throughout the design example. To distinguish between references to articles of the AASHTO-LRFD specifications and references to sections of the design example, the references to specification articles are preceded by the letter “S”. For example, S5.2 refers to Article 5.2 of AASHTO-LRFD specifications while 5.2 refers to Section 5.2 of the design example.

Two different forms of fonts are used throughout the example. Regular font is used for calculations and for text directly related to the example. Italic font is used for text that represents commentary that is general in nature and is used to explain the intent of some specifications provisions, explain a different available method that is not used by the example, provide an overview of general acceptable practices and/or present difference in application between different jurisdictions.

## 2.0 EXAMPLE BRIDGE

### 2.1 Bridge geometry and materials

#### 2.1.1 Bridge superstructure geometry

Superstructure type: Reinforced concrete deck supported on simple span prestressed girders made continuous for live load.

Spans: Two spans at 110 ft. each

Width: 55'-4 1/2" total  
52'-0" gutter line-to-gutter line (Three lanes 12'- 0" wide each, 10 ft. right shoulder and 6 ft. left shoulder. For superstructure design, the location of the driving lanes can be anywhere on the structure. For substructure design, the maximum number of 12 ft. wide lanes, i.e., 4 lanes, is considered)

Railings: Concrete Type F-Parapets, 1'- 8 1/4" wide at the base

Skew 20 degrees, valid at each support location

Girder spacing: 9'-8"

Girder type: AASHTO Type VI Girders, 72 in. deep, 42 in. wide top flange and 28 in. wide bottom flange (AASHTO 28/72 Girders)

Strand arrangement: Straight strands with some strands debonded near the ends of the girders

Overhang: 3'-6 1/4" from the centerline of the fascia girder to the end of the overhang

Intermediate diaphragms: For load calculations, one intermediate diaphragm, 10 in. thick, 50 in. deep, is assumed at the middle of each span.

Figures 2-1 and 2-2 show an elevation and cross-section of the superstructure, respectively. Figure 2-3 through 2-6 show the girder dimensions, strand arrangement, support locations and strand debonding locations.

Typically, for a specific jurisdiction, a relatively small number of girder sizes are available to select from. The initial girder size is usually selected based on past experience. Many jurisdictions have a design aid in the form of a table that determines the most likely girder size for each combination of span length and girder spacing.

The strand pattern and number of strands was initially determined based on past experience and subsequently refined using a computer design program. This design was refined using trial and error until a pattern produced stresses, at transfer and under service loads, which fell within the permissible stress limits and produced load resistances greater than the applied loads under the strength limit states. For debonded strands, S5.11.4.3 states that the number of partially debonded strands should not exceed 25 percent of the total number of strands. Also, the number of debonded strands in any horizontal row shall not exceed 40 percent of the strands in that row. The selected pattern has 27.2 percent of the total strands debonded. This is slightly higher than the 25 percent stated in the specifications, but is acceptable since the specifications require that this limit “should” be satisfied. Using the word “should” instead of “shall” signifies that the specifications allow some deviation from the limit of 25 percent.

Typically, the most economical strand arrangement calls for the strands to be located as close as possible to the bottom of the girders. However, in some cases, it may not be possible to satisfy all specification requirements while keeping the girder size to a minimum and keeping the strands near the bottom of the beam. This is more pronounced when debonded strands are used due to the limitation on the percentage of debonded strands. In such cases, the designer may consider the following two solutions:

- Increase the size of the girder to reduce the range of stress, i.e., the difference between the stress at transfer and the stress at final stage.
- Increase the number of strands and shift the center of gravity of the strands upward.

Either solution results in some loss of economy. The designer should consider specific site conditions (e.g., cost of the deeper girder, cost of the additional strands, the available under-clearance and cost of raising the approach roadway to accommodate deeper girders) when determining which solution to adopt.

### 2.1.2 Bridge substructure geometry

Intermediate pier: Multi-column bent (4 – columns spaced at 14'-1")  
Spread footings founded on sandy soil  
See Figure 2-7 for the intermediate pier geometry

End abutments: Integral abutments supported on one line of steel H-piles supported on bedrock. U-wingwalls are cantilevered from the fill face of the abutment. The approach slab is supported on the integral abutment at one end and a sleeper slab at the other end.  
See Figure 2-8 for the integral abutment geometry



### 2.1.3 Materials

#### Concrete strength

Prestressed girders:	Initial strength at transfer, $f'_{ci} = 4.8$ ksi
	28-day strength, $f'_c = 6$ ksi
Deck slab:	$f'_c = 4.0$ ksi
Substructure:	$f'_c = 3.0$ ksi
Railings:	$f'_c = 3.5$ ksi

#### Concrete elastic modulus S5.4.2.4

$$W_c = 0.150 \text{ kcf}$$

$$K_1 = 1$$

In the absence of measured data, the modulus of elasticity,  $E_c$ , for concretes with unit weights between 0.090 and 0.155 kcf and for normal weight concrete with specified compressive strengths up to 15 ksi may be taken as:

$$\text{Deck slab: } E_c = 120,000 \cdot K_1 \cdot (W_c^{2.0}) \cdot f'_c{}^{0.33} = 4266 \text{ ksi} \quad \text{S5.4.2.4-1}$$

$$\text{Girder at 28-day strength: } E_c = 120,000 \cdot K_1 \cdot (W_c^{2.0}) \cdot f'_c{}^{0.33} = 4877 \text{ ksi} \quad \text{S5.4.2.4-1}$$

$$\text{Girder at transfer: } E_c = 120,000 \cdot K_1 \cdot (W_c^{2.0}) \cdot f'_c{}^{0.33} = 4530 \text{ ksi} \quad \text{S5.4.2.4-1}$$

However, for normal weight concrete and specified compressive strengths up to 10 ksi,  $E_c$  may be taken as

$$\text{Deck slab: } E_c = 33,000 \cdot K_1 \cdot (W_c^{1.5}) \cdot \sqrt{f'_c} = 3834 \text{ ksi} \quad \text{SC5.4.2.4-2}$$

$$\text{Girder at 28-day strength: } E_c = 33,000 \cdot K_1 \cdot (W_c^{1.5}) \cdot \sqrt{f'_c} = 4696 \text{ ksi} \quad \text{SC5.4.2.4-2}$$

$$\text{Girder at transfer: } E_c = 33,000 \cdot K_1 \cdot (W_c^{1.5}) \cdot \sqrt{f'_c} = 4200 \text{ ksi} \quad \text{SC5.4.2.4-2}$$

Therefore, for this example use the following.

Deck slab elastic modulus, $E_s$	= 3,834 ksi
Girder final elastic modulus, $E_c$	= 4,696 ksi
Girder elastic modulus at transfer, $E_{ci}$	= 4,200 ksi

#### Reinforcing steel

Yield strength,  $f_y = 60$  ksi

#### Prestressing strands

0.5 inch diameter low relaxation strands Grade 270

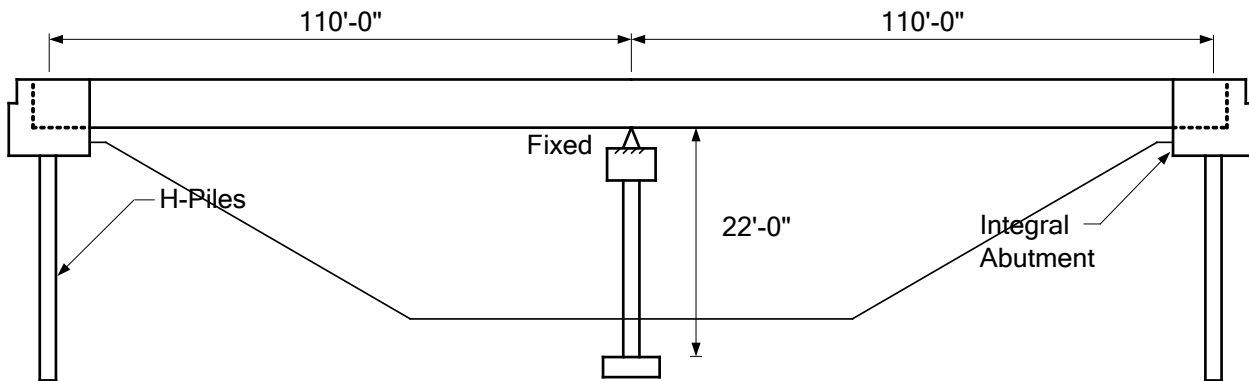
Strand area,  $A_{ps}$  = 0.153 in<sup>2</sup>

Steel yield strength,  $f_{py}$  = 243 ksi

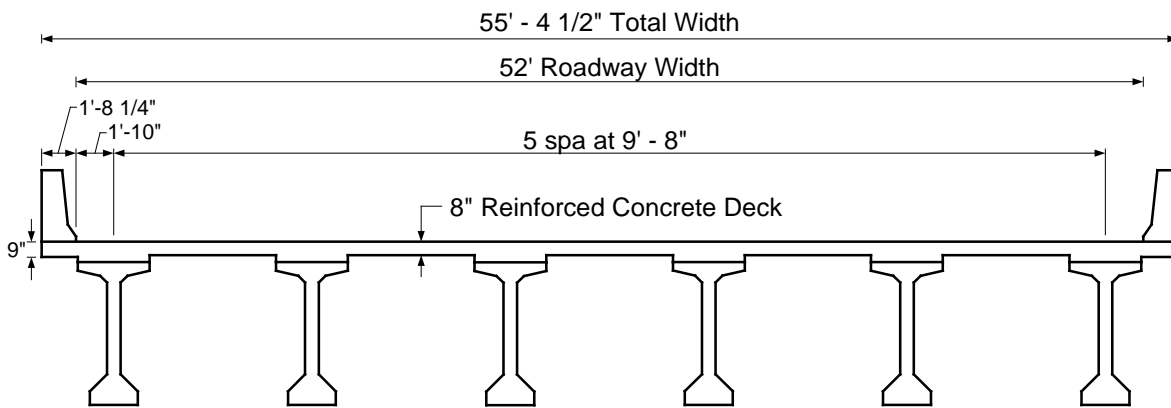
Steel ultimate strength,  $f_{pu}$  = 270 ksi  
 Prestressing steel modulus,  $E_p$  = 28,500 ksi

**2.1.4 Other parameters affecting girder analysis**

Time of Transfer = 1 day  
 Average Relative Humidity = 70%



**Figure 2-1 – Elevation View of the Example Bridge**



**Figure 2-2 – Bridge Cross-Section**

## 2.2 Girder geometry and section properties

### 2.2.1 Basic beam section properties

Beam length, L	= 110 ft. – 6 in.
Depth	= 72 in.
Thickness of web	= 8 in.
Area, $A_g$	= 1,085 in <sup>2</sup>
Moment of inertia, $I_g$	= 733,320 in <sup>4</sup>
N.A. to top, $y_t$	= 35.62 in.
N.A. to bottom, $y_b$	= 36.38 in.
Section modulus, $S_{TOP}$	= 20,588 in <sup>3</sup>
Section modulus, $S_{BOT}$	= 20,157 in <sup>3</sup>
CGS from bottom, at 0 ft.	= 5.375 in.
CGS from bottom, at 11 ft.	= 5.158 in.
CGS from bottom, at 54.5 ft.	= 5.0 in.
P/S force eccentricity at 0 ft., $e_{0'}$	= 31.005 in.
P/S force eccentricity at 11 ft., $e_{11'}$	= 31.222 in.
P/S force eccentricity at 54.5 ft., $e_{54.5'}$	= 31.380 in.

**2.2.2 Interior beam composite section properties**

Effective slab width = 116 in. (see calculations in Section 2.3)

Deck slab thickness = 8 in. (includes ½ in. integral wearing surface which is not included in the calculation of the composite section properties)

Haunch depth = 4 in. (maximum value - notice that the haunch depth varies along the beam length and, hence, is ignored in calculating section properties but is considered when determining dead load)

Moment of inertia,  $I_c$  = 1,402,024 in<sup>4</sup>

N.A. to slab top,  $y_{sc}$  = 27.54 in.

N.A. to beam top,  $y_{tc}$  = 20.04 in.

N.A. to beam bottom,  $y_{bc}$  = 51.96 in.

Section modulus,  $S_{TOP\ SLAB}$  = 50,902 in<sup>3</sup>

Section modulus,  $S_{TOP\ BEAM}$  = 69,949 in<sup>3</sup>

Section modulus,  $S_{BOT\ BEAM}$  = 26,985 in<sup>3</sup>

**2.2.3 Exterior beam composite section properties**

Effective Slab Width = 100.25 in. (see calculations in Section 2.3)

Deck slab thickness = 8 in. (includes ½ in. integral wearing surface which is not included in the calculation of the composite section properties)

Haunch depth = 4 in. (maximum value - notice that the haunch depth varies along the beam length and, hence, is ignored in calculating section properties but is considered when determining dead load)

Moment of inertia,  $I_c$  = 1,343,874 in<sup>4</sup>

N.A. to slab top,  $y_{sc}$  = 28.89 in.

N.A. to beam top,  $y_{tc}$  = 21.39 in.

N.A. to beam bottom,  $y_{bc}$  = 50.61 in.

Section modulus,  $S_{TOP\ SLAB}$  = 46,510 in<sup>3</sup>

Section modulus,  $S_{TOP\ BEAM}$  = 62,815 in<sup>3</sup>

Section modulus,  $S_{BOT\ BEAM}$  = 26,556 in<sup>3</sup>

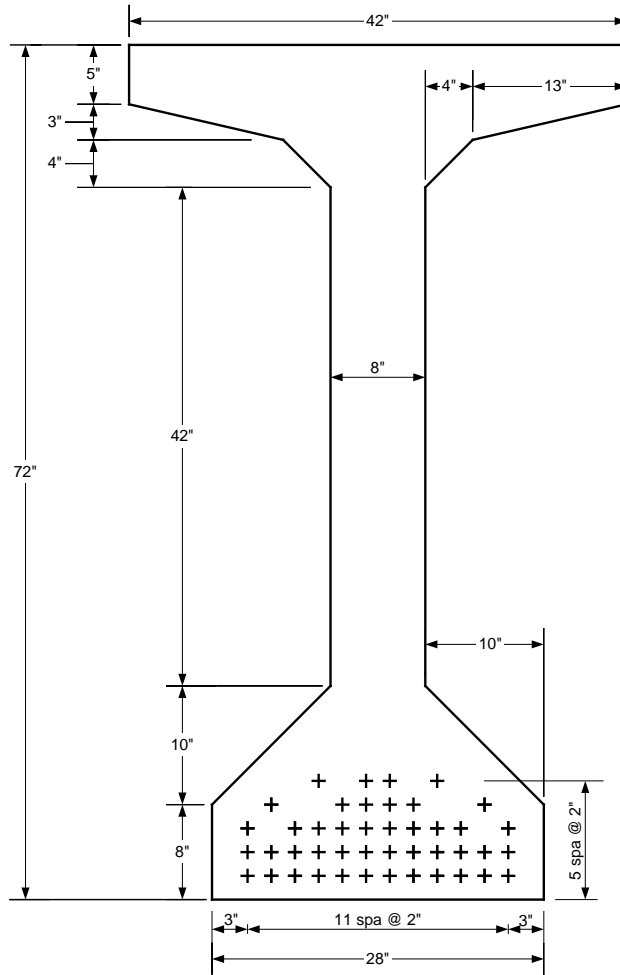


Figure 2-3 – Beam Cross-Section Showing 44 Strands

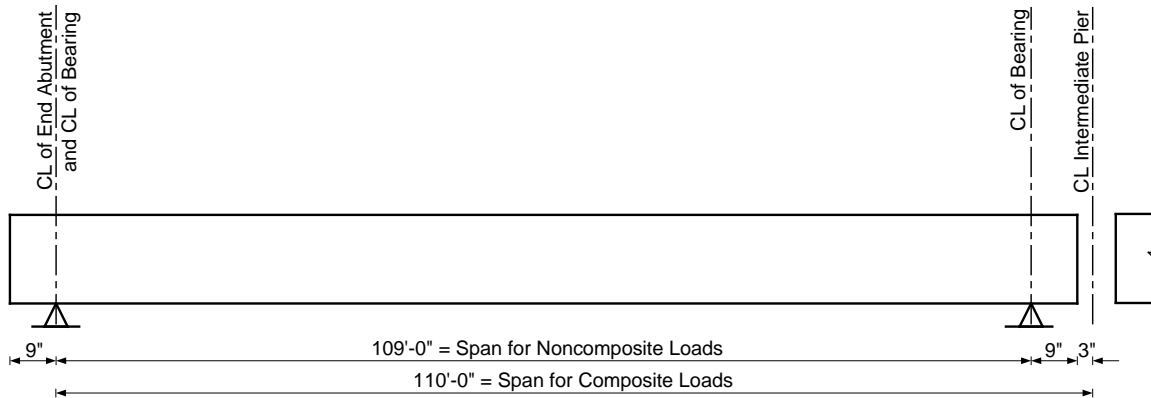


Figure 2-4 – General Beam Elevation

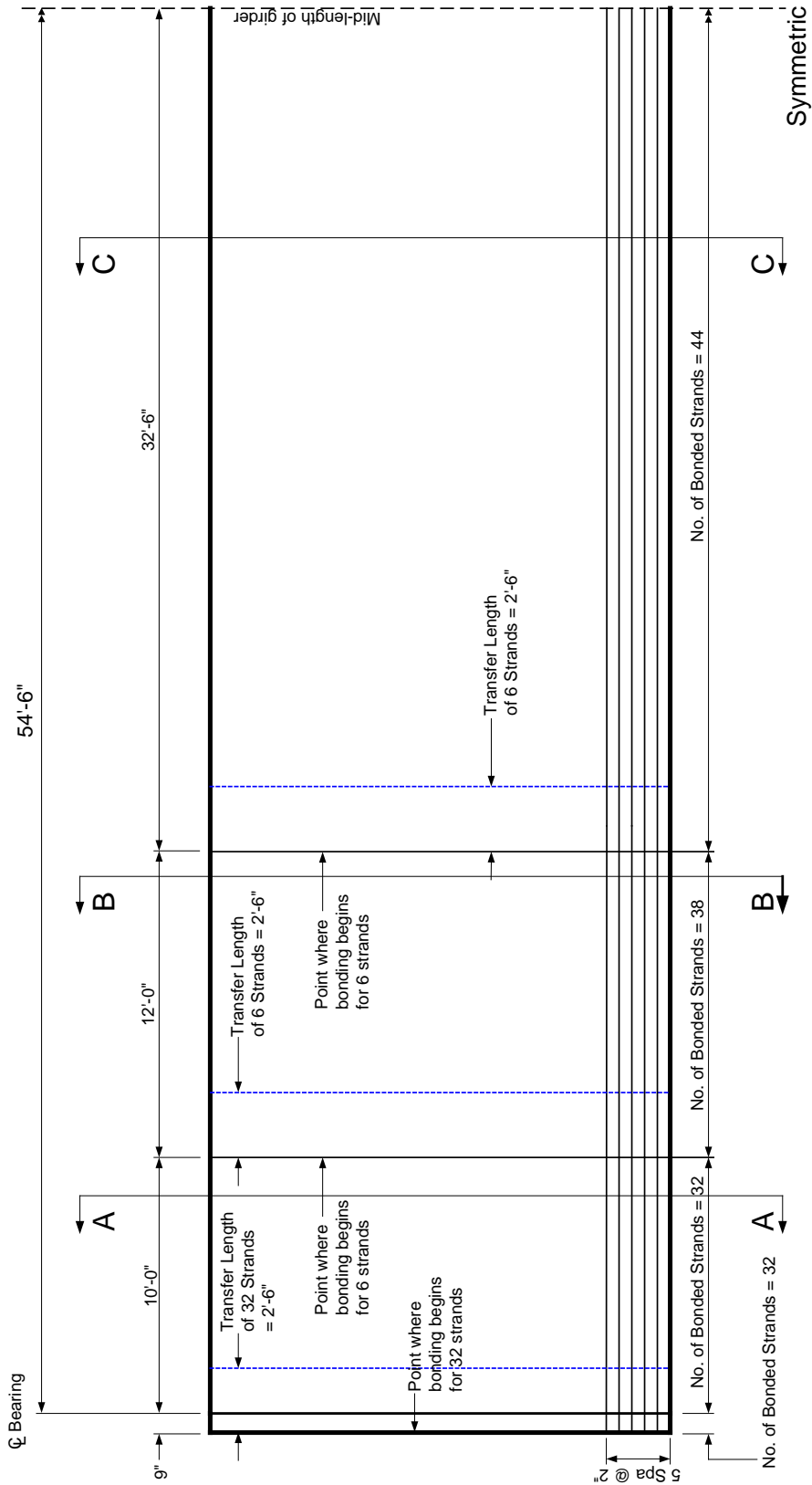
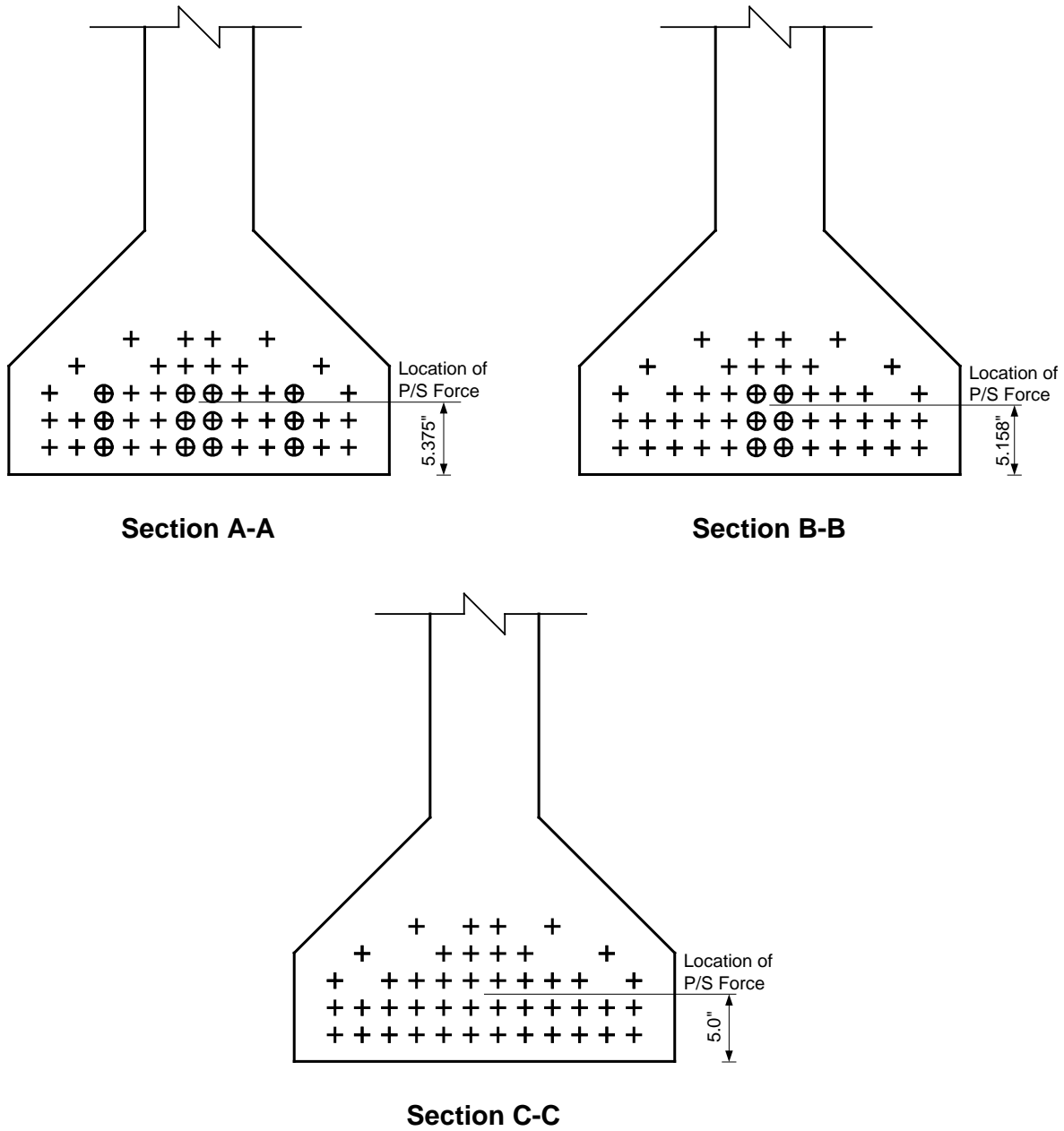


Figure 2-5 – Elevation View of Prestressing Strands



+ - Bonded Strand  
 ⊕ - Debonded Strand

For location of Sections A-A, B-B and C-C, see Figure 2-5

**Figure 2-6 – Beam at Sections A-A, B-B, and C-C**

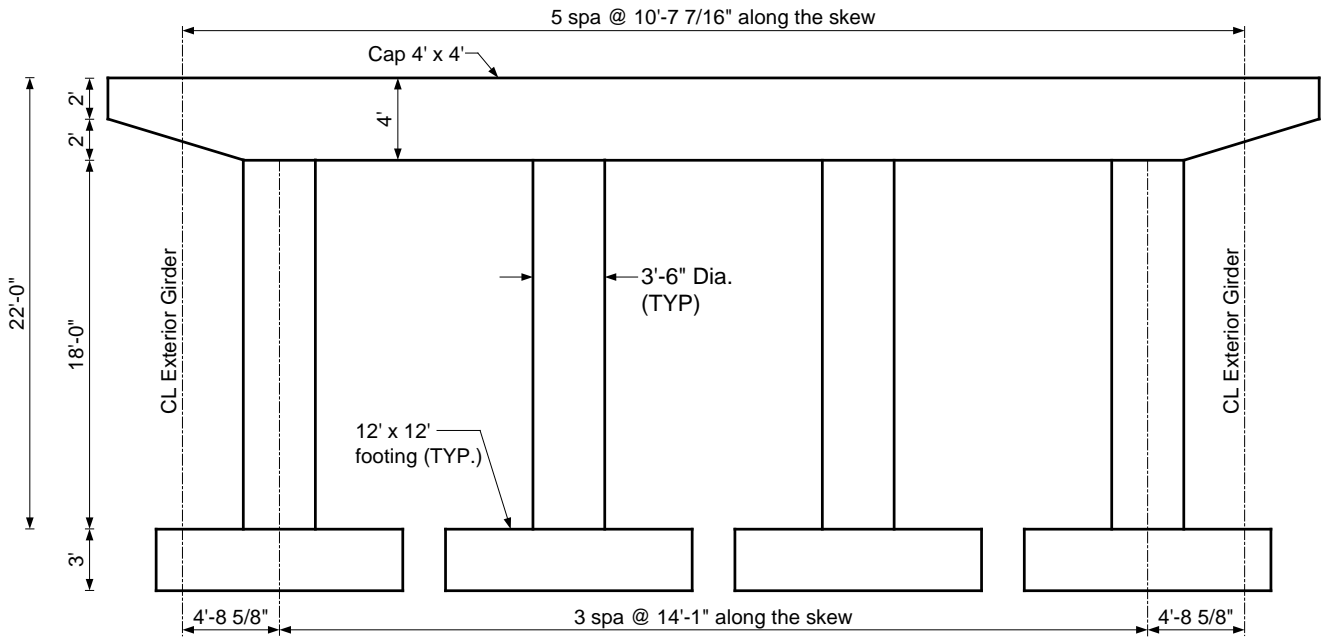


Figure 2-7 – Intermediate Bent

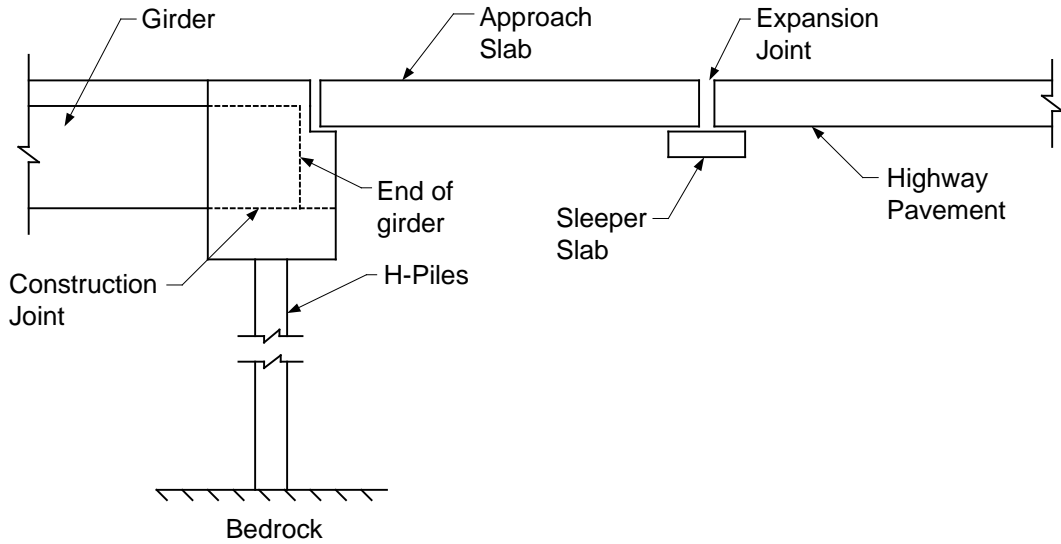


Figure 2-8 – Integral Abutment



### 2.3 Effective flange width (S4.6.2.6)

The effective flange width is calculated using the provisions of S4.6.2.6. According to S4.6.2.6.1, the effective flange width may be calculated as follows:

For interior girders:

The effective flange width is taken as:

The average spacing of adjacent beams = 9 ft. - 8 in. or 116 in.

For exterior girders:

The effective flange width is taken as:

One-half the distance to the adjacent interior girder plus the width of the overhang.

The width of the overhang = 3 ft.- 6 ¼ in. or 42.25 in.

Therefore, the effective flange width for the exterior girder is:

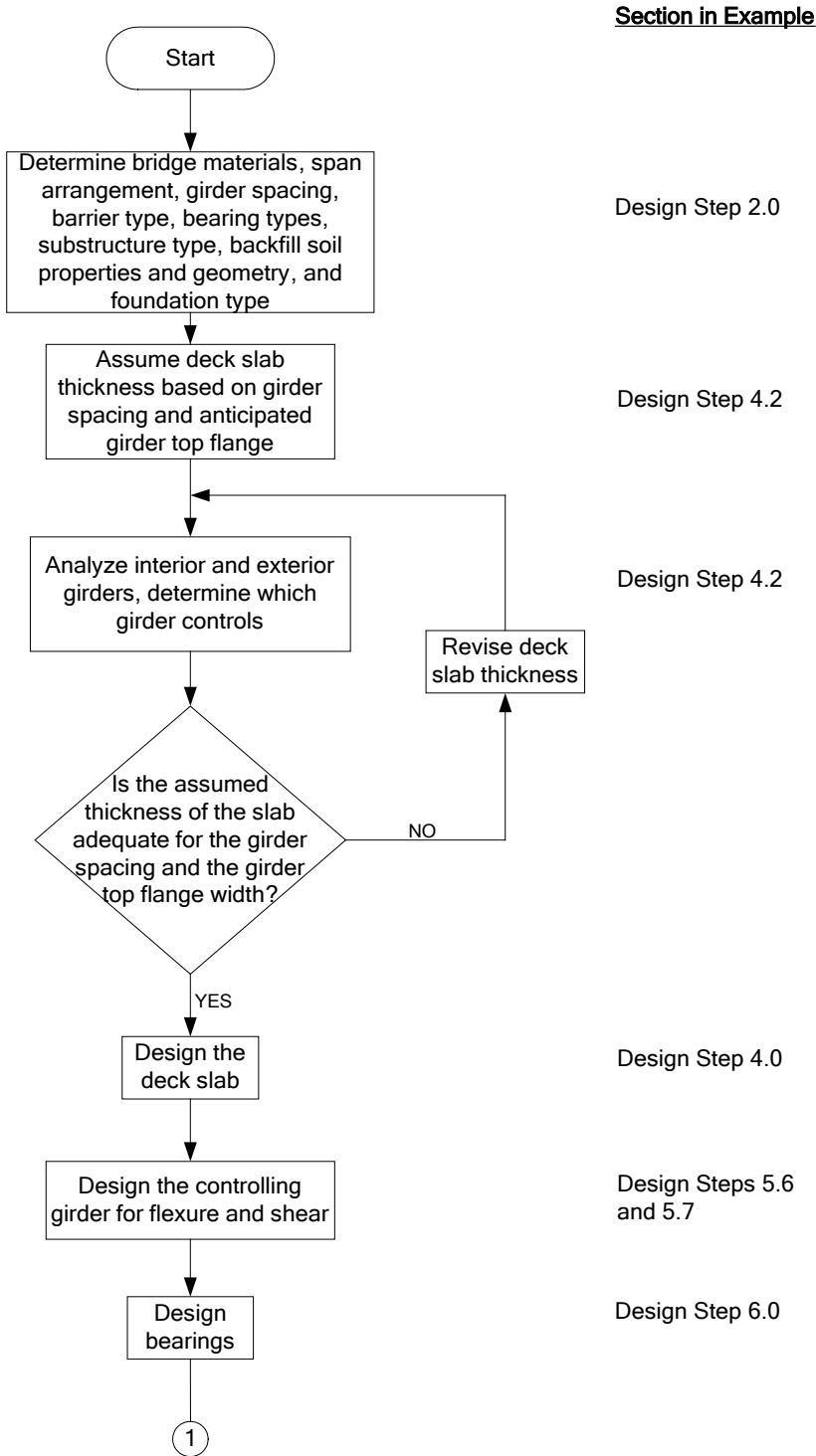
$$(116/2) + 42.25 = 100.25 \text{ in.}$$

S4.5 allows the consideration of continuous composite barriers when analyzing for service and fatigue limit states. S4.6.2.6 includes an approximate method of including the effect of the continuous composite barriers on the section by modifying the width of the overhang. Traditionally, the effect of the continuous composite barrier on the section is ignored in the design of new bridges and is ignored in this example. This effect may be considered when checking existing bridges with structurally sound continuous composite barriers.

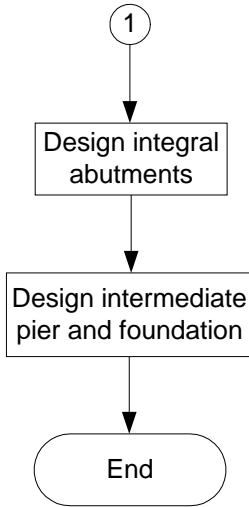
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### 3.0 FLOWCHARTS

#### Main Design Steps



**Main Design Steps (cont.)**

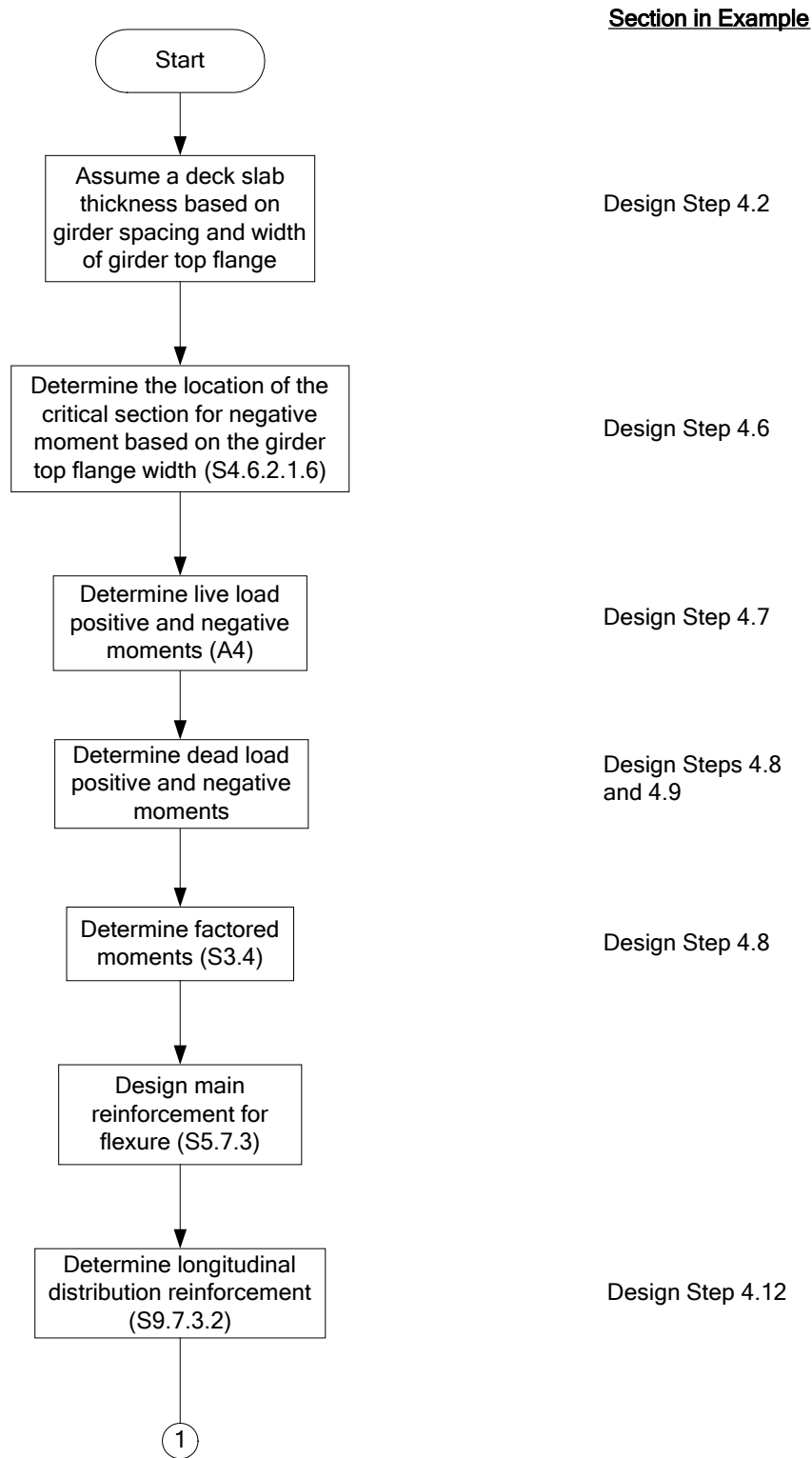


**Section in Example**

Design Step 7.1

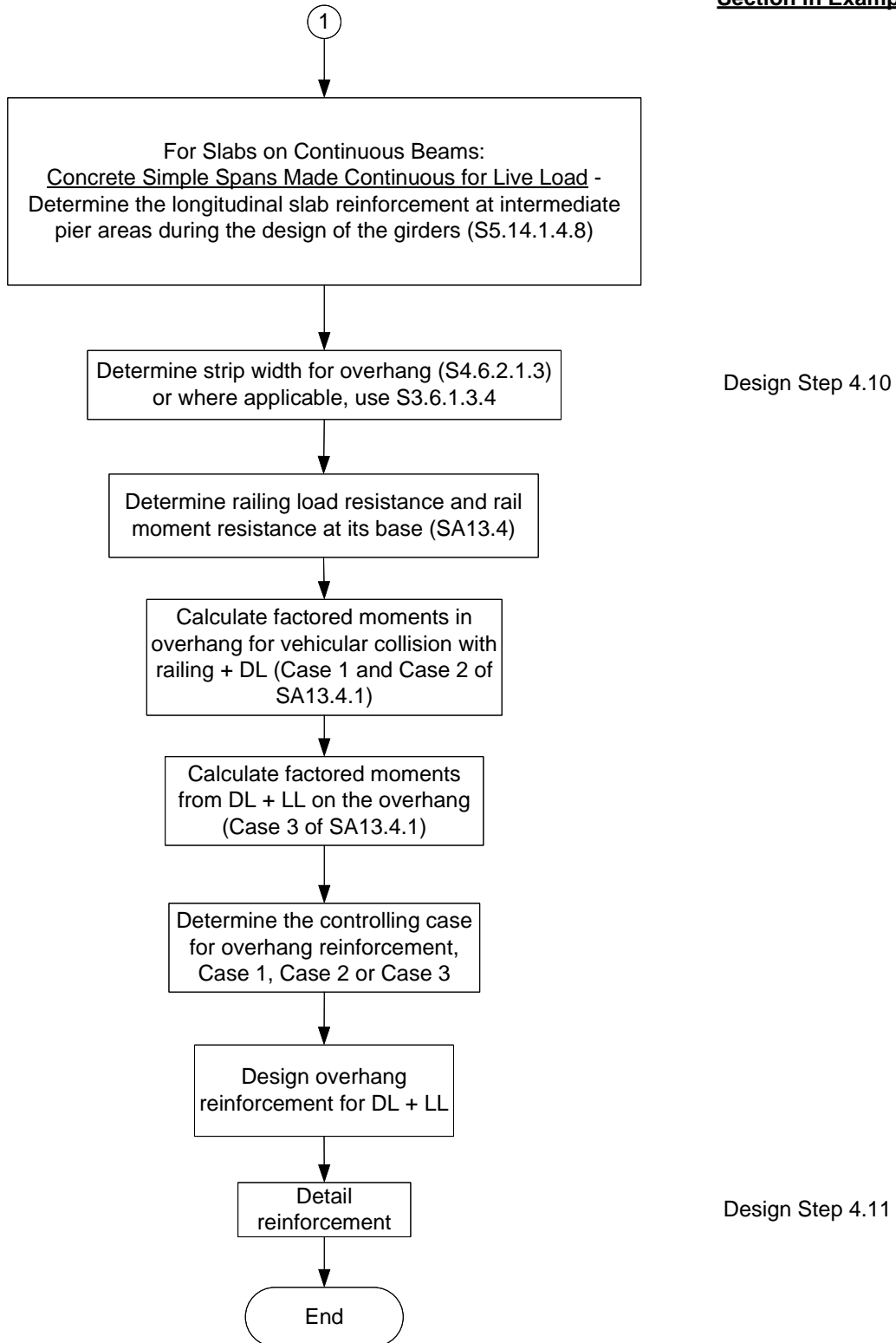
Design Step 7.2

Deck Slab Design



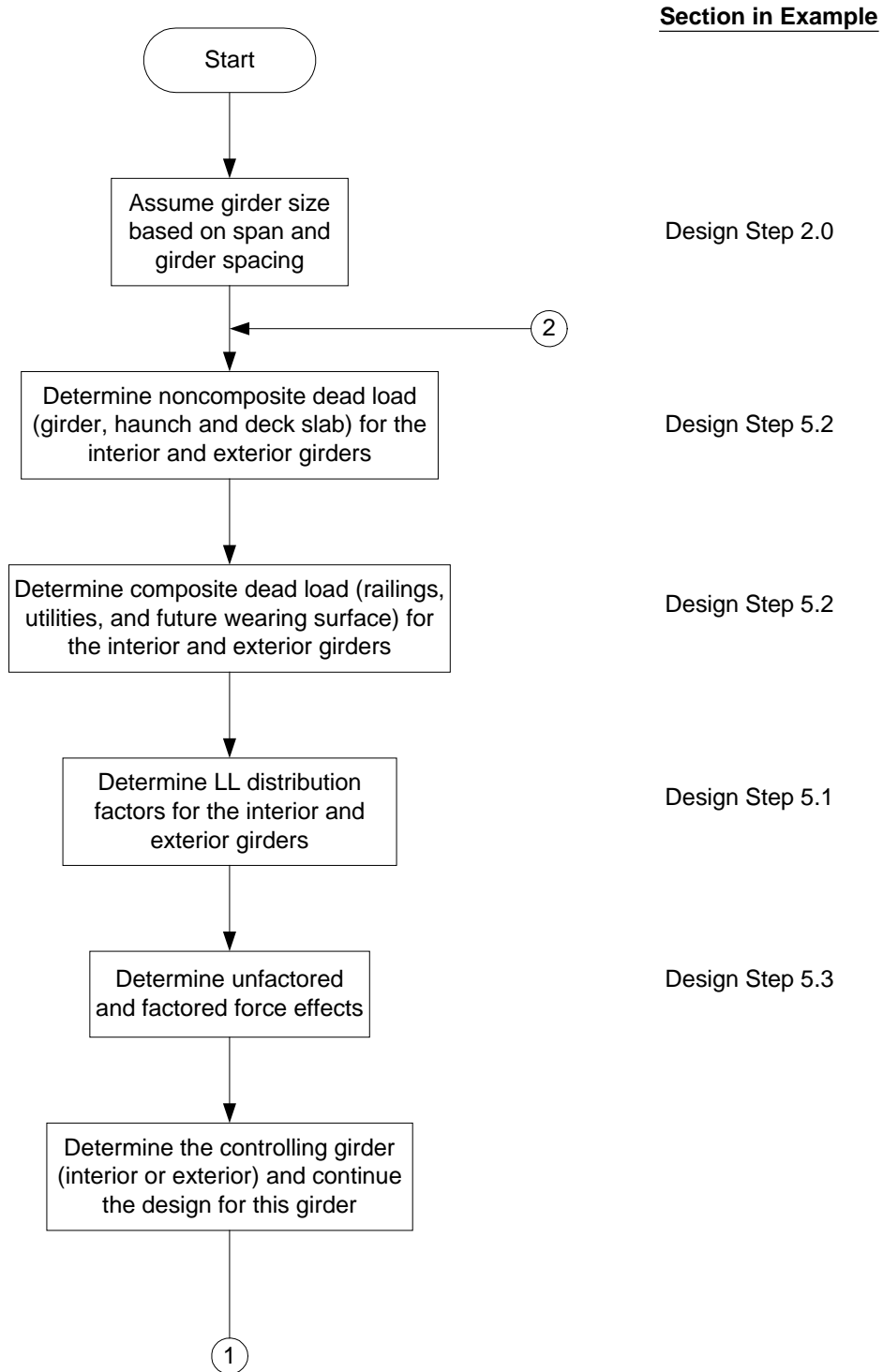
Deck Slab Design (cont.)

Section in Example

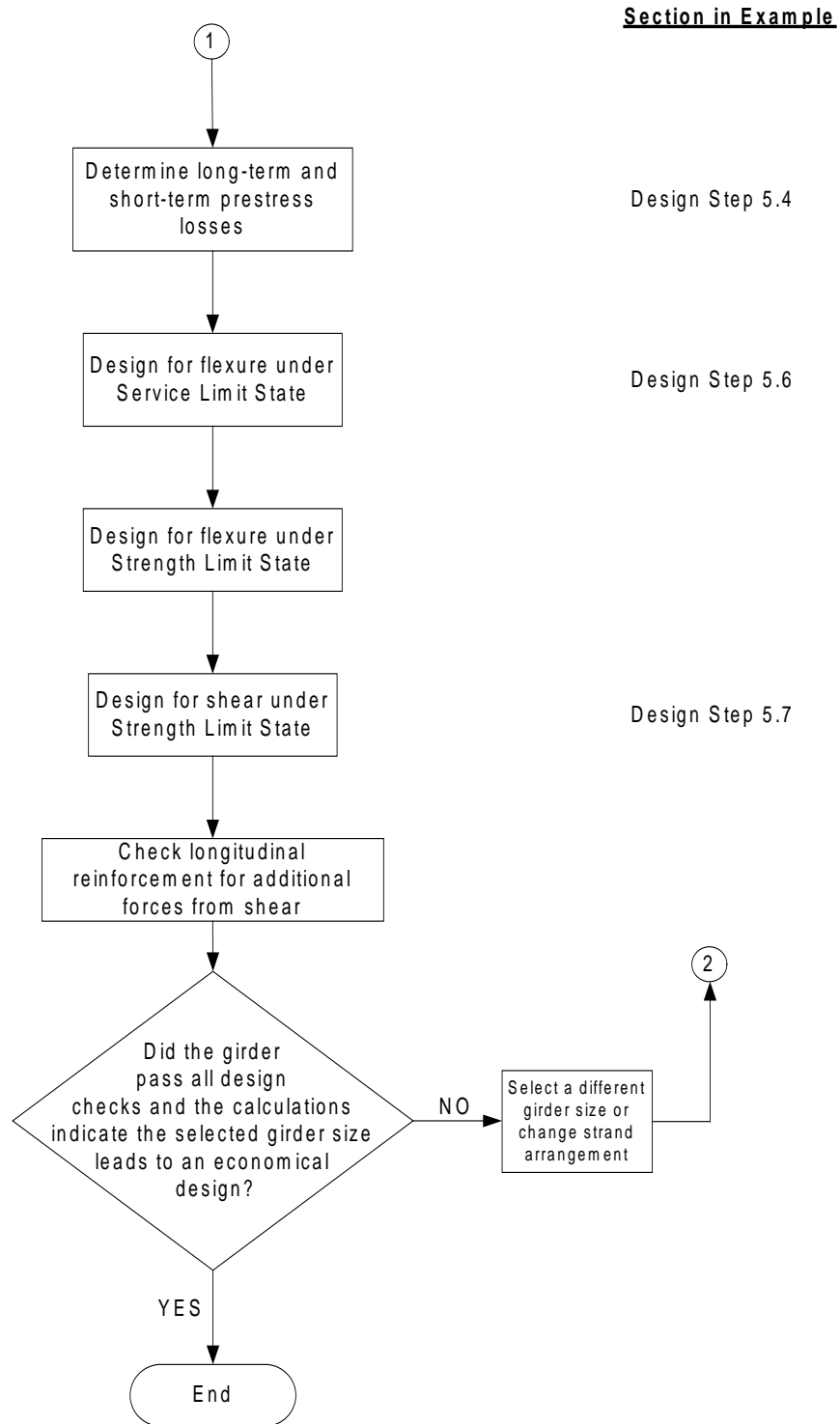


**General Superstructure Design**

(Notice that only major steps are presented in this flowchart. More detailed flowcharts of the design steps follow this flowchart)

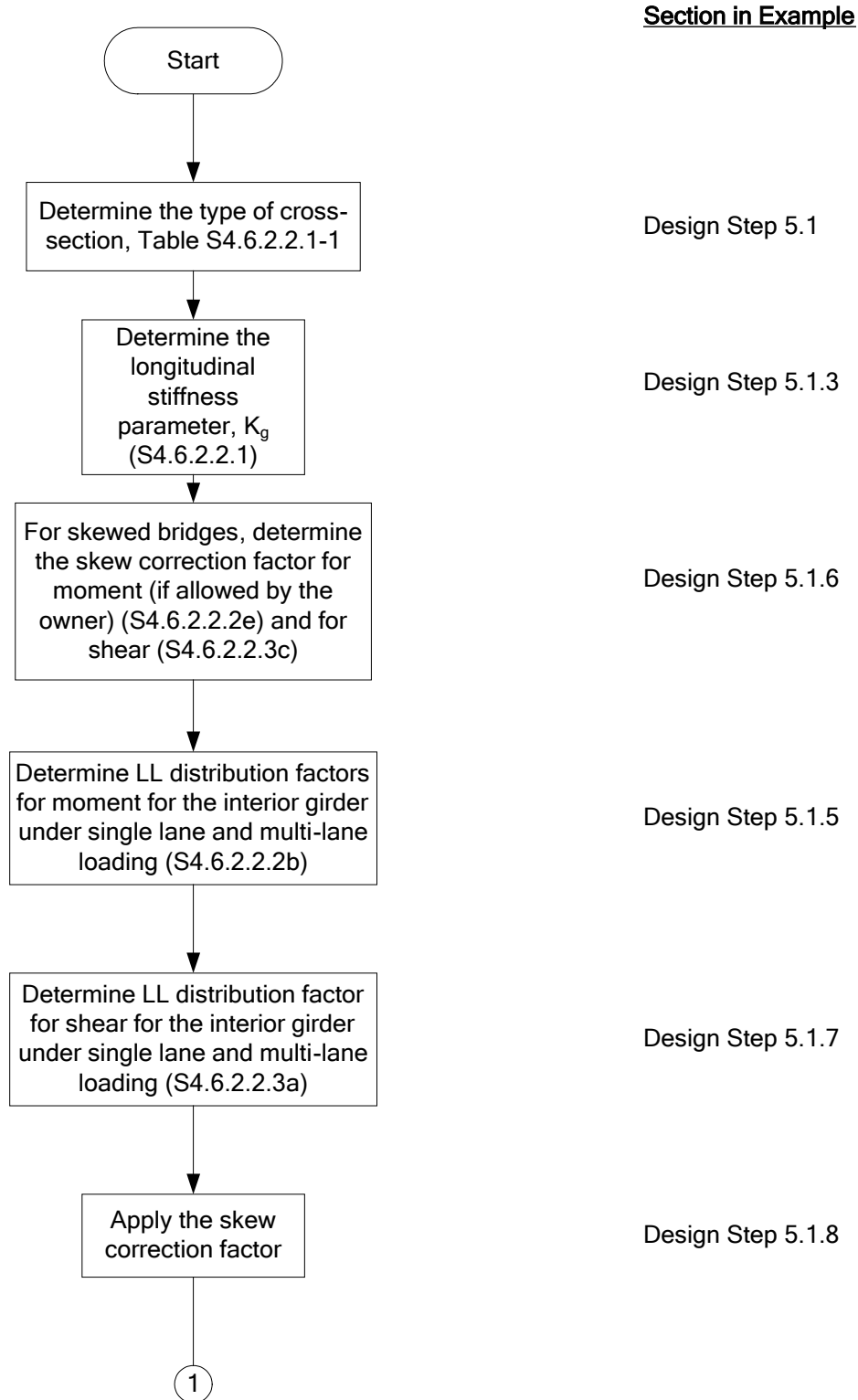


General Superstructure Design (cont.)

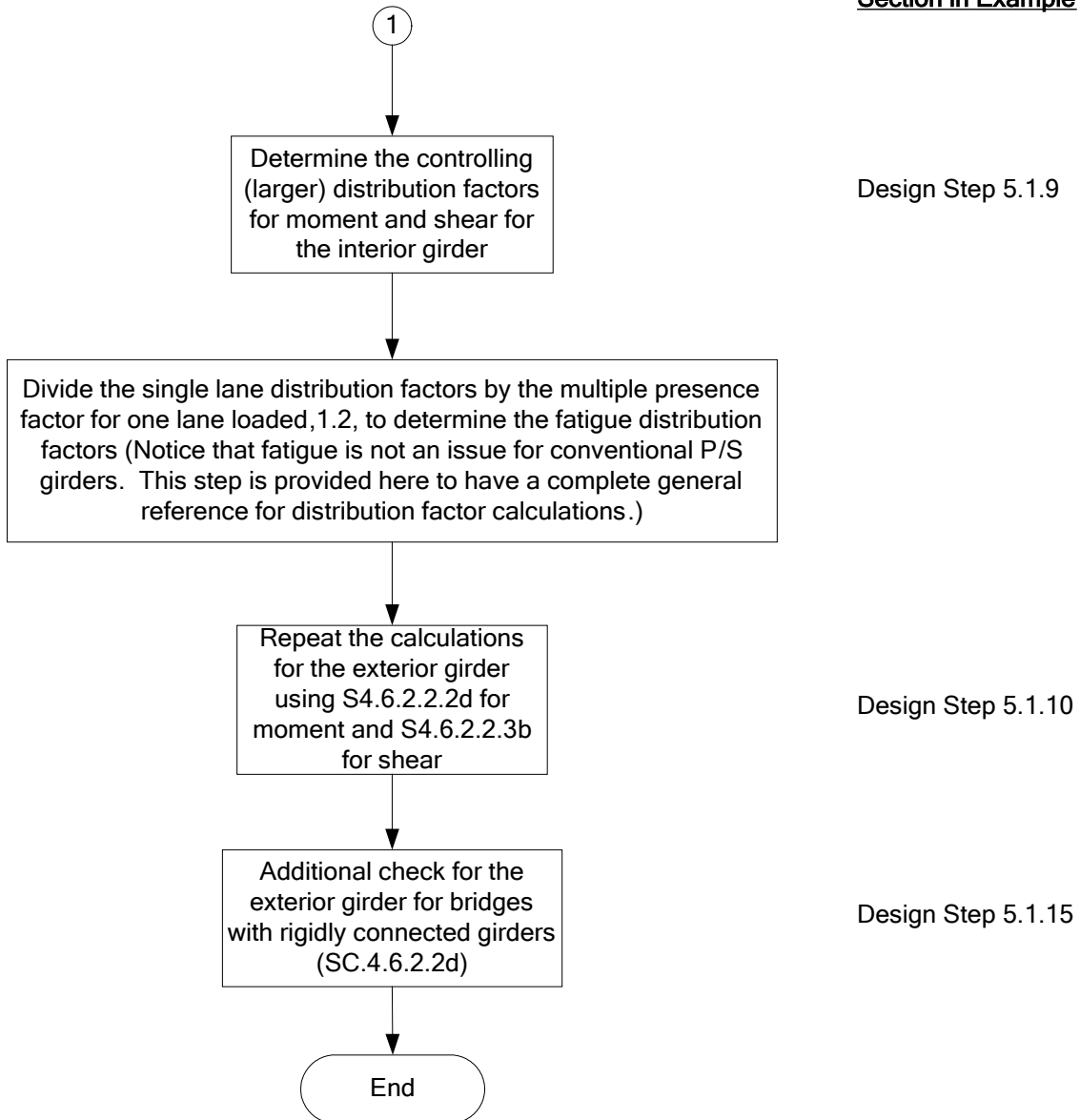




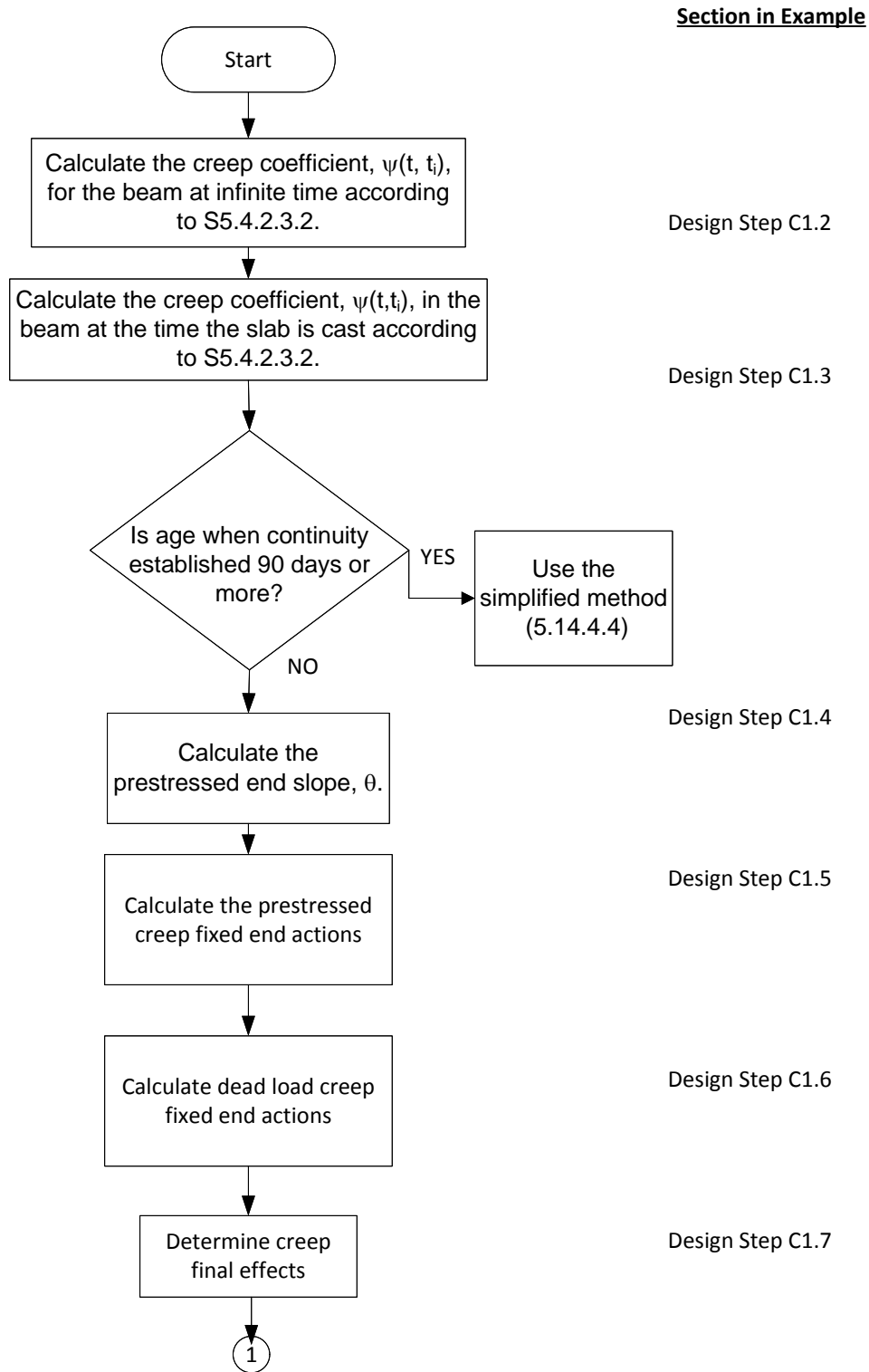
Live Load Distribution Factor Calculations



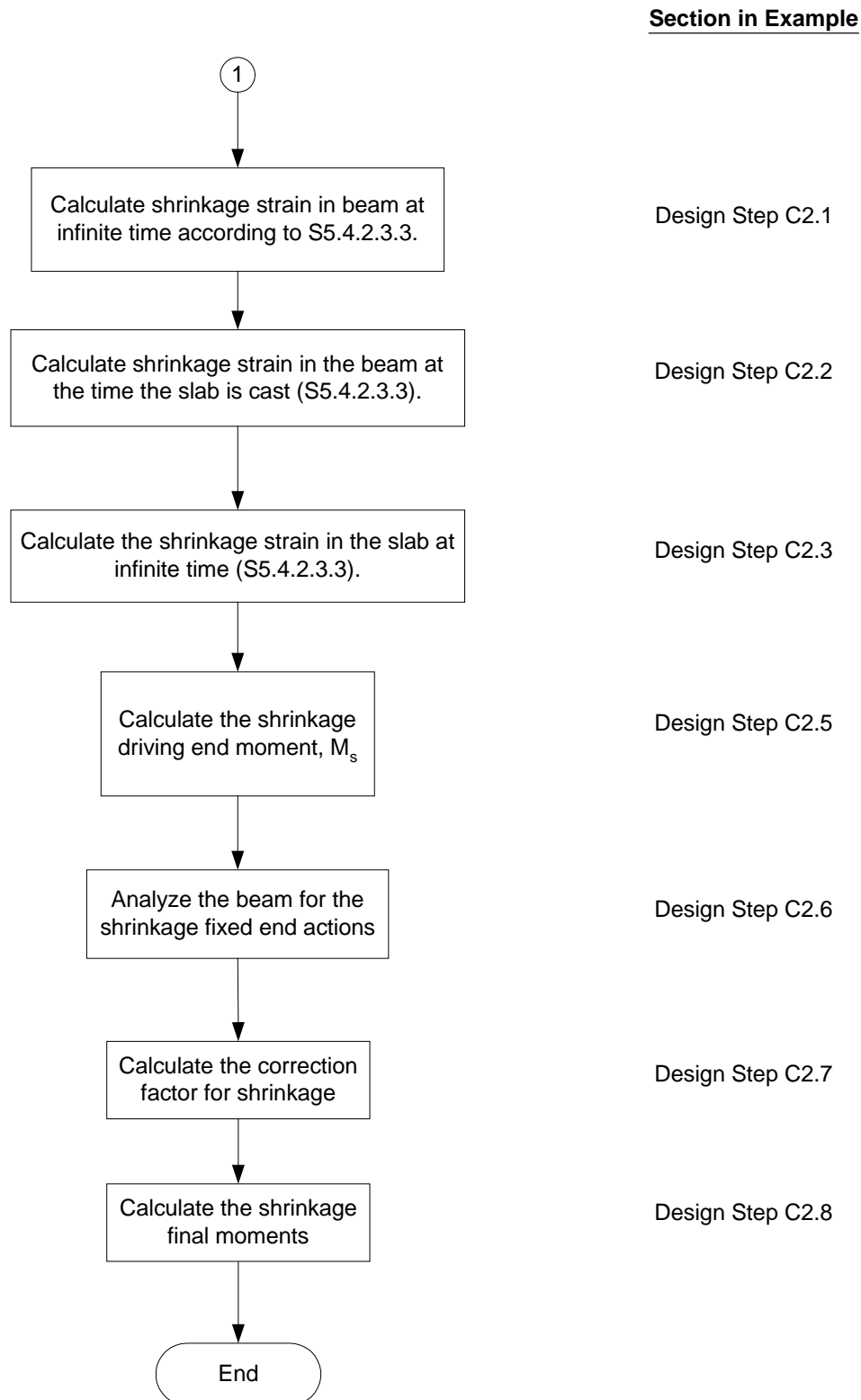
Live Load Distribution Factor Calculations (cont.)



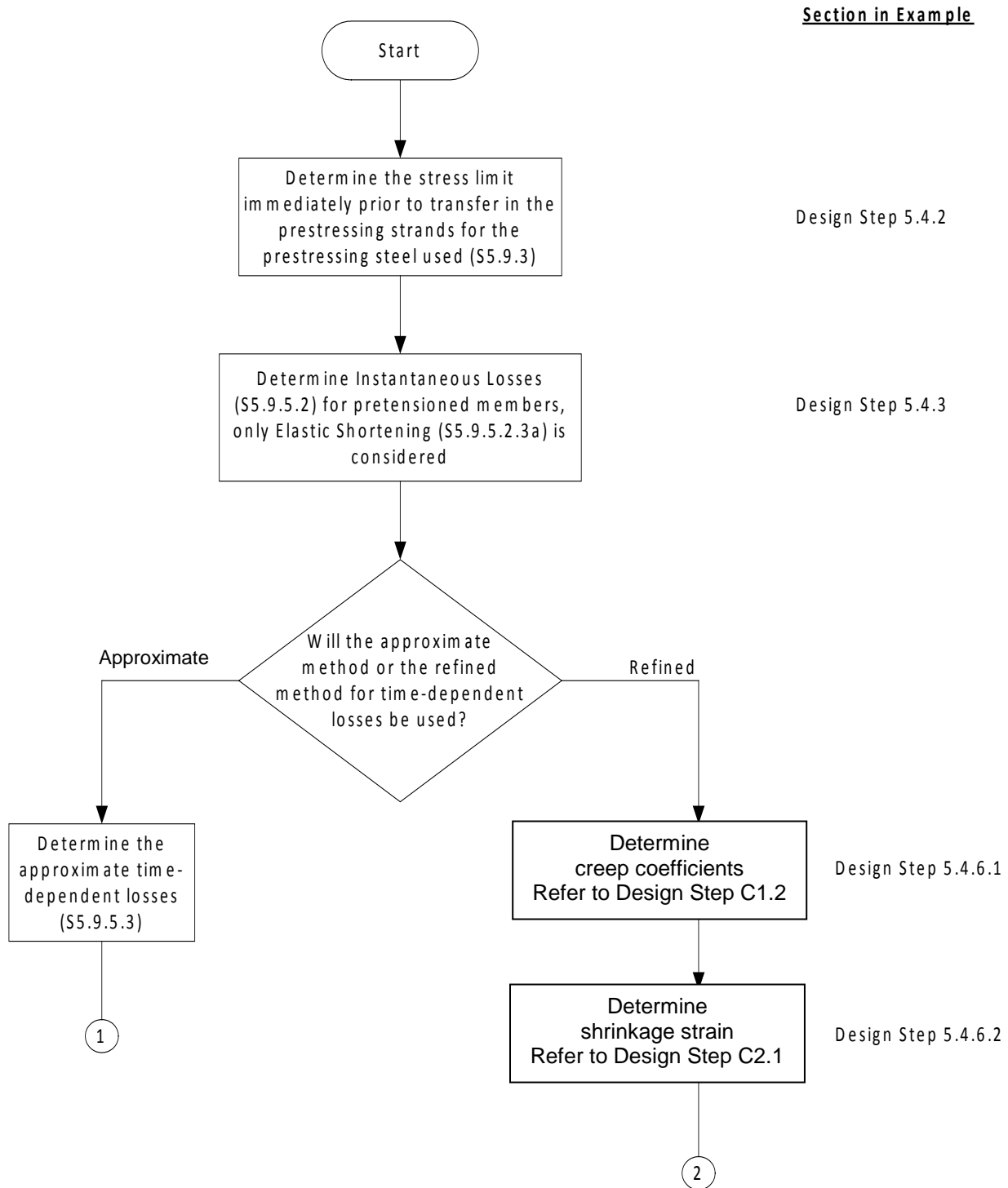
Creep and Shrinkage Calculations



Creep and Shrinkage Calculations (cont.)

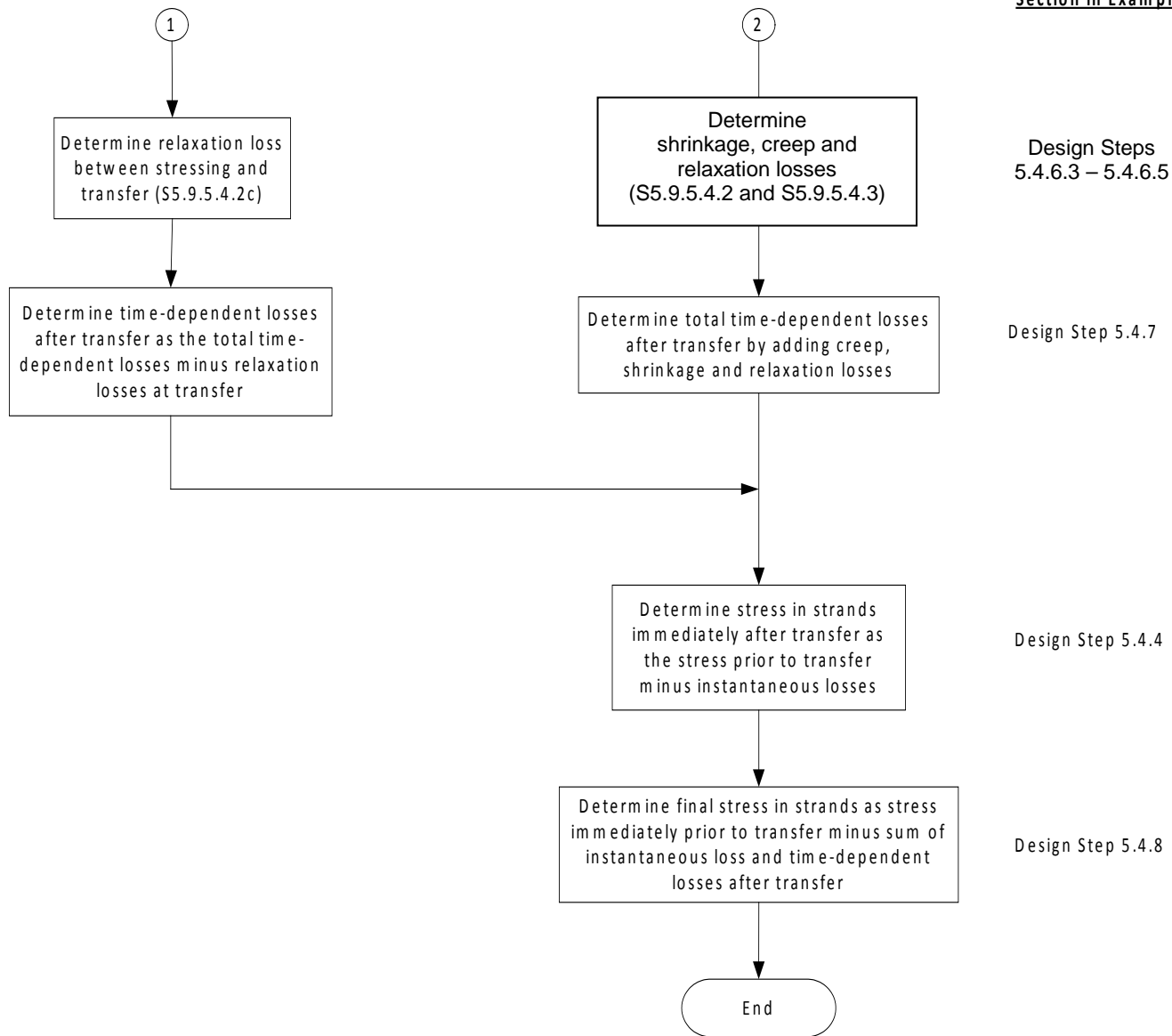


**Prestressing Losses Calculations**

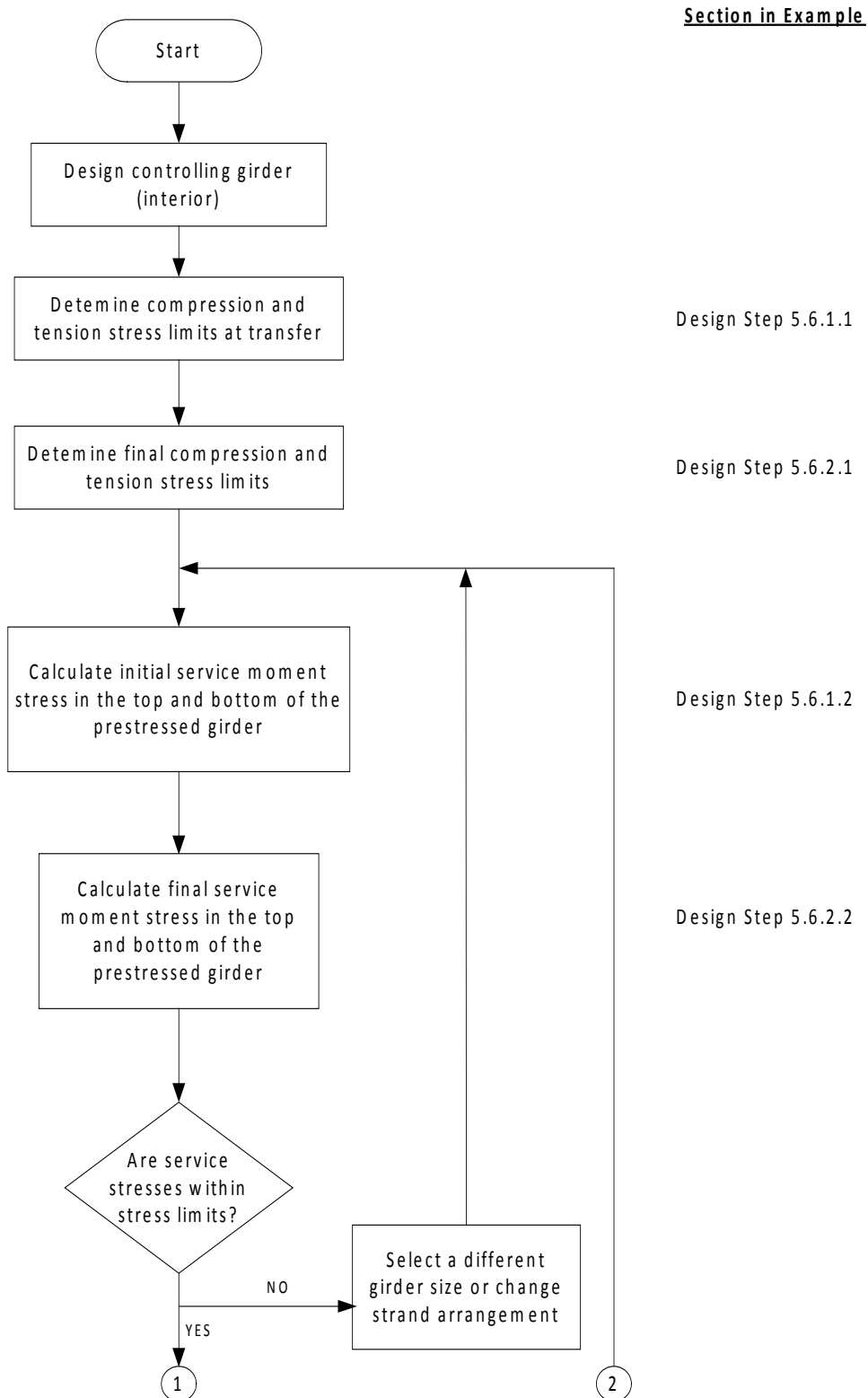


**Prestressing Losses Calculations (cont.)**

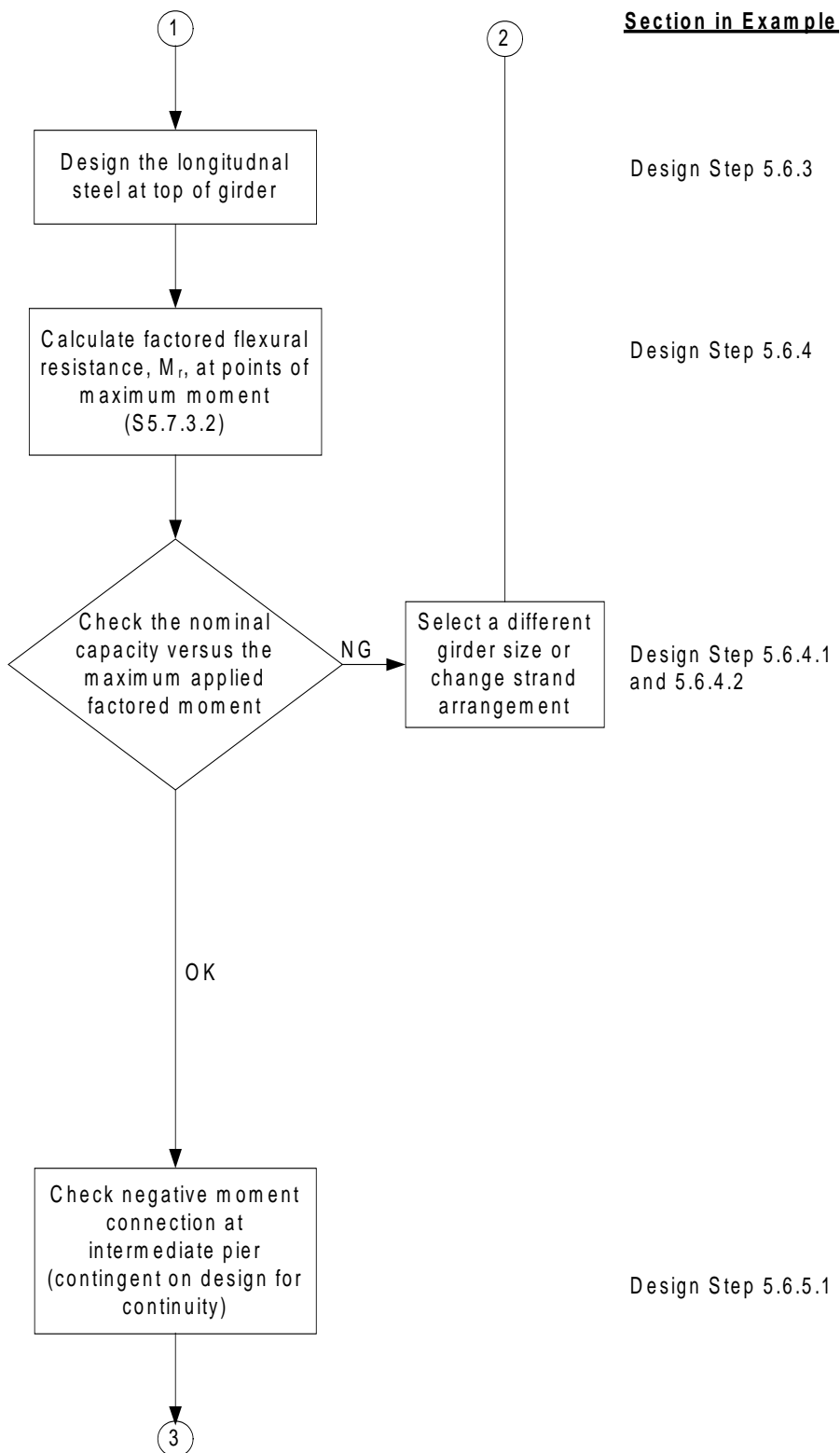
Section in Example



**Flexural Design**

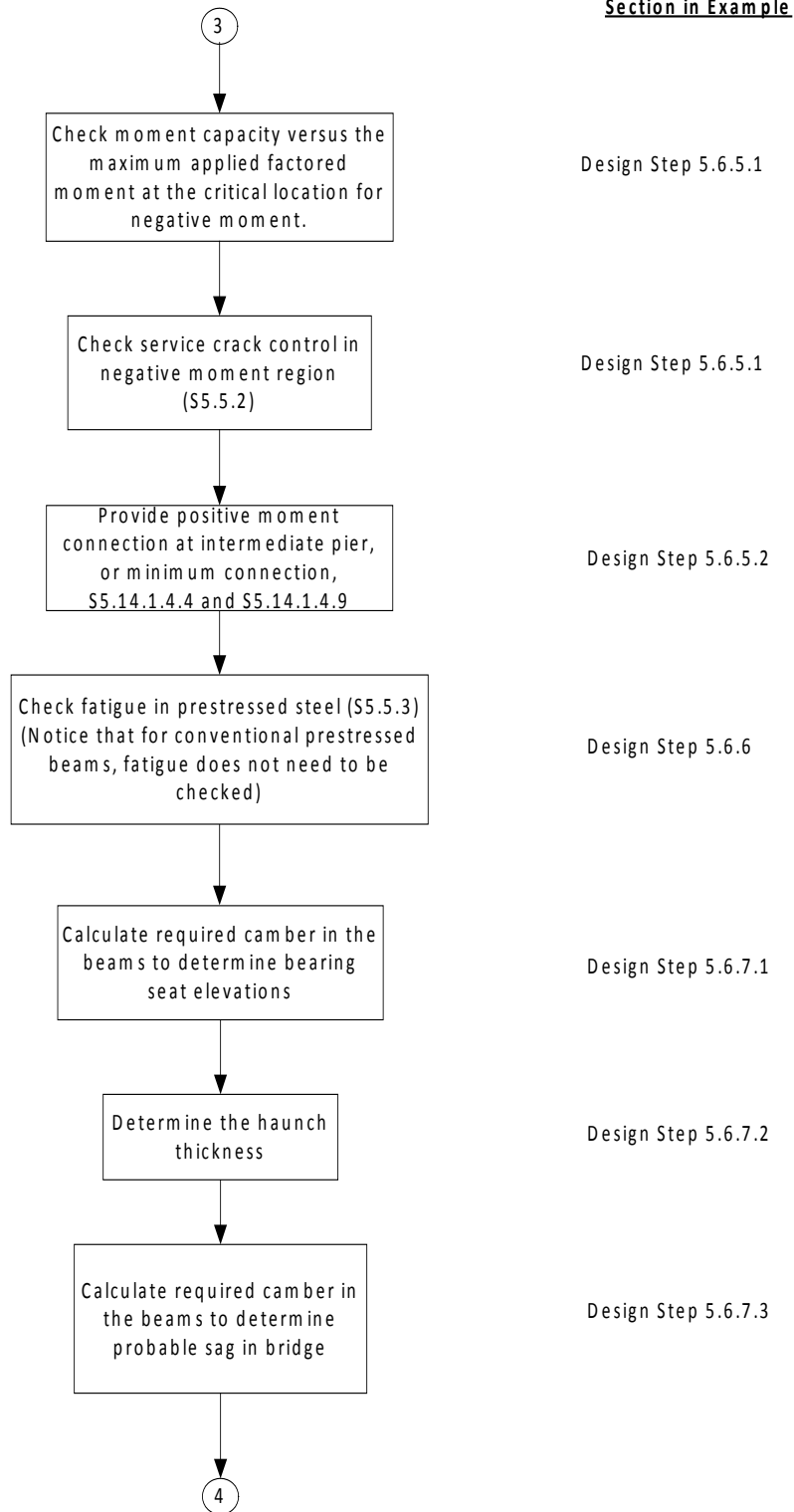


Flexural Design (cont.)

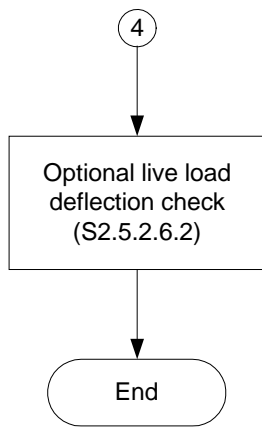




Flexural Design (cont.)



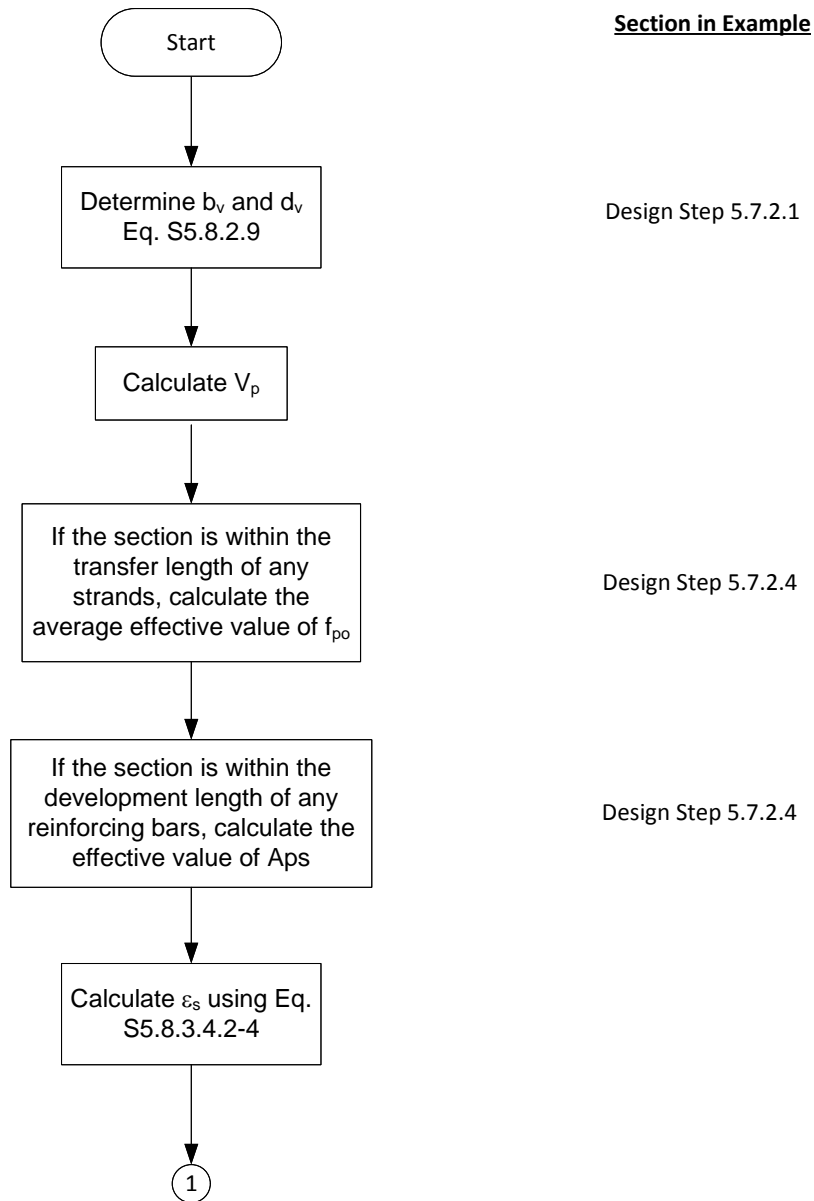
**Flexural Design (cont.)**



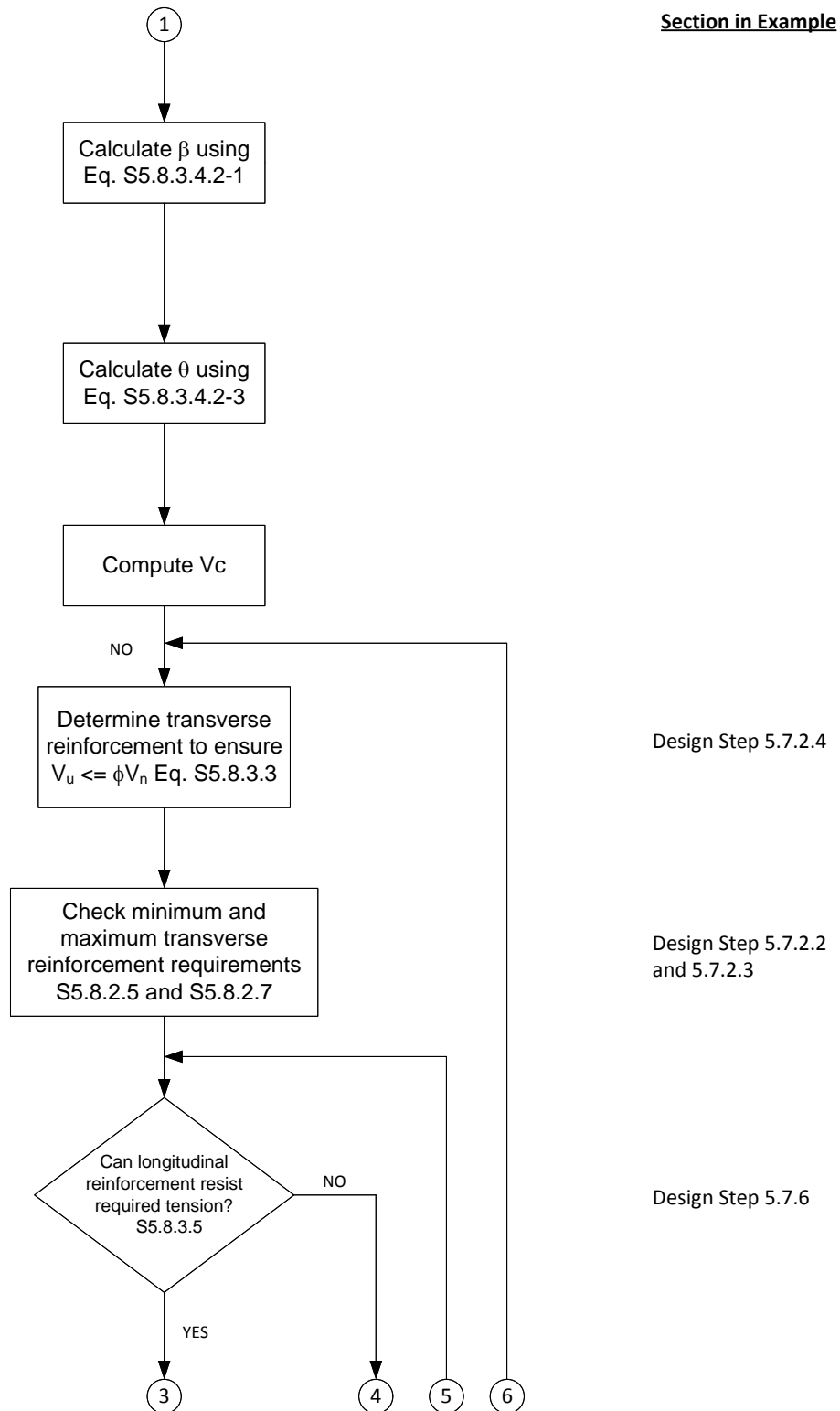
Section in Example

Design Step 5.6.8

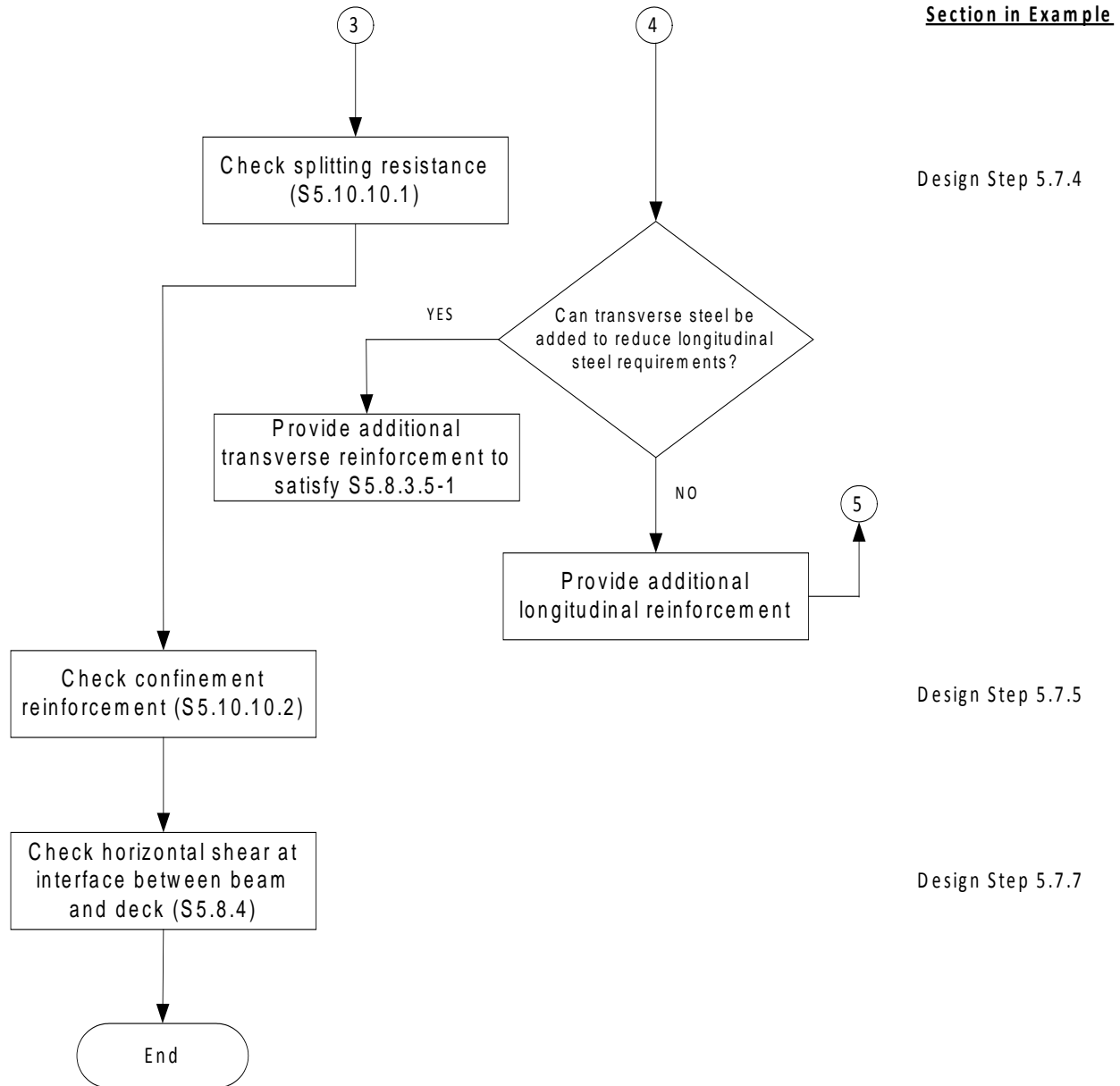
Shear Design – General Procedure, Angle  $\theta$



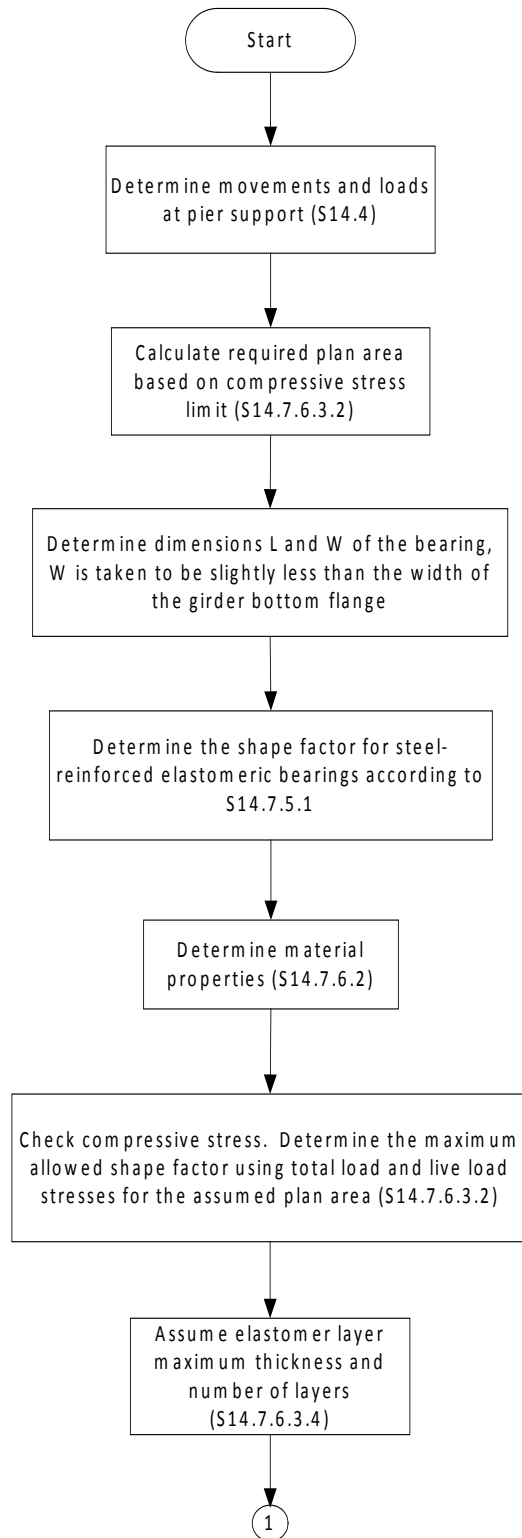
Shear Design (cont.)



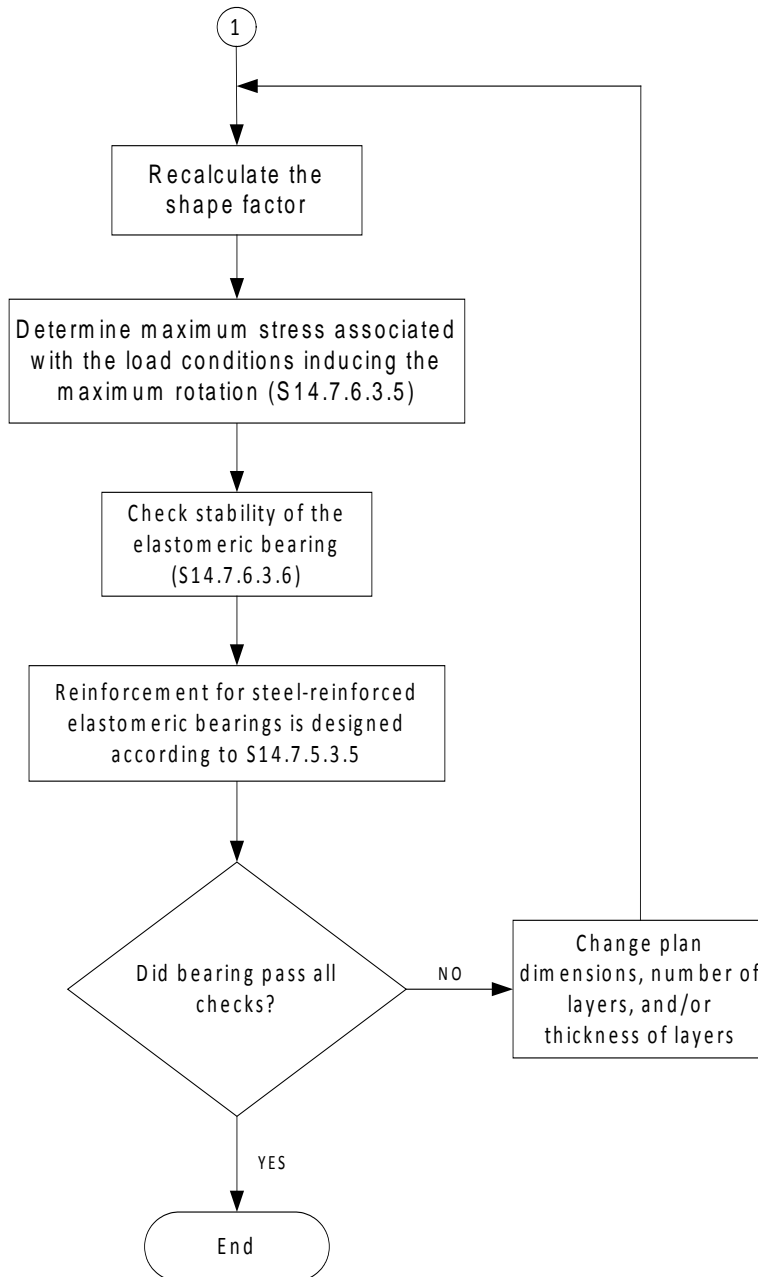
Shear Design (cont.)



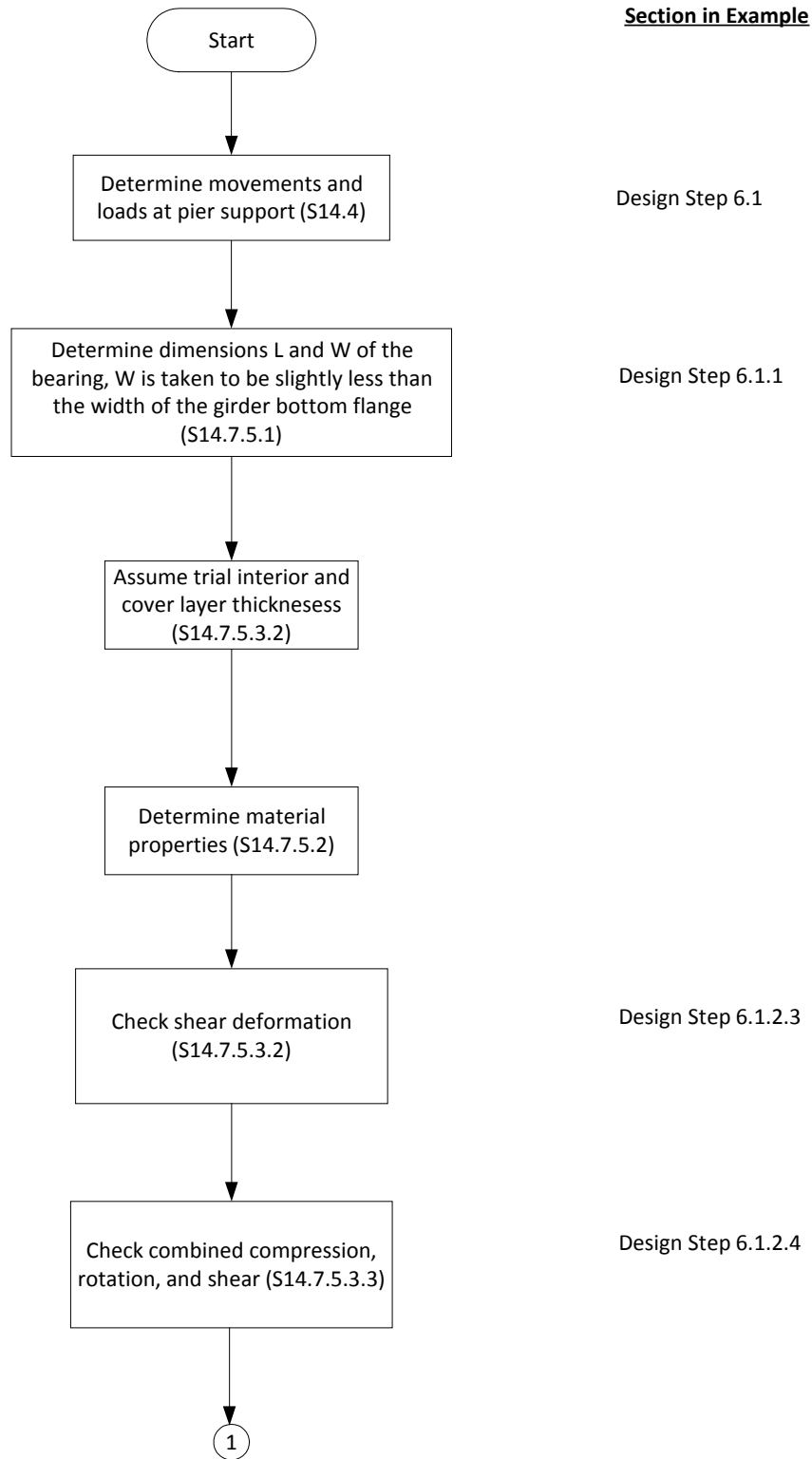
**Steel-Reinforced Elastomeric Bearing Design – Method A (Reference Only)**



Steel-Reinforced Elastomeric Bearing Design – Method A (Reference Only) (cont.)

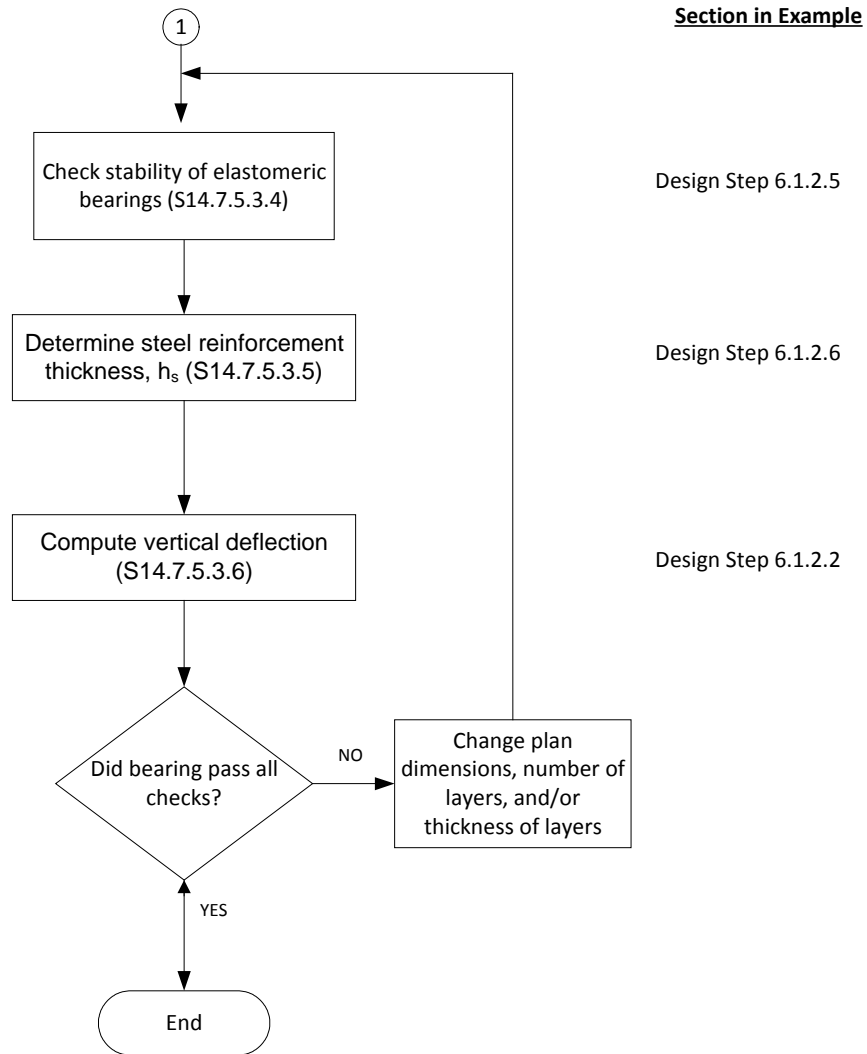


**Steel-Reinforced Elastomeric Bearing Design – Method B**



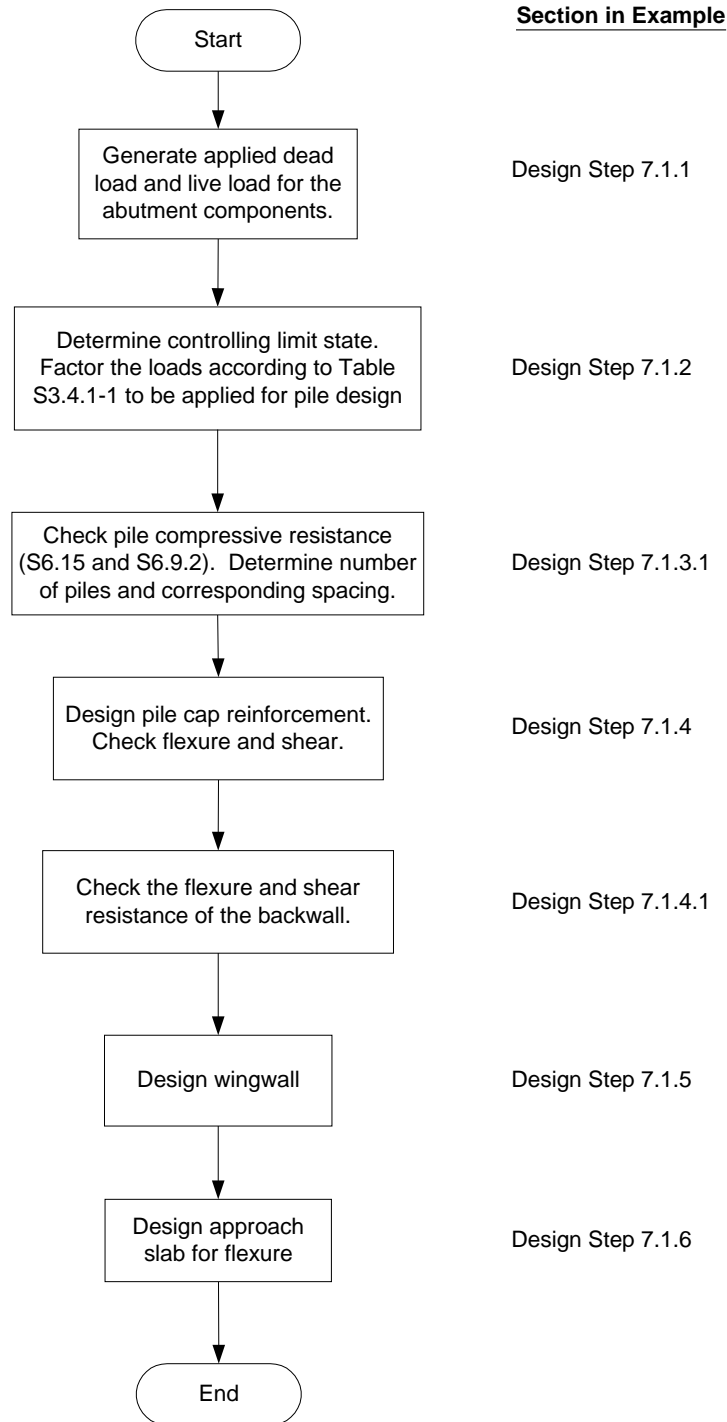


Steel-Reinforced Elastomeric Bearing Design – Method B (cont.)

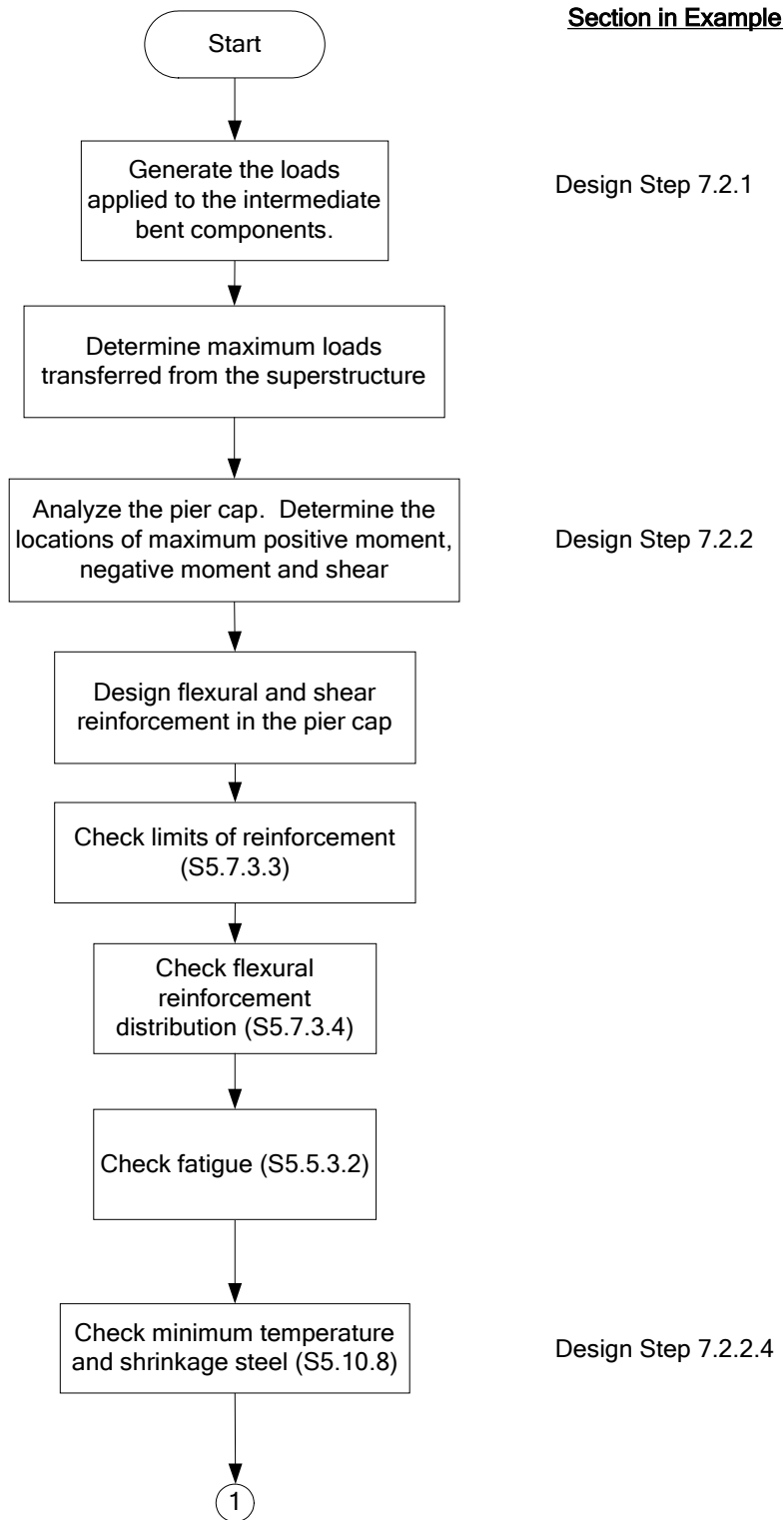


**SUBSTRUCTURE**

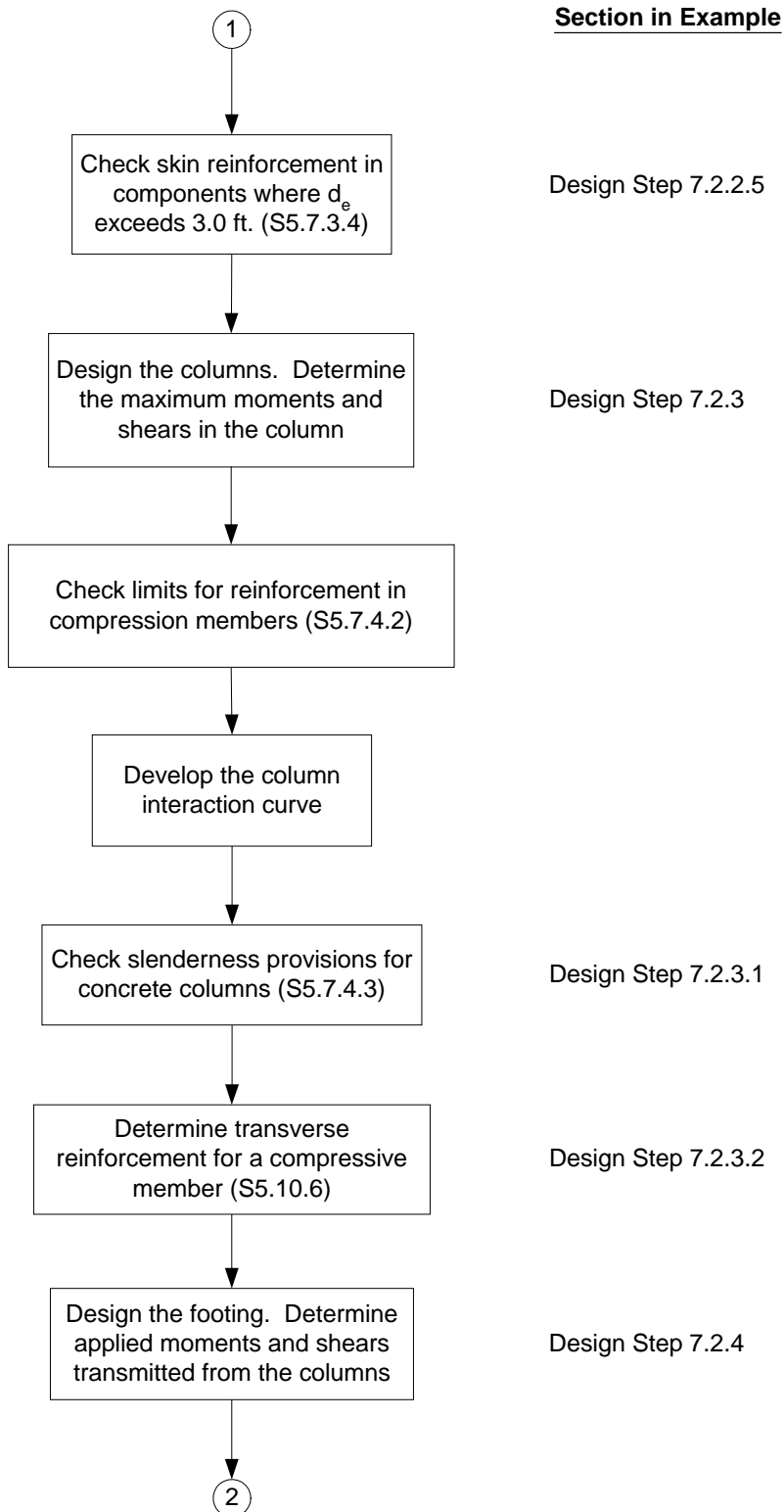
**Integral Abutment Design**



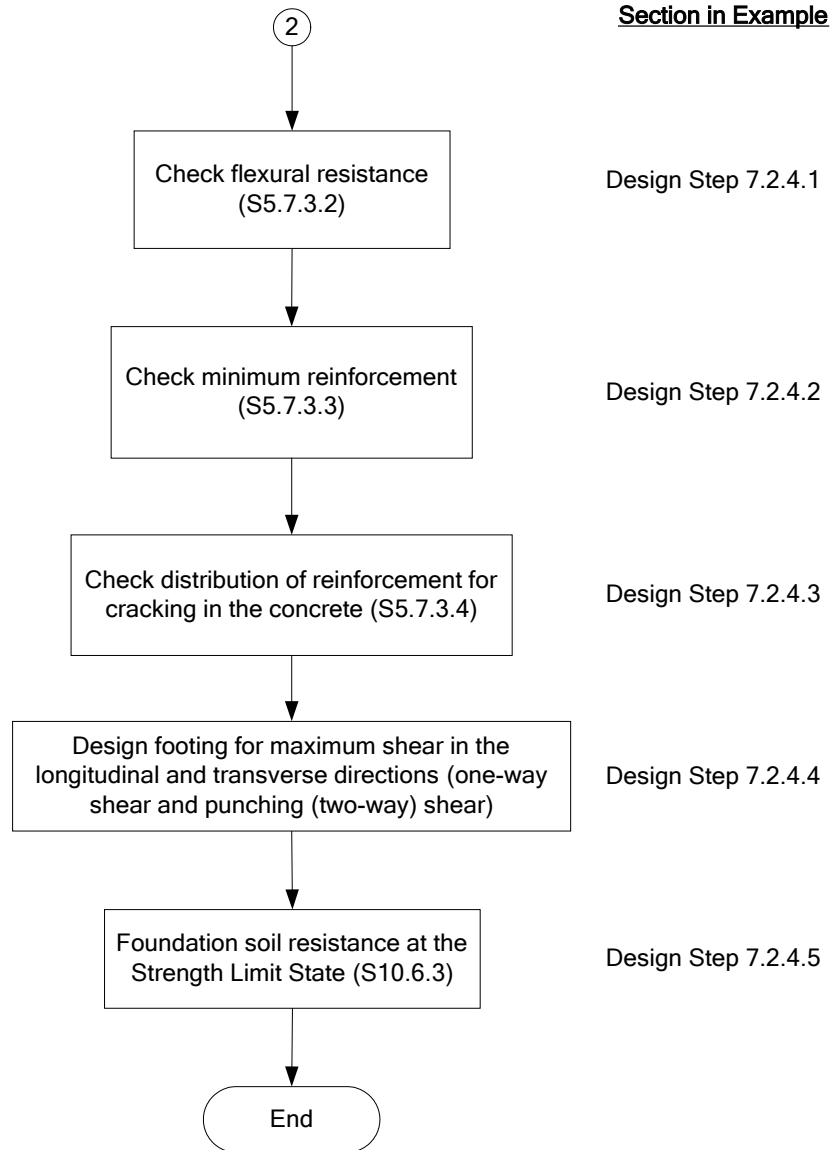
**Intermediate Bent Design**



Intermediate Bent Design (cont.)



Intermediate Bent Design (cont.)



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## 4.0 DESIGN OF DECK

### Design Step 4.1 Deck slab design

*In addition to designing the deck for dead and live loads at the strength limit state, the AASHTO-LRFD specifications require checking the deck for vehicular collision with the railing system at the extreme event limit state. The resistance factor at the extreme event limit state is taken as 1.0. This signifies that, at this level of loading, damage to the structural components is allowed and the goal is to prevent the collapse of any structural components.*

*The AASHTO-LRFD Specifications include two methods of deck design. The first method is called the approximate method of deck design (S4.6.2.1) and is typically referred to as the equivalent strip method. The second is called the Empirical Design Method (S9.7.2).*

*The equivalent strip method is based on the following:*

- *A transverse strip of the deck is assumed to support the truck axle loads.*
- *The strip is assumed to be supported on rigid supports at the center of the girders. The width of the strip for different load effects is determined using the equations in S4.6.2.1.*
- *The truck axle loads are moved laterally to produce the moment envelopes. Multiple presence factors and the dynamic load allowance are included. The total moment is divided by the strip distribution width to determine the live load per unit width.*
- *The loads transmitted to the bridge deck during vehicular collision with the railing system are determined.*
- *Design factored moments are then determined using the appropriate load factors for different limit states.*
- *The reinforcement is designed to resist the applied loads using conventional principles of reinforced concrete design.*
- *Shear and fatigue of the reinforcement need not be investigated.*

*The Empirical Design Method is based on laboratory testing of deck slabs. This testing indicates that the loads on the deck are transmitted to the supporting components mainly through arching action in the deck, not through shears and moments as assumed by traditional design. Certain limitations on the geometry of the deck are listed in S9.7.2. Once these limitations are satisfied, the specifications give reinforcement ratios for both the longitudinal and transverse reinforcement for both layers of deck reinforcement. No other design calculations are required for the interior portions of the deck. The overhang region is then designed for vehicular collision with the railing*

system and for dead and live loads acting on the deck. The Empirical Design Method requires less reinforcement in the interior portions of the deck than the Approximate Method.

For this example, the Approximate Method (Strip Width Method) is used.

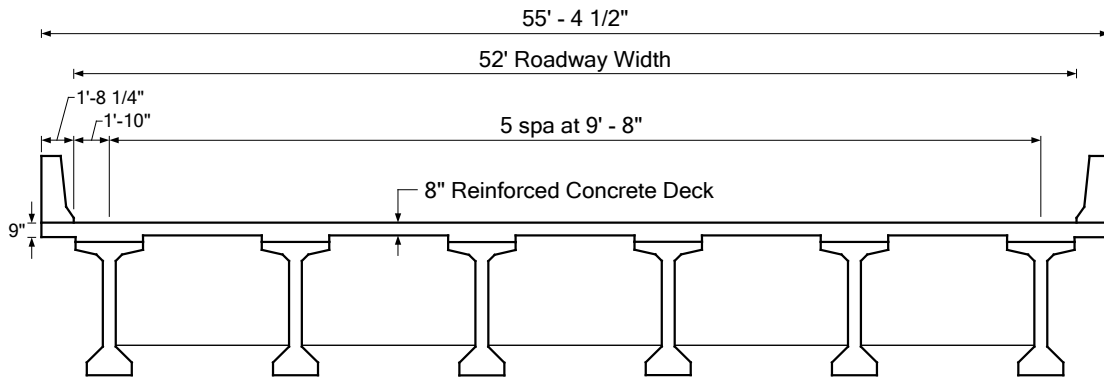


Figure 4-1 – Bridge Cross-Section

Required information:

Girder spacing	= 9 ft.- 8 in.
Top cover	= 2 ½ in. (S5.12.3) (includes ½ in. integral wearing surface)
Bottom cover	= 1 in. (S5.12.3)
Steel yield strength	= 60 ksi
Slab conc. compressive strength	= 4 ksi
Concrete density	= 150 pcf
Future wearing surface density	= 30 psf

**Design Step 4.2 Deck thickness**

The specifications require that the minimum thickness of a concrete deck, excluding any provisions for grinding, grooving and sacrificial surface, should not be less than 7 in. (S9.7.1.1). Thinner decks are acceptable if approved by the bridge owner. For slabs with depths less than 1/20 of the design span, consideration should be given to prestressing in the direction of that span in order to control cracking.

Most jurisdictions require a minimum deck thickness of 8 in., including the ½ inch integral wearing surface.

In addition to the minimum deck thickness requirements of S9.7.1.1, some jurisdictions check the slab thickness using the provisions of S2.5.2.6.3. The provisions in this article are meant for slab-



*type bridges and their purpose is to limit deflections under live loads. Applying these provisions to the design of deck slabs is not the common practice.*

For this example, a slab thickness of 8 in., including the ½ inch integral wearing surface, is assumed. The integral wearing surface is considered in the weight calculations. However, for resistance calculations, the integral wearing surface is assumed to not contribute to the section resistance, i.e., the section thickness for resistance calculations is assumed to be 7.5 in.

### Design Step 4.3 Overhang thickness

*For decks supporting concrete parapets, the minimum overhang thickness is 8 in. (S13.7.3.1.2), unless a lesser thickness is proven satisfactory through crash testing of the railing system. Using a deck overhang thickness of approximately ¾” to 1” thicker than the deck thickness has proven to be beneficial in past designs.*

For this example, an overhang thickness of 9 in., including the ½ in. sacrificial layer is assumed in the design.

### Design Step 4.4 Concrete parapet

A Type-F concrete parapet is assumed. The dimensions of the parapet are shown in Figure 4-2. The railing crash resistance was determined using the provisions of SA13.3.1. The characteristics of the parapet and its crash resistance are summarized below.

Concrete Parapet General Values and Dimensions:

Weight per unit length	= 650 lb/ft
Width at base	= 1 ft.- 8 ¼ in.
Moment capacity at the base calculated assuming the parapet acts as a vertical cantilever, $M_c/\text{length}$	= 17.83 k-ft/ft
Parapet height, H	= 42 in.
Length of parapet failure mechanism, $L_c$	= 235.2 in.
Collision load capacity, $R_w$	= 137.22 k

*Notice that each jurisdiction typically uses a limited number of railings. The properties of each parapet may be calculated once and used for all deck slabs.*

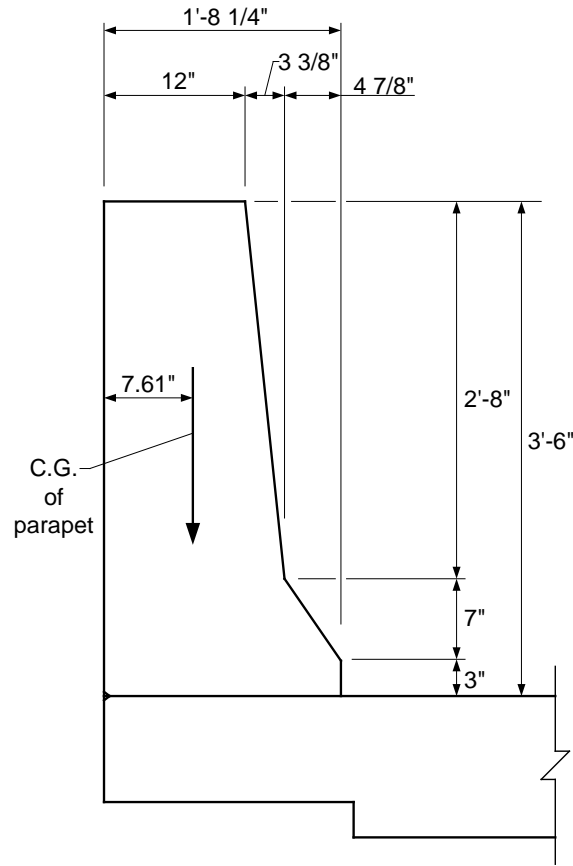


Figure 4-2 – Parapet Cross-Section

The load capacity of this parapet exceeds the minimum required by the Specifications. The deck overhang region is required to be designed to have a resistance larger than the computed resistance of the concrete parapet (SA13.4.2).

**Design Step 4.5 Equivalent strip method (S4.6.2)**

Moments are calculated for a deck transverse strip assuming rigid supports at web centerlines. The reinforcement is the same in all deck bays. The overhang is designed for cases of DL + LL at the Strength I Limit State and for collision with the railing system at the Extreme Event II Limit State.

**Design Step 4.5.1 Design dead load moments**

Strength I load factors (S3.4.1):

Slab and parapet:

Minimum = 0.9

Maximum = 1.25

Future wearing surface:

Minimum = 0.65

Maximum = 1.5

*It is not intended to maximize the load effects by applying the maximum load factors to some bays of the deck and the minimum load factors to others. Therefore, for deck slabs the maximum load factor controls the design and the minimum load factor may be ignored.*

*Dead loads represent a small fraction of the deck loads. Using a simplified approach to determine the deck dead load effects will result in a negligible difference in the total (DL + LL) load effects. Traditionally, dead load positive and negative moments in the deck, except for the overhang, for a unit width strip of the deck are calculated using the following approach:*

$$M = wl^2/c$$

where:

M = dead load positive or negative moment in the deck for a unit width strip (k-ft/ft)

w = dead load per unit area of the deck (ksf)

l = girder spacing (ft.)

c = constant, typically taken as 10 or 12

For this example, the dead load moments due to the self weight and future wearing surface are calculated assuming  $c = 10$ .

Self weight of the deck =  $8(150)/12 = 100$  psf

Unfactored self weight positive or negative moment =  $(100/1000)(9.66)^2/10$   
= 0.93 k-ft/ft

Future wearing surface = 30 psf

Unfactored FWS positive or negative moment =  $(30/1000)(9.66)^2/10$   
= 0.28 k-ft/ft

**Design Step 4.6 Distance from the center of the girder to the design section for negative moment**

*For precast I-shaped and T-shaped concrete beams, the distance from the centerline of girder to the design section for negative moment in the deck should be taken equal to one-third of the flange width from the centerline of the support (S4.6.2.1.6), but not to exceed 15 in.*

Girder top flange width = 42 in.

One-third of the girder top flange width = 14 in. < 15 in. (Use 14 in.)

**Design Step 4.7 Determining live load effects**

*Using the approximate method of deck analysis (S4.6.2), live load effects may be determined by modeling the deck as a beam supported on the girders. One or more axles may be placed side by side on the deck (representing axles from trucks in different traffic lanes) and move them transversely across the deck to maximize the moments (S4.6.2.1.6). To determine the live load moment per unit width of the bridge, the calculated total live load moment is divided by a strip width determined using the appropriate equation from Table S4.6.2.1.3-1. The following conditions have to be satisfied when determining live load effects on the deck:*

*Minimum distance from center of wheel to the inside face of parapet = 1 ft. (S3.6.1.3)*

*Minimum distance between the wheels of two adjacent trucks = 4 ft.*

*Dynamic load allowance = 33% (S3.6.2.1)*

*Load factor (Strength I) = 1.75 (S3.4.1)*

*Multiple presence factor (S3.6.1.1.2):*

*Single lane = 1.20*

*Two lanes = 1.00*

*Three lanes = 0.85*

*(Note: the “three lanes” situation never controls for girder spacings up to 16 ft.)*

*Trucks were moved laterally to determine extreme moments (S4.6.2.1.6)*

*Fatigue need not be investigated for concrete slabs in multi-girder bridges (S9.5.3 and S5.5.3.1)*

*Resistance factors,  $\phi$ , for moment: 0.9 for strength limit state (tension-controlled) (S5.5.4.2)  
1.0 for extreme event limit state (S1.3.2.1)*

*In lieu of this procedure, the specifications allow the live load moment per unit width of the deck to be determined using Table SA4-1. This table lists the positive and negative moment per unit width of decks with various girder spacings and with various distances from the design section to the*

*centerline of the girders for negative moment.* This table is based on the analysis procedure outlined above and will be used for this example. In addition, some states use their own slab design charts and tables.

Table SA4-1 does not include the girder spacing of 9'-8". It does include girder spacings of 9'-6" and 9'-9". Interpolation between the two girder spacings is allowed. However, due to the small difference between the values, the moments corresponding to the girder spacing of 9'-9" are used which gives slightly more conservative answers than interpolating. Furthermore, the table lists results for the design section for negative moment at 12 in. and 18 in. from the center of the girder. For this example, the distance from the design section for negative moment to the centerline of the girders is 14 in. Interpolation for the values listed for 12 in. and 18 in. is allowed. However, the value corresponding to the 12 in. distance may be used without interpolation resulting in a more conservative value. The latter approach is used for this example.

### Design Step 4.8 Design for positive moment in the deck

*The reinforcement determined in this section is based on the maximum positive moment in the deck. For interior bays of the deck, the maximum positive moment typically takes place at approximately the center of each bay. For the first deck bay, the bay adjacent to the overhang, the location of the maximum design positive moment varies depending on the overhang length and the value and distribution of the dead load. The same reinforcement is typically used for all deck bays.*

#### Factored loads

##### Live load

From Table SA4-1, for the girder spacing of 9'-9" (conservative):

Unfactored live load positive moment per unit width = 6.74 k-ft/ft

Maximum factored positive moment per unit width = 1.75(6.74)  
= 11.80 k-ft/ft

This moment is applicable to all positive moment regions in all bays of the deck (S4.6.2.1.1).

##### Deck weight

$1.25(0.93) = 1.16$  k-ft/ft

##### Future wearing surface

$1.5(0.28) = 0.42$  k-ft/ft

Dead load + live load design factored positive moment (Strength I limit state)

$$\begin{aligned} M_{DL+LL} &= 11.8 + 1.16 + 0.42 \\ &= 13.38 \text{ k-ft/ft} \end{aligned}$$

Notice that the total moment is dominated by the live load.

Resistance factor for flexure at the strength limit state,  $\phi = 0.90$  (tension-controlled) (S5.5.4.2.1)

*The flexural resistance equations in the AASHTO-LRFD Bridge Design Specifications are applicable to reinforced concrete and prestressed concrete sections. Depending on the provided reinforcement, the terms related to prestressing, tension reinforcing steel and/or compression reinforcing steel, are set to zero. The following text is further explanation on applying these provisions to reinforced concrete sections and the possible simplifications to the equations for this case.*

*For rectangular section behavior, the depth of the section in compression,  $c$ , is determined using Eq. S5.7.3.1.1-4:*

$$c = \frac{A_{ps} f_{pu} + A_s f_y - A'_s f'_y}{\alpha_1 f'_c \beta_1 b + k A_{ps} \frac{f_{pu}}{d_p}} \quad (\text{S5.7.3.1.1-4})$$

where:

$A_{ps}$  = area of prestressing steel (in<sup>2</sup>)

$f_{pu}$  = specified tensile strength of prestressing steel (ksi)

$f_{py}$  = yield strength of prestressing steel (ksi)

$A_s$  = area of mild steel tension reinforcement (in<sup>2</sup>)

$A'_s$  = area of compression reinforcement (in<sup>2</sup>)

$f_y$  = yield strength of tension reinforcement (ksi)

$f'_y$  = yield strength of compression reinforcement (ksi)

$b$  = width of rectangular section (in.)

$d_p$  = distance from the extreme compression fiber to the centroid of the prestressing tendons (in.)

$c$  = distance between the neutral axis and the compressive face (in.)

$\alpha_1$  = stress block factor specified in S5.7.2.2 where  $\alpha_1$  shall be taken as 0.85 for specified concrete compressive strengths not exceeding 10.0 ksi.

$\beta_1$  = stress block factor specified in S5.7.2.2

$$k = 2 \left( 1.04 - \frac{f_{py}}{f_{pu}} \right)$$

For reinforced concrete sections (no prestressing) without reinforcement on the compression side of the section (or if the compression reinforcement is ignored), the above equation is reduced to:

$$c = \frac{A_s f_y}{\alpha_1 f'_c \beta_1 b}$$

The depth of the compression block,  $a$ , may be calculated as:

$$a = c\beta_1$$

These equations for “ $a$ ” and “ $c$ ” are identical to those traditionally used in reinforced concrete design. Many text books use the following equation to determine the reinforcement ratio,  $\rho$ , and area of reinforcement,  $A_s$ :

$$k' = M_u / (\phi b d^2)$$

$$\rho = 0.85 \left( \frac{f'_c}{f_y} \right) \left[ 1.0 - \sqrt{1.0 - \frac{2k'}{0.85 f'_c}} \right]$$

$$A_s = \rho d_e$$

A different method to determine the required area of steel is based on using the above equation for “ $a$ ” and “ $c$ ” with the Eq. S5.7.3.2.2-1 as shown below. The nominal flexural resistance,  $M_n$ , may be taken as:

$$M_n = A_{ps} f_{ps} (d_p - a/2) + A_s f_y (d_s - a/2) - A'_s f'_y (d'_s - a/2) + \alpha_1 f'_c (b - b_w) h_f (a/2 - h_f/2) \quad (S5.7.3.2.2-1)$$

where:

$f_{ps}$  = average stress in prestressing steel at nominal bending resistance specified in Eq. S5.7.3.1.1-1 (ksi)

$d_s$  = distance from extreme compression fiber to the centroid of nonprestressed tensile reinforcement (in.)

$d'_s$  = distance from extreme compression fiber to the centroid of compression reinforcement (in.)

$b$  = width of the compression face of the member (in.)

$b_w$  = web width or diameter of a circular section (in.)

$h_f$  = compression flange depth of an I or T member (in.)

$\alpha_1$  = stress block factor specified in S5.7.2.2

For rectangular reinforced concrete sections (no prestressing) without reinforcement on the compression side of the section, the above equation is reduced to:

$$M_a = A_s f_y \left( d_s - \frac{a}{2} \right)$$

From the equations for “c” and “a” above, substituting for:

$$a = c\beta_1 = \frac{A_s f_y}{\alpha_1 f'_c b} \text{ in the equation for } M_a \text{ above yields:}$$

$$M_a = A_s f_y \left( d_s - \frac{a}{2} \right) = f_y d_s A_s - \left( \frac{f_y^2}{1.7 f'_c b} \right) A_s^2$$

Only  $A_s$  is unknown in this equation. By substituting for  $b = 12$  in., the required area of reinforcement per unit width can be determined by solving the equation.

Both methods outlined above yield the same answer. The first method is used throughout the following calculations.

For the positive moment section:

$$\begin{aligned} d_e &= \text{effective depth from the compression fiber to the centroid of the tensile} \\ &\quad \text{force in the tensile reinforcement (in.)} \\ &= \text{total thickness} - \text{bottom cover} - \frac{1}{2} \text{ bar diameter} - \text{integral wearing surface} \\ &= 8 - 1 - \frac{1}{2} (0.625) - 0.5 \\ &= 6.19 \text{ in.} \end{aligned}$$

$$\begin{aligned} k' &= M_u / (\phi b d^2) \\ &= 13.38 / [0.9(1.0)(6.19)^2] \\ &= 0.388 \text{ k/in}^2 \end{aligned}$$



$$\rho = 0.85 \left( \frac{f'_c}{f_y} \right) \left[ 1.0 - \sqrt{1.0 - \frac{2k'}{0.85f'_c}} \right]$$

$$= 0.00688$$

Therefore,

$$\text{Required } A_s = \rho d_e = 0.00688(6.19) = 0.0426 \text{ in}^2/\text{in.}$$

Required #5 bar spacing with bar area  $0.31 \text{ in}^2 = 0.31/0.0426 = 7.28 \text{ in.}$

Use #5 bars at 7 in. spacing

#### Check maximum and minimum reinforcement

*Based on past experience, maximum and minimum reinforcement requirements never control the deck slab design. The minimum reinforcement requirements are presented in S5.7.3.3.2. These provisions are illustrated later in this example.*

*Maximum reinforcement requirements were deleted from S5.7.3.3.1 in 2005. However, reinforced concrete sections in flexure should be checked for ductility in accordance with S5.7.2.1. The strain in the tension steel should be equal to or greater than 0.005 just as the concrete in compression reaches a strain of 0.003. This condition is called “tension-controlled”.*

Calculate depth of compression block:

$$\begin{aligned} T &= \text{tensile force in the tensile reinforcement (k)} \\ &= 0.31(60) \\ &= 18.6 \text{ k} \end{aligned}$$

$$\begin{aligned} a &= 18.6/[0.85(4)(7)] \\ &= 0.78 \text{ in.} \end{aligned}$$

$$\begin{aligned} \beta_1 &= \text{ratio of the depth of the equivalent uniformly stressed compression zone assumed} \\ &\quad \text{in the strength limit state to the depth of the actual compression zone} \\ &= 0.85 \text{ for } f'_c = 4 \text{ ksi (S5.7.2.2)} \end{aligned}$$

$$\begin{aligned} c &= 0.78/0.85 \\ &= 0.918 \text{ in.} \end{aligned}$$

Calculate the strain in the tension steel:

$$\epsilon_s = 0.003 \left( \frac{d_e}{c} - 1 \right)$$

$$\begin{aligned}
 &= 0.003 \left( \frac{6.19}{0.918} - 1 \right) \\
 &= 0.017 > 0.005 \quad \mathbf{OK}
 \end{aligned}$$

Check for cracking under Service I Limit State (S5.7.3.4)

Maximum spacing of tension steel using Eq. S5.7.3.4-1:

$$s \leq \frac{700\gamma_e}{\beta_s f_s} - 2d_c$$

in which:

$$\beta_s = 1 + \frac{d_c}{0.7(h - d_c)}$$

where:

$\gamma_e$  = exposure factor  
= 1.00 for Class 1 exposure condition

$d_c$  = thickness of concrete cover measured from extreme tension fiber to center of the flexural reinforcement located closest thereto (in.)  
= 1.3125 in.

$f_{ss}$  = tensile stress in steel reinforcement at the service limit state (ksi)  
= Maximum stress = 36 ksi

$h$  = overall thickness or depth of the component (in.)  
= 7.5 in.

By substituting  $d_c$  and  $h$ ,

$$\beta_s = 1 + \frac{1.3125}{0.7(7.5 - 1.3125)} = 1.303$$

Therefore,

$$s_{\text{required}} \leq \frac{700(1.00)}{(1.303)(36)} - 2(1.3125) = 12.3 \text{ in.}$$

$s_{\text{provided}} = 7 \text{ in.} < 12.3 \text{ in.} \quad \mathbf{OK}$

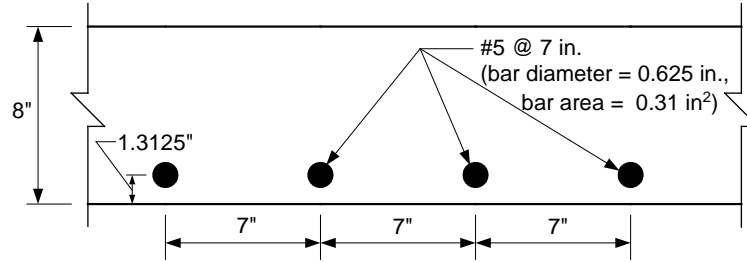


Figure 4-3 - Bottom Transverse Reinforcement

Stresses under service loads (S5.7.1)

*In calculating the transformed compression steel area, the Specifications require the use of two different values for the modular ratio when calculating the service load stresses caused by dead and live loads,  $2n$  and  $n$ , respectively. For deck design, it is customary to ignore the compression steel in the calculation of service load stresses and, therefore, this provision is not applicable. For tension steel, the transformed area is calculated using the modular ratio,  $n$ .*

Modular ratio for 4 ksi concrete,  $n = 8$

Assume stresses and strains vary linearly

Dead load service load moment =  $0.93 + 0.28 = 1.21$  k-ft/ft

Live load service load moment =  $6.74$  k-ft/ft

Dead load + live load service load positive moment =  $7.95$  k-ft/ft

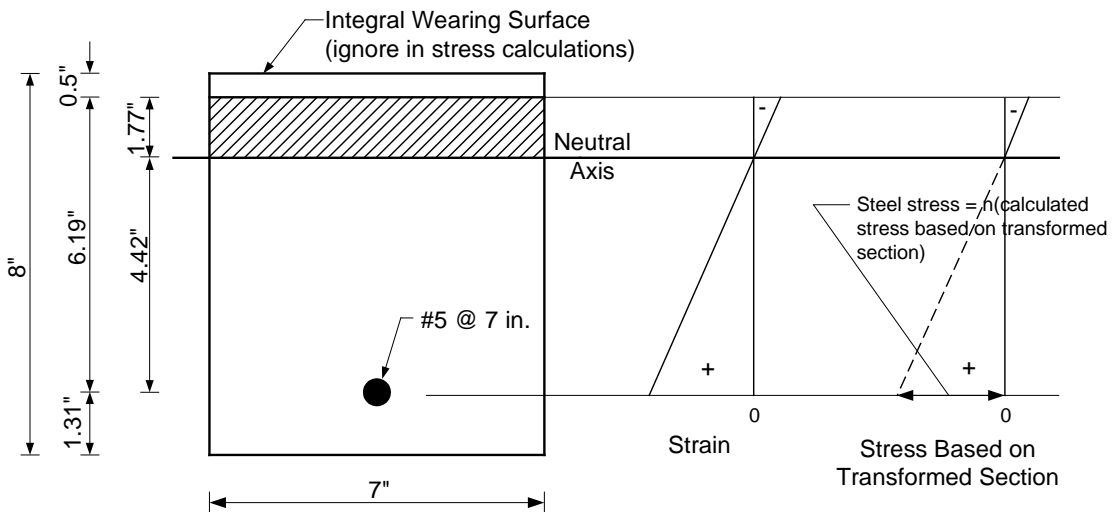


Figure 4-4 - Crack Control for Positive Moment Reinforcement Under Live Loads

*The transformed moment of inertia is calculated assuming elastic behavior, i.e. linear stress and strain distribution. In this case, the first moment of area of the transformed steel on the tension side about the neutral axis is assumed equal to that of the concrete in compression. The process of calculating the transformed moment of inertia is illustrated in Figure 4-4 and by the calculations below.*

For 4 ksi concrete, the modular ratio,  $n = 8$  (S6.10.1.1.1b or by dividing the modulus of elasticity of the steel by that of the concrete and rounding to the nearest integer number as required by S5.7.1).

Assume the neutral axis is at a distance “y” from the compression face of the section

Assume the section width equals the reinforcement spacing = 7 in.

The transformed steel area = (steel area)(modular ratio) =  $0.31(8) = 2.48 \text{ in}^2$

By equating the first moment of area of the transformed steel to that of the concrete, both about the neutral axis:

$$2.48(6.19 - y) = 7y(y/2)$$

Solving the equation results in  $y = 1.77 \text{ in.}$

$$\begin{aligned} I_{\text{transformed}} &= 2.48(6.19 - 1.77)^2 + 7(1.77)^3/3 \\ &= 61.4 \text{ in}^4 \end{aligned}$$

Stress in the steel,  $f_s = (Mc/I)n$ , where  $M$  is the moment acting on 7 in. width of the deck.

$$\begin{aligned} f_s &= [(7.95(12/12)(7)(4.42)]/61.4]8 \\ &= 32.05 \text{ ksi} \end{aligned}$$

Allowable service load stress = 36 ksi > 32.05 ksi **OK**

## Design Step 4.9 Design for negative moment at interior girders

### a. Live load

From Table SA4-1, for girder spacing of 9'-9" and the distance from the design section for negative moment to the centerline of the girder equal to 12 in. (see Design Step 4.7 for explanation):

Unfactored live load negative moment per unit width of the deck = 4.21 k-ft/ft

Maximum factored negative moment per unit width at the design section for negative moment =  $1.75(4.21) = 7.37 \text{ k-ft/ft}$

**b. Dead load**

Factored dead load moments at the design section for negative moment:

Dead weight

$$1.25(0.93) = 1.16 \text{ k-ft/ft}$$

Future wearing surface

$$1.5(0.28) = 0.42 \text{ k-ft/ft}$$

$$\begin{aligned} \text{Dead Load + live load design factored negative moment} &= 1.16 + 0.42 + 7.37 \\ &= 8.95 \text{ k-ft/ft} \end{aligned}$$

$d$  = distance from compression face to centroid of tension reinforcement (in.)  
 = total thickness – top cover –  $\frac{1}{2}$  bar diameter

Assume #5 bars; bar diameter = 0.625 in., bar area = 0.31 in<sup>2</sup>

$$\begin{aligned} d &= 8 - 2 \frac{1}{2} - \frac{1}{2} (0.625) \\ &= 5.19 \text{ in.} \end{aligned}$$

Required area of steel = 0.0339 in<sup>2</sup>/in.

Required spacing = 0.31/0.0339 = 9.15 in.

Use #5 at 9 in. spacing

As indicated earlier, checking the minimum and maximum reinforcement is not expected to control in deck slabs.

Check for cracking under service limit state (S5.7.3.4)

As explained earlier, service load stresses are calculated using a modular ratio,  $n = 8$ .

Dead load service load moment at the design section for negative moment near the middle = -1.21 k-ft/ft.

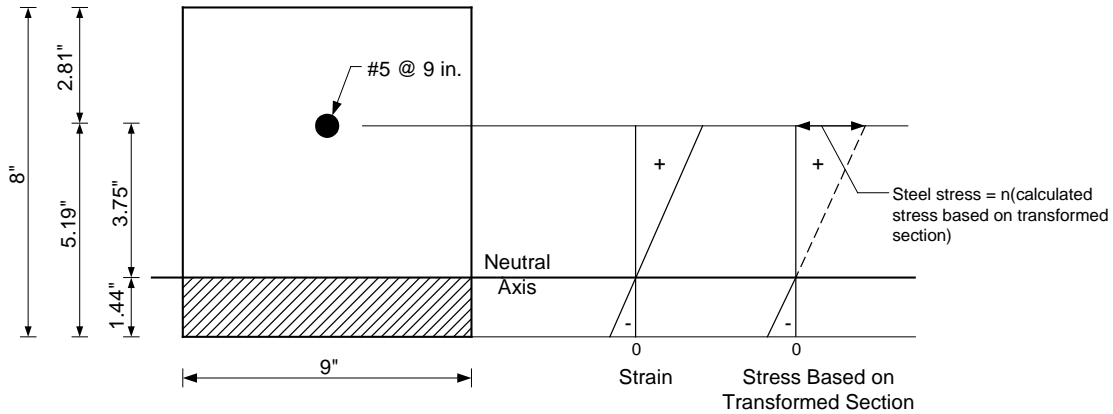


Figure 4-5a - Crack Control for Negative Moment Reinforcement Under Live Loads

Live load service load moment at the design section in the first interior bay near the first interior girder = -4.21 k-ft/ft.

Transformed section properties may be calculated as done for the positive moment section in Design Step 4.8. Refer to Figure 4-5a for the section dimensions and location of the neutral axis. The calculations are shown below.

Maximum dead load + live load service load moment = 5.42 k-ft/ft

$$n = 8$$

$$I_{\text{transformed}} = 43.83 \text{ in}^4$$

$$\begin{aligned} \text{Total DL + LL service load stresses} &= \left[ \frac{5.42(9)(3.75)}{43.83} \right] (8) \\ &= 33.39 \text{ ksi} \end{aligned}$$

Maximum spacing of tension steel:

$$s \leq \frac{700\gamma_e}{\beta_s f_s} - 2d_c$$

Concrete cover = 2 ½ in. – ½ in. integral wearing surface = 2 in.

where:

$$\begin{aligned} d_c &= \text{clear cover} + \frac{1}{2} \text{ bar diameter} \\ &= 2 + \frac{1}{2} (0.625) \\ &= 2.31 \text{ in.} \end{aligned}$$

$$f_s = 33.39 \text{ ksi}$$

$$\gamma_e = 1.00$$

$$h = 7.5 \text{ in.}$$

$$\beta_s = 1 + \frac{2.31}{0.7(7.5 - 2.31)} = 1.636$$

$$s = 9'' \geq \frac{700(1.00)}{(1.636)(33.39)} - 2(2.31) = 8.19'' \text{ NG}$$

To satisfy the crack control provisions, the most economical change is to replace the reinforcement bars by smaller bars at smaller spacing (area of reinforcement per unit width is the same). However, in this particular example, the #5 bar size cannot be reduced as this bar is customarily considered the minimum bar size for deck main reinforcement. Therefore, the bar diameter is kept the same and the spacing is reduced.

Assume reinforcement is #5 at 8 in. spacing (refer to Figure 4-5b).

$$I_{\text{transformed}} = 42.77 \text{ in}^4$$

$$\begin{aligned} \text{Total DL + LL service load stresses} &= [[5.42(8)(3.68)/42.77](8) \\ &= 29.85 \text{ ksi} \end{aligned}$$

$$s = 8'' \leq \frac{700(1.00)}{(1.636)(29.85)} - 2(2.31) = 9.71'' \text{ OK}$$

Use main negative moment reinforcement #5 at 8 in. spacing

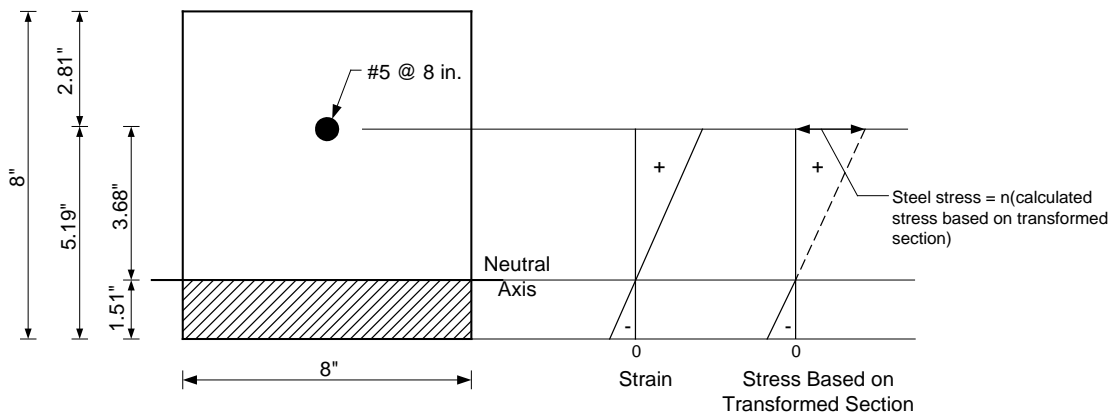


Figure 4-5b - Crack Control for Negative Moment Reinforcement Under Live Loads

Design Step 4.10 Design of the overhang

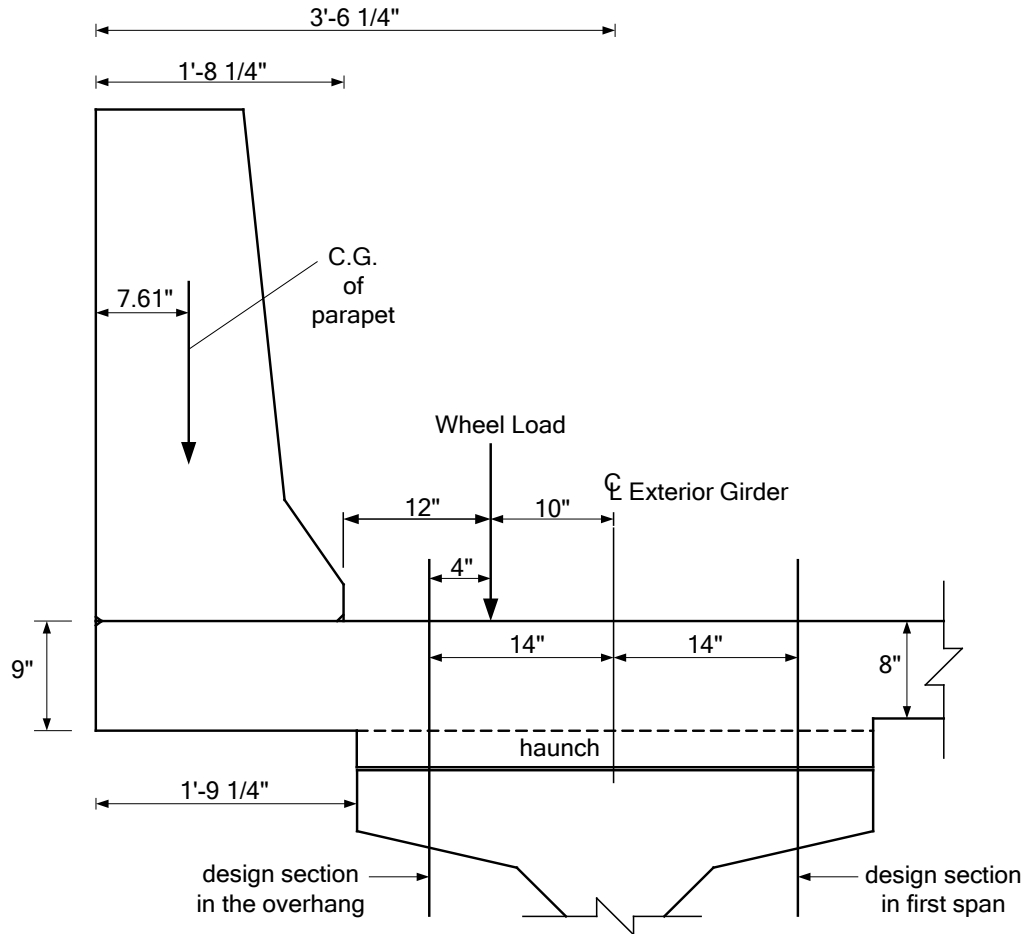


Figure 4-6 - Overhang Region, Dimensions and Truck Loading

Assume that the bottom of the deck in the overhang region is 1 inch lower than the bottom of other bays as shown in Figure 4-6. This results in a total overhang thickness equal to 9 in. This is usually beneficial in resisting the effects of vehicular collision. However, a section in the first bay of the deck, where the thickness is smaller than that of the overhang, must also be checked.

**Assumed loads**

Self weight of the slab in the overhang area = 0.1125 k/ft<sup>2</sup> of the deck overhang surface area

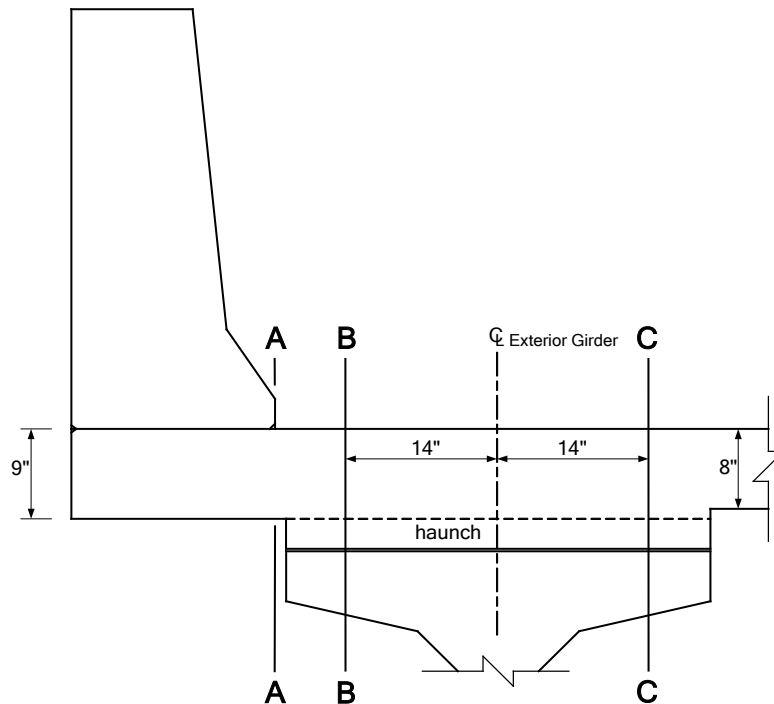
Weight of parapet = 0.650 k/ft of length of parapet



Future wearing surface = 0.030 k/ft<sup>2</sup> of deck surface area

As required by SA13.4.1, there are three design cases to be checked when designing the deck overhang regions.

**Design Case 1: Check overhang for horizontal vehicular collision load (SA13.4.1, Case 1)**



**Figure 4-7 - Design Sections in the Overhang Region**

*The overhang is designed to resist an axial tension force from vehicular collision acting simultaneously with the collision + dead load moment.*

*The resistance factor,  $\phi = 1.0$  for extreme event limit state (S1.3.2.1). The Specification requires that load effects in the extreme event limit state be multiplied by  $\eta_i \geq 1.05$  for bridges deemed important or  $\eta_i \geq 0.95$  for bridges deemed not important. For this example, a value of  $\eta_i = 1.0$  was used.*

**a. At inside face of parapet (Section A-A in Figure 4-7)**

(see Design Step 4.4 for parapet characteristics)

$M_c$  = moment capacity of the base of the parapet given as 17.83 k-ft/ft.

When this moment is transmitted to the deck overhang it subjects the deck to negative moment.

*For a complete railing design example that includes sample detailed calculations of railing parameters, see Lecture 16 of the participant notebook of the National Highway Institute Course No. 13061.*

$$\begin{aligned} M_{DL, \text{slab}} &= 0.1125(20.25/12)^2/2 \\ &= 0.16 \text{ k-ft/ft} \end{aligned}$$

$$\begin{aligned} M_{DL, \text{parapet}} &= 0.65(20.25 - 7.61)/12 \\ &= 0.68 \text{ k-ft/ft} \end{aligned}$$

$$\text{Design factored moment} = -17.83 - 1.25(0.16 + 0.68) = -18.88 \text{ k-ft/ft}$$

$$\begin{aligned} \text{Design axial tensile force (SA13.4.2)} &= R_w/(L_c + 2H) \\ &= 137.22/[(235.2 + 2(42))/12] \\ &= 5.16 \text{ k/ft} \end{aligned}$$

$$h \text{ slab} = 9 \text{ in.}$$

Assuming #5 reinforcement bars,

$$\begin{aligned} d &= \text{overhang slab thickness} - \text{top cover} - \frac{1}{2} \text{ bar diameter} \\ &= 9 - 2 \frac{1}{2} - \frac{1}{2}(0.625) \\ &= 6.19 \text{ in.} \end{aligned}$$

$$\text{Assume required area of steel} = 0.70 \text{ in}^2/\text{ft} \quad (1)$$

Effective depth of the section,  $d = 6.19$  in.

(Notice that the overhang has 1 inch additional thickness at its bottom)

For a section under moment and axial tension,  $P$ , the nominal resistance,  $M_n$ , may be calculated as:

$$M_n = T(d - a/2) - P(h/2 - a/2)$$

$$\text{Tension in reinforcement, } T = 0.70(60) = 42.0 \text{ k/ft}$$

$$\text{Compression in concrete, } C = 42.0 - 5.16 = 36.84 \text{ k/ft}$$

$$\begin{aligned} a &= C/b\beta f'_c \\ &= 36.84/[12(0.85)(4)] \\ &= 0.90 \text{ in.} \end{aligned}$$

$$c = a/\beta_1 = 0.90/0.85 = 1.05 \text{ in.}$$

$$\begin{aligned} M_n &= 42.0[6.19 - (0.9/2)] - 5.16[(6.19/2) - (0.9/2)] \\ &= 227.43/12 \\ &= 18.95 \text{ k-ft/ft} \end{aligned}$$

Notice that many designers determine the required reinforcement for sections under moment and axial tension,  $P$ , as the sum of two components:

- 1) the reinforcement required assuming the section is subjected to moment
- 2)  $P/f_y$

This approach is acceptable as it results in more conservative results, i.e., more reinforcement.

The ductility check is not expected to control. However, due to the additional reinforcement in the overhang, it is prudent to perform this check using the provisions of S5.7.2.1.

Resistance factor = 1.0 for extreme event limit state (S1.3.2.1)

$$\begin{aligned} M_r &= \phi M_n \\ &= 1.0(18.95) = 18.95 \text{ k-ft/ft} > M_u = 18.88 \text{ k-ft/ft} \quad \mathbf{OK} \end{aligned}$$

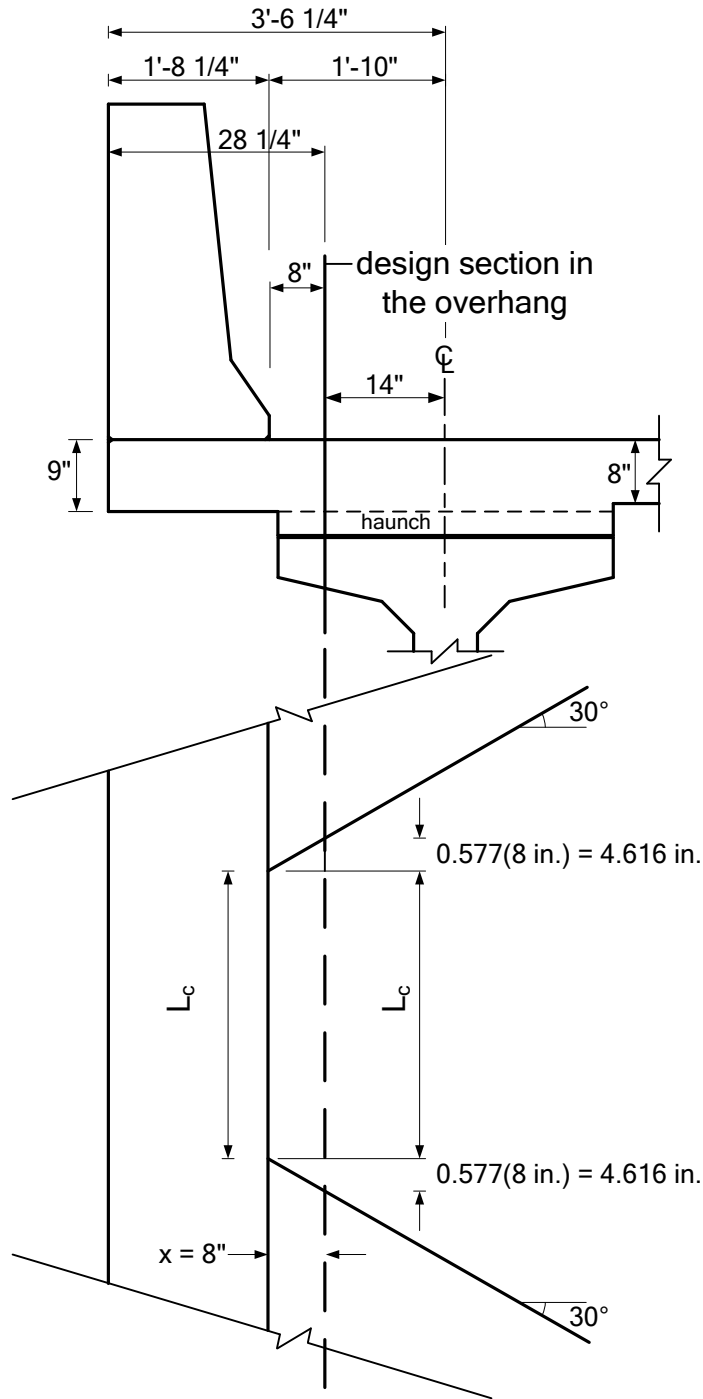
$$\epsilon_s = 0.003 \left( \frac{d}{c} - 1 \right) = 0.003 \left( \frac{6.19}{1.05} - 1 \right) = 0.0147 > 0.005 \quad \mathbf{OK}$$

Steel has sufficiently yielded before concrete crushes, i.e., the section is ductile.

#### **b. At design section in the overhang (Section B-B in Figure 4-7)**

Assume that the minimum haunch thickness is at least equal to the difference between the thickness of the interior regions of the slab and the overhang thickness, i.e., 1 in. This means that when designing the section in the overhang at 14 in. from the center of the girder, the total thickness of the slab at this point can be assumed to be 9 in. For thinner haunches, engineering judgment should be exercised to determine the thickness to be considered at this section.

*At the inside face of the parapet, the collision forces are distributed over a distance  $L_c$  for the moment, where  $L_c$  is the critical length of the yield line failure pattern, and  $L_c + 2H$  for axial force. It is reasonable to assume that the distribution length will increase as the distance from the section to the parapet increases. The value of the distribution angle is not specified in the specifications and is determined using engineering judgment. In this example, the distribution length was increased using a 30° angle from the base of the parapet (see Figure 4-8). Some designers assume a distribution angle of 45°, this angle would have also been acceptable.*



**Figure 4-8 - Assumed Distribution of Collision Moment Load in the Overhang**

Collision moment at the design section

$$= M_c L_c / [L_c + 2(0.577)X]$$

$$= -17.83(235.2) / [235.2 + 2(0.577)(8)]$$

$$= -17.16 \text{ k-ft/ft}$$

Dead load moment at the design section:

$$\begin{aligned} M_{DL, \text{slab}} &= 0.1125(28.25/12)^2/2 \\ &= 0.31 \text{ k-ft/ft} \end{aligned}$$

$$\begin{aligned} M_{DL, \text{parapet}} &= 0.65(28.25 - 7.61)/12 \\ &= 1.12 \text{ k-ft/ft} \end{aligned}$$

$$\begin{aligned} M_{DL, \text{FWS}} &= 0.03(8/12)^2/2 \\ &= 0.007 \text{ k-ft/ft} \end{aligned}$$

Factored design  $M = -17.16 - 1.25(0.31 + 1.12) - 1.5(0.007) = -18.96 \text{ k-ft/ft}$

$$\begin{aligned} \text{Design tensile force} &= R_w/[L_c + 2H + 2(0.577)X] \\ &= 137.22/[[235.2 + 2(42) + 2(0.577)(8)]/12] \\ &= 5.01 \text{ k/ft} \end{aligned}$$

$$h \text{ slab} = 9 \text{ in.}$$

By inspection, for Section A-A, providing an area of steel = 0.70 in<sup>2</sup>/ft resulted in a moment resistance of 18.95 k-ft/ft  $\cong$  the design moment for Section B-B.

Therefore, the required area of steel for Section B-B = 0.70 in<sup>2</sup>/ft (2)

**c. Check dead load + collision moments at design section in first span (Section C-C in Figure 4-7)**

*The total collision moment can be treated as an applied moment at the end of a continuous strip. The ratio of the moment  $M_2/M_1$  (see Figure 4-9) can be calculated for the transverse design strip. As an approximation, the ratio  $M_2/M_1$  may be taken equal to 0.4. This approximation is based on the fact that  $M_2/M_1 = 0.5$  if the rotation at the first interior girder is restrained. Since this rotation is not restrained, the value of  $M_2$  will be less than  $0.5M_1$ . Thus, the assumption that  $M_2/M_1 = 0.4$  seems to be reasonable. The collision moment per unit width at the section under consideration can then be determined by dividing the total collision moment by the distribution length. The distribution length may be determined using the 30° distribution as illustrated in Figure 4-8 except that the distance “X” will be 36 in. for Section C.*

*The dead load moments at the design section for negative moment to the inside of the exterior girder may be determined by superimposing two components: (1) the moments in the first deck span due to the dead loads acting on the overhang (see Figure 4-10), and (2) the effect of the dead loads acting on the first span of the deck (see Figure 4-11).*

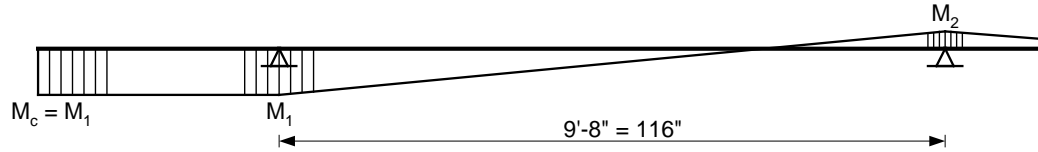


Figure 4-9 – Assumed Distribution of the Collision Moment Across the Width of the Deck

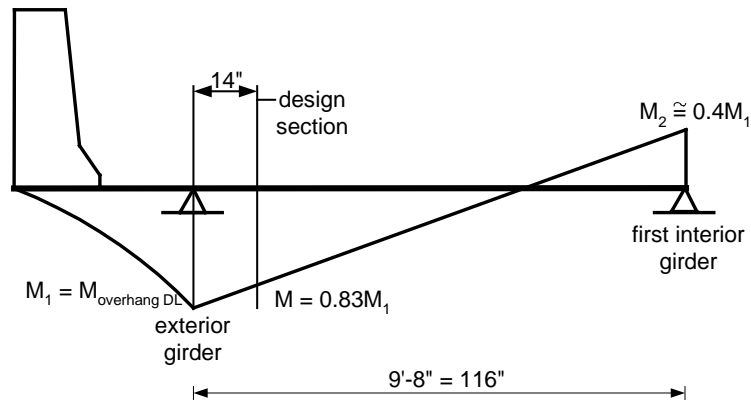


Figure 4-10 - Dead Load Moment at Design Section Due to Dead Loads on the Overhang

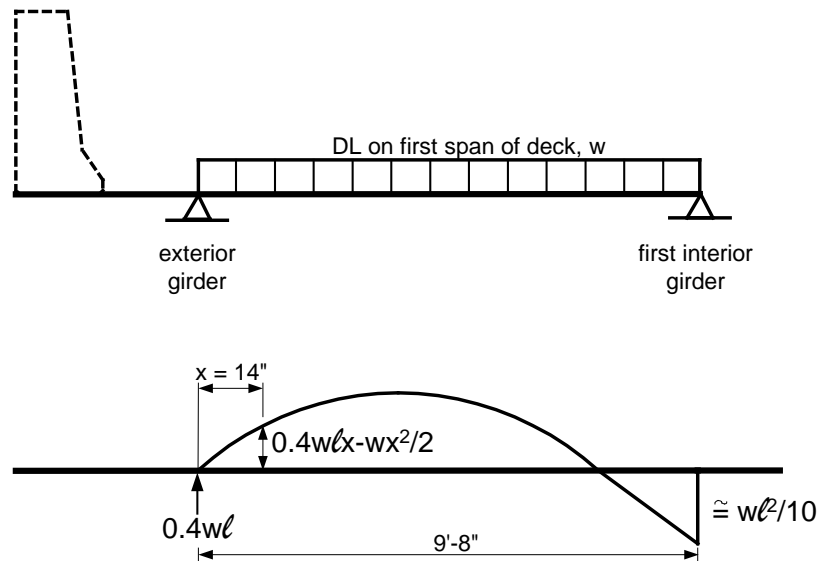


Figure 4-11 - Dead Load Moment at Design Section Due to Dead Loads on the First Deck Span

Collision moment at exterior girder,  $M_1 = -17.83$  k-ft/ft

Collision moment at first interior girder,  $M_2 = 0.4(17.83) = 7.13$  k-ft/ft

By interpolation for a section in the first interior bay at 14 in. from the exterior girder:

$$\text{Total collision moment} = -17.83 + 14(17.83 + 7.13)/116 = -14.81 \text{ k-ft/ft}$$

Using the 30° angle distribution, as shown in Figure 4-8:

$$\text{Design collision moment} = -14.81L_c/[L_c + 2(0.577)(22 + 14)] = -12.59 \text{ k-ft/ft}$$

$$\text{where } L_c = 235.2 \text{ in.}$$

Dead load moment at the centerline of the exterior girder:

$$\begin{aligned} M_{DL, \text{Slab}} &= -0.1125(42.25/12)^2/2 \\ &= -0.70 \text{ k-ft/ft} \end{aligned}$$

$$\begin{aligned} M_{DL, \text{Parapet}} &= -0.65 (42.25 - 7.61)/12 \\ &= -1.88 \text{ k-ft/ft} \end{aligned}$$

$$\begin{aligned} M_{DL, \text{FWS}} &= -0.03[(42.25 - 20.25)/12]^2/2 \\ &= -0.05 \text{ k-ft/ft} \end{aligned}$$

Factored dead load moment at the centerline of the exterior girder:

$$\begin{aligned} M_{FDL} &= 1.25(-0.70) + 1.25(-1.88) + 1.5(-0.05) \\ &= -3.3 \text{ k-ft/ft} \end{aligned}$$

Based on Figure 4-10

The design factored dead load moment at the design section due to loads on the overhang is:

$$\begin{aligned} M_{FDL,O} &= 0.83(-3.3) \\ &= -2.74 \text{ k-ft/ft} \end{aligned}$$

From Figure 4-11, the dead load design factored moment due to DL on the first deck span is:

$$\begin{aligned} M &= 1.25[0.1125[0.4(9.66)(14/12) - (14/12)^2/2]] + 1.5[0.03[0.4(9.66)(14/12) - (14/12)^2/2]] \\ &= 0.71 \text{ k-ft/ft} \end{aligned}$$

Total design dead load + collision moment:

$$\begin{aligned} M_{DL+C} &= -12.59 - 2.74 + 0.71 \\ &= -14.62 \text{ k-ft/ft} \end{aligned}$$

Resistance factor = 1.0 for extreme event limit state (S1.3.2.1)

Assuming the slab thickness at this section equals 8 in. and the effective depth equals 5.19 in.;

Required area of steel = 0.62 in<sup>2</sup>/ft (3)

### **Design Case 2: Vertical collision force (SA13.4.1, Case 2)**

*For concrete parapets, the case of vertical collision never controls*

### **Design Case 3: Check DL + LL (SA13.4.1, Case 3)**

*Except for decks supported on widely spaced girders (approximately 12 ft. and 14 ft. girder spacing for girders with narrow flanges and wide flanges, respectively), Case 3 does not control the design of decks supporting concrete parapets. Widely spaced girders allow the use of wider overhangs which in turn may lead to live load moments that may exceed the collision moment and, thus, control the design. The deck of this example is highly unlikely to be controlled by Case 3. However, this case is checked to illustrate the complete design process.*

Resistance factor = 0.9 for strength limit state (S5.5.4.2.1).

#### **a. Design section in the overhang (Section B-B in Figure 4-7)**

The live load distribution width equations for the overhang (S4.6.2.1.3) are based on assuming that the distance from the design section in the overhang to the face of the parapet exceeds 12 in. such that the concentrated load representing the truck wheel is located closer to the face of the parapet than the design section. As shown in Figure 4-12, the concentrated load representing the wheel load on the overhang is located to the inside of the design section for negative moment in the overhang. This means that the distance "X" in the distribution width equation is negative which was not intended in developing this equation. This situation is becoming common as prestressed girders with wide top flanges are being used more frequently. In addition, Figure 4-6 may be wrongly interpreted as that there is no live load negative moment acting on the overhang. This would be misleading since the wheel load is distributed over the width of the wheels in the axle. Live load moment in these situations is small and is not expected to control design. *For such situations, to determine the live load design moment in the overhang, either of the following two approaches may be used:*

- 1) *The design section may be conservatively assumed at the face of the girder web, or*
- 2) *The wheel load may be distributed over the width of the wheels as shown in Figure 4-12 and the moments are determined at the design section for negative moment. The distribution width may be calculated assuming "X" as the distance from the design section to the edge of the wheel load nearest the face of the parapet.*



The latter approach is used in this example. The wheel load is assumed to be distributed over a tire width of 20 in. as specified in S3.6.1.2.5.

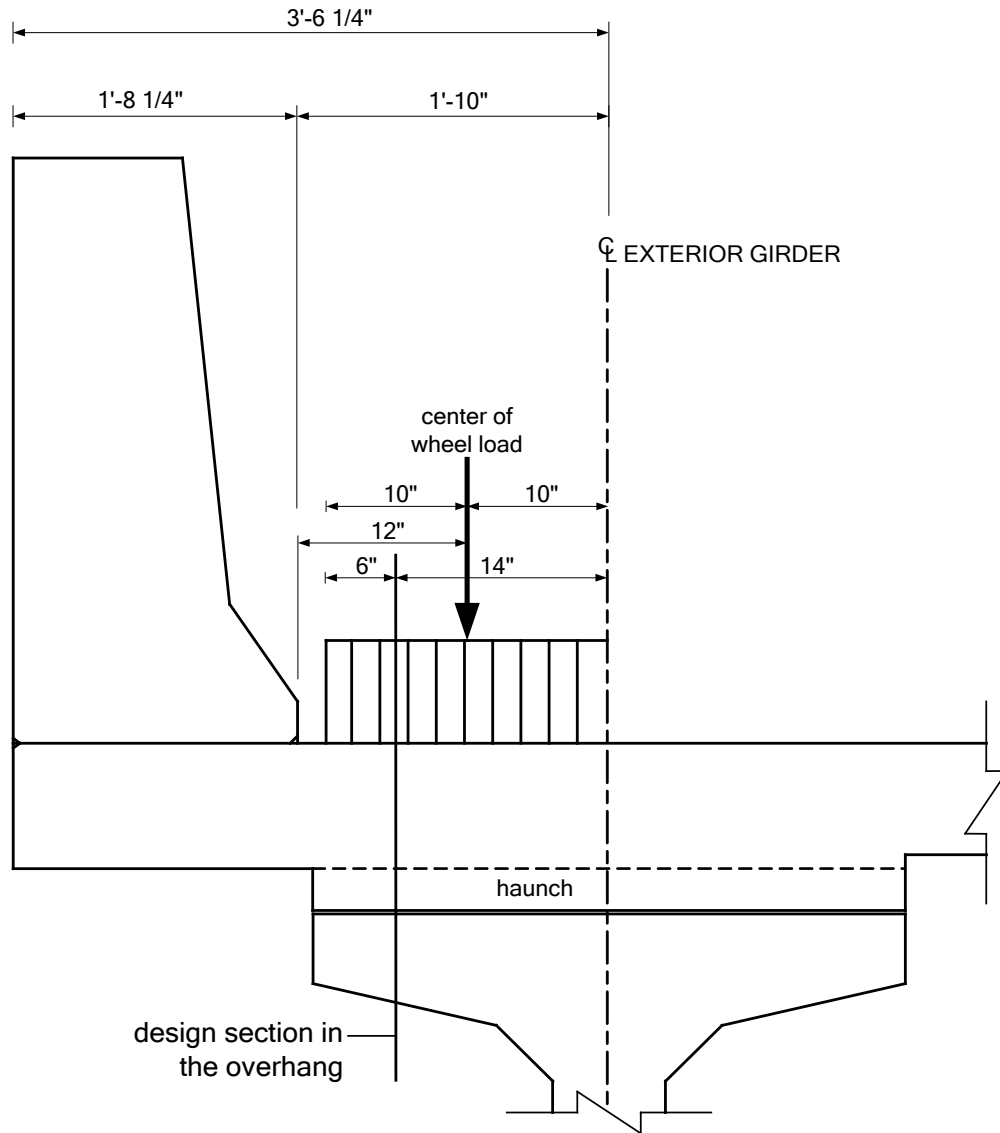


Figure 4-12 – Overhang Live Load - Distributed Load

Using the multiple presence factor for a single truck = 1.2 (S3.6.1.1.2) and dynamic load allowance for truck loading = 1.33 (S3.6.2.1), live load moment may be determined.

$$\begin{aligned} \text{Equivalent strip width for live load} &= 45 + 10(6/12) \\ &= 50 \text{ in. (S4.6.2.1.3)} \end{aligned}$$

Design factored moment:

$$\begin{aligned}
 M_n &= -1.25(0.1125)[(42.25 - 14)/12]^2/2 \\
 &\quad -1.25(0.65)(42.25 - 14 - 7.61)/12 \\
 &\quad -1.5(0.03)[(42.25 - 20.25 - 14)/12]^2/2 \\
 &\quad -1.75(1.33)(1.2)[16/(20/12)[((6/12)^2/2)/(50/12)]] \\
 &= -2.60 \text{ k-ft/ft}
 \end{aligned}$$

d = 6.19 in.

Required area of steel = 0.09 in<sup>2</sup>/ft (4)

b. Check dead load + live load moments at design section in first span (Section C-C in Figure 4-7)

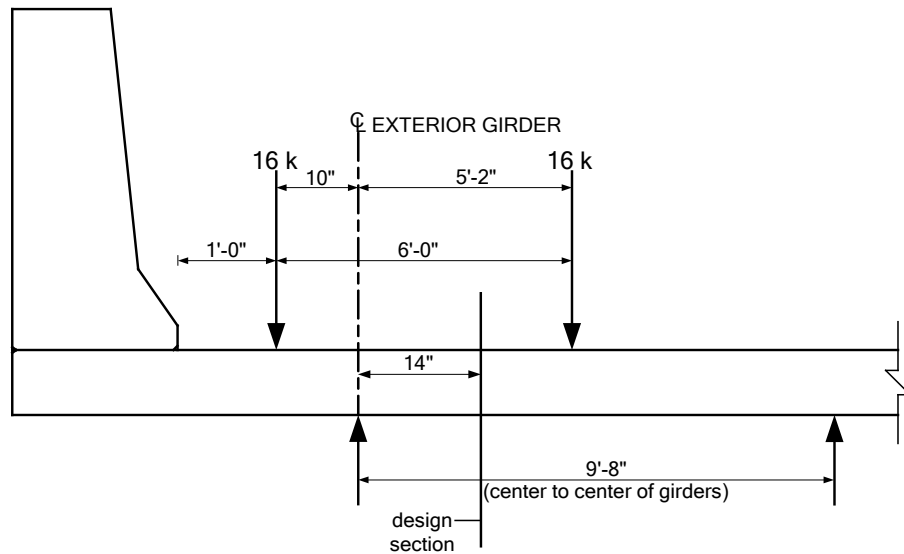


Figure 4-13 – Overhang Live Load

Assume slab thickness at this section = 8 in. (conservative to ignore the haunch)

Based on the earlier calculations for this section under collision + DL, DL factored moment at the section = -2.74 k-ft/ft.

Determining live load at this section may be conducted by modeling the deck as a beam supported on the girders and by moving the design load across the width of the deck to generate the moment envelopes. However, this process implies a degree of accuracy that may not be possible to achieve

due to the approximate nature of the distribution width and other assumptions involved, e.g., the girders are not infinitely rigid and the top flange is not a point support. An approximate approach suitable for hand calculations is illustrated in Figure 4-13. In this approximate approach, the first axle of the truck is applied to a simply supported beam that consists of the first span of the deck and the overhang. The negative moment at the design section is then calculated. The multiple presence factor for a single lane (1.2) and dynamic load allowance (33%) are also applied. Based on the dimensions and the critical location of the truck axle shown in Figure 4-13, the unfactored live load moment at the design section for negative moment is 3.03 k-ft.

Live load moment (including the load factor, dynamic load allowance and multiple presence factor) =  $3.03(1.75)(1.33)(1.2) = 8.46$  k-ft

Since the live load negative moment is produced by a load on the overhang, use the overhang strip width at the girder centerline.

Equivalent strip width =  $45 + 10(10/12) = 53.3$  in. (S4.6.2.1.3)

Design factored moment (DL + LL) =  $2.74 + 8.46/(53.3/12)$   
= 4.65 k-ft/ft

Required area of steel =  $0.19$  in<sup>2</sup>/ft (5)

#### Design Step 4.11 Detailing of overhang reinforcement

From the different design cases of the overhang and the adjacent region of the deck, the required area of steel in the overhang is equal to the largest of (1), (2), (3), (4) and (5) =  $0.7$  in<sup>2</sup>/ft

The provided top reinforcement in the slab in regions other than the overhang region is: #5 at 8 in. =  $0.31(12/8) = 0.465$  in<sup>2</sup>/ft

$0.465$  in<sup>2</sup>/ft provided <  $0.7$  in<sup>2</sup>/ft required, therefore, additional reinforcement is required in the overhang.

Bundle one #4 bar to each top bar in the overhang region of the deck.

Provided reinforcement =  $(0.2 + 0.31)(12/8) = 0.76$  in<sup>2</sup>/ft >  $0.7$  in<sup>2</sup>/ft required **OK**.

*Notice that many jurisdictions require a #5 minimum bar size for the top transverse reinforcement. In this case, the #4 bars used in this example would be replaced by #5 bars. Alternatively, to reduce the reinforcement area, a #5 bar may be added between the alternating main bars if the main bar spacing would allow adding bars in between without resulting in congested reinforcement.*

Check the depth of the compression block:

$$\begin{aligned} T &= 60(0.76) \\ &= 45.6 \text{ kips} \end{aligned}$$

$$\begin{aligned} a &= 45.6/[0.85(4)(12)] \\ &= 1.12 \text{ in.} \end{aligned}$$

$$\beta_1 = 0.85 \text{ for } f'_c = 4 \text{ ksi (S5.7.2.2)}$$

$$\begin{aligned} c &= 1.12/0.85 \\ &= 1.32 \text{ in.} \end{aligned}$$

Among Sections A, B and C of Figure 4-7, Section C has the least slab thickness. Hence, the ratio  $d_e/c$  is more critical at this section, making the strain in the reinforcing steel more critical.

$$d_e \text{ at Section C-C} = 5.19 \text{ in.}$$

$$\text{Minimum } \epsilon_s = 0.003 \left( \frac{d_e}{c} - 1 \right) = 0.003 \left( \frac{5.19}{1.32} - 1 \right) = 0.0088 > 0.005 \text{ OK (S5.7.2.1)}$$

The above calculation shows that the section is tension controlled.

Cracking under service load in the overhang needs to be checked. The reinforcement area in the overhang is 65% larger than the negative moment reinforcement in the interior portions of the deck, yet the applied service moment ( $2.74 + 3.03 = 5.77$  k-ft/ft) is 6% larger than the service moment at interior portions of the deck (5.42 k-ft/ft from Step 4.9). By inspection, cracking under service load does not control.

Determine the point in the first bay of the deck where the additional bars are no longer needed by determining the point where both (DL + LL) moment and (DL + collision) moments are less than or equal to the moment of resistance of the deck slab without the additional top reinforcement. By inspection, the case of (DL + LL) does not control and only the case of (DL + collision) needs to be checked.

Negative moment resistance of the deck slab reinforced with #5 bars at 8 in. spacing is 10.15 k-ft/ft for strength limit state (resistance factor = 0.9), or 11.27 k-ft/ft for the extreme event limit state (resistance factor = 1.0). By calculating the moments at different points along the deck first span in the same manner they were calculated for Section C-C for (DL + collision), it was determined that the design negative moment is less than 11.27 k-ft/ft at a point approximately 25 in. from the centerline of the exterior girder.

The theoretical termination point from the centerline of the exterior girder is 25 in.

Extend the additional bars beyond this point for a distance equal to the cut-off length. In addition, check that the provided length measured beyond the design section for moment is larger than the development length (S5.11.1.2.1).

#### Cut-off length requirement (S5.11.1.2.1)

Checking the three requirements of S5.11.1.2.1, the cut-off length is controlled by 15 times the bar diameter.

$$\text{Cut-off length} = 15(0.625) = 9.375 \text{ in.}$$

$$\begin{aligned} \text{Required length past the centerline of the exterior girder} &= 25 + 9.375 \\ &= 34.375 \text{ in.} \end{aligned}$$

#### Development length (S5.11.2)

The basic development length,  $\ell_{db}$ , is taken as the larger of:

$$\frac{1.25 A_b f_y}{\sqrt{f'_c}} = \frac{1.25(0.31)(60)}{\sqrt{4}} = 11.625 \text{ in.}$$

OR

$$0.4 d_b f_y = 0.4(0.625)(60) = 15 \text{ in.}$$

OR

$$12 \text{ in.}$$

Therefore, the basic development length = 15 in.

Correction factors:

$$\text{Epoxy-coated bars} = 1.2 \text{ (S5.11.2.1.2)}$$

$$\text{Two bundled bars} = 1.0 \text{ (S5.11.2.3)}$$

$$\text{Spacing} > 6 \text{ in.} = 0.8 \text{ (S5.11.2.1.3)}$$

$$\text{Development length} = 15(1.2)(1.0)(0.8) = 14.4 \text{ in.}$$

Required length of the additional bars past the centerline of the exterior girder = 14 + 14.4 = 28.4 in. < 34.375 in. (needed to be satisfy cut off requirements) **OK**

Extend the additional bars in the overhang a minimum of 34.375 in. (say 3 ft.) beyond the centerline of the exterior girder.

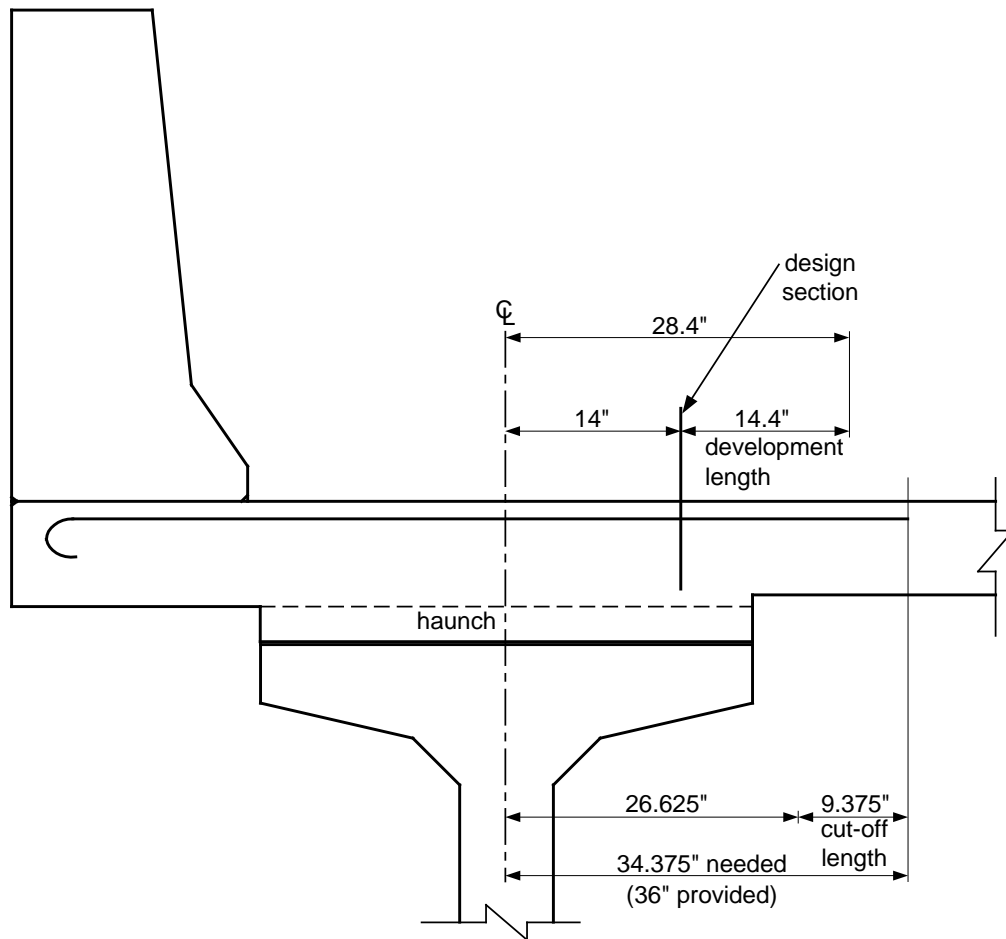


Figure 4-14 - Length of the Overhang Additional Bars

**Design Step 4.12 Longitudinal reinforcement**

**Bottom distribution reinforcement (S9.7.3.2)**

$$\text{Percentage of longitudinal reinforcement} = \frac{220}{\sqrt{S}} \leq 67\%$$

where:

- S = the effective span length taken as equal to the effective length specified in S9.7.2.3 (ft.); the distance between sections for negative moment and sections at the ends of one deck span
- = (116 – 14 – 14)/(12)
- = 7.33 ft.

$$\text{Percentage} = \frac{220}{\sqrt{7.33}} = 81\% > 67\%$$

Use 67% of transverse reinforcement

Transverse reinforcement = #5 at 7 in. spacing = 0.53 in<sup>2</sup>/ft

Required longitudinal reinforcement = 0.67(0.53) = 0.36 in<sup>2</sup>/ft

Use #5 bars; bar diameter = 0.625 in., bar area = 0.31 in<sup>2</sup>

Required spacing = 0.31/0.36 = 0.86 ft. (10.375 in.)

Use #5 bars at 10 in. spacing

### **Top longitudinal reinforcement**

There are no specific requirements to determine this reinforcement. Many jurisdictions use #4 bars at 12 in. spacing for the top longitudinal reinforcement.

### **Design Step 4.13 Deck top longitudinal reinforcement in the girder negative moment region**

This design step computes the deck top longitudinal reinforcement in the girder negative moment region, i.e., over the intermediate supports of the girders.

For simple span precast girders made continuous for live load: design according to S5.14.1.4.

(Notice that for continuous steel girders, this reinforcement is designed according to S6.10.1.7).

The required reinforcement area is determined during girder design. See Section 5.6 for the calculations for this reinforcement.

Use #6 bars at 4.5 in. spacing in the top layer  
#6 bars at 8.5 in. spacing in the bottom layer

### **Design Step 4.14 Check shrinkage and temperature reinforcement according to S5.10.8**

Reinforcement for shrinkage and temperature stresses is provided near surfaces of concrete exposed to daily temperature changes. Shrinkage and temperature reinforcement is added to ensure that the total reinforcement on exposed surfaces is not less than the following:

$$A_s \geq (1.30bh)/(2(b+h)f_y) \quad (S5.10.8-1)$$

$$0.11 \leq A_s \leq 0.60 \quad (S5.10.8-2)$$

For members less than 18.0 in. thick, temperature and shrinkage reinforcement is not required.

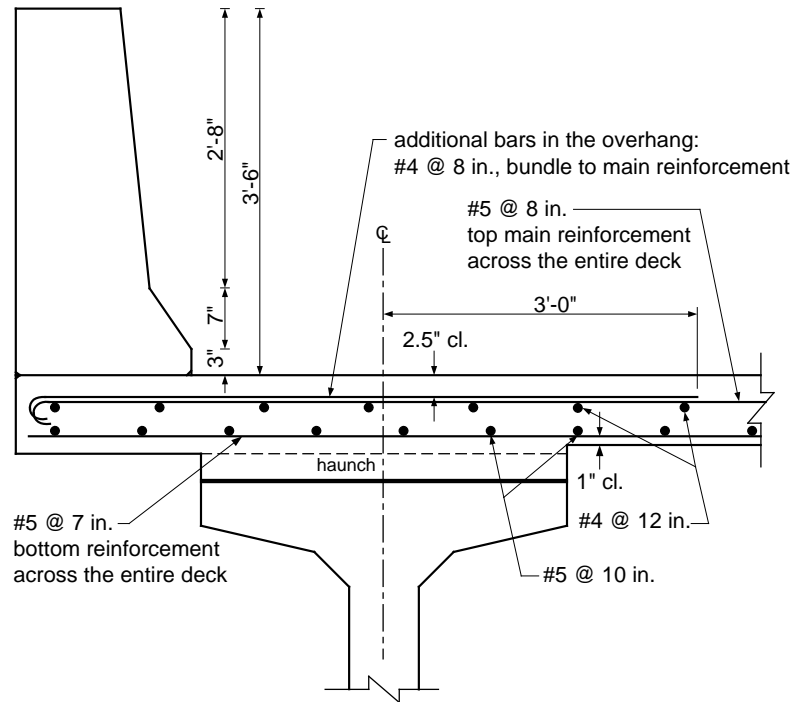


Figure 4-15 - Deck Reinforcement at Midspan of Girders



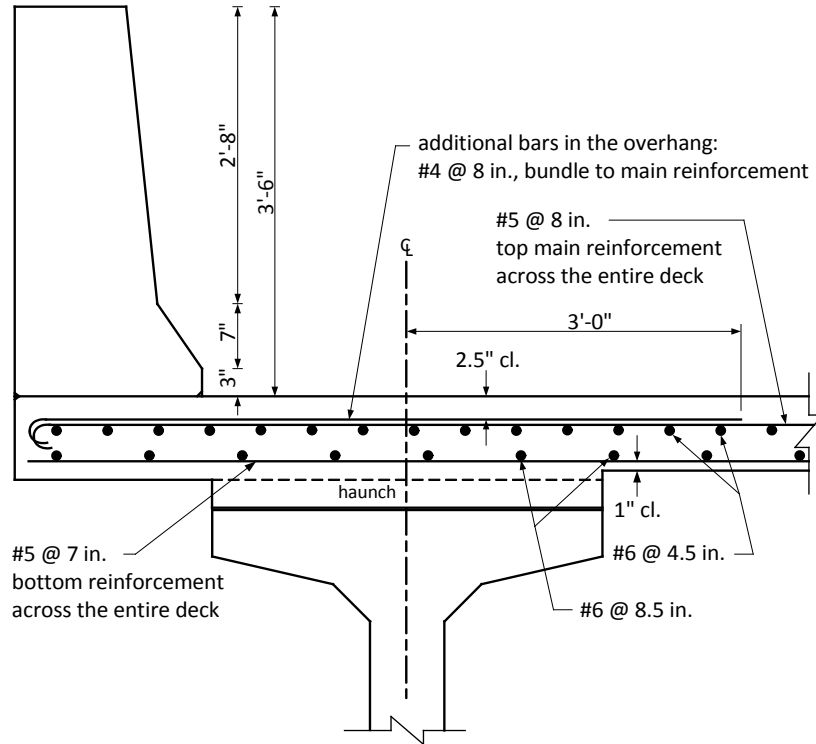


Figure 4-16 - Deck Reinforcement at Intermediate Pier

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**Design Step 5.1 LIVE LOAD DISTRIBUTION FACTORS (S4.6.2.2)**

*The AASHTO-LRFD Specifications allow the use of advanced methods of analysis to determine the live load distribution factors. However, for typical bridges, the specifications list equations to calculate the distribution factors for different types of bridge superstructures. The types of superstructures covered by these equations are described in Table S4.6.2.2.1-1. From this table, bridges with concrete decks supported on precast concrete I or bulb-tee girders are designated as cross-section “K”. Other tables in S4.6.2.2.2 list the distribution factors for interior and exterior girders including cross-section “K”. The distribution factor equations are largely based on work conducted in the NCHRP Project 12-26 and have been verified to give accurate results compared to 3-dimensional bridge analysis and field measurements. The multiple presence factors are already included in the distribution factor equations except when the tables call for the use of the lever rule. In these cases, the computations need to account for the multiple presence factors. Notice that the distribution factor tables include a column with the heading “range of applicability”. The ranges of applicability listed for each equation are based on the range for each parameter used in the study leading to the development of the equation. When the girder spacing exceeds the listed value in the “range of applicability” column, the specifications require the use of the lever rule (S4.6.2.2.1). One or more of the other parameters may be outside the listed range of applicability. In this case, the equation could still remain valid, particularly when the value(s) is(are) only slightly out of the range of applicability. However, if one or more of the parameters greatly exceed the range of applicability, engineering judgment needs to be exercised.*

*Article S4.6.2.2.2d of the specifications states: “In beam-slab bridge cross-sections with diaphragms or cross-frames, the distribution factor for the exterior beam shall not be taken less than that which would be obtained by assuming that the cross-section deflects and rotates as a rigid cross-section”. This provision was added to the specifications because the original study that developed the distribution factor equations did not consider intermediate diaphragms. Application of this provision requires the presence of a sufficient number of intermediate diaphragms whose stiffness is adequate to force the cross section to act as a rigid section. For prestressed girders, different jurisdictions use different types and numbers of intermediate diaphragms. Depending on the number and stiffness of the intermediate diaphragms, the provisions of S4.6.2.2.2d may not be applicable. For this example, one deep reinforced concrete diaphragm is located at the midspan of each span. The stiffness of the diaphragm was deemed sufficient to force the cross-section to act as a rigid section, therefore, the provisions of S4.6.2.2.2d apply.*

For this example, the distribution factors listed in S4.6.2.2.2 will be used.

*Notice that fatigue in the prestressing steel need not be checked for conventional prestressed girders (S5.5.3) when maximum stress in the concrete at Service III limit state is taken according to Table S5.9.4.2.2-1. This statement is valid for this example. The fatigue distribution factors are calculated in the following sections to provide the user with a complete reference for the application of the LRFD distribution factors.*

Required information:

AASHTO Type I-Beam (28/72)	
Noncomposite beam area, $A_g$	= 1,085 in <sup>2</sup>
Noncomposite beam moment of inertia, $I_g$	= 733,320 in <sup>4</sup>
Deck slab thickness, $t_s$	= 8 in.
Span length, $L$	= 110 ft.
Girder spacing, $S$	= 9 ft. - 8 in.
Modulus of elasticity of the beam, $E_B$	4,696 ksi (S5.4.2.4)
Modulus of elasticity of the deck, $E_D$	= 3,834 ksi (refer to Design Example Section 2.1.3)
C.G. to top of the basic beam	= 35.62 in.
C.G. to bottom of the basic beam	= 36.38 in.

**Design Step 5.1.1**

Calculate  $n$ , the modular ratio between the beam and the deck.

$$\begin{aligned}
 n &= E_B/E_D && \text{(S4.6.2.2.1-2)} \\
 &= 4,696/3,834 \\
 &= 1.225
 \end{aligned}$$

**Design Step 5.1.2**

Calculate  $e_g$ , the distance between the center of gravity of the noncomposite beam and the deck. Ignore the thickness of the haunch in determining  $e_g$ . It is also possible to ignore the integral wearing surface, i.e., use  $t_s = 7.5$  in. However the difference in the distribution factor will be minimal.

$$\begin{aligned}
 e_g &= NA_{YT} + t_s/2 \\
 &= 35.62 + 8/2 \\
 &= 39.62 \text{ in.}
 \end{aligned}$$

**Design Step 5.1.3**

Calculate  $K_g$ , the longitudinal stiffness parameter.

$$\begin{aligned}
 K_g &= n(I + Ae_g^2) && \text{(S4.6.2.2.1-1)} \\
 &= 1.225[733,320 + 1,085(39.62)^2] \\
 &= 2,984,704 \text{ in}^4
 \end{aligned}$$

**Design Step 5.1.4 Interior girder**

Calculate the moment distribution factor for an interior beam with two or more design lanes loaded using Table S4.6.2.2.2b-1.

$$\begin{aligned}
 g_M &= 0.075 + (S/9.5)^{0.6} (S/L)^{0.2} (K_g/12.0Lt_s^3)^{0.1} \\
 &= 0.075 + (9.667/9.5)^{0.6} (9.667/110)^{0.2} [2,984,704/[12(110)(8)^3]]^{0.1} \\
 &= 0.796 \text{ lane} && (1)
 \end{aligned}$$

**Design Step 5.1.5**

According to S4.6.2.2.2e, a skew correction factor for moment may be applied for bridge skews greater than 30 degrees. The bridge in this example is skewed 20 degrees, and, therefore, no skew correction factor for moment is allowed.

Calculate the moment distribution factor for an interior beam with one design lane loaded using Table S4.6.2.2.2b-1.

$$\begin{aligned} g_M &= 0.06 + (S/14)^{0.4} (S/L)^{0.3} (K_g/12.0L_t^3)^{0.1} \\ &= 0.06 + (9.667/14)^{0.4} (9.667/110)^{0.3} [2,984,704/[12(110)(8)^3]]^{0.1} \\ &= 0.542 \text{ lane} \quad (2) \end{aligned}$$

Notice that the distribution factor calculated above for a single lane loaded already includes the 1.2 multiple presence factor for a single lane, therefore, this value may be used for the service and strength limit states. However, multiple presence factors should not be used for the fatigue limit state. Therefore, the multiple presence factor of 1.2 for the single lane is required to be removed from the value calculated above to determine the factor used for the fatigue limit state.

For single-lane loading to be used for fatigue design, remove the multiple presence factor of 1.2.

$$\begin{aligned} g_M &= 0.542/1.2 \\ &= 0.452 \text{ lane} \quad (3) \end{aligned}$$

**Design Step 5.1.6 Skew correction factor for shear**

According to S4.6.2.2.3c, a skew correction factor for support shear at the obtuse corner must be applied to the distribution factor of all skewed bridges. The value of the correction factor is calculated using Table S4.6.2.2.3c-1

$$\begin{aligned} SC &= 1.0 + 0.20(12.0L_t^3/K_g)^{0.3} \tan \theta \\ &= 1.0 + 0.20[[12.0(110)(8)^3]/2,984,704]^{0.3} \tan 20 \\ &= 1.047 \end{aligned}$$

**Design Step 5.1.7**

Calculate the shear distribution factor for an interior beam with two or more design lanes loaded using Table S4.6.2.2.3a-1.

$$\begin{aligned} g_V &= 0.2 + (S/12) - (S/35)^2 \\ &= 0.2 + (9.667/12) - (9.667/35)^2 \\ &= 0.929 \text{ lane} \end{aligned}$$

Apply the skew correction factor:

$$\begin{aligned} g_v &= 1.047(0.929) \\ &= 0.973 \text{ lane} \quad (4) \end{aligned}$$

### Design Step 5.1.8

Calculate the shear distribution factor for an interior beam with one design lane loaded using Table S4.6.2.2.3a-1.

$$\begin{aligned} g_v &= 0.36 + (S/25.0) \\ &= 0.36 + (9.667/25.0) \\ &= 0.747 \text{ lane} \end{aligned}$$

Apply the skew correction factor:

$$\begin{aligned} g_v &= 1.047(0.747) \\ &= 0.782 \text{ lane} \quad (5) \end{aligned}$$

For single-lane loading to be used for fatigue design, remove the multiple presence factor of 1.2.

$$\begin{aligned} g_v &= 0.782/1.2 \\ &= 0.652 \text{ lane} \quad (6) \end{aligned}$$

### Design Step 5.1.9

From (1) and (2), the service and strength limit state moment distribution factor for the interior girder is equal to the larger of 0.796 and 0.542 lane. Therefore, the moment distribution factor is 0.796 lane.

From (3):

The fatigue limit state moment distribution factor is 0.452 lane

From (4) and (5), the service and strength limit state shear distribution factor for the interior girder is equal to the larger of 0.973 and 0.782 lane. Therefore, the shear distribution factor is 0.973 lane.

From (6):

The fatigue limit state shear distribution factor is 0.652 lane

Design Step 5.1.10 Exterior girder

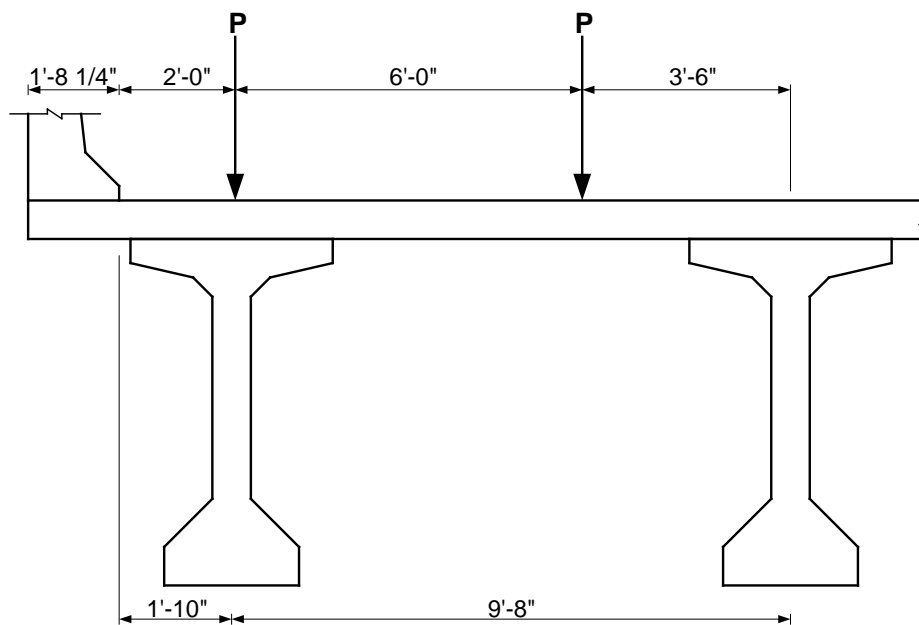


Figure 5.1-1 – Lever Rule

Design Step 5.1.11

Calculate the moment distribution factor for an exterior beam with two or more design lanes using Table S4.6.2.2.2d-1.

$$g_M = e g_{M\text{Interior}}$$

$$e = 0.77 + d_e/9.1$$

where  $d_e$  is the distance from the exterior web of the exterior girder to the inside face of the curb or barrier (conservatively use the centerline of web for this example).

$$e = 0.77 + 1.83/9.1$$

$$= 0.97$$

$$g_M = 0.97(0.796)$$

$$= 0.772 \text{ lane} \quad (7)$$

Design Step 5.1.12

Calculate the moment distribution factor for an exterior beam with one design lane using the lever rule as per Table S4.6.2.2.2d-1.

$$\begin{aligned}
 g_M &= [(3.5 + 6) + 3.5]/9.667 \\
 &= 1.344 \text{ wheels/2} \\
 &= 0.672 \text{ lane} \quad (8) \quad (\text{Fatigue})
 \end{aligned}$$

*Notice that this value does not include the multiple presence factor, therefore, it is adequate for use with the fatigue limit state. For service and strength limit states, the multiple presence factor for a single lane loaded needs to be included.*

$$\begin{aligned}
 g_M &= 0.672(1.2) \\
 &= 0.806 \text{ lane} \quad (9) \quad (\text{Strength and Service})
 \end{aligned}$$

### Design Step 5.1.13

Calculate the shear distribution factor for an exterior beam with two or more design lanes loaded using Table S4.6.2.2.3b-1.

$$g_v = e g_{v\text{interior}}$$

where:

$$\begin{aligned}
 e &= 0.6 + d_c/10 \\
 &= 0.6 + 1.83/10 \\
 &= 0.783
 \end{aligned}$$

$$\begin{aligned}
 g_v &= 0.783(0.973) \\
 &= 0.762 \text{ lane} \quad (10)
 \end{aligned}$$

### Design Step 5.1.14

Calculate the shear distribution factor for an exterior beam with one design lane loaded using the lever rule as per Table S4.6.2.2.3b-1. This value will be the same as the moment distribution factor with the skew correction factor applied.

$$\begin{aligned}
 g_v &= 1.047(0.672) \\
 &= 0.704 \text{ lane} \quad (11) \quad (\text{Fatigue})
 \end{aligned}$$

$$\begin{aligned}
 g_v &= 1.047(0.806) \\
 &= 0.845 \text{ lane} \quad (12) \quad (\text{Strength and Service})
 \end{aligned}$$

*Notice that S4.6.2.2.2d includes additional requirements for the calculation of the distribution factors for exterior girders when the girders are connected with relatively stiff cross-frames that force the cross-section to act as a rigid section. As indicated in Design Step 5.1, these provisions are applied to this example; the calculations are shown below.*



**Design Step 5.1.15 Additional check for rigidly connected girders (S4.6.2.2.2d)**

The multiple presence factor,  $m$ , is applied to the reaction of the exterior beam (Table S3.6.1.1.2-1)

$$m_1 = 1.20$$

$$m_2 = 1.00$$

$$m_3 = 0.85$$

$$R = N_L/N_b + X_{\text{ext}}(\Sigma e)/\Sigma X^2 \quad (\text{SC4.6.2.2.2d-1})$$

where:

$R$  = reaction on exterior beam in terms of lanes

$N_L$  = number of loaded lanes under consideration

$e$  = eccentricity of a design truck or a design lane load from the center of gravity of the pattern of girders (ft.)

$x$  = horizontal distance from the center of gravity of the pattern of girders to each girder (ft.)

$X_{\text{ext}}$  = horizontal distance from the center of gravity of the pattern to the exterior girder (ft.)

See Figure 5.1-2 for dimensions.

One lane loaded (only the leftmost lane applied):

$$\begin{aligned} R &= 1/6 + 24.167(21)/[2(24.167^2 + 14.5^2 + 4.833^2)] \\ &= 0.1667 + 0.310 \\ &= 0.477 \quad (\text{Fatigue}) \end{aligned}$$

Add the multiple presence factor of 1.2 for a single lane:

$$\begin{aligned} R &= 1.2(0.477) \\ &= 0.572 \quad (\text{Strength}) \end{aligned}$$

Two lanes loaded:

$$\begin{aligned} R &= 2/6 + 24.167(21 + 9)/[2(24.167^2 + 14.5^2 + 4.833^2)] \\ &= 0.333 + 0.443 \\ &= 0.776 \end{aligned}$$

Add the multiple presence factor of 1.0 for two lanes loaded:

$$R = 1.0(0.776) = 0.776 \quad \text{(Strength)}$$

Three lanes loaded:

$$R = 3/6 + 24.167(21 + 9 - 3)/[2(24.167^2 + 14.5^2 + 4.833^2)] = 0.5 + 0.399 = 0.899$$

Add the multiple presence factor of 0.85 for three or more lanes loaded:

$$R = 0.85(0.899) = 0.764 \quad \text{(Strength)}$$

These values do not control over the distribution factors summarized in Design Step 5.1.16.

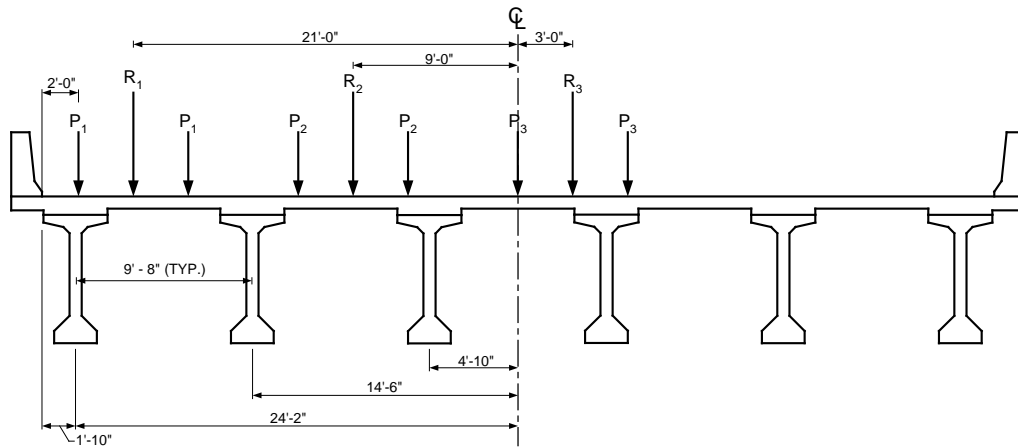


Figure 5.1-2 - General Dimensions

**Design Step 5.1.16**

From (7) and (9), the service and strength limit state moment distribution factor for the exterior girder is equal to the larger of 0.772 and 0.806 lane. Therefore, the moment distribution factor is 0.806 lane.

From (8):

The fatigue limit state moment distribution factor is 0.672 lane

From (10) and (12), the service and strength limit state shear distribution factor for the exterior girder is equal to the larger of 0.762 and 0.845 lane. Therefore, the shear distribution factor is 0.845 lane.

From (11):

The fatigue limit state shear distribution factor is 0.704 lane

**Table 5.1-1 – Summary of Service and Strength Limit State Distribution Factors**

Distribution Factor Method	Load Case	Moment interior beams	Moment exterior beams	Shear interior beams	Shear exterior beams
Distribution factors from Tables in S4.6.2.2.2	Multiple lanes loaded	0.796	0.772	0.973	0.762
	Single lane loaded	0.542	0.806	0.782	0.845
Additional check for rigidly connected girders	Multiple lanes loaded	NA	0.776	NA	0.776
	Single lane loaded	NA	0.572	NA	0.572
Design value		0.796	0.806	0.973	0.845

**Table 5.1-2 – Summary of Fatigue Limit State Distribution Factors**

Distribution Factor Method	Load Case	Moment interior beams	Moment exterior beams	Shear interior beams	Shear exterior beams
Distribution factors from Tables in S4.6.2.2.2	Multiple lanes loaded	NA	NA	NA	NA
	Single lane loaded	0.452	0.672	0.652	0.704
Additional check for rigidly connected girders	Multiple lanes loaded	NA	NA	NA	NA
	Single lane loaded	NA	0.477	NA	0.477
Design value		0.452	0.672	0.652	0.704

**Design Step 5.2 DEAD LOAD CALCULATION**

Calculate the dead load of the bridge superstructure components for the interior girder, which is designed in this example. Values for the exterior girder have also been included for reference. The girder, slab, haunch, and exterior diaphragm loads are applied to the noncomposite section; the parapets and future wearing surface are applied to the composite section.

**Interior girder**Girder weight

$$DC_{\text{girder (I)}} = A_g(\gamma_{\text{girder}})$$

where:

$$\begin{aligned} A_g &= \text{beam cross-sectional area (in}^2\text{)} \\ &= 1,085 \text{ in}^2 \end{aligned}$$

$$\begin{aligned} \gamma &= \text{unit weight of beam concrete (kcf)} \\ &= 0.150 \text{ kcf} \end{aligned}$$

$$\begin{aligned} DC_{\text{girder (I)}} &= (1,085/144)(0.150) \\ &= 1.13 \text{ k/ft/girder} \end{aligned}$$

Deck slab weight

The total thickness of the slab is used in calculating the weight.

$$\begin{aligned} \text{Girder spacing} &= 9.667 \text{ ft.} \\ \text{Slab thickness} &= 8 \text{ in.} \end{aligned}$$

$$\begin{aligned} DC_{\text{slab (I)}} &= 9.667(8/12)(0.150) \\ &= 0.967 \text{ k/ft/girder} \end{aligned}$$

**Exterior girder**Girder weight

$$DC_{\text{girder (E)}} = 1.13 \text{ k/ft/girder}$$

Deck slab weight

$$\begin{aligned} \text{Slab width} &= \text{overhang width} + \frac{1}{2} \text{ girder spacing} \\ &= 3.521 + \frac{1}{2}(9.667) \\ &= 8.35 \text{ ft.} \end{aligned}$$

$$\text{Slab thickness} = 8 \text{ in.}$$

$$\begin{aligned} DC_{\text{slab (E)}} &= 8.35(8/12)(0.150) \\ &= 0.835 \text{ k/ft/girder} \end{aligned}$$

**Haunch weight**

$$\begin{aligned} \text{Width} &= 42 \text{ in.} \\ \text{Thickness} &= 4 \text{ in.} \end{aligned}$$

$$\begin{aligned} DC_{\text{haunch}} &= [42(4)/144](0.150) \\ &= 0.175 \text{ k/ft/girder} \end{aligned}$$

*Notice that the haunch weight in this example is assumed as a uniform load along the full length of the beam. This results in a conservative design as the haunch typically have a variable thickness that decreases toward the middle of the span length. Many jurisdictions calculate the haunch load effects assuming the haunch thickness to vary parabolically along the length of the beam. The location of the minimum thickness varies depending on the grade of the roadway surface at bridge location and the presence of a vertical curve. The use of either approach is acceptable and the difference in load effects is typically negligible. However, when analyzing existing bridges, it may be necessary to use the variable haunch thickness in the analysis to accurately represent the existing situation*

**Concrete diaphragm weight**

A concrete diaphragm is placed at one-half the noncomposite span length.

Location of the diaphragms:

$$\begin{aligned} \text{Span 1} &= 54.5 \text{ ft. from centerline of end bearing} \\ \text{Span 2} &= 55.5 \text{ ft. from centerline of pier} \end{aligned}$$

For this example, arbitrarily assume that the thickness of the diaphragm is 10 in. The diaphragm spans from beam to beam minus the web thickness and has a depth equal to the distance from the top of the beam to the bottom of the web. Therefore, the concentrated load to be applied at the locations above is:

$$\begin{aligned} DC_{\text{diaphragm}} &= 0.15(10/12)[9.667 - (8/12)](72 - 18)/12 \\ &= 5.0625 \text{ k/girder} \end{aligned}$$

The exterior girder only resists half of this loading.

**Parapet weight**

According to the S4.6.2.2.1, the parapet weight may be distributed equally to all girders in the cross section.

Parapet cross-sectional area = 4.33 ft<sup>2</sup>

$$\begin{aligned} DC_{\text{parapet}} &= 4.33(0.150) = 0.650 \text{ k/ft} \\ &= 0.650/6 \text{ girders} \\ &= 0.108 \text{ k/ft/girder for one parapet} \end{aligned}$$

Therefore, the effect of two parapets yields:

$$DC_{\text{parapet}} = 0.216 \text{ k/ft per girder}$$

Note that some jurisdictions distribute parapet loads to the 2 or 3 most exterior girders on each side of the cross section.

### Future wearing surface

#### Interior girder

$$\begin{aligned} \text{Weight/ft}^2 &= 0.030 \text{ k/ft}^2 \\ \text{Width} &= 9.667 \text{ ft.} \\ \\ DW_{\text{FWS (I)}} &= 0.030(9.667) \\ &= 0.290 \text{ k/ft/girder} \end{aligned}$$

#### Exterior Girder

$$\begin{aligned} \text{Weight/ft}^2 &= 0.030 \text{ k/ft}^2 \\ \text{Width} &= \text{slab width} - \text{parapet width} \\ &= 8.35 - 1.6875 \\ &= 6.663 \text{ ft.} \\ \\ DW_{\text{FWS (E)}} &= 0.030(6.663) \\ &= 0.200 \text{ k/ft/girder} \end{aligned}$$

*Notice that some jurisdictions divide the weight of the future wearing surface equally between all girders (i.e. apply a uniform load of 0.26 k/ft to all girders). Article S4.6.2.2.1 states that permanent loads of and on the deck may be distributed uniformly among the beams. This method would also be acceptable and would minimally change the moments and shears given in the tables in Design Step 5.3.*

## Design Step 5.3 UNFACTORED AND FACTORED LOAD EFFECTS

## Design Step 5.3.1 Summary of loads

The dead load moments and shears were calculated based on the loads shown in Design Step 5.2. The live load moments and shears were calculated using a generic live load analysis computer program. The live load distribution factors from Design Step 5.1 are applied to these values.

**Table 5.3-1 - Summary of Unfactored Moments**

Interior girder, Span 1 shown, Span 2 symmetric

Location*	Noncomposite Moments					Composite Moments		Live Load + IM Moments	
	Girder		Slab and Haunch	Diaphragm	Total Noncomp.	Parapet	FWS	Positive HL-93	Negative HL-93
	**	***							
(ft.)	(k-ft)	(k-ft)	(k-ft)	(k-ft)	(k-ft)	(k-ft)	(k-ft)	(k-ft)	(k-ft)
0	47	0	0	0	0	0	0	0	0
1.0	108	61	62	3	125	9	12	92	-11
5.5	368	322	325	14	661	46	62	476	-58
11.0	656	609	615	28	1,252	85	114	886	-116
16.5	909	863	871	42	1,776	118	158	1,230	-174
22.0	1,128	1,082	1,093	56	2,230	144	193	1,509	-233
27.5	1,313	1,267	1,279	70	2,616	164	220	1,724	-291
33.0	1,464	1,417	1,432	84	2,933	177	237	1,882	-349
38.5	1,580	1,534	1,549	98	3,181	183	246	1,994	-407
44.0	1,663	1,616	1,633	111	3,360	183	246	2,047	-465
49.5	1,711	1,664	1,681	125	3,471	177	237	2,045	-523
54.5	1,725	1,679	1,696	138	3,512	165	222	2,015	-576
55.0	1,725	1,678	1,695	137	3,511	164	220	2,010	-581
60.5	1,705	1,658	1,675	123	3,456	144	194	1,927	-640
66.0	1,650	1,604	1,620	109	3,333	118	159	1,794	-698
71.5	1,562	1,515	1,531	95	3,141	86	115	1,613	-756
77.0	1,439	1,392	1,407	81	2,880	46	62	1,388	-814
82.5	1,282	1,236	1,248	67	2,551	1	1	1,124	-872
88.0	1,091	1,044	1,055	53	2,152	-52	-69	825	-1,124
93.5	865	819	827	39	1,686	-110	-148	524	-1,223
99.0	606	560	565	25	1,150	-176	-236	297	-1,371
104.5	312	266	268	11	546	-248	-332	113	-1,663
108.0	110	61	62	3	125	-297	-398	33	-1,921
109.0	47	0	0	0	0	-311	-418	15	-2,006
Span 2 - 0	-	0	0	0	0	-326	-438	0	-2,095

\* Distance from the centerline of the end bearing

\*\* Based on the simple span length of 110.5 ft. and supported at the ends of the girders. These values are used to calculate stresses at transfer.

\*\*\* Based on the simple span length of 109 ft. and supported at the centerline of bearings. These values are used to calculate the final stresses.

**Table 5.3-2 – Summary of Factored Moments**

Interior girder, Span 1 shown, Span 2 symmetric

Location*	Strength I (k-ft)	Service I Moments**		Service III Moments **	
		NC (k-ft)	Comp. (k-ft)	NC (k-ft)	Comp. (k-ft)
0	0	0	0	0	0
1.0	346	125	112	125	94
5.5	1,809	661	584	661	488
11.0	3,394	1,252	1,085	1,252	908
16.5	4,756	1,776	1,506	1,776	1,260
22.0	5,897	2,230	1,846	2,230	1,544
27.5	6,821	2,616	2,108	2,616	1,763
33.0	7,536	2,933	2,296	2,933	1,920
38.5	8,063	3,181	2,423	3,181	2,024
44.0	8,381	3,360	2,477	3,360	2,067
49.5	8,494	3,471	2,459	3,471	2,050
54.5	8,456	3,512	2,402	3,512	1,999
55.0	8,440	3,511	2,394	3,511	1,992
60.5	8,163	3,456	2,265	3,456	1,880
66.0	7,690	3,333	2,070	3,333	1,712
71.5	7,027	3,141	1,813	3,141	1,490
77.0	6,181	2,880	1,497	2,880	1,219
82.5	5,158	2,551	1,126	2,551	901
88.0	3,967	2,152	-1,245	2,152	-1,020
93.5	2,664	1,686	-1,481	1,686	-1,237
99.0	-1,535	1,150	-1,783	1,150	-1,509
104.5	-3,035	546	-2,242	546	-1,910
108.0	-4,174	125	-2,616	125	-2,232
109.0	-4,525	0	-2,734	0	-2,333
Span 2 - 0	-4,729	0	-2,858	0	-2,439

Load Factor Combinations

Strength I = 1.25(DC) + 1.5(DW) + 1.75(LL + IM)

Service I = 1.0[DC + DW + (LL + IM)]

Service III = 1.0(DC + DW) + 0.8(LL + IM)

\* Distance from the centerline of the end bearing

\*\* For service limit states, moments are applied to the section of the girder, i.e. noncomposite or composite, that resists these moments. Hence, noncomposite and composite moments have to be separated for service load calculations.



**Table 5.3-3 - Summary of Unfactored Shear**  
Interior girder, Span 1 shown, Span 2 symmetric

Location*	Noncomposite Shear				Composite Shear		Live Load + IM Shear	
	Girder	Slab and Haunch	Diaphragm	Total Noncomp.	Parapet	FWS	Positive HL-93	Negative HL-93
(ft.)	(k)	(k)	(k)	(k)	(k)	(k)	(k)	(k)
0	61.6	62.2	2.5	126.4	8.9	12.0	113.3	-12.9
1.0	60.5	61.1	2.5	124.1	8.7	11.7	111.7	-12.9
5.5	55.4	55.9	2.5	113.9	7.7	10.4	104.3	-13.0
11.0	49.2	49.7	2.5	101.4	6.5	8.8	95.5	-13.4
16.5	43.0	43.4	2.5	88.9	5.4	7.2	86.9	-15.9
22.0	36.7	37.1	2.5	76.4	4.2	5.6	78.7	-20.6
27.5	30.5	30.8	2.5	63.9	3.0	4.0	70.8	-26.0
33.0	24.3	24.6	2.5	51.4	1.8	2.4	63.1	-32.8
38.5	18.1	18.3	2.5	38.9	0.6	0.8	55.9	-39.8
44.0	11.9	12.0	2.5	26.4	-0.6	-0.8	48.9	-46.8
49.5	5.7	5.7	2.5	13.9	-1.8	-2.4	42.4	-54.0
54.5	0	0	-2.5	-2.5	-2.9	-3.8	36.8	-60.5
55.0	-0.6	-0.6	-2.5	-3.7	-3.0	-4.0	36.2	-61.2
60.5	-6.8	-6.9	-2.5	-16.2	-4.2	-5.6	30.4	-68.4
66.0	-13.0	-13.1	-2.5	-28.7	-5.3	-7.2	25.0	-75.7
71.5	-19.2	-19.4	-2.5	-41.2	-6.5	-8.8	20.0	-82.9
77.0	-25.4	-25.7	-2.5	-53.7	-7.7	-10.4	15.4	-90.1
82.5	-31.7	-32.0	-2.5	-66.1	-8.9	-12.0	11.3	-97.3
88.0	-37.9	-38.3	-2.5	-78.6	-10.1	-13.6	8.2	-104.3
93.5	-44.1	-44.5	-2.5	-91.1	-11.3	-15.1	5.5	-111.3
99.0	-50.3	-50.8	-2.5	-103.6	-12.5	-16.7	3.2	-118.0
104.5	-56.5	-57.1	-2.5	-116.1	-13.7	-18.3	1.2	-124.7
108.0	-60.5	-61.1	-2.5	-124.1	-14.4	-19.4	0.4	-128.7
109.0	-61.6	-62.2	-2.5	-126.4	-14.6	-19.6	0.2	-129.9
Span 2 - 0	0	0	0	0	-14.8	-19.9	0	-131.1

\* Distance from the centerline of the end bearing

**Table 5.3-4 – Summary of Factored Shear**  
Interior girder, Span 1 shown, Span 2 symmetric

Location*	Strength I Shear	Service I Shear	Service III Shear
(ft.)	(k)	(k)	(k)
0	385.4	260.6	237.9
1.0	379.0	256.2	233.8
5.5	350.0	236.2	215.4
11.0	315.1	212.1	193.0
16.5	280.7	188.3	170.9
22.0	246.8	164.8	149.1
27.5	213.4	141.6	127.5
33.0	180.6	118.7	106.1
38.5	148.3	96.2	85.0
44.0	116.7	74.0	64.2
49.5	85.7	52.1	43.6
54.5	-118.4	-69.7	-57.6
55.0	-121.3	-71.8	-59.6
60.5	-153.5	-94.3	-80.6
66.0	-185.7	-116.9	-101.7
71.5	-217.9	-139.4	-122.8
77.0	-250.0	-161.8	-143.8
82.5	-282.0	-184.3	-164.8
88.0	-313.8	-206.6	-185.7
93.5	-345.4	-228.8	-206.6
99.0	-376.8	-250.9	-227.3
104.5	-407.9	-272.8	-247.8
108.0	-427.4	-286.6	-260.8
109.0	-433.0	-290.5	-264.5
Span 2 - 0	-277.8	-165.8	-139.6

Load Factor Combinations

$$\text{Strength I} = 1.25(\text{DC}) + 1.5(\text{DW}) + 1.75(\text{LL} + \text{IM})$$

$$\text{Service I} = 1.0[\text{DC} + \text{DW} + (\text{LL} + \text{IM})]$$

$$\text{Service III} = 1.0(\text{DC} + \text{DW}) + 0.8(\text{LL} + \text{IM})$$

\* Distance from the centerline of the end bearing

**Table 5.3-5 - Summary of Unfactored Moments**

Exterior girder, Span 1 shown, Span 2 symmetric

Location*	Noncomposite Moments					Composite Moments		Live Load + IM Moments	
	Girder		Slab and Haunch	Diaphragm	Total Noncomp.	Parapet	FWS	Positive HL-93	Negative HL-93
	**	***							
(ft.)	(k-ft)	(k-ft)	(k-ft)	(k-ft)	(k-ft)	(k-ft)	(k-ft)	(k-ft)	(k-ft)
0	47	0	0	0	0	0	0	0	0
1.0	108	61	55	1	117	9	8	93	-11
5.5	368	322	288	7	616	46	41	482	-59
11.0	656	609	545	14	1,168	85	77	897	-118
16.5	909	863	771	21	1,655	118	106	1,245	-177
22.0	1,128	1,082	967	28	2,076	144	130	1,528	-236
27.5	1,313	1,267	1,132	35	2,434	164	148	1,746	-294
33.0	1,464	1,417	1,267	42	2,726	177	160	1,906	-353
38.5	1,580	1,534	1,371	49	2,954	183	165	2,019	-412
44.0	1,663	1,616	1,445	56	3,117	183	166	2,073	-471
49.5	1,711	1,664	1,488	63	3,215	177	160	2,071	-530
54.5	1,725	1,679	1,501	69	3,248	165	149	2,041	-583
55.0	1,725	1,678	1,501	68	3,247	164	148	2,035	-589
60.5	1,705	1,658	1,482	61	3,202	144	130	1,951	-648
66.0	1,650	1,604	1,434	54	3,092	118	107	1,816	-706
71.5	1,562	1,515	1,355	48	2,917	86	77	1,633	-765
77.0	1,439	1,392	1,245	41	2,678	46	42	1,406	-824
82.5	1,282	1,236	1,105	34	2,374	1	1	1,139	-883
88.0	1,091	1,044	934	27	2,005	-52	-47	836	-1,138
93.5	865	819	732	20	1,571	-110	-100	531	-1,238
99.0	606	560	500	13	1,072	-176	-159	300	-1,389
104.5	312	266	238	6	509	-248	-224	114	-1,683
108.0	110	61	55	1	117	-297	-268	33	-1,945
109.0	47	0	0	0	0	-311	-281	15	-2,031
Span 2 - 0	-	0	0	0	0	-326	-294	0	-2,121

\* Distance from the centerline of the end bearing

\*\* Based on the simple span length of 110.5 ft. and supported at the ends of the girders. These values are used to calculate stresses at transfer.

\*\*\* Based on the simple span length of 109 ft. and supported at the centerline of bearings. These values are used to calculate the final stresses.

**Table 5.3-6 – Summary of Factored Moments**  
Exterior girder, Span 1 shown, Span 2 symmetric

Location*	Strength I (ft.)	Service I Moments**		Service III Moments**	
		NC (k-ft)	Comp. (k-ft)	NC (k-ft)	Comp. (k-ft)
0	0	0	0	0	0
1.0	331	117	110	117	91
5.5	1,734	616	570	616	473
11.0	3,251	1,168	1,059	1,168	879
16.5	4,554	1,655	1,469	1,655	1,220
22.0	5,644	2,076	1,801	2,076	1,496
27.5	6,524	2,434	2,057	2,434	1,708
33.0	7,203	2,726	2,242	2,726	1,861
38.5	7,702	2,954	2,368	2,954	1,964
44.0	8,001	3,117	2,422	3,117	2,007
49.5	8,103	3,215	2,407	3,215	1,993
54.5	8,061	3,248	2,355	3,248	1,947
55.0	8,047	3,247	2,347	3,247	1,940
60.5	7,793	3,202	2,226	3,202	1,836
66.0	7,351	3,092	2,041	3,092	1,678
71.5	6,727	2,917	1,796	2,917	1,469
77.0	5,928	2,678	1,494	2,678	1,213
82.5	4,961	2,374	1,140	2,374	912
88.0	3,834	2,005	-1,237	2,005	-1,009
93.5	2,605	1,571	-1,448	1,571	-1,201
99.0	-1,547	1,072	-1,723	1,072	-1,445
104.5	-2,954	509	-2,154	509	-1,818
108.0	-4,031	117	-2,510	117	-2,121
109.0	-4,364	0	-2,623	0	-2,217
Span 2 - 0	-4,560	0	-2,741	0	-2,317

#### Load Factor Combinations

$$\text{Strength I} = 1.25(\text{DC}) + 1.5(\text{DW}) + 1.75(\text{LL} + \text{IM})$$

$$\text{Service I} = 1.0[\text{DC} + \text{DW} + (\text{LL} + \text{IM})]$$

$$\text{Service III} = 1.0(\text{DC} + \text{DW}) + 0.8(\text{LL} + \text{IM})$$

\* Distance from the centerline of the end bearing

\*\* For service limit states, moments are applied to the section of the girder, i.e. noncomposite or composite, that resists these moments. Hence, noncomposite and composite moments have to be separated for service load calculations.

**Table 5.3-7 - Summary of Unfactored Shear**  
Exterior girder, Span 1 shown, Span 2 symmetric

Location*	Noncomposite Shear				Composite Shear		Live Load + IM Shear	
	Girder	Slab and Haunch	Diaphragm	Total Noncomp.	Parapet	FWS	Positive HL-93	Negative HL-93
(ft.)	(k)	(k)	(k)	(k)	(k)	(k)	(k)	(k)
0	61.6	55.1	1.3	117.9	8.9	8.1	98.4	-11.2
1.0	60.5	54.1	1.3	115.8	8.7	7.9	97.0	-11.2
5.5	55.4	49.5	1.3	106.2	7.7	7.0	90.6	-11.3
11.0	49.2	44.0	1.3	94.4	6.5	5.9	82.9	-11.6
16.5	43.0	38.4	1.3	82.6	5.4	4.8	75.5	-13.8
22.0	36.7	32.8	1.3	70.8	4.2	3.8	68.3	-17.9
27.5	30.5	27.3	1.3	59.1	3.0	2.7	61.4	-22.6
33.0	24.3	21.7	1.3	47.3	1.8	1.6	54.8	-28.5
38.5	18.1	16.2	1.3	35.5	0.6	0.5	48.5	-34.5
44.0	11.9	10.6	1.3	23.7	-0.6	-0.5	42.5	-40.7
49.5	5.7	5.1	1.3	12.0	-1.8	-1.6	36.8	-46.9
54.5	0	0	-1.3	-1.3	-2.9	-2.6	31.9	-52.6
55.0	-0.6	-0.5	-1.3	-2.3	-3.0	-2.7	31.4	-53.1
60.5	-6.8	-6.1	-1.3	-14.1	-4.2	-3.8	26.4	-59.4
66.0	-13.0	-11.6	-1.3	-25.9	-5.3	-4.8	21.7	-65.7
71.5	-19.2	-17.2	-1.3	-37.7	-6.5	-5.9	17.4	-72.0
77.0	-25.4	-22.7	-1.3	-49.4	-7.7	-7.0	13.4	-78.3
82.5	-31.7	-28.3	-1.3	-61.2	-8.9	-8.0	9.8	-84.5
88.0	-37.9	-33.9	-1.3	-73.0	-10.1	-9.1	7.2	-90.6
93.5	-44.1	-39.4	-1.3	-84.8	-11.3	-10.2	4.8	-96.6
99.0	-50.3	-45.0	-1.3	-96.5	-12.5	-11.3	2.8	-102.5
104.5	-56.5	-50.5	-1.3	-108.3	-13.7	-12.3	1.0	-108.3
108.0	-60.5	-54.1	-1.3	-115.8	-14.4	-13.0	0.4	-111.8
109.0	-61.6	-55.1	-1.3	-117.9	-14.6	-13.2	0.2	-112.8
Span 2 - 0	0	0	0	0	-14.8	-13.4	0	-113.8

\* Distance from the centerline of the end bearing

**Table 5.3-8 – Summary of Factored Shear**  
Exterior girder, Span 1 shown, Span 2 symmetric

Location*	Strength I Shear	Service I Shear	Service III Shear
(ft.)	(k)	(k)	(k)
0	342.9	233.3	213.7
1.0	337.2	229.4	210.0
5.5	311.3	211.4	193.3
11.0	280.1	189.7	173.2
16.5	249.3	168.3	153.2
22.0	219.0	147.1	133.4
27.5	189.1	126.2	113.9
33.0	159.7	105.5	94.6
38.5	130.9	85.2	75.5
44.0	102.5	65.1	56.6
49.5	74.8	45.4	38.0
54.5	-101.0	-59.3	-48.7
55.0	-103.6	-61.1	-50.5
60.5	-132.4	-81.4	-69.5
66.0	-161.3	-101.8	-88.6
71.5	-190.1	-122.1	-107.7
77.0	-218.8	-142.4	-126.7
82.5	-247.5	-162.6	-145.7
88.0	-276.0	-182.8	-164.7
93.5	-304.4	-202.8	-183.5
99.0	-332.5	-222.8	-202.2
104.5	-360.4	-242.5	-220.9
108.0	-377.9	-255.0	-232.7
109.0	-382.9	-258.6	-236.0
Span 2 - 0	-237.8	-142.1	-119.3

Load Factor Combinations

Strength I =  $1.25(\text{DC}) + 1.5(\text{DW}) + 1.75(\text{LL} + \text{IM})$

Service I =  $1.0[\text{DC} + \text{DW} + (\text{LL} + \text{IM})]$

Service III =  $1.0(\text{DC} + \text{DW}) + 0.8(\text{LL} + \text{IM})$

\* Distance from the centerline of the end bearing

Based on the analysis results, the interior girder controls the design. The remaining sections covering the superstructure design are based on the interior girder analysis. The exterior girder calculations would be identical.

**Design Step 5.3.2 ANALYSIS OF CREEP AND SHRINKAGE EFFECTS****Design Step 5.3.2.1 Creep effects**

*The compressive stress in the beams due to prestressing causes the prestressed beams to creep. For simple span pretensioned beams under dead loads, the highest compression in the beams is typically at the bottom, therefore, creep causes the camber to increase, i.e., causes the upward deflection of the beam to increase. This increased upward deflection of the simple span beam is not accompanied by stresses in the beam since there is no rotational restraint of the beam ends. When simple span beams are made continuous through a connection at the intermediate support, the rotation at the ends of the beam due to creep taking place after the connection is established are restrained by the continuity connection. This results in the development of fixed end moments (FEM) that maintain the ends of the beams as flat. As shown schematically in Figure 5.3-1 for a two-span bridge, the initial deformation is due to creep that takes place before the continuity connection is established. If the beams were left as simple spans, the creep deformations would increase; the deflected shape would appear as shown in part “b” of the figure. However, due to the continuity connection, fixed end moments at the ends of the beam will be required to restrain the end rotations after the continuity connection is established as shown in part “c” of the figure. The beam is analyzed under the effects of the fixed end moments to determine the final creep effects.*

*Similar effects, albeit in the opposite direction, take place under permanent loads. For ease of application, the effect of the dead load creep and the prestressing creep are analyzed separately. Figures 5.3-2 and 5.3-3 show the creep moment for a two-span bridge with straight strands. Notice that the creep due to prestressing and the creep due to dead load result in restrained moments of opposite sign. The creep from prestressing typically has a larger magnitude than the creep from dead loads.*

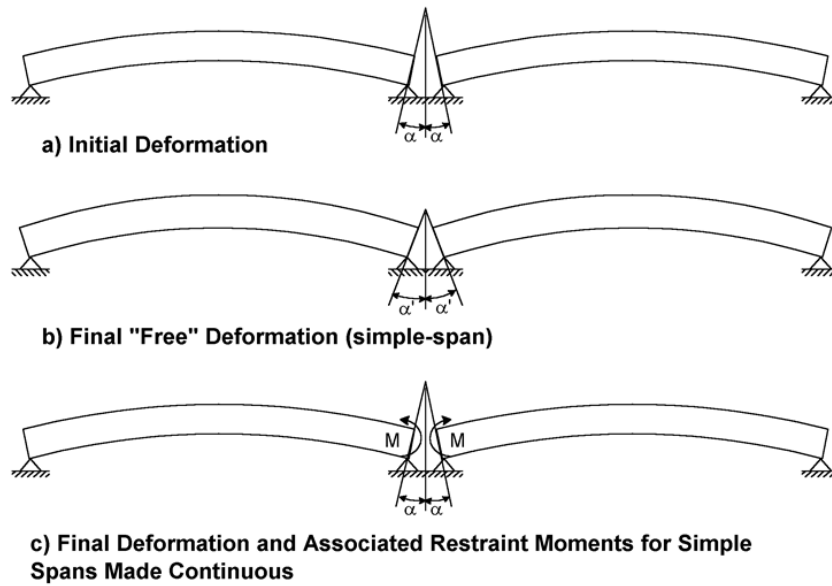


Figure 5.3-1 - Prestressed Creep Deformations and Restraint Moments

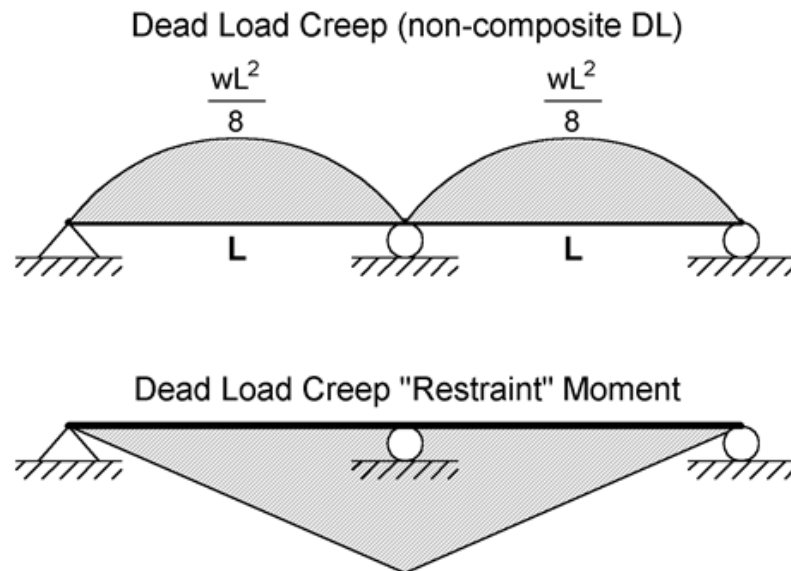


Figure 5.3-2 - Dead Load Creep Moment



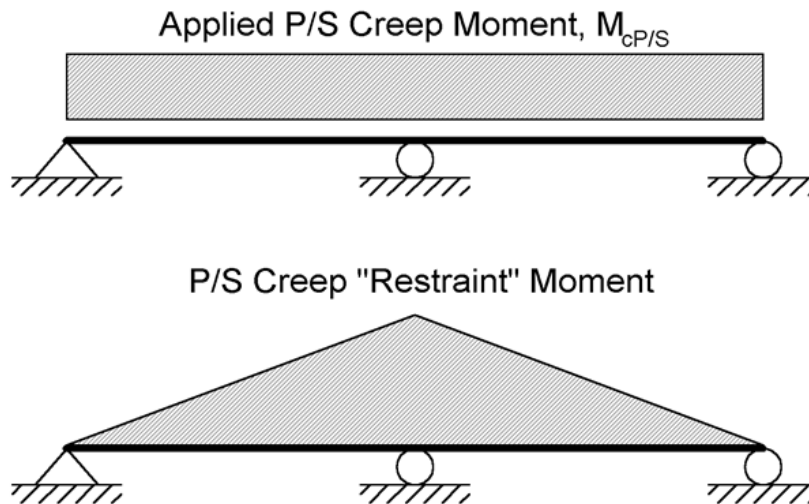
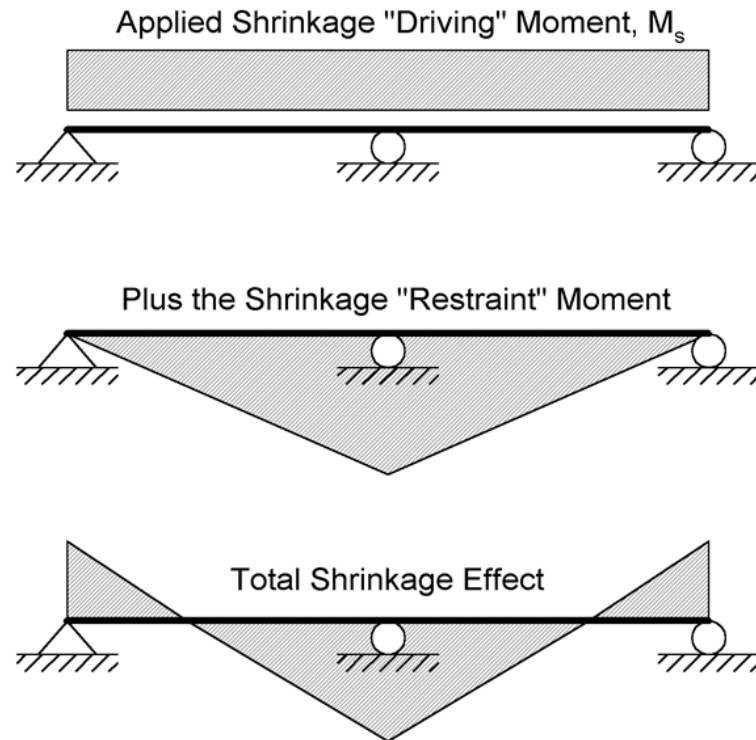


Figure 5.3-3 - Prestressed Creep Moment

#### Design Step 5.3.2.2 Shrinkage effects

*The shrinkage of the pretensioned beams is different from the shrinkage of the deck slab. This is due to the difference in the age, concrete strength, and method of curing of the two concretes. Unlike creep, differential shrinkage induces stresses in all prestressed composite beams, including simple spans. The larger shrinkage of the deck causes the composite beams to sag as shown in Figure 5.3-4. The restraint and final moments are also shown schematically in the figure.*



**Figure 5.3-4 - Shrinkage Moment**

### Calculations of creep and shrinkage effects

*The effect of creep and shrinkage should be determined using the methods outlined AASHTO LRFD 5.14.1.4 and NCHRP Report 519, 2004.*

### Design Step 5.3.2.3 Effect of beam age at the time of the continuity connection application

*The age of the beam at the time of application of the continuity connection has a great effect on the final creep and shrinkage moments. AASHTO LRFD 5.14.1.4.4 through 5.14.1.4.10 provides further discussion on this subject.*

For this example, creep and shrinkage effects were ignored.

**Design Step 5.4 LOSS OF PRESTRESS (S5.9.5)****Design Step 5.4.1 General**

*Loss of prestress can be characterized as that due to instantaneous loss and time-dependent loss. Losses due to anchorage set, friction and elastic shortening are instantaneous. Losses due to creep, shrinkage and relaxation are time-dependent.*

*For pretensioned members, prestress loss is due to elastic shortening, shrinkage, creep of concrete and relaxation of steel. For members constructed and prestressed in a single stage, relative to the stress immediately before transfer, the loss may be taken as:*

$$\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pLT} \quad (\text{S5.9.5.1-1})$$

*where:*

$\Delta f_{pES}$  = *sum of all losses or gains due to elastic shortening or extension at the time of application of prestress and/or external loads (ksi)*

$\Delta f_{pLT}$  = *losses due to long-term shrinkage and creep of concrete, and relaxation of the steel (ksi)*

*Notice that an additional loss occurs during the time between jacking of the strands and transfer. This component is the loss due to the relaxation of steel at transfer.*

*The stress limit for prestressing strands of pretensioned members given in S5.9.3 is for the stress immediately prior to transfer referred to as  $f_{pbt}$  in the LRFD Specifications. To determine the jacking stress, the loss due to relaxation at transfer needs to be added to the stress limits in S5.9.3. Practices differ from state to state as what strand stress is to be shown on the contract drawings. The Specifications assume that the designer will determine the stress in the strands immediately before transfer. The fabricator is responsible for determining the jacking force by adding the relaxation loss at transfer, jacking losses and seating losses to the Engineer-determined stress immediately prior to transfer. The magnitude of the jacking and seating losses depends on the jacking equipment and anchorage hardware used in the precasting yard. It is recommended that the Engineer conduct preliminary calculations to determine the anticipated jacking stress.*

*Accurate estimation of the total prestress loss requires recognition that the time-dependent losses resulting from creep and relaxation are interdependent. If required, rigorous calculation of the prestress losses should be made in accordance with a method supported by research data. However, for conventional construction, such a refinement is seldom warranted or even possible at the design stage, since many of the factors are either unknown or beyond the designer's control. Thus, three methods of estimating time-dependent losses are provided in the LRFD Specifications: (1) the approximate estimate, (2) a refined estimate, and (3) the background necessary to perform a rigorous time-step analysis.*

*The Approximate Method for calculating the time-dependent losses is presented in S5.9.5.3. The values obtained from this method include the loss due to relaxation at transfer,  $\Delta f_{pR1}$ . To*

determine the time-dependent loss after transfer for pretensioned members,  $\Delta f_{pRI}$  needs to be estimated and deducted from the total time-dependent losses calculated using S5.9.5.3. The refined method of calculating time-dependent losses is presented in S5.9.5.4. The method described above is used in this example.

#### Design Step 5.4.2 Calculate the initial stress in the tendons immediately prior to transfer (S5.9.3).

$$\begin{aligned} f_{pbt} &= 0.75f_{pu} \\ &= 0.75(270) \\ &= 202.5 \text{ ksi} \end{aligned}$$

#### Design Step 5.4.3 Determine the instantaneous losses (S5.9.5.2)

##### Friction (S5.9.5.2.2)

The only friction loss possible in a pretensioned member is at hold-down devices for draping or harping tendons. The LRFD Specifications only specify the consideration of these losses.

For this example, all strands are straight strands and hold-down devices are not used.

##### Elastic Shortening, $\Delta f_{pES}$ (S5.9.5.2.3)

The prestress loss due to elastic shortening in pretensioned members is taken as the concrete stress at the centroid of the prestressing steel at transfer,  $f_{cgp}$ , multiplied by the ratio of the modulus of elasticities of the prestressing steel and the concrete at transfer. This is presented in Eq. S5.9.5.2.3a-1.

$$\Delta f_{pES} = (E_p/E_{ct})f_{cgp} \quad (\text{S5.9.5.2.3a-1})$$

where:

$f_{cgp}$  = sum of concrete stresses at the center of gravity of prestressing tendons due to the prestressing force immediately after transfer and the self-weight of the member at the sections of maximum moment (ksi)

$E_p$  = modulus of elasticity of the prestressing steel (ksi)

$E_{ct}$  = modulus of elasticity of the concrete at transfer (ksi)

Applying this equation requires estimating the stress in the strands after transfer. Proposed estimates for pretensioned members are given in S5.9.5.2.3a. For information on additional elastic shortening gains/losses due to strand elongation/shortening caused by external loading and discussion on non-transformed vs. transformed beam sections, see SC5.9.5.2.3a.

Alternatively, the loss due to elastic shortening may be calculated using Eq. C5.9.5.2.3a-1:

$$\Delta f_{pES} = \frac{A_{ps} f_{pbt} (I_g + e_{54.5}^2 A_g) - e_{54.5} M_g A_g}{A_{ps} (I_g + e_{54.5}^2 A_g) + \frac{A_g I_g E_{ci}}{E_p}} \quad (SC5.9.5.2.3a-1)$$

where:

$e_{54.5}$  = average prestressing steel eccentricity at midspan (in.)

$f_{pbt}$  = stress in prestressing steel immediately prior to transfer;  $0.75f_{pu}$  (ksi)

$M_g$  = midspan moment due to member self-weight (k-in)

The alternative approach is used for this example.

$$\Delta f_{pES} = \frac{44(0.153)[0.75(270)]\left[733,320 + 31.38^2(1,085)\right] - 31.38(20,142)(1,085)}{44(0.153)\left[733,320 + 31.38^2(1,085)\right] + \frac{1,085(733,320)(4,200)}{28,500}}$$

$$\Delta f_{pES} = 13.7 \text{ ksi}$$

**Design Step 5.4.4 Calculate the prestressing stress immediately after transfer**

$$\begin{aligned} f_{pi} &= \text{Stress immediately prior to transfer} - \Delta f_{pES} \\ &= f_{pbt} - \Delta f_{pES} \\ &= 202.5 - 13.7 \\ &= 188.8 \text{ ksi} \end{aligned}$$

**Design Step 5.4.5 Calculate the prestressing force at transfer**

$$\begin{aligned} P_i &= N_{strands}(A_{ps})(f_{pi}) \\ &= 44(0.153)(188.8) \\ &= 1,271 \text{ kips (initial loss} = 6.77\%) \end{aligned}$$

**Design Step 5.4.6 Time-dependent losses after transfer, refined method (S5.9.5.4)**

*Refined estimated time-dependent losses are specified in S5.9.5.4. The refined method can provide a better estimate of total losses than the Approximate Method of S5.9.5.3.*

The variables used in these calculations and their values are repeated or calculated here for convenience:

$$\begin{aligned} f'_c &= 6.0 \text{ ksi} = \text{concrete compressive strength of beam at 28 days} \\ f'_{ci} &= 4.8 \text{ ksi} = \text{concrete compressive strength of beam at transfer} \end{aligned}$$

$A_g = 1,085 \text{ in}^2 =$  gross area of beam

$I_g = 733,320 \text{ in}^4 =$  moment of inertia of beam

$e_{s4.5} = 31.380 \text{ in.} =$  prestressing eccentricity at 54.5 ft.

$A_c = 1,795 \text{ in}^2 =$  area of composite section

$I_c = 1,402,024 \text{ in}^4 =$  moment of inertia of composite section

$e_{pc} = 46.96 \text{ in.} =$  prestressing eccentricity in composite section

$(V/S)_b = A_g / \text{perimeter} = 1085/246.2 = 4.406 \text{ in.} =$  volume-to-surface ratio of beam

$(V/S)_d = 4.0 \text{ in.} =$  volume-to-surface ratio of deck

$t_i = 1 \text{ day} =$  equivalent age at transfer (steam cure)

$t_d = 90 \text{ days} =$  age at deck placement

$t_f = 10,000 \text{ days} =$  final time

$e_d = 23.79 \text{ in.} =$  eccentricity of deck with respect to gross composite section, positive in  
typical construction where deck is above girder (in.)

$M_g = 20,142 \text{ k-in.} =$  mid-span moment due to member self-weight

44 – 0.5 in. diameter, low relaxation, Grade 270 strands

$A_{ps} = 0.153 \text{ in}^2 =$  area per strand (based on 8" slab (includes 1/2" wearing surface used for  
shrinkage calculations later)

$f_{py} = 243 \text{ ksi} =$  prestressing steel yield strength

$f_{pu} = 270 \text{ ksi} =$  prestressing steel ultimate strength

$E_p = 28,500 \text{ ksi} =$  prestressing steel modulus

$A_d = 928 \text{ in}^2 =$  area of deck concrete

$f'_{cs} = 4.0 \text{ ksi} =$  slab concrete strength at 28 days

$f'_{cis} = 3.2 \text{ ksi} =$  slab concrete strength at loading (set at 80% of  $f'_{cs}$ )

$E_c = 4696 \text{ ksi} =$  girder final elastic modulus

$E_{ci} = 4,200 \text{ ksi} =$  girder elastic modulus at transfer

$E_{cd} = 3,834 \text{ ksi} =$  slab elastic modulus

*The average annual ambient relative humidity may be obtained from local weather statistics or taken from the map of Figure S5.4.2.3.3-1 shown below.*

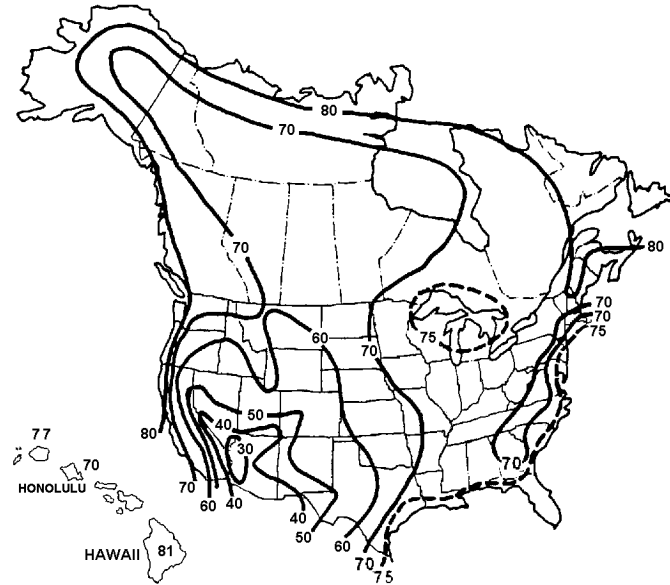


Figure S5.4.2.3.3-1 – Annual Average Ambient Relative Humidity in Percent

For the Atlanta, Georgia area, where the example bridge is assumed, the average relative humidity may be taken as 70%.

**Design Step 5.4.6.1 Creep coefficients (S5.4.2.3.2)**

The creep coefficients are used in the expression for creep and shrinkage losses in S5.9.5.4 and are calculated from Eq. S5.4.2.3.2-1:

$$\Psi(t, t_i) = 1.9k_s k_{hc} k_f k_{td} t_i^{-0.118} \tag{S5.4.2.3.2-1}$$

in which:

$$k_s = 1.45 - 0.13(V/S) \geq 1.0 \tag{S5.4.2.3.2-2}$$

$$k_{hc} = 1.56 - 0.008H \tag{S5.4.2.3.2-3}$$

$$k_f = \frac{5}{1 + f'_{ci}} \tag{S5.4.2.3.2-4}$$

$$k_{td} = \left( \frac{t}{12((100 - 4f'_{ci})/(f'_{ci} + 20)) + t} \right) \tag{S5.4.2.3.2-5}$$

where:

$k_s$  = factor for the effect of the volume-to-surface ratio of the component

$k_{hc}$  = humidity factor for creep

$k_f$  = factor for the effect of concrete strength

$k_{td}$  = time development factor

For the girder concrete,

$$k_s = 1.45 - 0.13(4.406) = 0.877 \text{ (Must be } \geq 1.0; \text{ therefore, use } k_s = 1.0)$$

$$k_{hc} = 1.56 - 0.008(70) = 1.00$$

$$k_f = \frac{5}{1 + 4.8} = 0.862$$

For final age,  $t = t_f - t_i = 10,000 \text{ days} - 1 \text{ day} = 9,999 \text{ days}$ ,

$$k_{td} = \frac{9,999}{12((100 - 4(4.8))/(4.8 + 20)) + 9,999} = 1.0$$

For age at deck placement,  $t = t_d - t_i = 90 - 1 = 89 \text{ days}$ ,

$$k_{td} = \frac{89}{12((100 - 4(4.8))/(4.8 + 20)) + 89} = 0.695$$

Substituting the above factors into the creep coefficient equation results in the following values:

Between age at transfer,  $t_i = 1 \text{ day}$  and final age,  $t_f = 10,000 \text{ days}$ ,

$$\Psi_b(t_f, t_i) = 1.9(1.00)(1.00)(0.862)(1.0)(1)^{-0.118} = 1.638$$

Between age at transfer,  $t_i = 1 \text{ day}$  and age at deck placement,  $t_d = 90 \text{ days}$ ,

$$\Psi_b(t_d, t_i) = 1.9(1.00)(1.00)(0.862)(0.695)(1)^{-0.118} = 1.138$$

Between age at deck placement,  $t_d = 90 \text{ days}$  and final age,  $t_f = 10,000 \text{ days}$ ,

$$\Psi_b(t_f, t_d) = 1.9(1.00)(1.00)(0.862)(1.0)(90)^{-0.118} = 0.963$$

For the deck concrete,

$$k_s = 1.45 - 0.13(4.0) = 0.93 \text{ (Must be } \geq 1.0; \text{ therefore, use } k_s = 1.0)$$

$$k_{hc} = 1.56 - 0.008(70) = 1.00$$



$$k_f = \frac{5}{1 + 3.2} = 1.19$$

For final age,  $t_{fs} = t_{fb} - t_d = 10,000 - 90 = 9910$  days,

$$k_{td} = \frac{9,910}{12((100 - 4(3.2))/(3.2 + 20)) + 9,910} = 1.0$$

Between age of deck at deck placement,  $t_d = 1$  day and final age,  $t_{fs} = 9,910$  days,

$$\Psi_d(t_{fs}, t_d) = 1.9(1.00)(1.00)(1.19)(1.0)(1)^{-0.118} = 2.261$$

### Design Step 5.4.6.2 Shrinkage Strain (S5.4.2.3.3)

The shrinkage strains are used in the expressions for shrinkage losses in S5.9.5.4 and are calculated from Eq. S5.4.2.3.3-1:

$$\varepsilon_{sh} = k_s k_{hs} k_f k_{td} 0.48 \times 10^{-3} \quad (\text{S5.4.2.3.3-1})$$

in which the equations for  $k_{vs}$ ,  $k_f$  and  $k_{td}$  are the same as those given in the previous step for creep coefficient calculations, and:

$$k_{hs} = 2.00 - 0.014H \quad (\text{S5.4.2.3.3-2})$$

where:

$k_{hs}$  = humidity factor for shrinkage

For the girder concrete,

$$k_{hs} = 2.00 - 0.014(70) = 1.02$$

Substituting the above factor and the values for  $k_{vs}$ ,  $k_f$  and  $k_{td}$  previously calculated ( $k_{vs} = 1.00$ ,  $k_f = 0.862$ ,  $k_{td} = 1.0$  for  $t_f = 10,000$  days and  $k_{td} = 0.695$  for  $t_d = 90$  days) results in the following values for shrinkage strain:

Between age at transfer,  $t_i = 1$  day and final age,  $t_f = 10,000$  days,

$$\varepsilon_{bif} = (1.00)(1.02)(0.862)(1.0)(0.48 \times 10^{-3}) = 0.000422$$

Between age at transfer,  $t_i = 1$  day and age at deck placement,  $t_d = 90$  days,

$$\varepsilon_{bid} = (1.00)(1.02)(0.862)(0.695)(0.48 \times 10^{-3}) = 0.000293$$

Between age at deck placement,  $t_d = 90$  days and final age,  $t_f = 10,000$  days,

$$\varepsilon_{bdf} = 0.000422 - 0.000293 = 0.000129$$

For the deck concrete, between age of deck at deck placement,  $t_d = 1$  day and final age,  $t_{fs} = 9,910$  days, the previously calculated values for the factors may be used ( $k_{vs} = 1.00$ ,  $k_{hs} = 1.02$ ,  $k_f = 1.19$ , and  $k_{td} = 1.0$  for  $t_f = 10,000$  days). This results in the following shrinkage strain:

$$\varepsilon_{ddf} = 1.00(1.02)(1.19)(1.0)(0.48 \times 10^{-3}) = 0.000583$$

**Design Step 5.4.6.3 Shrinkage Losses (S5.9.5.4.2a, S5.9.5.4.3a, and S5.9.5.4.3d)**

The prestress loss due to shrinkage of the girder concrete between time of transfer and deck placement is given as Eq. S5.9.5.4.2a-1:

$$\Delta f_{pSR} = \varepsilon_{bid} E_p K_{id} \tag{S5.9.5.4.2a-1}$$

in which:

$$K_{id} = \frac{1}{1 + \frac{E_p}{E_{ci}} \frac{A_{ps}}{A_g} \left( 1 + \frac{A_g e_{pg}^2}{I_g} \right) [1 + 0.7 \Psi_b(t_f, t_i)]} \tag{S5.9.5.4.2a-2}$$

where:

$K_{id}$  = transformed section coefficient that accounts for time-dependent interaction between concrete and bonded steel in the section being considered for time period between transfer and deck placement

Substituting the values given previously in this section,

$$K_{id} = \frac{1}{1 + \frac{28,500}{4,200} \frac{44(0.153)}{1,085} \left( 1 + \frac{1,085(31.38)^2}{733,320} \right) [1 + 0.7(1.638)]} = 0.818$$

The loss due to shrinkage of girder concrete between transfer and deck placement is, therefore

$$\Delta f_{pSR} = (0.000293)(28,500)(0.818) = 6.831 \text{ ksi}$$

The prestress loss due to shrinkage of the girder concrete between time of deck placement and final time is given as Eq. S5.9.5.4.3a-1:

$$\Delta f_{pSD} = \epsilon_{bdf} E_p K_{df} \quad (\text{S5.9.5.4.3a-1})$$

in which:

$$K_{df} = \frac{1}{1 + \frac{E_p}{E_{ci}} \frac{A_{ps}}{A_c} \left( 1 + \frac{A_c e_{pc}^2}{I_c} \right) [1 = 0.7\Psi_b(t_f, t_i)]} \quad (\text{S5.9.5.4.3a-2})$$

where:

$K_{df}$  = transformed section coefficient that accounts for time-dependent interaction between concrete and bonded steel in the section being considered for time period between deck placement and final time

Substituting the values given previously in this section,

$$K_{df} = \frac{1}{1 + \frac{28,500}{4,200} \frac{44(0.153)}{1,795} \left( 1 + \frac{1,795(46.96)^2}{1,402,024} \right) [1 + 0.7(1.638)]} = 0.827$$

The loss due to shrinkage of girder concrete between deck placement and final time is, therefore

$$\Delta f_{pSD} = (0.000129)(28,500)(0.827) = 3.040 \text{ ksi}$$

The prestress loss due to shrinkage of the deck composite section is given as Eq. S5.9.5.4.3d-1:

$$\Delta f_{pSS} = \frac{E_p}{E_c} \Delta f_{cdf} K_{df} [1 + 0.7\Psi_b(t_f, t_d)] \quad (\text{S5.9.5.4.3d-1})$$

in which:

$$\Delta f_{cdf} = \left[ \frac{\epsilon_{ddf} A_d E_{cd}}{1 + 0.7\Psi_d(t_f, t_d)} \right] \left( \frac{1}{A_c} - \frac{e_{pc} e_d}{I_c} \right) \quad (\text{S5.9.5.4.3d-2})$$

where:

$\Delta f_{cdf}$  = change in concrete stress at centroid of prestressing strands due to shrinkage of deck concrete

Substituting the values given previously in this section,

$$\Delta f_{cdf} = \frac{(0.000583)(928)(3,834)}{[1 + 0.7(2.261)]} \left( \frac{1}{1,795} - \frac{(46.96)(23.79)}{1,402,024} \right) = -0.192 \text{ ksi}$$

The loss due to shrinkage of the deck is, therefore

$$\Delta f_{pSS} = \frac{28,500}{4,696} (-0.192)(0.827)[1 + 0.7(0.963)] = -1.613 \text{ ksi}$$

The negative sign implies a gain in tension in the strands, offsetting the losses but at the same time causing bottom fiber tensile stress increases.

**Design Step 5.4.6.4 Creep Losses (S5.9.5.4.2b and S5.9.5.4.3b)**

The prestress loss due to creep of girder concrete between time of transfer and deck placement is given as Eq. S5.9.5.4.2b-1:

$$\Delta f_{pCR} = \frac{E_p}{E_{ci}} f_{cgp} \Psi_b(t_d, t_i) K_{id} \tag{S5.9.5.4.2b-1}$$

in which:

$f_{cgp}$  = sum of concrete stresses at the center of gravity of prestressing tendons due to the prestressing force after jacking and the self-weight of the member at the sections of maximum moment

$$\begin{aligned} f_{cgp} &= \frac{P_i}{A_g} + \frac{P_i e_{54.5}^2}{I_g} - \frac{M_g e_{54.5}}{I_g} \\ &= \frac{1,271}{1,085} + \frac{1,271(31.38)^2}{733,320} - \frac{20,142(31.38)}{733,320} \\ &= 2.016 \text{ ksi} \end{aligned}$$

The loss due to creep of the girder between transfer and deck placement is, therefore

$$\Delta f_{pCR} = \frac{28,500}{4,200} (2.016)(1.138)(0.818) = 12.734 \text{ ksi}$$

The prestress loss due to creep of girder concrete between time of deck placement and final time is given as Eq. S5.9.5.4.3b-1:

$$\Delta f_{pCD} = \frac{E_p}{E_{ci}} f_{cgp} [\Psi_b(t_f, t_i) - \Psi_b(t_d, t_i)] K_{df} + \frac{E_p}{E_c} \Delta f_{cd} \Psi_b(t_f, t_d) K_{df} \quad (S5.9.5.4.3b-1)$$

in which:

$\Delta f_{cd}$  = change in concrete stress at centroid of prestressing strands due to long-term losses between transfer and deck placement, combined with deck weight and superimposed loads

Assuming relaxation loss,  $\Delta f_{pRI} = 1.2$ ksi as allowed by S5.9.5.4.2c, the total long-term losses between transfer and deck placement are

$$\Delta f_p(t_d, t_i) = \Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pRI} = 6.831 + 12.734 + 1.2 = 20.765 \text{ ksi}$$

The loss of force in the strands is, therefore

$$P_{loss}(t_d, t_i) = A_{ps} \Delta f_p(t_d, t_i) = 44(0.153)(20.765) = 139.8 \text{ kips}$$

As previously calculated,

$$M_{deck} = M_{slab} + M_{diaph} = 1,696 + 138 = 1,834 \text{ k-ft}$$

$$M_{SDL} = M_{parapet} + M_{FWS} = 165 + 222 = 387 \text{ k-ft}$$

Therefore, the change in concrete stress is

$$\begin{aligned} \Delta f_{cd} &= \frac{-P_{loss}}{A_g} - \frac{P_{loss} e_{54.5}^2}{I_g} - \frac{M_{deck} e_{54.5}}{I_g} - \frac{M_{SDL} e_{pc}}{I_c} \\ &= \frac{-139.8}{1,085} - \frac{139.8(31.38)^2}{733,320} - \frac{1,834(12)(31.38)}{733,320} - \frac{(387)(12)(46.96)}{1,402,024} \\ &= -1.413 \text{ ksi} \end{aligned}$$

The loss due to creep of the girder between deck placement and final time is, therefore

$$\begin{aligned} \Delta f_{pCD} &= \frac{28,500}{4,200} (2.016)(1.638 - 1.138)(0.827) + \frac{28,500}{4,696} (-1.413)(0.963)(0.827) \\ &= -1.175 \text{ ksi} \end{aligned}$$

**Design Step 5.4.6.5 Relaxation Losses (S5.9.5.4.2c and S5.9.5.4.3c)**

The prestress loss due to relaxation of prestressing strands between time of transfer and deck placement is given as Eq. S5.9.5.4.2c-1:

$$\Delta f_{pR1} = \frac{f_{pi}}{K_L} \left( \frac{f_{pi}}{f_{py}} - 0.55 \right) \quad (S5.9.5.4.2c-1)$$

in which:

$$K_L = 30 \text{ for low-relaxation strands}$$

The loss due to relaxation between transfer and deck placement is, therefore

$$\Delta f_{pR1} = \frac{188.8}{30} \left( \frac{188.8}{243} - 0.55 \right) = 1.428 \text{ ksi}$$

The prestress loss due to relaxation of prestressing strands in the composite section between time of deck placement and final time is given as Eq. S5.9.5.4.3c-1:

$$\Delta f_{pR2} = \Delta f_{pR1} = 1.428 \text{ ksi} \quad (S5.9.5.4.3c-1)$$

**Design Step 5.4.7 Calculate total time dependent loss after transfer**

$$\begin{aligned} \Delta f_{pLT} &= (\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1})_{jd} + (\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} - \Delta f_{pSS})_{df} \\ &= (6.831 + 12.734 + 1.428) + (3.040 - 1.175 + 1.428 - 1.613) \\ &= 22.67 \text{ ksi} \end{aligned} \quad (S5.9.5.4.1-1)$$

$$\begin{aligned} \Delta f_{pT} &= \Delta f_{pES} + \Delta f_{pLT} \\ &= 13.70 + 22.67 \\ &= 36.37 \end{aligned} \quad (S5.9.5.1-1)$$

If the approximate method is used,  $\Delta f_{pT}$  is approximately equal to 37.37 ksi. The approximate method is simpler and generally validates the more refined method. The refined method is likely to be used in software due to its ability to rapidly calculate the various quantities needed for loss evaluation.

**Design Step 5.4.8 Calculate the final effective prestress responses**

$$\begin{aligned} \text{Max } f_{pe} &= 0.80f_{py} \text{ (Table S5.9.3-1 – Stress Limits for Prestressing Tendons at the} \\ &\text{Service Limit State after all losses)} \\ &= 0.8(243) \\ &= 194.4 \text{ ksi} \end{aligned}$$

Calculate the actual effective prestress stress after all losses

$$\begin{aligned} f_{pe} &= 0.75f_{pu} - \Delta f_{pT} \\ &= 0.75(270) - 36.37 \\ &= 166.13 \text{ ksi} < 194.4 \text{ ksi} \quad \mathbf{OK} \end{aligned}$$

Calculate the actual effective prestress force after all losses

$$\begin{aligned} P_e &= N_{\text{strands}}(A_{ps})(f_{pe}) \\ &= 44(0.153)(166.13) \\ &= 1,118 \text{ kips (total loss = 18\%)} \end{aligned}$$

#### Design Step 5.4.9 Calculate jacking stress, $f_{pj}$

As indicated earlier, the Fabricator is responsible for calculation of the jacking force. The calculations presented below are for reference purposes.

As shown earlier, the stress in the prestressing strands immediately prior to transfer is 202.5 ksi.

The Jacking Stress,  $f_{pj}$  = Stress immediately prior to transfer + Relaxation loss at transfer

Relaxation at transfer was calculated in Design Step 5.4.6.5 and is as follows:

$$\Delta f_{pR1} = 1.428 \text{ ksi}$$

Therefore, the jacking stress is

$$f_{pj} = 202.5 + 1.428 = 203.93 \text{ ksi}$$

**Design Step 5.5 STRESS IN PRESTRESSING STRANDS****Design Step 5.5.1 Stress in prestressing strands at nominal flexural resistance**

The stress in prestressing steel at nominal flexural resistance may be determined using stress compatibility analysis. In lieu of such analysis a simplified method is presented in S5.7.3.1.1. This method is applicable to rectangular or flanged sections subjected to flexure about one axis where the Whitney stress block stress distribution specified in S5.7.2.2 is used and for which  $f_{pe}$ , the effective prestressing steel stress after losses, is not less than  $0.5f_{pu}$ . The average stress in prestressing steel,  $f_{ps}$ , may be taken as:

$$f_{ps} = f_{pu}[1 - k(c/d_p)] \quad (S5.7.3.1.1-1)$$

where:

$$k = 2(1.04 - f_{py}/f_{pu}) \quad (S5.7.3.1.1-2)$$

The value of “k” may be calculated using the above equation based on the type and properties of prestressing steel used or it may be obtained from Table SC5.7.3.1.1-1.

The distance from the neutral axis to the compression face of the member may be determined as follows:

for T-section behavior (Eq. S5.7.3.1.1-3):

$$c = \frac{A_{ps} f_{pu} + A_s f_y - A'_s f'_y - \alpha_1 f'_c (b - b_w) h_f}{\alpha_1 f'_c \beta_1 b_w + k A_{ps} \frac{f_{pu}}{d_p}}$$

for rectangular section behavior (Eq. S5.7.3.1.1-4):

$$c = \frac{A_{ps} f_{pu} + A_s f_y - A'_s f'_y}{\alpha_1 f'_c \beta_1 b + k A_{ps} \frac{f_{pu}}{d_p}}$$

T-sections where the neutral axis lies in the flange, i.e., “c” is less than the slab thickness, are considered rectangular sections.

From Table SC5.7.3.1.1-1:

$$k = 0.28 \text{ for low relaxation strands}$$



Assuming rectangular section behavior with no compression steel or mild tension reinforcement:

$$c = A_{ps}f_{pu} / [\alpha_1 f'_c \beta_1 b + kA_{ps}(f_{pu}/d_p)]$$

For the midspan section

$$\begin{aligned} \text{Total section depth, } h &= \text{girder depth} + \text{structural slab thickness} \\ &= 72 + 7.5 \\ &= 79.5 \text{ in.} \end{aligned}$$

$$\begin{aligned} d_p &= h - (\text{distance from bottom of beam to location of P/S steel force}) \\ &= 79.5 - 5.0 \\ &= 74.5 \text{ in.} \end{aligned}$$

$$\beta_1 = 0.85 \text{ for 4 ksi slab concrete (S5.7.2.2)}$$

$$\begin{aligned} b &= \text{effective flange width (calculated in Section 2 of this example)} \\ &= 116 \text{ in.} \end{aligned}$$

$$\begin{aligned} c &= 6.73(270) / [0.85(4)(0.85)(116) + 0.28(6.73)(270/74.5)] \\ &= 5.31 \text{ in.} < \text{structural slab thickness} = 7.5 \text{ in.} \end{aligned}$$

The assumption of the section behaving as a rectangular section is correct.

*Notice that if “c” from the calculations above was greater than the structural slab thickness (the integral wearing surface is ignored), the calculations for “c” would have to be repeated assuming a T-section behavior following the steps below:*

1) *Assume the neutral axis lies within the precast girder flange thickness and calculate “c”. For this calculation, the girder flange width and area should be converted to their equivalent in slab concrete by multiplying the girder flange width by the modular ratio between the precast girder concrete and the slab concrete. The web width in the equation for “c” will be substituted for using the effective converted girder flange width. If the calculated value of “c” exceeds the sum of the deck thickness and the precast girder flange thickness, proceed to the next step. Otherwise, use the calculated value of “c”.*

2) *Assume the neutral axis is below the flange of the precast girder and calculate “c”. The term “ $0.85 f'_c(b - b_w)$ ” in the calculations should be broken into two terms, one refers to the contribution of the deck to the composite section flange and the second refers to the contribution of the precast girder flange to the composite girder flange.*

$$\begin{aligned} f_{ps} &= f_{pu}[1 - k(c/d_p)] && \text{(S5.7.3.1.1-1 )} \\ &= 270[1 - 0.28(5.31/74.5)] \\ &= 264.6 \text{ ksi} \end{aligned}$$

**Design Step 5.5.2 Transfer and development length**

$$\begin{aligned} \text{Transfer Length} &= 60(\text{Strand diameter}) && (\text{S5.11.4.1}) \\ &= 60(0.5 \text{ in.}) \\ &= 30 \text{ in.} \end{aligned}$$

$$\text{Development Length} = l_d \geq \kappa[f_{ps} - (2/3)f_{pe}]d_b \quad (\text{S5.11.4.2-1})$$

From earlier calculations:

$$\begin{aligned} f_{ps} &= 264.6 \text{ ksi (Design Step 5.5.1)} \\ f_{pe} &= 166.13 \text{ ksi (Design Step 5.4.8)} \end{aligned}$$

From S5.11.4.2,  $\kappa = 1.6$  for fully bonded strands

From S5.11.4.3,  $\kappa = 2.0$  for partially debonded strands

For fully bonded strands (32 strands):

$$l_d \geq 1.6[264.6 - (2/3)166.13](0.5) = 123.08 \text{ in. (10.26 ft. or } 10'-4'')$$

For partially debonded strands (two groups of 6-strands each):

$$l_d \geq 2.0[264.6 - (2/3)166.13](0.5) = 153.85 \text{ in. (12.82 ft. or } 12'-9 \frac{13}{16}'')$$

**Design Step 5.5.3 Variation in stress in prestressing steel along the length of the girders**

*According to S5.11.4.1, the prestressing force,  $f_{pe}$ , may be assumed to vary linearly from 0.0 at the point where bonding commences to the effective stress after losses,  $f_{pe}$ , at the transfer length. Between the transfer length and the development length, the strand force may be assumed to increase linearly, reaching the stress at nominal resistance,  $f_{ps}$  at the development length.*

As shown in Figures 2-5 and 2-6, each beam contains three groups of strands:

Group 1: 32 strands fully bonded, i.e., bonded length starts 9 in. outside the centerline of bearings of the noncomposite beam (note that the bond length starts at the end of the beam)

Group 2: 6 strands. Bonded length starts 10 ft. from the centerline of bearings of the noncomposite beam, i.e., 10'-9" from the end of the beam

Group 3: 6 strands. Bonded length starts 22 ft. from the centerline of bearings of the noncomposite beam, i.e., 22'-9" from the end of the beam

For each group, the stress in the prestressing strands is assumed to increase linearly from 0.0 at the point where bonding commences to  $f_{pe}$ , over the transfer length, i.e., over 30 inches. The stress is

also assumed to increase linearly from  $f_{pe}$  at the end of the transfer length to  $f_{ps}$  at the end of the development length. Table 5.5-1 shows the strand forces at the service limit state (maximum strand stress =  $f_{pe}$ ) and at the strength limit state (maximum strand stress =  $f_{ps}$ ) at different sections along the length of the beams. To facilitate the calculations, the forces are calculated for each of the three groups of strands separately and sections at the points where bonding commences, end of transfer length and end of development length for each group are included in the tabulated values. Figure 5.5-1 is a graphical representation of Table 5.5-1.

**Table 5.5-1 – Prestressing Strand Forces**

Dist. from Grdr End (ft)	Dist. from CL of Brg (ft)	Initial Prestressing Force at Transfer			
		Group 1 (k)	Group 2 (k)	Group 3 (k)	Total (k)
0*	-0.75*	0.0			0.0
0.75	0.00	277.3			277.3
2.50	1.75	924.4			924.4
7.75	7.00	924.4			924.4
10.39	9.64	924.4			924.4
10.75**	10.00**	924.4	0.0		924.4
11.75	11.00	924.4	69.3		993.7
13.25	12.50	924.4	173.3		1,097.7
17.25	16.50	924.4	173.3		1,097.7
22.75***	22.00***	924.4	173.3	0.0	1,097.7
23.73	22.98	924.4	173.3	67.9	1,165.6
25.25	24.50	924.4	173.3	173.3	1,271.0
28.25	27.50	924.4	173.3	173.3	1,271.0
33.75	33.00	924.4	173.3	173.3	1,271.0
35.73	34.98	924.4	173.3	173.3	1,271.0
39.25	38.50	924.4	173.3	173.3	1,271.0
44.75	44.00	924.4	173.3	173.3	1,271.0
50.25	49.50	924.4	173.3	173.3	1,271.0
55.25	54.50	924.4	173.3	173.3	1,271.0
55.75	55.00	924.4	173.3	173.3	1,271.0
61.25	60.50	924.4	173.3	173.3	1,271.0
66.75	66.00	924.4	173.3	173.3	1,271.0
72.25	71.50	924.4	173.3	173.3	1,271.0
74.77	74.02	924.4	173.3	173.3	1,271.0
77.75	77.00	924.4	173.3	173.3	1,271.0
83.25	82.50	924.4	173.3	173.3	1,271.0
85.25	84.50	924.4	173.3	173.3	1,271.0
86.77	86.02	924.4	173.3	67.9	1,165.6
87.75+++	87.00+++	924.4	173.3	0.0	1,097.7
88.75	88.00	924.4	173.3		1,097.7
94.25	93.50	924.4	173.3		1,097.7
97.25	96.50	924.4	173.3		1,097.7
99.75++	99.00++	924.4	0.0		924.4
100.11	99.36	924.4			924.4
103.25	102.50	924.4			924.4
108.00	107.25	924.4			924.4
109.75	109.00	277.3			277.3
110.5+	109.75+	0.0			0.0

\*, \*\*, \*\*\* - Point where bonding commences for strand Groups 1, 2, and 3, respectively  
 +, ++, +++ - Point where bonding ends for strand Groups 1, 2, and 3, respectively

Table 5.5-1 (cont.) – Prestressing Strand Forces

Dist. from Grdr End	Dist. from CL of Brg	Prestressing Force After Losses				Force at the Nominal Flexural Resistance			
		Group 1	Group 2	Group 3	Total	Group 1	Group 2	Group 3	Total
(ft)	(ft)	(k)	(k)	(k)	(k)	(k)	(k)	(k)	(k)
0*	-0.75*	0.0			0.0	0.0			0.0
0.75	0.00	244.0			244.0	244.0			244.0
2.50	1.75	813.4			813.4	813.4			813.4
7.75	7.00	813.4			813.4	1132.1			1132.1
10.44	9.69	813.4			813.4	1295.5			1295.5
10.75**	10.00**	813.4	0.0		813.4	1295.5	0.0		1295.5
11.75	11.00	813.4	61.0		874.4	1295.5	61.0		1356.5
13.25	12.50	813.4	152.5		965.9	1295.5	152.5		1448.0
17.25	16.50	813.4	152.5		965.9	1295.5	186.8		1482.3
22.75***	22.00***	813.4	152.5	0.0	965.9	1295.5	233.9	0.0	1529.4
23.80	23.05	813.4	152.5	64.1	1029.9	1295.5	242.9	64.1	1602.4
25.25	24.50	813.4	152.5	152.5	1118.4	1295.5	242.9	152.5	1690.9
28.25	27.50	813.4	152.5	152.5	1118.4	1295.5	242.9	178.2	1716.6
33.75	33.00	813.4	152.5	152.5	1118.4	1295.5	242.9	225.3	1763.7
35.80	35.05	813.4	152.5	152.5	1118.4	1295.5	242.9	242.9	1781.3
39.25	38.50	813.4	152.5	152.5	1118.4	1295.5	242.9	242.9	1781.3
44.75	44.00	813.4	152.5	152.5	1118.4	1295.5	242.9	242.9	1781.3
50.25	49.50	813.4	152.5	152.5	1118.4	1295.5	242.9	242.9	1781.3
55.25	54.50	813.4	152.5	152.5	1118.4	1295.5	242.9	242.9	1781.3
55.75	55.00	813.4	152.5	152.5	1118.4	1295.5	242.9	242.9	1781.3
61.25	60.50	813.4	152.5	152.5	1118.4	1295.5	242.9	242.9	1781.3
66.75	66.00	813.4	152.5	152.5	1118.4	1295.5	242.9	242.9	1781.3
72.25	71.50	813.4	152.5	152.5	1118.4	1295.5	242.9	242.9	1781.3
74.70	73.95	813.4	152.5	152.5	1118.4	1295.5	242.9	242.9	1781.3
77.75	77.00	813.4	152.5	152.5	1118.4	1295.5	242.9	216.8	1755.2
83.25	82.50	813.4	152.5	152.5	1118.4	1295.5	242.9	169.6	1708.0
85.25	84.50	813.4	152.5	152.5	1118.4	1295.5	242.9	152.5	1690.9
86.70	85.95	813.4	152.5	64.1	1029.9	1295.5	242.9	64.1	1602.4
87.75+++	87.00+++	813.4	152.5	0.0	965.9	1295.5	233.9	0.0	1529.4
88.75	88.00	813.4	152.5		965.9	1295.5	225.3		1520.8
94.25	93.50	813.4	152.5		965.9	1295.5	178.2		1473.7
97.25	96.50	813.4	152.5		965.9	1295.5	152.5		1448.0
99.75++	99.00++	813.4	0.0		813.4	1295.5	0.0		1295.5
100.06	99.31	813.4			813.4	1295.5			1295.5
103.25	102.50	813.4			813.4	1101.8			1101.8
108.00	107.25	813.4			813.4	813.4			813.4
109.75	109.00	244.0			244.0	244.0			244.0
110.5+	109.75+	0.0			0.0	0.0			0.0

\*, \*\*, \*\*\* - Point where bonding commences for strand Groups 1, 2, and 3, respectively

+, ++, +++ - Point where bonding ends for strand Groups 1, 2, and 3, respectively

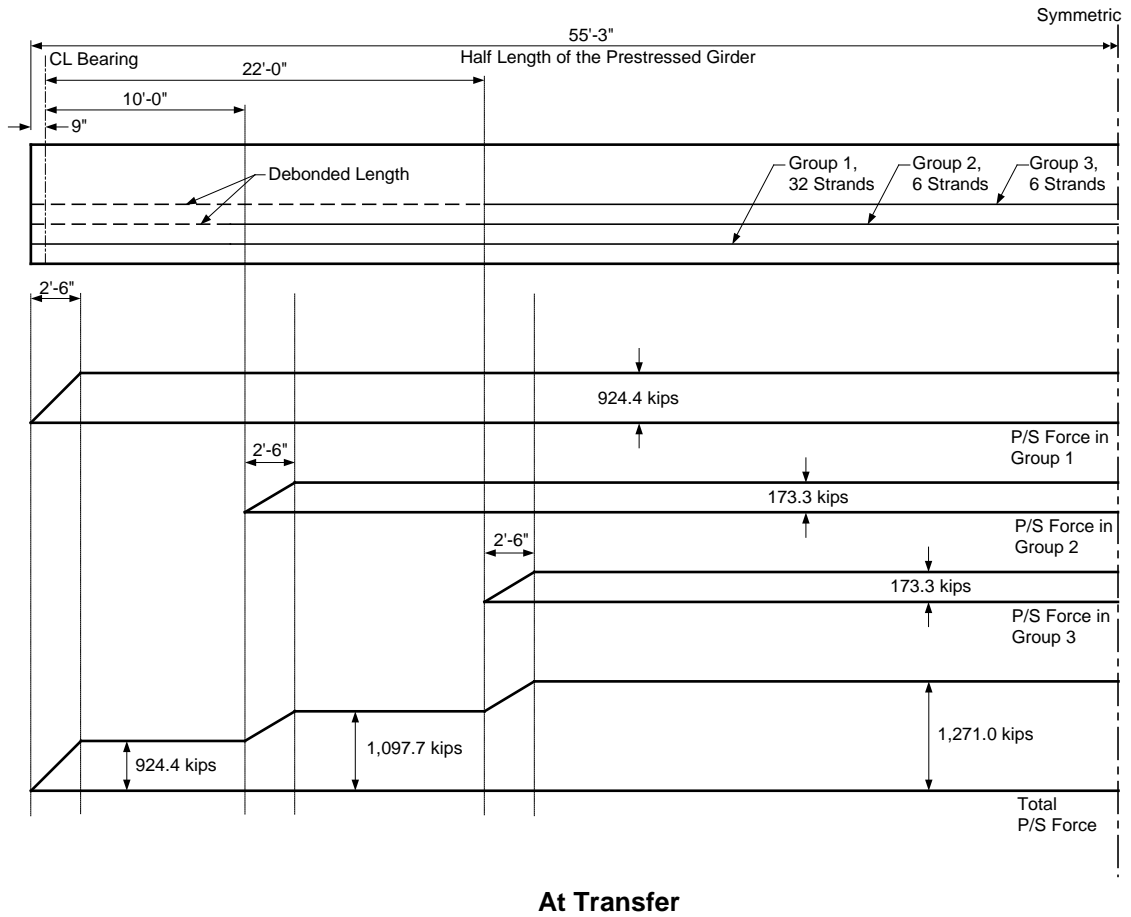
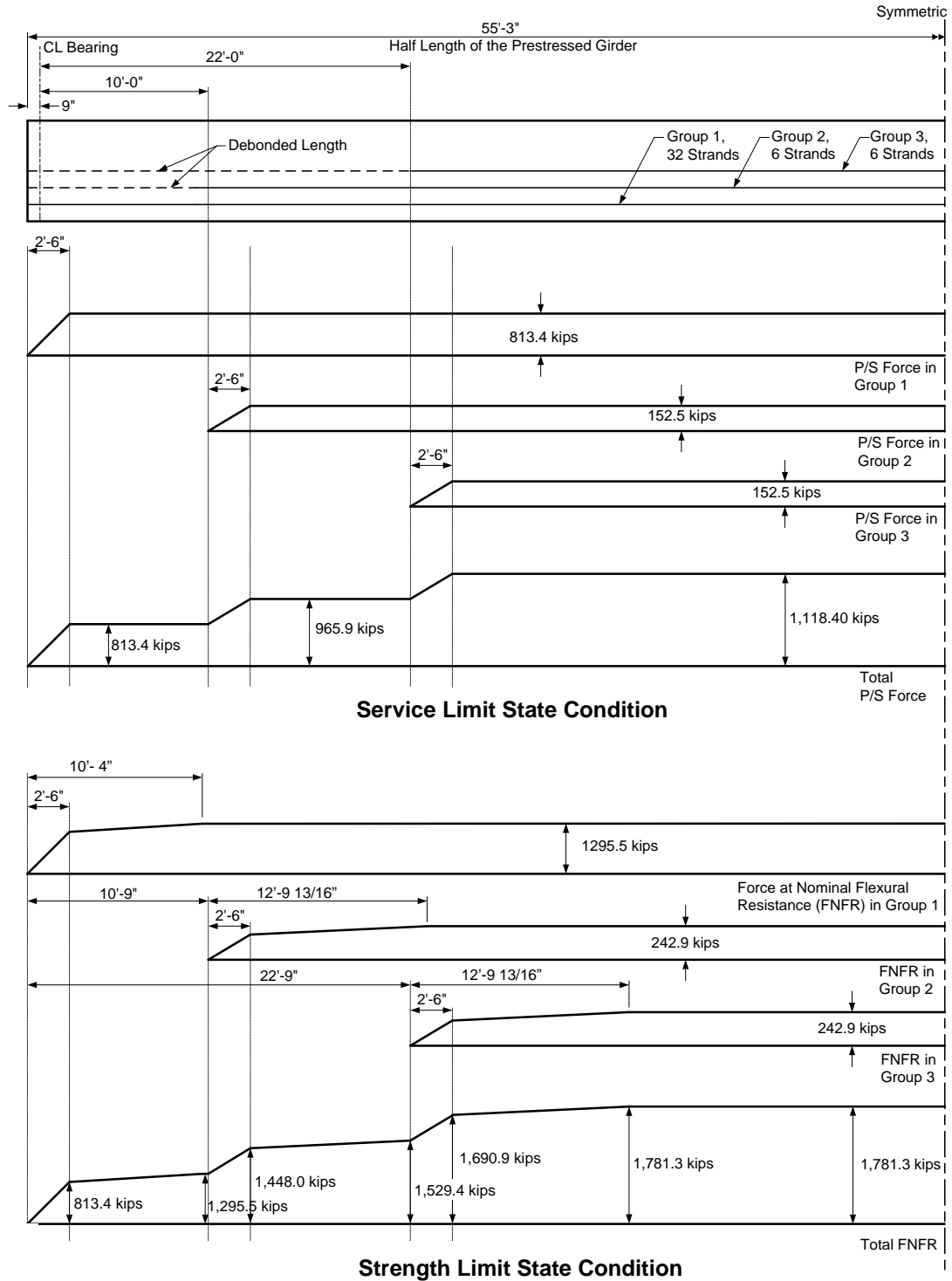


Figure 5.5-1 – Prestressing Strand Forces Shown Graphically (Service Limit State)



Transfer length = 30 in.  
 Development length, fully bonded = 123.08 in.  
 Development length, debonded = 153.85 in.

Figure 5.5-1 (cont.) – Prestressing Strand Forces Shown Graphically

**Design Step 5.5.4 Sample strand stress calculations**Prestress force at centerline of end bearing after losses under Service or Strength

Only Group 1 strands are bonded at this section. Ignore Group 2 and 3 strands.

Distance from the point bonding commences for Group 1 strands = 0.75 ft < transfer length

$$\begin{aligned} \text{Percent of prestressing force developed in Group 1 strands} &= 0.75/\text{transfer length} \\ &= (0.75/2.5)(100) = 30\% \end{aligned}$$

$$\text{Stress in strands} = 0.3(166.13) = 49.8 \text{ ksi}$$

$$\text{Force in strands at the section} = 32(0.153)(49.8) = 244.0 \text{ kips}$$

Prestress force at a section 11 ft. from the centerline of end bearing after losses under Service conditions

Only strands in Group 1 and 2 are bonded at this section. Ignore Group 3 strands.

The bonded length of Group 1 strands before this section is greater than the transfer length. Therefore, the full prestressing force exists in Group 1 strands.

$$\text{Force in Group 1 strands} = 32(0.153)(166.13) = 813.4 \text{ kips}$$

Distance from the point bonding commences for Group 2 strands = 1.0 ft. < transfer length

$$\begin{aligned} \text{Percent of prestressing force developed in Group 2 strands} &= 1.0/\text{transfer length} \\ &= (1.0/2.5)(100) = 40\% \end{aligned}$$

$$\text{Stress in Group 2 strands} = 0.4(166.13) = 66.4 \text{ ksi}$$

$$\text{Force in Group 2 strands at the section} = 6(0.153)(66.4) = 61.0 \text{ kips}$$

$$\begin{aligned} \text{Total prestressing force at this section} &= \text{force in Group 1} + \text{force in Group 2} \\ &= 813.4 + 61.0 = 874.4 \text{ kips} \end{aligned}$$

Strands maximum resistance at nominal flexural capacity at a section 7.0 ft. from the centerline of end bearing

Only Group 1 strands are bonded at this section. Ignore Group 2 and 3 strands.

Distance from the point bonding commences for Group 1 strands, i.e., distance from end of beam = 7.75 ft. (7'- 9")



This distance is greater than the transfer length (2.5 ft.) but less than the development length of the fully bonded strands (10.26 ft.). Therefore, the stress in the strand is assumed to reach  $f_{pe}$ , 166.13 ksi, at the transfer length then increases linearly from  $f_{pe}$  to  $f_{ps}$ , 264.6 ksi, between the transfer length and the development length.

$$\begin{aligned}\text{Stress in Group 1 strands} &= 166.13 + (264.6 - 166.13)[(7.75 - 2.5)/(10.26 - 2.5)] \\ &= 232.70 \text{ ksi}\end{aligned}$$

$$\begin{aligned}\text{Force in Group 1 strands} &= 32(0.153)(232.70) \\ &= 1,139.5 \text{ kips}\end{aligned}$$

Strands maximum resistance at nominal flexural capacity at a section 22 ft. from centerline of end bearing

Only strands in Group 1 and 2 are bonded at this section. Ignore Group 3 strands.

The bonded length of Group 1 strands before this section is greater than the development length for Group 1 (fully bonded) strands. Therefore, the full force exists in Group 1 strands.

$$\text{Force in Group 1 strands} = 32(0.153)(264.6) = 1,295.5 \text{ kips}$$

The bonded length of Group 2 at this section = 22 – 10 = 12 ft.

$$\begin{aligned}\text{Stress in Group 2 strands} &= 166.13 + (264.6 - 166.13)[(12 - 2.5)/(12.82 - 2.5)] \\ &= 256.8 \text{ ksi}\end{aligned}$$

$$\text{Force in Group 2 strands} = 6(0.153)(256.8) = 235.7 \text{ kips}$$

$$\begin{aligned}\text{Total prestressing force at this section} &= \text{force in Group 1} + \text{force in Group 2} \\ &= 1,295.5 + 235.7 \\ &= 1,531.2 \text{ kips}\end{aligned}$$

**Design Step 5.6 FLEXURE DESIGN****Design Step 5.6.1 Flexural stress at transfer****Design Step 5.6.1.1**Stress limits at transfer

Compression stress:

The allowable compression stress limit for pretensioned concrete components is calculated according to S5.9.4.1.1.

$$\begin{aligned}f_{\text{Compression}} &= -0.60(f'_{ci}) \\ &= -0.60(4.8 \text{ ksi}) \\ &= -2.88 \text{ ksi}\end{aligned}$$

Tension stress:

From Table S5.9.4.1.2-1, the stress limit in areas with bonded reinforcement sufficient to resist the tension force in the cracked concrete computed on the basis of an uncracked section is calculated as:

$$\begin{aligned}f_{\text{Tension}} &= 0.24\sqrt{f'_{ci}} \\ &= 0.24\sqrt{4.8} \\ &= 0.53 \text{ ksi}\end{aligned}$$

## Design Step 5.6.1.2 Stress calculations at transfer

Table 5.6-1 – Stresses at Top and Bottom of Beam at Transfer

Location (ft.) <sup>(1)</sup>	Girder Self Weight Moment (k-ft) <sup>(2)</sup>	$F_{ps}$ at Transfer (kips) <sup>(3)</sup>	Stress at Transfer	
			Top of Beam (ksi)	Bottom of Beam (ksi)
0	47	277.3	0.135	-0.654
1.75	153	924.4	0.451	-2.183
5.5	368	924.4	0.326	-2.055
11.0	656	993.7	0.209	-2.065
16.5	909	1,097.7	0.123	-2.171
22.0	1,128	1,097.7	-0.005	-2.040
27.5	1,313	1,271.0	-0.009	-2.358
33.0	1,464	1,271.0	-0.097	-2.269
38.5	1,580	1,271.0	-0.155	-2.209
44.0	1,663	1,271.0	-0.203	-2.160
49.5	1,711	1,271.0	-0.231	-2.132
54.5	1,725	1,271.0	-0.240	-2.120
55.0	1,725	1,271.0	-0.240	-2.123
60.5	1,705	1,271.0	-0.228	-2.135
66.0	1,650	1,271.0	-0.196	-2.168
71.5	1,562	1,271.0	-0.144	-2.220
77.0	1,439	1,271.0	-0.083	-2.284
82.5	1,282	1,271.0	0.009	-2.377
88.0	1,091	1,097.7	0.017	-2.063
93.5	865	1,097.7	0.149	-2.197
99.0	606	924.4	0.197	-1.923
104.5	312	924.4	0.358	-2.105
107.25	153	924.4	0.451	-2.200
109.0	47	277.3	0.135	-0.660

Notes:

- 1 - Distance measured from the centerline of the bearing of the simple span girder  
2 - See Section 5.3, based on 110.5 ft. length  
3 - See Section 5.5 for prestressing forces

**Sample Calculations for Flexural Stresses at Transfer**

Definitions:

 $P_i$  = Initial prestressing force taken from Table 5.5-1 (kips) $A_g$  = Gross area of the basic beam (in<sup>2</sup>) $e$  = Distance between the neutral axis of the noncomposite girder and the center of gravity of the prestressing steel (in.)

$S_t$  = Section moduli, top of noncomposite beam (in<sup>3</sup>)

$S_b$  = Section moduli, bottom of noncomposite beam (in<sup>3</sup>)

$M_g$  = Moment due to the girder self weight only (k-ft)

See Section 2.2 for section properties.

Sample Calculations at 1 ft. – 9 in. From CL of Bearing (2 ft. – 6 in. From Girder End)

Girder top stress:

$$\begin{aligned} f_{\text{top}} &= -P_i/A_g + P_i e_0'/S_t - M_g/S_t \\ &= \frac{-924.4}{1,085} + \frac{924.4(31.005)}{20,588} - \frac{153(12)}{20,588} \\ &= 0.451 \text{ ksi} < \text{Stress limit for tension (0.53 ksi)} \quad \mathbf{OK} \end{aligned}$$

Girder bottom stress:

$$\begin{aligned} f_{\text{bottom}} &= -P_i/A_g - P_i e_0'/S_b + M_g/S_b \\ &= \frac{-924.4}{1,085} - \frac{924.4(31.005)}{20,157} + \frac{153(12)}{20,157} \\ &= -2.183 \text{ ksi} < \text{Stress limit for compression (-2.88 ksi)} \quad \mathbf{OK} \end{aligned}$$

Sample Calculations at 11 ft. From the CL of Bearing (11 ft. – 9 in. From Girder End)

Girder top stress:

$$\begin{aligned} f_{\text{top}} &= -P_i/A_g + P_i e_{11}'/S_t - M_g/S_t \\ &= \frac{-993.7}{1,085} + \frac{993.7(31.222)}{20,588} - \frac{656(12)}{20,588} \\ &= 0.209 \text{ ksi} < \text{Stress limit for tension (0.53 ksi)} \quad \mathbf{OK} \end{aligned}$$

Girder bottom stress:

$$\begin{aligned} f_{\text{bottom}} &= -P_i/A_g - P_i e_{11}'/S_b + M_g/S_b \\ &= \frac{-993.7}{1,085} - \frac{993.7(31.222)}{20,157} + \frac{656(12)}{20,157} \\ &= -2.064 \text{ ksi} < \text{Stress limit for compression (-2.88 ksi)} \quad \mathbf{OK} \end{aligned}$$

Sample Calculations at 54 ft. – 6 in. From the CL of Bearing (55 ft. – 3 in. From Girder End) –  
Midspan of Noncomposite Beam

Girder top stress:

$$\begin{aligned}
 f_{\text{top}} &= -P_i/A_g + P_i e_{54.5'} / S_t - M_g / S_t \\
 &= \frac{-1,271.0}{1,085} + \frac{1,271.0(31.38)}{20,588} - \frac{1,725(12)}{20,588} \\
 &= -0.239 \text{ ksi} < \text{Stress limit for compression (-2.88 ksi)} \quad \mathbf{OK}
 \end{aligned}$$

Girder bottom stress:

$$\begin{aligned}
 f_{\text{bottom}} &= -P_i/A_g - P_i e_{54.5'} / S_b + M_g / S_b \\
 &= \frac{-1,271.0}{1,085} - \frac{1,271.0(31.38)}{20,157} + \frac{1,725(12)}{20,157} \\
 &= -2.123 \text{ ksi} < \text{Stress limit for compression (-2.88 ksi)} \quad \mathbf{OK}
 \end{aligned}$$

### Design Step 5.6.2 Final flexural stress under service limit state

*Maximum compression is checked under Service I limit state and maximum tension is checked under Service III limit state for jurisdictions using gross section analysis. The difference between Service I and Service III limit states is that Service I has a load factor of 1.0 for live load while Service III has a load factor of 0.8.*

*As indicated in Section 5.3, many jurisdictions do not include creep and shrinkage restraint moment effects in designing a pretensioned girder bridge. The calculations presented herein do not include creep and shrinkage moments. If creep and shrinkage are required by a specific jurisdiction, then their effects should be included. See Section 5.3 and Appendix C for calculations and values of creep and shrinkage effects for the example bridge.*

**Design Step 5.6.2.1**Stress limits

Compression stress:

From Table S5.9.4.2.1-1, the stress limit due to the sum of the effective prestress, permanent loads, and transient loads and during shipping and handling is taken as  $0.6\phi_w f'_c$  (where  $\phi_w$  equals 1.0 for solid sections such as pretensioned I-beams).

For prestressed concrete beams ( $f'_c = 6.0$  ksi)

$$\begin{aligned} f_{\text{Comp, beam1}} &= -0.6(6.0 \text{ ksi}) \\ &= -3.6 \text{ ksi} \end{aligned}$$

For deck slab ( $f'_c = 4.0$  ksi)

$$\begin{aligned} f_{\text{Comp, slab}} &= -0.6(4.0 \text{ ksi}) \\ &= -2.4 \text{ ksi} \end{aligned}$$

From Table S5.9.4.2.1-1, the stress limit in prestressed concrete at the service limit state after losses for fully prestressed components in bridges other than segmentally constructed due to the sum of effective prestress and permanent loads shall be taken as:

$$\begin{aligned} f_{\text{Comp, beam 2}} &= -0.45(f'_c) \\ &= -0.45(6.0) \\ &= -2.7 \text{ ksi} \end{aligned}$$

From Section 5.5.3.1, for prestressed components in other than segmentally constructed bridges, the compressive stress due to the Fatigue I load combination and one-half the sum of the effective prestress and permanent loads after losses shall not exceed:

$$\begin{aligned} f_{\text{Comp, beam 3}} &= -0.40(f'_c) \\ &= -0.40(6.0) \\ &= -2.4 \text{ ksi} \end{aligned}$$

Tension stress:

From Table S5.9.4.2.2-1, the stress limit in prestressed concrete at the service limit state after losses for fully prestressed components in bridges other than segmentally constructed, which include bonded prestressing tendons and are subjected to not worse than moderate corrosion conditions shall be taken as the following:

$$\begin{aligned} f_{\text{Tensile}} &= 0.19\sqrt{f'_c} \\ &= 0.19\sqrt{6} \\ &= 0.465 \text{ ksi} \end{aligned}$$

**Table 5.6-2 – Stresses in the Prestressed Beam**

Location	Girder Noncomposite Moment	$F_{ps}$ After Losses	Composite Dead Load Moment	Live load Positive Moment
(ft.) <sup>(1)</sup>	(k-ft) <sup>(2)</sup>	(kips) <sup>(3)</sup>	(k-ft) <sup>(2)</sup>	(k-ft) <sup>(2)</sup>
0	0	244.0	0	0
1.75	217	813.4	36	170
5.5	661	813.4	108	476
11.0	1,252	874.4	199	886
16.5	1,776	965.9	276	1,230
22.0	2,230	965.9	337	1,509
27.5	2,616	1118.4	384	1,724
33.0	2,933	1118.4	414	1,882
38.5	3,181	1118.4	429	1,994
44.0	3,360	1118.4	429	2,047
49.5	3,471	1118.4	414	2,045
54.5	3,512	1118.4	387	2,015
55.0	3,511	1118.4	384	2,010
60.5	3,456	1118.4	338	1,927
66.0	3,333	1118.4	277	1,794
71.5	3,141	1118.4	201	1,613
77.0	2,880	1118.4	108	1,388
82.5	2,551	1118.4	2	1,124
88.0	2,152	965.9	-121	825
93.5	1,686	965.9	-258	524
99.0	1,150	813.4	-452	297
104.5	546	813.4	-580	113
107.25	217	813.4	-670	58
109.0	0	244.0	-729	15

Notes:

- 1 - Distance measured from the centerline of the bearing of the end abutment
- 2 - See Section 5.3 for load effects based on 110.5 ft. girder length
- 3 - See Section 5.5 for prestressing forces based on 110.5 ft. girder length
- 4 - Service I limit state for compression
- 5 - Service III limit state for tension

Table 5.6-2 – Stresses in the Prestressed Beam (cont.)

Location (ft.) <sup>(1)</sup>	Final Stress under PS & DL		Stress Under 1/2 (DL + P/S) + Live Load (ksi) <sup>(4)</sup>	Final Stress Under All Loads		
	Top of Beam (ksi) <sup>(4)</sup>	Bottom of Beam (ksi) <sup>(4)</sup>		Top of Beam (ksi) <sup>(4)</sup>	Bottom of Beam (ksi) <sup>(5)</sup>	Top of Slab (ksi) <sup>(4)</sup>
0	0.143	-0.600	0.071	0.143	-0.600	0.000
1.75	0.343	-1.856	0.142	0.313	-1.795	-0.040
5.5	0.071	-1.559	-0.046	-0.010	-1.390	-0.112
11.0	-0.244	-1.326	-0.274	-0.396	-1.011	-0.209
16.5	-0.508	-1.206	-0.465	-0.719	-0.769	-0.290
22.0	-0.783	-0.909	-0.650	-1.042	-0.372	-0.355
27.5	-0.917	-1.044	-0.754	-1.213	-0.430	-0.406
33.0	-1.107	-0.842	-0.876	-1.430	-0.172	-0.442
38.5	-1.254	-0.687	-0.969	-1.596	0.022	-0.466
44.0	-1.358	-0.581	-1.030	-1.709	0.147	-0.476
49.5	-1.420	-0.521	-1.061	-1.771	0.206	-0.473
54.5	-1.440	-0.509	-1.065	-1.785	0.208	-0.462
55.0	-1.438	-0.511	-1.064	-1.783	0.204	-0.461
60.5	-1.399	-0.564	-1.030	-1.729	0.121	-0.436
66.0	-1.316	-0.664	-0.966	-1.624	-0.026	-0.399
71.5	-1.191	-0.813	-0.872	-1.468	-0.239	-0.349
77.0	-1.023	-1.009	-0.750	-1.261	-0.515	-0.288
82.5	-0.813	-1.252	-0.600	-1.006	-0.852	-0.217
88.0	-0.659	-1.159	-0.471	-0.801	-0.865	-0.135
93.5	-0.364	-1.497	-0.272	-0.454	-1.311	-0.051
99.0	-0.117	-1.517	-0.110	-0.168	-1.411	0.030
104.5	0.257	-1.934	0.109	0.237	-1.893	0.090
107.25	0.464	-2.169	0.222	0.454	-2.149	0.118
109.0	0.268	-0.924	0.131	0.265	-0.919	0.137

Notes:

- 1 - Distance measured from the centerline of the bearing of the end abutment
- 2 - See Section 5.3 for load effects based on 110.5 ft. girder length
- 3 - See Section 5.5 for prestressing forces based on 110.5 ft. girder length
- 4 - Service I limit state for compression
- 5 - Service III limit state for tension

## Definitions:

$P_e$  = Final prestressing force taken from Design Step 5.4 (kips)

$S_{tc}$  = Section moduli, top of the beam of the composite section – gross section (in<sup>3</sup>)

$S_{bc}$  = Section moduli, bottom of the beam of the composite section – gross section (in<sup>3</sup>)

$S_{tsc}$  = Section moduli, top of slab of the composite beam (in<sup>3</sup>)

$M_{DNC}$  = Moment due to the girder, slab, haunch and interior diaphragm (k-ft)



$M_{DC}$  = Total composite dead load moment, includes parapets and future wearing surface (k-ft)  
 $M_{LLC}$  = Live load moment (k-ft)

*All tension stresses and allowables use positive sign convention. All compression stresses and allowables use negative sign convention. All loads are factored according to Table 3.4.1-1 in the AASHTO LRFD Specifications for Service I and Service III limit states as applicable.*

### Design Step 5.6.2.2

#### Sample Calculations at 11 ft. From the CL of Bearing (11 ft. – 9 in. From Girder End)

Girder top stress after losses under sum of all loads (Service I):

$$\begin{aligned} f_{top} &= -P_e/A_g + P_e e_{11}/S_t - M_{DNC}/S_t - M_{DC}/S_{tc} - M_{LLC}/S_{tc} \\ &= \frac{-874.4}{1,085} + \frac{874.4(31.222)}{20,588} - \frac{1,252(12)}{20,588} - \frac{199(12)}{69,949} - \frac{886(12)}{69,949} \\ &= -0.806 + 1.326 - 0.730 - 0.034 - 0.152 \\ &= -0.396 \text{ ksi} < \text{Stress limit for compression under full load (-3.6 ksi) } \mathbf{OK} \end{aligned}$$

Girder top stress under prestressing and dead load after losses:

$$\begin{aligned} f_{top} &= -P_e/A_g + P_e e_{11}/S_t - M_{DNC}/S_t - M_{DC}/S_{tc} \\ &= \frac{-874.4}{1,085} + \frac{874.4(31.222)}{20,588} - \frac{1,252(12)}{20,588} - \frac{199(12)}{69,949} \\ &= -0.806 + 1.326 - 0.730 - 0.034 \\ &= -0.244 \text{ ksi} < \text{Stress limit for compression under permanent load (-2.7 ksi) } \mathbf{OK} \end{aligned}$$

Girder top stress under LL + ½(PS + DL) after losses (Fatigue I with 1.5 Load Factor):

$$\begin{aligned} f_{top} &= -P_e/A_g + P_e e_{11}/S_t - M_{DNC}/S_t - M_{DC}/S_{tc} - M_{LL}/S_{tc} \\ &= \frac{-874.4}{1,085(2)} + \frac{874.4(31.222)}{20,588(2)} - \frac{1,252(12)}{20,588(2)} - \frac{199(12)}{69,949(2)} - \frac{886(12)}{69,949} \end{aligned}$$

$$\begin{aligned}
 &= -0.403 + 0.663 - 0.365 - 0.017 - 0.151 \\
 &= -0.273 \text{ ksi} * 1.5 = -0.410 < \text{Stress limit for compression under LL} + \frac{1}{2}(\text{DL} + \text{PS}) \\
 &\quad \text{load (-2.4 ksi) } \mathbf{OK}
 \end{aligned}$$

Girder bottom stress under all loads (Service III):

$$\begin{aligned}
 f_{\text{bottom}} &= -P_e/A_g - P_e e_{11} / S_b + M_{\text{DNC}}/S_b + M_{\text{DC}}/S_{bc} + M_{\text{LLC}}/S_{bc} \\
 &= \frac{-874.4}{1,085} - \frac{874.4(31.222)}{20,157} + \frac{1,252(12)}{20,157} + \frac{199(12)}{26,985} + \frac{0.8(886)(12)}{26,985} \\
 &= -0.806 - 1.354 + 0.745 + 0.088 + 0.315 \\
 &= -1.012 \text{ ksi} < \text{Stress limit for compression under full} \\
 &\quad \text{load (-2.7 ksi) } \mathbf{OK}
 \end{aligned}$$

*Notice that the gross concrete composite section properties are typically used for the stress calculations due to all load components. However, some jurisdictions use the transformed section properties in calculating the stress due to live load. The transformed section properties are listed in Section 2. In this example, the gross section properties are used for this calculation.*

Girder bottom stress under prestressing and dead load after losses:

$$\begin{aligned}
 f_{\text{bottom}} &= -P_e/A_g - P_e e_{11} / S_b + M_{\text{DNC}}/S_b + M_{\text{DC}}/S_{bc} \\
 &= \frac{-874.4}{1,085} - \frac{874.4(31.222)}{20,157} + \frac{1,252(12)}{20,157} + \frac{199(12)}{26,985} \\
 &= -0.806 - 1.354 + 0.745 + 0.088 \\
 &= -1.327 \text{ ksi} < \text{Stress limit for compression under prestress and permanent loads} \\
 &\quad \text{(-2.7 ksi) } \mathbf{OK}
 \end{aligned}$$

Sample Calculations at 54 ft. – 6 in. From the CL of Bearing (55 ft. – 3 in. From Girder End) – Midspan of Noncomposite Girder

Girder top stress after losses under sum of all loads (Service I):

$$f_{\text{top}} = -P_e/A_g + P_e e_{54.5} / S_t - M_{\text{DNC}}/S_t - M_{\text{DC}}/S_{tc} - M_{\text{LLC}}/S_{tc}$$

$$\begin{aligned}
 &= \frac{-1,118.4}{1,085} + \frac{1,118.4(31.38)}{20,588} - \frac{3,512(12)}{20,588} - \frac{387(12)}{69,949} - \frac{2,015(12)}{69,949} \\
 &= -1.031 + 1.705 - 2.047 - 0.066 - 0.346 \\
 &= -1.785 \text{ ksi} < \text{Stress limit for compression} \\
 &\quad \text{under full load (-3.6 ksi) } \mathbf{OK}
 \end{aligned}$$

Girder top stress after losses under prestress and permanent loads:

$$\begin{aligned}
 f_{\text{top}} &= -P_e/A_g + P_e e_{54.5}/S_t - M_{\text{DNC}}/S_t - M_{\text{DC}}/S_{\text{tc}} \\
 &= \frac{-1,118.4}{1,085} + \frac{1,118.4(31.38)}{20,588} - \frac{3,512(12)}{20,588} - \frac{387(12)}{69,949} \\
 &= -1.031 + 1.705 - 2.047 - 0.066 \\
 &= -1.439 \text{ ksi} < \text{Stress limit for compression under prestress and permanent loads} \\
 &\quad \text{(-2.7 ksi) } \mathbf{OK}
 \end{aligned}$$

Girder top stress under LL + ½(PS + DL) after losses (Fatigue I with 1.5 Load Factor):

$$\begin{aligned}
 f_{\text{top}} &= -P_e/A_g + P_e e_{54.5}/S_t - M_{\text{DNC}}/S_t - M_{\text{DC}}/S_{\text{tc}} - M_{\text{LL}}/S_{\text{tc}} \\
 &= \frac{-1,118.4}{1,085(2)} + \frac{1,118.4(31.38)}{20,588(2)} - \frac{3,512(12)}{20,588(2)} - \frac{387(12)}{69,949(2)} - \frac{2,015(12)}{69,949} \\
 &= -0.515 + 0.852 - 1.024 - 0.033 - 0.346 \\
 &= -1.066 \text{ ksi} < \text{Stress limit for compression under LL + ½(DL + PS) load (-2.4} \\
 &\quad \text{ksi) } \mathbf{OK}
 \end{aligned}$$

Girder bottom stress (Service III):

$$\begin{aligned}
 f_{\text{bottom}} &= -P_e/A_g - P_e e_{54.5}/S_b + M_{\text{DNC}}/S_b + M_{\text{DC}}/S_{\text{bc}} + M_{\text{LLC}}/S_{\text{bc}} \\
 &= \frac{-1,118.4}{1,085} - \frac{1,118.4(31.38)}{20,157} + \frac{3,512(12)}{20,157} + \frac{387(12)}{26,985} + \frac{0.8(2,015)(12)}{26,985} \\
 &= -1.031 - 1.741 + 2.091 + 0.172 + 0.717 \\
 &= 0.208 \text{ ksi} < \text{Stress limit for tension (0.465 ksi) } \mathbf{OK}
 \end{aligned}$$

*Notice that the stresses are calculated without including creep and shrinkage. Jurisdictions that do not include creep and shrinkage typically design the girders for a reduced tensile stress limit or for zero tension at final condition. Including creep and shrinkage would normally result in additional tensile stress at the bottom of the beam at the midspan section.*

Girder bottom stress after losses under prestress and dead load:

$$\begin{aligned}
 f_{\text{bottom}} &= -P_e/A_g - P_e e_{54.5'} / S_b + M_{\text{DNC}} / S_b + M_{\text{DC}} / S_{bc} \\
 &= \frac{-1,118.4}{1,085} - \frac{1,118.4(31.38)}{20,157} + \frac{3,512(12)}{20,157} + \frac{387(12)}{26,985} \\
 &= -1.031 - 1.741 + 2.091 + 0.172 \\
 &= -0.509 \text{ ksi} < \text{Stress limit for compression (-2.7 ksi)} \quad \mathbf{OK}
 \end{aligned}$$

Deck slab top stress under full load:

$$\begin{aligned}
 f_{\text{top slab}} &= (-M_{\text{DC}} / S_{\text{tsc}} - M_{\text{LLC}} / S_{\text{tsc}}) / \text{modular ratio between beam and slab} \\
 &= \left( -\frac{387(12)}{50,902} - \frac{1.0(2,015)(12)}{50,902} \right) / \left( \frac{4,696}{3,834} \right) \\
 &= (-0.091 - 0.475) / 1.225 \\
 &= -0.462 \text{ ksi} < \text{Stress limit for compression in slab (-2.4 ksi)} \quad \mathbf{OK}
 \end{aligned}$$

### Stresses at service limit state for sections in the negative moment region

*Sections in the negative moment region may crack under service limit state loading due to high negative composite dead and live loads. The cracking starts in the deck and as the loads increase the cracks extend downward into the beam. The location of the neutral axis for a section subject to external moments causing compressive stress at the side where the prestressing force is located may be determined using a trial and error approach as follows:*

1. Assume the location of the neutral axis.
2. Assume a value for the compressive strain at the extreme compression fiber (bottom of the beam). Calculate the tensile strain in the longitudinal reinforcement of the deck assuming the strain varies linearly along the height of the section and zero strain at the assumed location of the neutral axis.
3. Calculate the corresponding tension in the deck reinforcement based on the assumed strain.

4. Calculate the compressive force in the concrete.
5. Check the equilibrium of the forces on the section (prestressing, tension in deck steel and compression in the concrete). Change the assumed strain at the bottom of beam until the force equilibrium is achieved.
6. After the forces are in equilibrium, check the equilibrium of moments on the section (moment from prestressing, external moment and moment from internal compression and tension).
7. If moment equilibrium is achieved, the assumed location of the neutral axis and strains are correct. If the moments are not in equilibrium, change the assumed location of the neutral axis and go to Step 2 above.
8. After both force and moment equilibriums are achieved, calculate the maximum stress in the concrete as the product of the maximum concrete strain and the concrete modulus of elasticity.

Notice that when additional compression is introduced into the concrete due to external applied forces, the instantaneous stress in the prestressing steel is decreased by the modular ratio multiplied by the additional compressive stress in the surrounding concrete. The change in the prestressing steel force is typically small and was ignored in the following calculations.

Sample Calculations for a Section in the Negative Moment Region Under Service Limit State, Section at 107 ft. – 3 in. From the CL of End Bearing (108 ft. From Girder End)

From Table 5.3-1,

Location (ft.)	Noncomposite Moments				Composite Moments		Live Load + IM Moments	
	Girder* (k-ft)	Slab and Haunch (k-ft)	Exterior Diaphragm (k-ft)	Total Noncomp. (k-ft)	Parapet (k-ft)	FWS (k-ft)	Positive HL-93 (k-ft)	Negative HL-93 (k-ft)
104.5	266	268	11	546	-248	-332	113	-1,663
108.0	61	62	3	125	-297	-398	33	-1,921

\* Based on the simple span length of 109 ft.

$$\begin{aligned} \text{Maximum negative moment at the section at 104.5 ft} &= 546 - 248 - 332 - 1,663 \\ &= -1,697 \text{ k-ft} \end{aligned}$$

$$\begin{aligned} \text{Maximum negative moment at the section at 108.0 ft} &= 125 - 297 - 398 - 1,921 \\ &= -2,491 \text{ k-ft} \end{aligned}$$

By interpolation, the maximum Service I negative moment at the section under consideration is:

$$\begin{aligned} M_{\text{neg}} &= -2,491 - (-2,491 + 1,697)[(108 - 107.25)/(108 - 104.5)] \\ &= -2,321 \text{ k-ft} \end{aligned}$$

Trial and error approach (see above) was applied to determine the location of the neutral axis. The calculations of a cycle of the process is shown below.

Referring to Figure 5.6-1:

Assume neutral axis at 32.5 inches from the bottom of beam

Assume maximum concrete compressive strain = 0.00079 in./in.

Tensile strain in deck reinforcement =  $0.00079(75.52 - 32.5)/32.5 = 0.001046$  in./in.

Modulus of elasticity of concrete beam = 4,696 ksi (see Section 2)

Concrete stress at bottom of beam =  $0.00079(4,696) = 3.71$  ksi

Area of deck longitudinal reinforcement =  $15.31 \text{ in}^2$  (see Section 5.6.5.1 for calculation)

Force in deck steel =  $15.31(0.001046)(29,000) = 464.5$  k

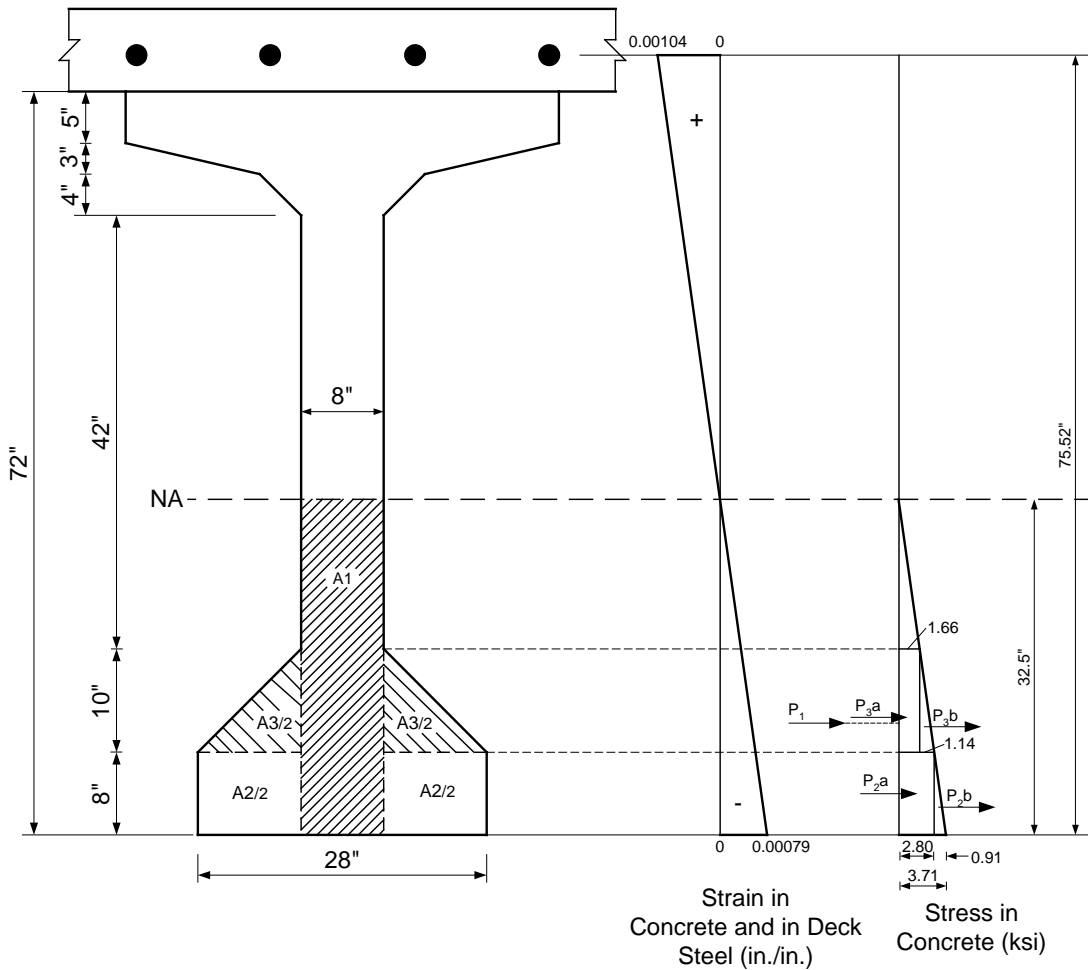
Force in prestressing steel = 813.4 k (see Table 5.5-1)

### **Compressive forces in the concrete:**

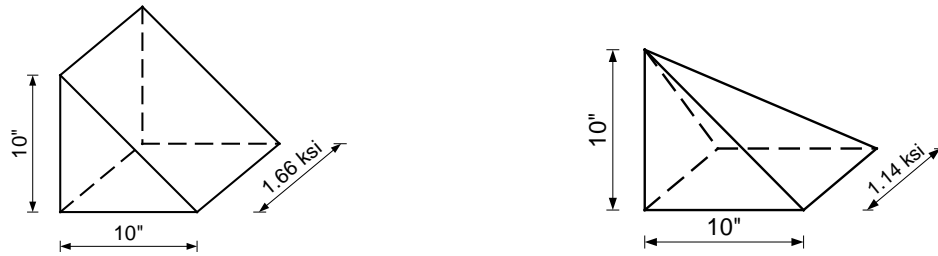
Considering Figure 5.6-1, by calculating the forces acting on different areas as the volume of the stress blocks for areas A1, A2 and A3 as the volume of a wedge, prism or pyramid, as appropriate, the forces in Table 5.6-3 may be calculated. Recall that the centers of gravity of a wedge, a prism with all rectangular faces, a prism with a triangular vertical face and a pyramid are at one-third, one-half, one-third and one-quarter the height, respectively. The location of the centers of gravity shown in the figure may also be calculated. The moment from internal compressive concrete forces shown in Table 5.6-3 is equal to the force multiplied by the distance from the neutral axis to the location of the force.

**Table 5.6-3 – Forces in Concrete Under Service Load in Negative Moment Regions (Section at 107'-3" from the end bearing)**

Area Designation	Force Designation	Area	Stress	Force	Distance from bot. of beam	Distance to N/A	Moment at N/A
		(in <sup>2</sup> )	(ksi)	(kips)	(in)	(in)	(k-ft)
A1	P <sub>1</sub>	260	3.71	482.3	10.83	21.67	871.0
A2	P <sub>2</sub> a	160	2.80	448.0	4.00	28.50	1,064.0
A2	P <sub>2</sub> b	160	0.91	72.8	2.67	29.83	181.0
A3 *	P <sub>3</sub> a	100	1.66	166.0	11.33	21.17	292.9
A3 *	P <sub>3</sub> b	100	1.14	76.0	10.50	22.00	139.3
Total				1,245.1			2,548.2



**Figure 5.6-1 – Compressive Force in the Concrete**



a) Rectangular Stress Distribution

b) Triangular Stress Distribution

**\*Figure 5.6-1a – Shapes Used in Determining Forces for A3**

Sample force calculations for area A3.

Two components of stress act on area A3. The first component is a rectangular stress distribution with an intensity of 1.66 ksi. The second component is a triangular stress distribution with an intensity of 1.14 ksi.

Force due to the rectangular stress distribution:

$$F_{\text{rectangular}} = 2[0.5(10)(10)](1.66) = 166.0 \text{ k}$$

The volume used to determine the effect of the triangular stress distribution is calculated using geometry of a pyramid.

$$F_{\text{triangular}} = 2 \text{ triangles} \left( \frac{1}{3} \text{ pyramid base} \right) (\text{pyramid height}) = 2 \left( \frac{1}{3} \right) (10)(1.14)(10) = 76.0 \text{ k}$$

**Check force equilibrium:**

$$\begin{aligned} \text{Net force on section} &= P/S \text{ steel force} + \text{concrete compressive force} + \text{deck steel force} \\ &= 813.4 + (-1,245.1) + 464.5 \\ &= 32.8 \text{ kips } \mathbf{OK} \text{ (Slightly unbalanced but based on the concrete compressive force of } \\ &\quad \text{1245 kips, the engineer decided not to do another iteration and proceeded.)} \end{aligned}$$

**Check moment equilibrium:**

$$\begin{aligned} \text{Net M on the section} &= \text{external moment} + \text{prestressing force moment} + \text{deck slab force} \\ &\quad \text{moment} + \text{concrete compression moment} \\ &= 2,321 + 813.4 (32.5 - 5.375)/12 - 464.5(75.52 - 32.5)/12 - 2,548.2 \\ &= -53.8 \text{ k-ft } \approx 0 \mathbf{OK} \end{aligned}$$



From Table 5.6-3, the maximum stress in the concrete is 3.71 ksi. The stress limit for compression under all loads (Table S5.9.4.2.1-1) under service condition is  $0.6f'_c$  (where  $f'_c$  is the compressive strength of the girder concrete). For this example, the stress limit equals 3.6 ksi.

The calculated stress equals 3.71 ksi or is 3% overstressed. However, as explained above, the stress in the prestressing steel should decrease due to compressive strains in the concrete caused by external loads, i.e., prestressing steel force less than 813.4 k and the actual stress is expected to be lower than the calculated stress, and the above difference (3%) is considered within the acceptable tolerance.

*Notice that the above calculations may be repeated for other cases of loading in Table S5.9.4.2.1-1 and the resulting applied stress is compared to the respective stress limit. However, the case of all loads applied typically controls. Strain compatibility software could be used to perform the above calculations which would simplify the task but the entire procedure was shown here for completeness.*

### Design Step 5.6.3 Longitudinal steel at top of girder

*The tensile stress limit at transfer used in this example requires the use of steel at the tension side of the beam to resist the tensile force in the concrete calculated based on an uncracked section (Table S5.9.4.1.2-1) where reinforcement is proportioned using a stress of  $0.5f_y$ , not to exceed 30 ksi. The sample calculations are shown for the section in Table 5.6-1 with the highest tensile stress at transfer, i.e., the section at 1.75 ft. from the centerline of the end bearing.*

By integrating the tensile stress in Figure 5.6-2 over the corresponding area of the beam, the tensile force may be calculated as:

$$\begin{aligned} \text{Tensile force} &= 5(42)(0.451 + 0.268)/2 + 7.33(8.0)(0.268 + 0.0)/2 + 2[4(3)](0.268 + 0.158)/2 + \\ & 2[4(4)/2][0.012 + (0.158 - 0.012)(2/3)] + 2[3(13)/2][0.158 + (0.268 - 0.158)(2/3)] \\ &= 99.2 \text{ k} \end{aligned}$$

$$\begin{aligned} \text{Required area of steel} &= 99.2/0.5f_y \\ &= 99.2/(0.5)(60) \\ &= 3.31 \text{ in}^2 \end{aligned}$$

$$\begin{aligned} \text{Required number of \#5 bars} &= 3.31/0.31 \\ &= 10.68 \text{ bars} \end{aligned}$$

Use 10 #5 bars as shown in Figure 5.6-3, for a stress in the reinforcement of  $99.2/(10)(0.31) = 32$  ksi, a slight overstress.

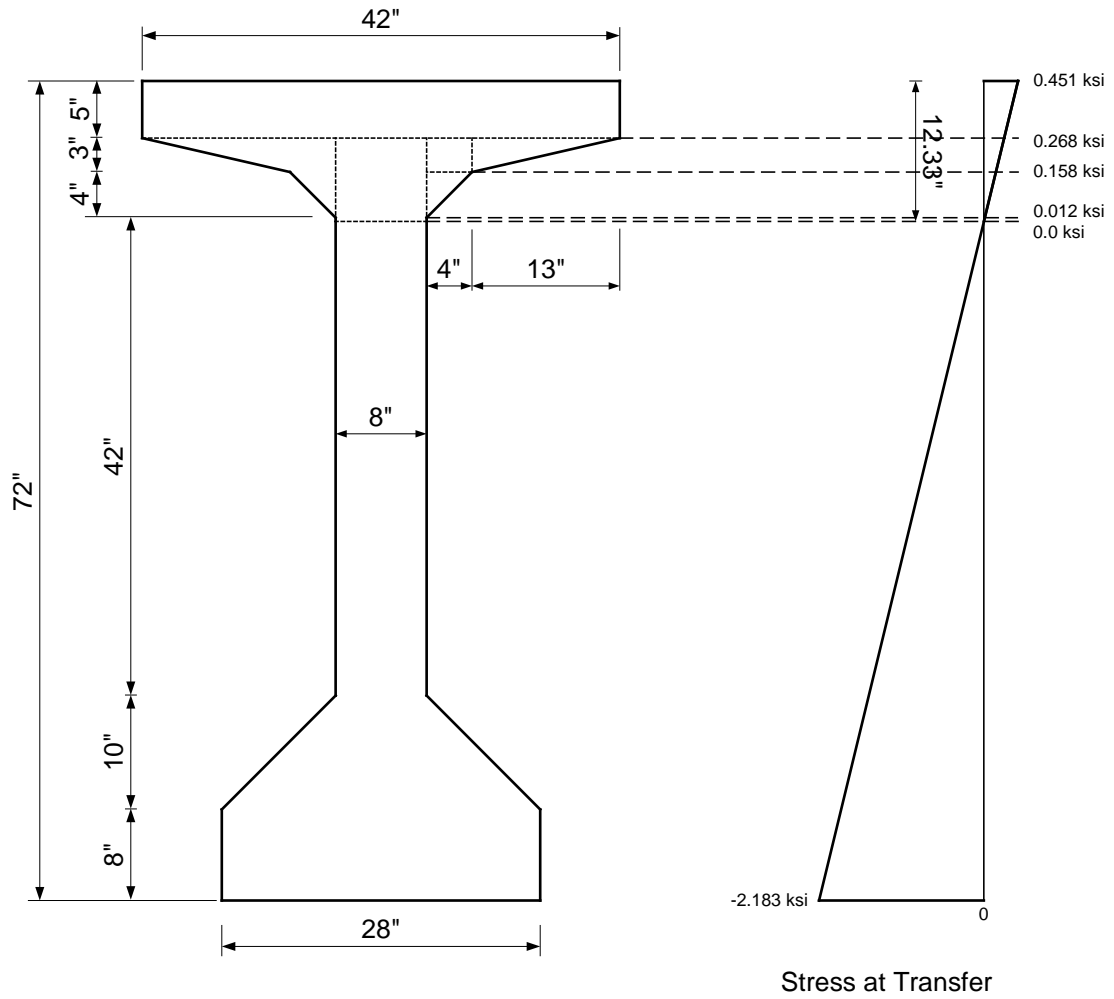


Figure 5.6-2 – Stress at Location of Maximum Tensile Stress at Transfer

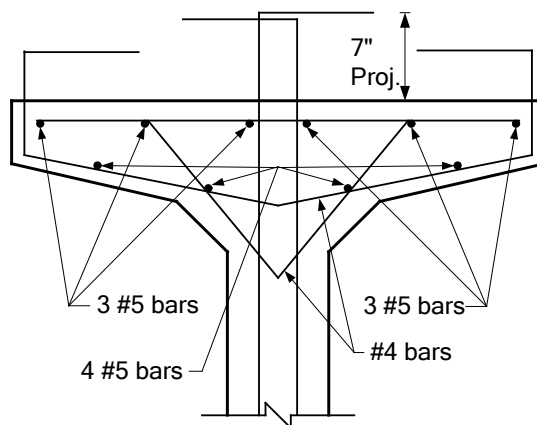


Figure 5.6-3 – Longitudinal Reinforcement of Girder Top Flange

Design Step 5.6.4 Flexural resistance at the strength limit state in positive moment region (S5.7.3.1)

Sample calculations at midspan

$c$  = distance between the neutral axis and the compressive face at the nominal flexural resistance (in.)

$c$  = 5.31 in., which is less than the slab thickness, therefore, the neutral axis is in the slab and section is treated as a rectangular section. (See Design Step 5.5.1 for commentary explaining how to proceed if “ $c$ ” is greater than the deck thickness.)

$f_{ps}$  = stress in the prestressing steel at the nominal flexural resistance (ksi)

$f_{ps}$  = 264.6 ksi

The factored flexural resistance,  $M_r$ , shall be taken as  $\phi M_a$ , where  $M_a$  is determined using Eq. S5.7.3.2.2-1.

Factored flexural resistance in flanged sections (S5.7.3.2.2)

$$M_a = A_{ps}f_{ps}(d_p - a/2) + A_s f_y (d_s - a/2) - A'_s f'_y (d'_s - a/2) + \alpha_1 f'_c (b - b_w) h_f (a/2 - h_f/2) \quad (S5.7.3.2.2-1)$$

The definition of the variables in the above equation and their values for this example are as follows:

$$A_{ps} = \text{area of prestressing steel (in}^2\text{)} \\ = 6.73 \text{ in}^2$$

$$\begin{aligned}
 f_{ps} &= \text{average stress in prestressing steel at nominal bending resistance specified in Eq. S5.7.3.1.1-1 (ksi)} \\
 &= 264.6 \text{ ksi}
 \end{aligned}$$

$$\begin{aligned}
 d_p &= \text{distance from extreme compression fiber to the centroid of prestressing tendons (in.)} \\
 &= 74.5 \text{ in.}
 \end{aligned}$$

$$\begin{aligned}
 A_s &= \text{area of nonprestressed tension reinforcement (in}^2\text{)} \\
 &= 0.0 \text{ in}^2
 \end{aligned}$$

$$\begin{aligned}
 f_y &= \text{specified yield strength of reinforcing bars (ksi)} \\
 &= 60 \text{ ksi}
 \end{aligned}$$

$$\begin{aligned}
 d_s &= \text{distance from extreme compression fiber to the centroid of nonprestressed tensile reinforcement (in.), NA}
 \end{aligned}$$

$$\begin{aligned}
 A'_s &= \text{area of compression reinforcement (in}^2\text{)} \\
 &= 0.0 \text{ in}^2
 \end{aligned}$$

$$\begin{aligned}
 f'_y &= \text{specified yield strength of compression reinforcement (ksi), NA}
 \end{aligned}$$

$$\begin{aligned}
 d'_s &= \text{distance from the extreme compression fiber to the centroid of compression reinforcement (in.), NA}
 \end{aligned}$$

$$\begin{aligned}
 f'_c &= \text{specified compressive strength of concrete at 28 days, unless another age is specified (ksi)} \\
 &= 4.0 \text{ ksi (slab)}
 \end{aligned}$$

$$\begin{aligned}
 b &= \text{width of the effective compression block of the member (in.)} \\
 &= \text{width of the effective flange} = 116 \text{ in. (See Design Step 5.5.1 for commentary for the determination of the effective width, } b, \text{ when the calculations indicate that the compression block depth is larger than the flange thickness.)}
 \end{aligned}$$

$$\begin{aligned}
 b_w &= \text{web width taken equal to the section width “} b \text{” for a rectangular section (in.), NA}
 \end{aligned}$$

$$\begin{aligned}
 \alpha_1 &= \text{stress block factor specified in S5.7.2.2 where } \alpha_1 \text{ shall be taken as 0.85 for specified concrete compressive strengths not exceeding 10.0 ksi.}
 \end{aligned}$$

$$\begin{aligned}
 \beta_1 &= \text{stress block factor specified in S5.7.2.2, NA}
 \end{aligned}$$

$$\begin{aligned}
 h_f &= \text{compression flange depth of an I or T member (in.), NA}
 \end{aligned}$$

$$\begin{aligned}
 a &= \beta_{1c}; \text{ depth of the equivalent stress block (in.)} \\
 &= 0.85(5.31) \\
 &= 4.51 \text{ in.}
 \end{aligned}$$

The second, third and fourth terms in Eq. S5.7.3.2.2-1 are equal to zero for this example.

Substituting,

$$\begin{aligned} M_a &= 6.73(264.6)[74.5 - (4.51/2)] \\ &= 128,651/12 \\ &= 10,721 \text{ k-ft} \end{aligned}$$

Therefore, the factored flexural resistance,  $M_r$ , shall be taken as:

$$M_r = \phi M_a \quad (\text{S5.7.3.2.1-1})$$

where:

$$\begin{aligned} \phi &= \text{resistance factor as specified in S5.5.4.2 for flexure in prestressed concrete,} \\ &\quad \text{tension-controlled sections} \\ &= 1.0 \end{aligned}$$

$$\begin{aligned} M_r &= 1.0(10,721 \text{ k-ft}) \\ &= 10,721 \text{ k-ft} \end{aligned}$$

The maximum factored applied moment for Strength I limit state is 8,456 k-ft (see Table 5.3-2)

$$M_r = 10,721 \text{ k-ft} > M_u = 8,456 \text{ k-ft} \quad \mathbf{OK}$$

### Design Step 5.6.4.1

#### Check if section is tension-controlled

The section is tension-controlled if the strain in the reinforcement is greater than or equal to 0.005 when the concrete in compression reaches a strain of 0.003.

The strain in the prestressed reinforcement, found by proportioning the linear strain diagram is:

$$\epsilon_s = 0.003 \left( \frac{d_e}{c} - 1 \right)$$

where:

$$c = 5.31 \text{ in. (see Section 5.5.1)}$$

$$d_e = 74.5 \text{ in. (Conservatively use } d_p \text{ although S5.7.2.1 states that the distance can be to the extreme tension steel)}$$

$$\epsilon_s = 0.003 \left( \frac{74.5}{5.31} - 1 \right) = 0.039 > 0.005 \quad \mathbf{OK}$$

## Design Step 5.6.4.2

Check minimum required reinforcement (S5.7.3.3.2)

Critical location is at the midspan of the continuous span = 54.5 ft. from the end bearing.

All strands are fully bonded at this location.

According to S5.7.3.3.2, unless otherwise specified, at any section of a flexural component, the amount of prestressed and nonprestressed tensile reinforcement shall be adequate to develop a factored flexural resistance,  $M_r$ , at least equal to the lesser of:

1.33 times the factored moment required by the applicable strength load combinations specified in Table 3.4.1-1.

AND

The cracking moment strength as determined by S5.7.3.3.2-1

$$M_{cr} = \gamma_3 \left[ (\gamma_1 f_r + \gamma_2 f_{cpe}) S_c - M_{dnc} \left( \frac{S_c}{S_{nc}} - 1 \right) \right]$$

where:

$$\begin{aligned} M_{DNC} &= \text{factored using Service I limit state, see Table 5.3-1} \\ &= 3,511 \text{ k-ft} \end{aligned}$$

$$\begin{aligned} S_b &= 20,157 \text{ in}^3 \\ S_{bc} &= 26,985 \text{ in}^3 \\ F_{cpe} &= 0.545 \text{ ksi (from Design Step 5.6.2.2)} \end{aligned}$$

$$\begin{aligned} f_r &= 0.37 \sqrt{f'_c} \quad (\text{S5.4.2.6}) \\ &= 0.37 \sqrt{6} \\ &= 0.906 \text{ ksi} \end{aligned}$$

$$\begin{aligned} \gamma_1 &= 1.6 \text{ for all concrete structures} \\ \gamma_2 &= 1.1 \text{ for bonded tendons} \\ \gamma_3 &= 1.0 \text{ for prestressed concrete structures} \end{aligned}$$

$$\begin{aligned} M_{cr} &= 1.0 \left[ (1.6 * 0.906 + 1.1 * 0.545) 26,985 - 3,511 * 12 \left( \frac{26,985}{20,157} - 1 \right) \right] \\ M_{cr} &= 3,418 \text{ k-ft} \end{aligned}$$

The applied factored moment,  $M_u$ , taken from Table 5.3-2 is 8,456 k-ft (Strength I)

$$1.33(8,456) = 11,246 \text{ k-ft}$$

$M_r$  has to be greater than the lesser of  $M_{cr}$  and  $1.33M_u$ , i.e., 3,418 k-ft.

$M_r$  also has to be greater than the applied factored load  $M_u = 8,456$  k-ft (strength requirement)

$M_r = 10,697$  k-ft, therefore, both provisions are **OK**

### Design Step 5.6.5 Continuity connection at intermediate support

#### Design Step 5.6.5.1

##### Negative moment connection at the Strength limit state

Determine the deck steel at the intermediate pier.

Based on preliminary calculations, the top and bottom longitudinal reinforcement of the deck are assumed to be #6 bars at 5.5 in. spacing and #6 bars at 8.5 in. spacing, respectively.

Calculate the total area of steel per unit width of slab:

$$A_s = A_{\text{bar}}/\text{spacing} \text{ (in}^2\text{/in.)}$$

$$\begin{aligned} \text{For top row of bars: } A_{s \text{ top}} &= 0.44/5.5 \\ &= 0.08 \text{ in}^2\text{/in.} \end{aligned}$$

$$\begin{aligned} \text{For bottom row of bars: } A_{s \text{ bot}} &= 0.44/8.5 \\ &= 0.052 \text{ in}^2\text{/in.} \end{aligned}$$

$$\begin{aligned} \text{Therefore, } A_s &= 0.08 + 0.052 \\ &= 0.132 \text{ in}^2\text{/in.} \end{aligned}$$

Calculate the center of gravity of the slab steel from the top of the slab. Calculations are made from the top of the total thickness and include the integral wearing surface in the total thickness of slab. (See Figure 4-16)

$$\begin{aligned} \text{Top mat (B1): } CGS_{\text{top}} &= \text{Cover}_{\text{top}} + \text{Dia}_{\#5 \text{ main rein.}} + \frac{1}{2} \text{Dia}_{\#6} \\ &= 2.5 + 0.625 + \frac{1}{2} (0.75) \\ &= 3.5 \text{ in.} \end{aligned}$$

$$\begin{aligned} \text{Bot. mat (B2): } CGS_{\text{bot}} &= t_{\text{slab}} - \text{Cover}_{\text{bot}} - \text{Dia}_{\#5 \text{ main rein.}} - \frac{1}{2} \text{Dia}_{\#6} \\ &= 8 - 1 - 0.625 - \frac{1}{2} (0.75) \\ &= 6 \text{ in.} \end{aligned}$$

Center of gravity of the deck longitudinal reinforcement from the top of the deck:

$$\begin{aligned} \text{CGS} &= [A_s \text{ top}(\text{CGS}_{\text{top}}) + A_s \text{ bot}(\text{CGS}_{\text{bot}})]/A_s \\ &= [0.08(3.5) + 0.052(6)]/0.132 \\ &= 4.48 \text{ in. from the top of slab (3.98 in. from the top of the structural thickness)} \end{aligned}$$

Calculate the depth to the slab steel from the bottom of the beam. The haunch depth is ignored in the following calculations.

$$\begin{aligned} d_s &= \text{girder} + \text{slab} - \text{CGS} \\ &= 72 + 8 - 4.48 \\ &= 75.52 \text{ in.} \end{aligned}$$

*The specification is silent about the strength of the concrete in the connection zone. Many jurisdictions use the girder concrete strength for these calculations. This reflects observations made during girder tests in the past. In these tests, the failure always occurred in the girder. This behavior is due to the confinement of the diaphragm concrete in the connection zone provided by the surrounding concrete. This confinement increases the apparent strength of the diaphragm concrete compared to the unconfined strength measured during typical testing of concrete cylinders.*

Assume the neutral axis is in the bottom flange (rectangular behavior), therefore,

$$\begin{aligned} f'_c &= f'_{c, \text{beam}} = 6.0 \text{ ksi} \\ \beta_1 &= \beta_{1, \text{beam}} = 0.75 \text{ (corresponds to the 6.0 ksi concrete, S5.7.2.2)} \\ \alpha_1 &= \alpha_{1, \text{beam}} = 0.85 \text{ (corresponds to the 6.0 ksi concrete, S5.7.2.2)} \\ b &= \text{width of section} = \text{width of girder bottom flange} = 28 \text{ in.} \end{aligned}$$

Calculate  $c$ ,

$$c = A_s f_y / \alpha_1 \beta_1 f'_c b \quad (\text{S5.7.3.1.1-4, modified})$$

where:

$$\begin{aligned} A_s &= \text{area of reinforcement within the effective flange} \\ &\quad \text{width of 116 in. (in}^2\text{)} \\ &= A_s b_{\text{slab}} \\ &= (0.132)(116) \\ &= 15.31 \text{ in}^2 \\ f_y &= 60 \text{ ksi} \\ f'_c &= 6.0 \text{ ksi} \\ \beta_1 &= 0.75 \\ \alpha_1 &= 0.85 \\ b &= 28 \text{ in.} \end{aligned}$$



$$c = 15.31(60)/[0.85(0.75)(6.0)(28)]$$

= 8.58 in., which is approximately equal to the thickness of the bottom flange of the beam (8 in.), therefore, the section is checked as a rectangular section. If “c” was significantly larger than the thickness of the bottom flange, a reduction in the section width should be considered.

Calculate the nominal flexural resistance according to S5.7.3.2.1 and the provisions for a rectangular section.

$$M_n = A_s f_y (d_s - a/2)$$

where:

$$\begin{aligned} a &= \beta_1 c \\ &= 0.75(8.58) \\ &= 6.43 \text{ in.} \end{aligned}$$

$$d_s = 75.52 \text{ in.}$$

$$\begin{aligned} M_n &= 15.31(60)[75.52 - (6.43/2)]/12 \\ &= 5,535 \text{ k-ft} \end{aligned}$$

The factored flexural resistance,  $M_r$ , is

$$M_r = \phi_f M_n \quad (\text{S5.7.3.2.1-1})$$

where:

$$\phi_f = 0.9 \text{ for flexure in reinforced concrete (S5.5.4.2.1)}$$

$$\begin{aligned} M_r &= 0.9(5,535) \\ &= 4,982 \text{ k-ft} \end{aligned}$$

### Check moment capacity versus the maximum applied factored moment at the critical location

Critical location is at the centerline of pier.

Strength I limit state controls.

$$|M_u| = 4,729 \text{ k-ft (see Table 5.3-2)} < M_r = 4,982 \text{ k-ft OK}$$

**Check service crack control (S5.5.2)**

*Actions to be considered at the service limit state are cracking, deformations, and concrete stresses, as specified in Articles S5.7.3.4, S5.7.3.6, and S5.9.4, respectively. The cracking stress is taken as the modulus of rupture specified in S5.4.2.6.*

The maximum spacing of the mild steel reinforcement for control of cracking at the service limit state shall satisfy:

$$s \leq \frac{700\gamma_e}{\beta_s f_s} - 2d_c$$

in which:

$$\beta_s = 1 + \frac{d_c}{0.7(h - d_c)}$$

where:

$$\begin{aligned} \gamma_e &= \text{exposure factor} \\ &= 1.00 \text{ for Class 1 exposure condition} \end{aligned}$$

$$\begin{aligned} d_c &= \text{thickness of concrete cover measured from extreme tension fiber to} \\ &\quad \text{center of the flexural reinforcement located closest thereto (in.)} \\ &= 2 + 0.625 + \frac{1}{2}(0.75) = 3 \text{ in. (see Figure 5.6-4)} \end{aligned}$$

$$\begin{aligned} h &= \text{overall thickness or depth of the component (in.)} \\ &= 72 + 7.5 = 79.5 \text{ in.} \end{aligned}$$

By substituting  $d_c$  and  $h$ ,

$$\beta_s = 1 + \frac{3.0}{0.7(79.5 - 3.0)} = 1.06$$

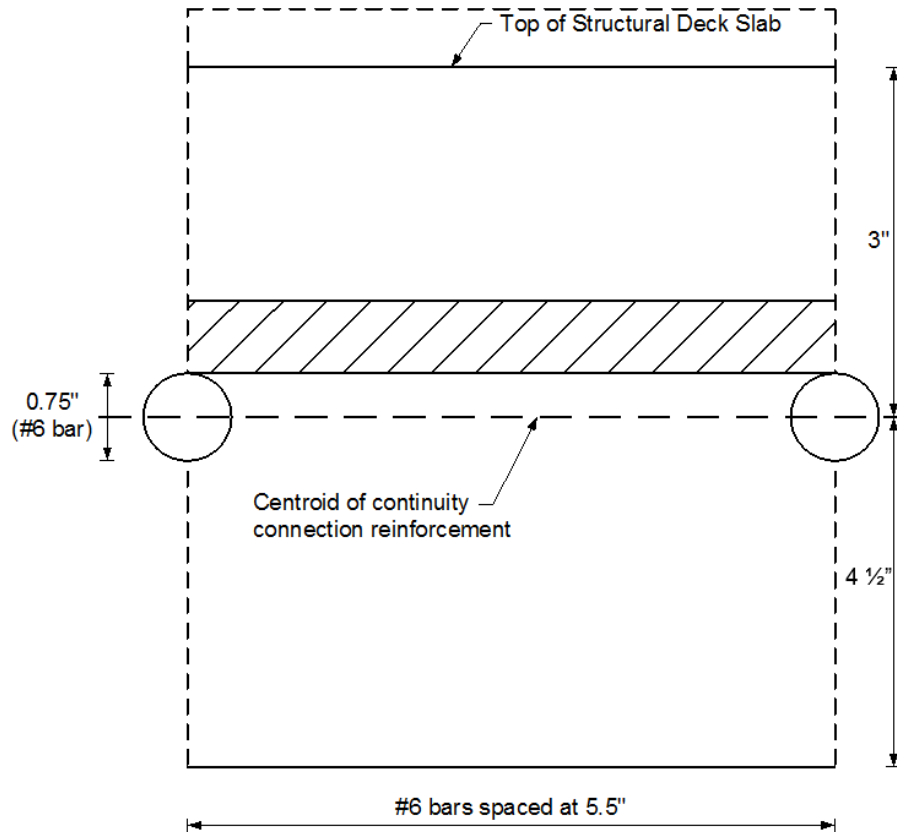


Figure 5.6-4 – Dimensions for Calculation of the Area, A

Connection moment at Service I limit state is 2,858 k-ft (see Table 5.3-2)

Assuming: Section width is equal to beam bottom flange width = 28 in.

Modular ratio = 6 for 6 ksi concrete

Area of steel = 15.31 in<sup>2</sup>

At service limit state, the depth of the neutral axis and the transformed moment of inertia under service loads may be calculated using the same procedure used earlier in the example (Section 4). The neutral axis is 18.86 in. from the bottom of the beam.

Maximum service stress in the steel =  $f_s = 33.74$  ksi

Therefore:

$$S_{\text{required}} \leq \frac{700(1.00)}{(1.06)(33.74)} - 2(3.0) = 13.6 \text{ in.}$$

$$S_{\text{provided}} = 5.5 \text{ in.} < 13.6 \text{ in.} \quad \mathbf{OK}$$

So, use #6 bars @ 5.5 in. to satisfy crack control.

Total area of steel per unit width of slab:

$$A_{s \text{ top}} = \frac{0.44}{5.5} = 0.080 \text{ in}^2/\text{in.}$$

$$A_{s \text{ bot}} = \frac{0.44}{8.5} = 0.052 \text{ in}^2/\text{in.}$$

Therefore,

$$A_s = 0.080 + 0.052 = 0.132 \text{ in}^2/\text{in.}$$

$$\begin{aligned} A_s &= \text{area of reinforcement within the effective flange width of 116 in. (in}^2\text{)} \\ &= A_s b_{\text{slab}} \\ &= (0.132)(116) = 15.31 \text{ in}^2 \end{aligned}$$

Center of gravity of the deck longitudinal reinforcement from the top of the deck:

$$\begin{aligned} \text{CGS} &= [A_{s \text{ top}}(\text{CGS}_{\text{top}}) + A_{s \text{ bot}}(\text{CGS}_{\text{bot}})]/A_s \\ &= [0.080(3.5) + (0.052)(6)]/0.132 = 4.48 \text{ in. (3.87 in. from the top of the} \\ &\quad \text{structural thickness)} \end{aligned}$$

### Design Step 5.6.5.2

#### Positive moment connection

*For jurisdictions that consider creep and shrinkage in the design, it is likely that positive moment will develop at intermediate piers under the effect of prestressing, permanent loads and creep and shrinkage. These jurisdictions provide reinforcement at the bottom of the beams at intermediate diaphragms to resist the factored positive moment at these locations.*

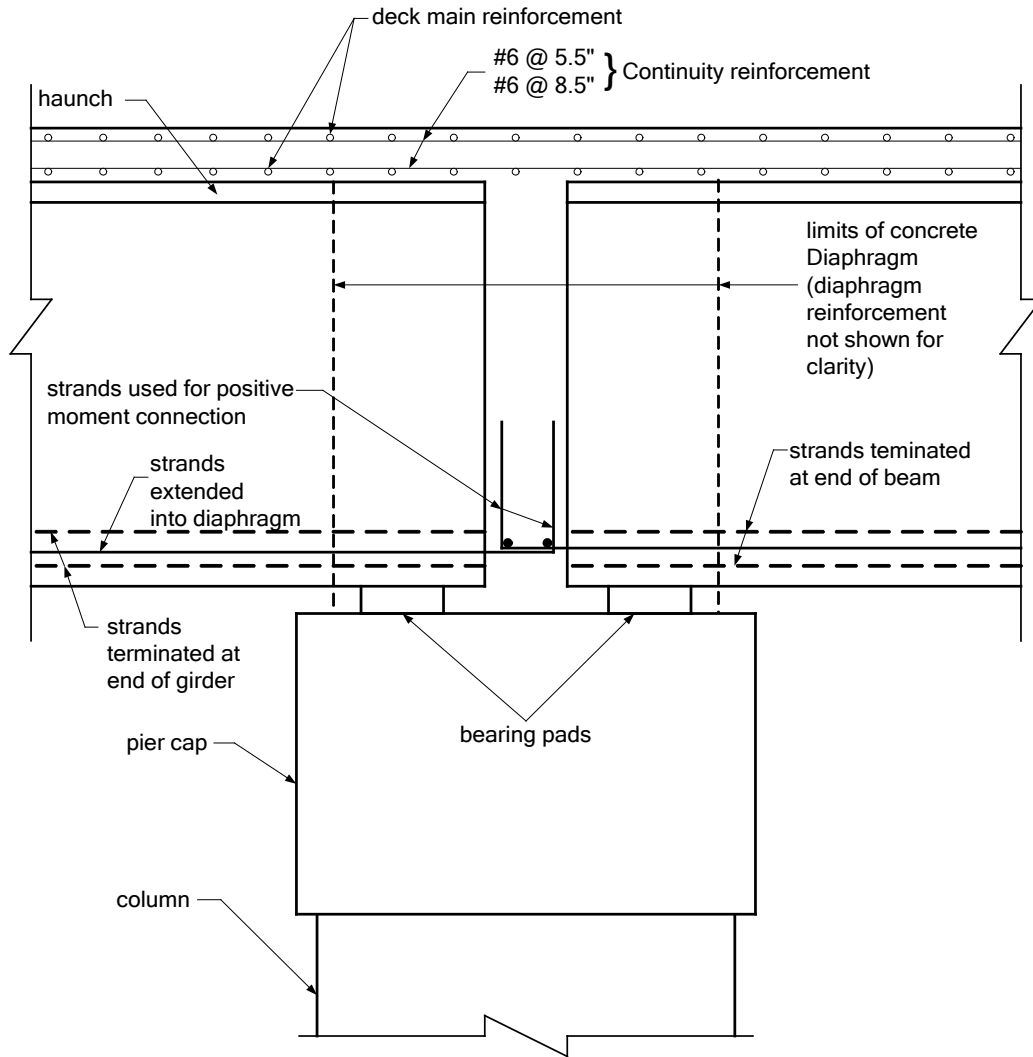
*For jurisdictions that do not consider creep and shrinkage in the design, it is unlikely that live load positive moments at intermediate supports will exceed the negative moments from composite permanent loads at these locations. This suggests that there is no need for the positive moment connection. However, in recognition of the presence of creep and shrinkage effects, most jurisdictions specify some reinforcement to resist positive moments.*

*Two forms of the connection have been in use:*

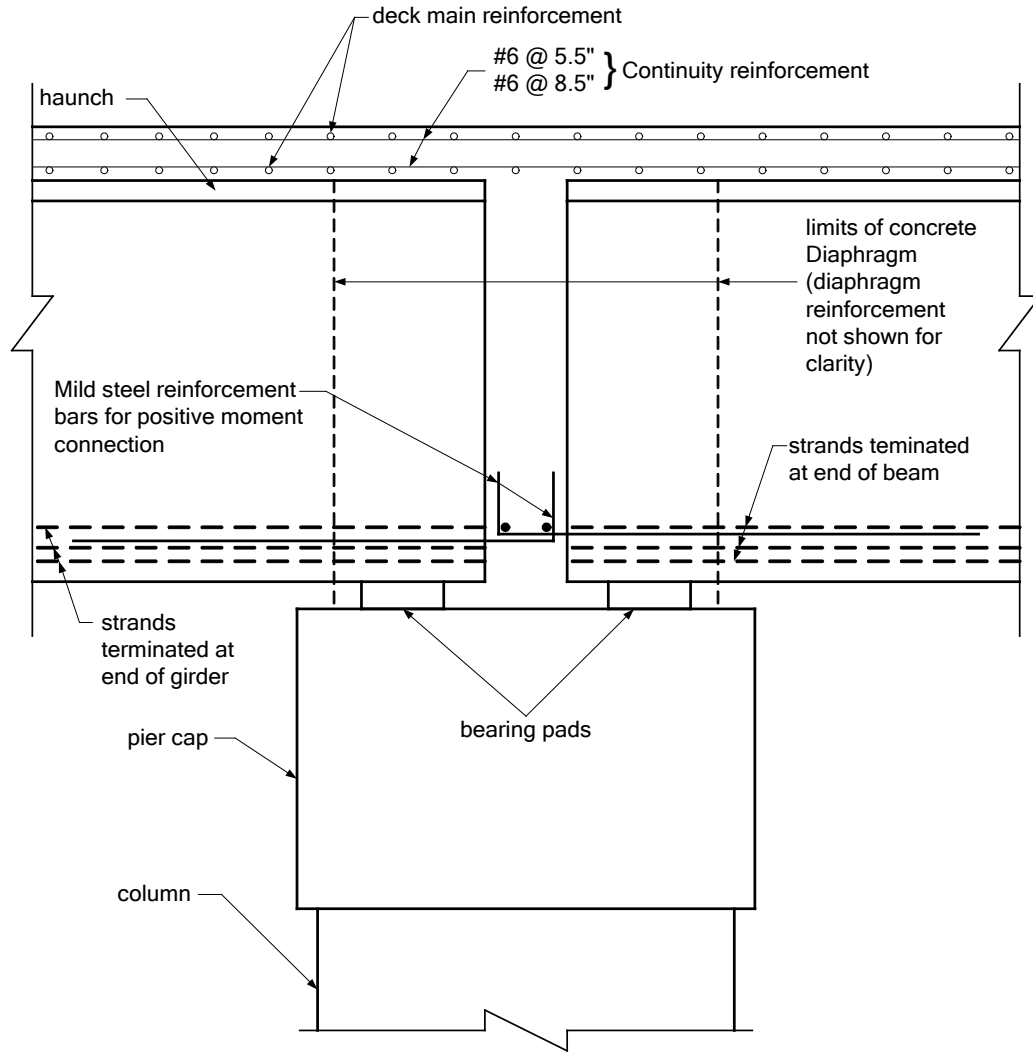
- 1) Figure 5.6-5 shows one alternative that requires extending some of the prestressing strands at the end of the girder into the intermediate diaphragm. Due to the small space between girders, these strands are bent upwards into the diaphragm to provide adequate anchorage. Only strands that are fully bonded are used for the positive moment connection.*

2) The second alternative requires adding mild reinforcement bars as shown in Figure 5.6-6. This alternative may lead to congestion at the end of the beam due to the presence of the prestressing strands at these locations.

Typical details of the top of the pier cap for expansion and fixed bearings are shown schematically in Figures 5.6-7 and 5.6-8.



**Figure 5.6-5 – Continuity Connection Alternative 1: Strands Used for Positive Moment Connection**



**Figure 5.6-6 – Continuity Connection Alternative 2: Reinforcement Bars Used for Positive Moment Connection**

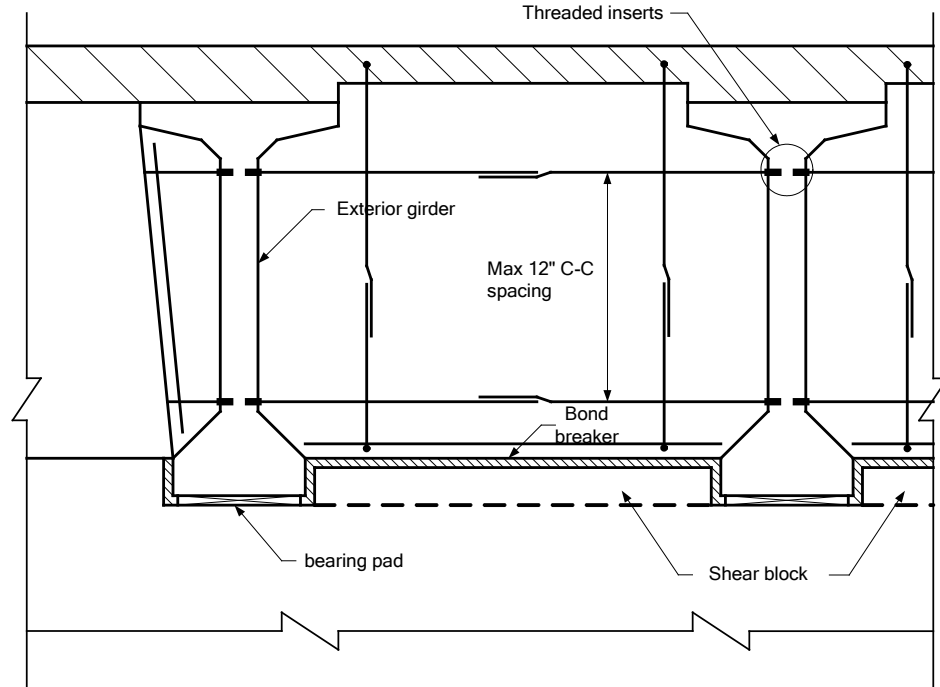


Figure 5.6-7 – Typical Diaphragm at Intermediate Pier (Expansion Bearing)

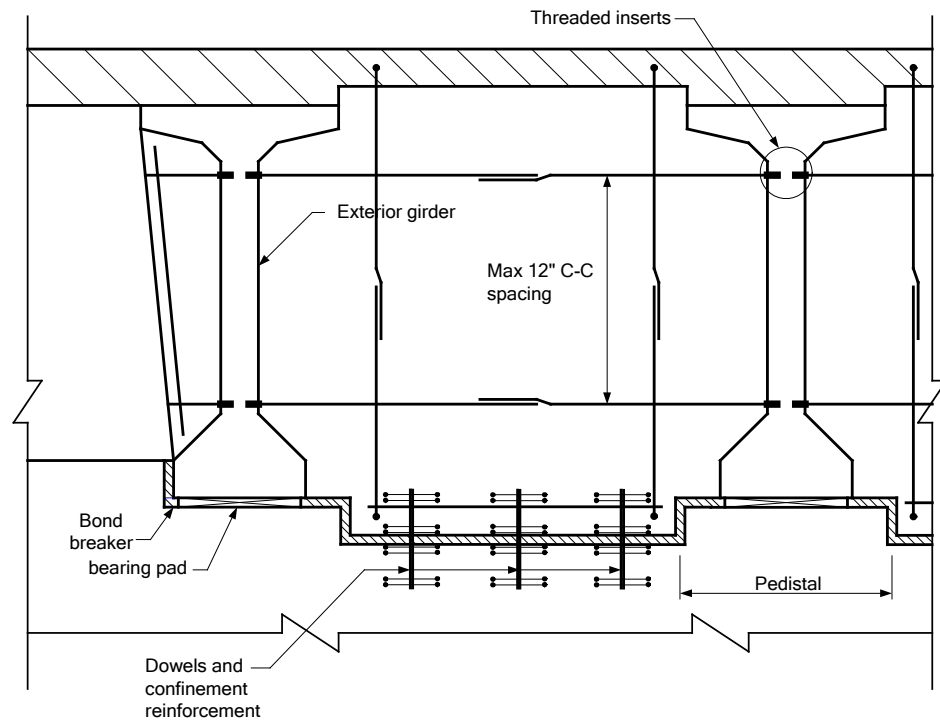


Figure 5.6-8 – Typical Diaphragm at Intermediate Pier (Fixed Bearing)

**Design Step 5.6.6 Fatigue in prestressed steel (S5.5.3)**

*Article S5.5.3 states that fatigue need not be checked when the maximum tensile stress in the concrete under Service III limit state is taken according to the stress limits of Table S5.9.4.2.2-1. The stress limit in this table was used in this example and, therefore, fatigue of the prestressing steel need not be checked.*

**Design Step 5.6.7 Camber (S5.7.3.6)**

*The provisions of S2.5.2.6 shall be considered.*

*Deflection and camber calculations shall consider dead load, live load, prestressing, erection loads, concrete creep and shrinkage, and steel relaxation. For determining deflection and camber, the provisions of Articles S4.5.2.1, S4.5.2.2, and S5.9.5.5 shall apply.*

*Instantaneous deflections are computed using the modulus of elasticity for concrete as specified in S5.4.2.4 and taking the gross moment of inertia,  $I_g$ , as allowed by S5.7.3.6.2.*

*Deflection values are computed based on prestressing, girder self-weight, slab, formwork, exterior diaphragm weight, and superimposed dead load weight. Camber values are computed based on initial camber, initial camber adjusted for creep, and final camber. Typically, these calculations are conducted using a computer program. Detailed calculations are presented below.*

Deflection due to initial prestressing is computed as:

$$\Delta_{P/S} = -(P_i e_s L^2) / (8 E_{ci} I_g) \quad (\text{for straight bonded strands})$$

$$\Delta_{P/S} = -P_i e_s [L^2 - (L_t + 2L_x)^2] / (8 E_{ci} I_g) \quad (\text{for debonded strands})$$

where:

$P_i$  = applied load acting on the section (kips)

$e_s$  = eccentricity of the prestressing force with respect to the centroid of the cross section at the midspan of the beam (in.)

$L$  = span length (ft.)

$L_t$  = transfer length of the strands (in.)

$L_x$  = distance from end of beam to point where bonding commences (in.)

$E_{ci}$  = modulus of elasticity of concrete at transfer (ksi)

$I_g$  = moment of inertia (in<sup>4</sup>)

*The negative sign indicates upward deflection.*

*Computer software is generally used to determine the deflections due to each loading. However, sample calculations are provided for this example.*



See Table 5.5-1 for prestressing forces.

Group 1 strands: 32 fully bonded strands

Initial prestressing force = 924.4 k

Distance from bottom of the beam to the neutral axis = 36.38 in.

Distance from the bottom of the beam to the centroid of Group 1 strands = 5.375 in.

Deflection due to Group 1 strands:

$$\begin{aligned}\Delta_{P/S 1} &= -(P_i e_s L^2)/(8E_{ci} I_g) \\ &= -[924.4(36.38 - 5.375)[109(12)]^2]/[8(4,200)(733,320)] \\ &= -1.99 \text{ in. (upward deflection)}\end{aligned}$$

Group 2 strands: 6 strands debonded for 10 ft. from centerline of bearings

Transfer length = 30 in.

Initial prestressing force = 173.3 k

From Figures 2-5 and 2-6, the distance from the bottom of the beam to the centroid of Group 2 is 4.0 in.

Deflection due to Group 2 strands:

$$\begin{aligned}\Delta_{P/S 2} &= -P_i e_s [L^2 - (L_t + 2L_x)^2]/(8E_{ci} I_g) \\ &= -173.3(36.38 - 4.0)[[109(12)]^2 - [30 + 2(10)(12)]^2]/[8(4,200)(733,320)] \\ &= -0.37 \text{ in. (upward deflection)}\end{aligned}$$

Group 3 strands:

6 strands debonded for 22 ft. from centerline of bearings

Transfer length = 30 in.

Initial prestressing force = 173.3 k

From Figures 2-5 and 2-6, the distance from the bottom of the beam to the centroid of Group 3 is 4.0 in.

Deflection due to Group 3 strands:

$$\begin{aligned}\Delta_{P/S 3} &= -P_i e_s [L^2 - (L_t + 2L_x)^2]/(8E_{ci} I_g) \\ &= -173.3(36.38 - 4.0)[[109(12)]^2 - [30 + 2(22)(12)]^2]/[8(4,200)(733,320)] \\ &= -0.32 \text{ in. (upward deflection)}\end{aligned}$$

Total initial deflection due to prestressing:

$$\begin{aligned}\Delta_{P/S Tot} &= -1.99 - 0.37 - 0.32 \\ &= -2.68 \text{ in. (upward deflection)}\end{aligned}$$

*Notice that for camber calculations, some jurisdictions assume that some of the prestressing force is lost and only consider a percentage of the value calculated above (e.g. Pennsylvania uses 90% of the above value to compute beam seat elevations and 85% to compute probable beam sag). In the following calculations the full value is used. The user may revise these values to match any reduction required by the bridge owner's specification. The reader may also refer to the PCI Bridge Design Manual for additional methods for estimating long term deflections.*

Using conventional beam theory to determine deflection of simple span beams under uniform load or concentrated loads and using the loads calculated in Section 5.2, using noncomposite and composite girder properties for loads applied before and after the slab is hardened, respectively, the following deflections may be calculated:

$$\begin{aligned}\Delta_{sw} &= \text{deflection due to the girder self-weight} \\ &= 1.16 \text{ in.}\end{aligned}$$

$$\begin{aligned}\Delta_s &= \text{deflection due to the slab, formwork, and exterior diaphragm weight} \\ &= 1.12 \text{ in.}\end{aligned}$$

$$\begin{aligned}\Delta_{SDL} &= \text{deflection due to the superimposed dead load weight} \\ &= 0.104 \text{ in.}\end{aligned}$$

All deflection from dead load is positive (downward).

### Design Step 5.6.7.1

#### Camber to determine bridge seat elevations

Initial camber,  $C_i$ :

$$\begin{aligned}C_i &= \Delta_{P/S \text{ Tot}} + \Delta_{sw} \\ &= -2.68 + 1.16 \\ &= -1.52 \text{ in. (upward deflection)}\end{aligned}$$

Initial camber adjusted for creep,  $C_{iA}$ :

$$C_{iA} = C_i C_r$$

where:

$$C_r = \text{constant to account for creep in camber} \quad (\text{S5.4.2.3.2})$$

$$= 1.9k_{vs}k_{hc}k_fk_{td}t_i^{-0.118} \quad (\text{S5.4.2.3.2-1})$$

$$C_r = 1.436 \text{ from Section 5.4.6.1}$$

Therefore, the initial camber,  $C_{iA}$  is:

$$\begin{aligned} C_{iA} &= -1.52(1.436) \\ &= -2.18 \text{ in. (upward deflection)} \end{aligned}$$

Final camber,  $C_F$ :

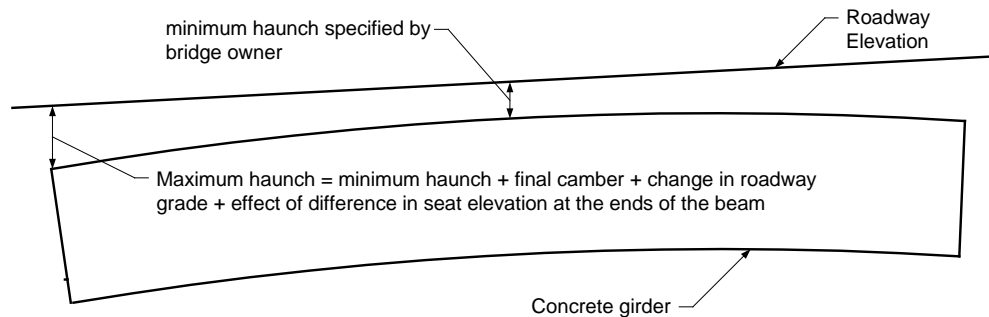
$$\begin{aligned} C_F &= C_{iA} + \Delta_s + \Delta_{SDL} \\ &= -2.18 + 1.12 + 0.104 \\ &= -0.96 \text{ in. (upward deflection)} \end{aligned}$$

This camber is used to determine bridge seat elevation.

**Design Step 5.6.7.2**

Haunch thickness

The haunch thickness is varied along the length of the girders to provide the required roadway elevation. For this example, the roadway grade is assumed to be 0.0. Therefore, the difference between the maximum haunch thickness at the support and the minimum haunch thickness at the center of the beam should equal the final camber, i.e., 0.96 in. in this example. Minimum haunch thickness is not included in the specifications and is typically specified by the bridge owner. Figure 5.6-9 shows schematically the variation in haunch thickness. Haunch thickness at intermediate points is typically calculated using a computer program.



**Figure 5.6-9 – Schematic View of Haunch**

**Design Step 5.6.7.3**Camber to determine probable sag in bridge

*To eliminate the possibility of sag in the bridge under permanent loads, some jurisdictions require that the above calculations for  $C_F$  be repeated assuming a further reduction in the initial P/S camber. The final  $C_F$  value after this reduction should show upward deflection.*

**Design Step 5.6.8 Optional live load deflection check**

*Service load deformations may cause deterioration of wearing surfaces and local cracking in concrete slabs and in metal bridges which could impair serviceability and durability, even if self limiting and not a potential source of collapse.*

*As early as 1905, attempts were made to avoid these effects by limiting the depth-to-span ratios of trusses and girders, and starting in the 1930's, live load deflection limits were prescribed for the same purpose. In a study of deflection limitations of bridges ASCE (1958), an ASCE committee, found numerous shortcomings in these traditional approaches and noted them. For example:*

*"The limited survey conducted by the Committee revealed no evidence of serious structural damage that could be attributed to excessive deflection. The few examples of damaged stringer connections or cracked concrete floors could probably be corrected more effectively by changes in design than by more restrictive limitations on deflection. On the other hand, both the historical study and the results from the survey indicate clearly that unfavorable psychological reaction to bridge deflection is probably the most frequent and important source of concern regarding the flexibility of bridges. However, those characteristics of bridge vibration which are considered objectionable by pedestrians or passengers in vehicles cannot yet be defined."*

*Since that time, there has been extensive research on human response to motion, and it is now generally agreed that the primary factor affecting human sensitivity is acceleration as opposed to deflection, velocity, or the rate of change of acceleration for bridge structures, but the problem is a difficult subjective one. Thus, to this point in history there are no simple definitive guidelines for the limits of tolerable static deflection or dynamic motion. Among current specifications, the Ontario Highway Bridge Design Code of 1983 contains the most comprehensive provisions regarding vibrations tolerable to humans.*

*The deflection criteria in S2.5.2.6.2 is considered optional. The bridge owner may select to invoke this criteria if desired. If an Owner chooses to invoke deflection control, the following principles may apply:*

- *when investigating the maximum absolute deflection for straight girder systems, all design lanes should be loaded, and all supporting components should be assumed to deflect equally,*

- *for curved steel box and I-girder systems, the deflection of each girder should be determined individually based on its response as part of a system,*
- *for composite design, the stiffness of design cross-section used for determination of deflection should include the entire width of the roadway and the structurally continuous portions of the railings, sidewalks and median barriers,*
- *for straight girder systems, the composite bending stiffness of an individual girder may be taken as the stiffness determined as specified above, divided by the number of girders,*
- *when investigating maximum relative displacements, the number and position of loaded lanes should be selected to provide the worst differential effect,*
- *the live load portion of load combination Service I of Table S3.4.1-1 should be used, including the dynamic load allowance, IM*
- *the live load shall be taken from S3.6.1.3.2,*
- *the provisions of S3.6.1.1.2 should apply,*
- *for skewed bridges, a right cross-section may be used; for curved and curved skewed bridges a radial cross-section may be used.*

*If the Owner invokes the optional live load deflection criteria, the deflection should be taken as the larger of:*

- *That resulting from the design truck alone, or*
- *That resulting from 25 percent of the design truck taken together with the design lane load.*

According to S2.5.2.6.2, the deflection criteria for vehicular live load limits deflection to L/800 (not including pedestrian loads).

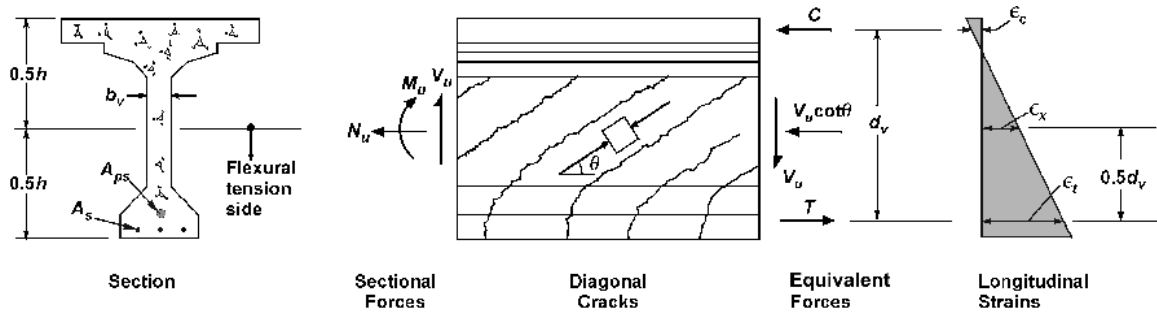
$$110(12)/800 = 1.65 \text{ in.}$$

The calculated live load deflection determined by using computer software is 0.324 in.

$$0.324 \text{ in.} < 1.65 \text{ in.} \quad \mathbf{OK}$$

Design Step 5.7 SHEAR DESIGN (S5.8)

Shear design in the AASHTO-LRFD Specifications is based on the modified compression field theory. This method takes into account the effect of the axial force on the shear behavior of the section. The angle of the shear cracking,  $\theta$ , and the shear constant,  $\beta$ , are both functions of the level of applied shear stress and the axial strain of the section. Figure S5.8.3.4.2-1 (reproduced below) illustrates the shear parameters.



**Figure S5.8.3.4.2-1 - Illustration of Shear Parameters for Section Containing at Least the Minimum Amount of Transverse Reinforcement,  $V_p = 0$ .**

The transverse reinforcement (stirrups) along the beam is shown in Figure 5.7-1. Table 5.7-1 lists the variables required to be calculated at several sections along the beam for shear analysis.

A sample calculation for shear at several sections follows the table.

*Notice that many equations contain the term  $V_p$ , the vertical component of the prestressing force. Since draped strands do not exist in the example beams, the value of  $V_p$  is taken as zero.*

Table 5.7-1 Shear Analysis at Different Sections

Dist. <sup>(1)</sup>	A <sub>ps</sub> <sup>(2)</sup>	A <sub>s</sub> <sup>(3)</sup>	CGS <sup>(4)</sup>	d <sub>e</sub> <sup>(5)</sup>	c (Rectangular behavior) <sup>(6)</sup>	c (T-section behavior) <sup>(7)</sup>	d <sub>e</sub> – β <sub>1</sub> c/2	0.9d <sub>e</sub>	d <sub>v</sub> <sup>(8)</sup>	V <sub>u</sub> <sup>(9)</sup>	V <sub>p</sub> <sup>(9,10)</sup>	v <sub>u</sub> <sup>(11)</sup>	M <sub>u</sub> <sup>(9,12)</sup>	M <sub>u</sub> /d <sub>v</sub>
(ft.)	(in <sup>2</sup> )	(in <sup>2</sup> )	(in.)	(in.)	(in.)	(in.)	(in.)	(in.)	(in.)	(kips)	(kips)	(ksi)	(kip-ft)	(kips)
7.00	4.90		5.375	74.125	3.89	N/A	72.5	66.7	72.5	340.4	0.0	0.652	2,241	370.92
11.00	5.26		5.279	74.221	4.17	N/A	72.4	66.8	72.4	315.1	0.0	0.604	3,393	562.38
16.50	5.81		5.158	74.342	4.60	N/A	72.4	66.9	72.4	280.7	0.0	0.539	4,755	788.12
22.00	5.81		5.158	74.342	4.60	N/A	72.4	66.9	72.4	246.7	0.0	0.473	5,897	977.40
27.50	6.73		5.000	74.500	5.31	N/A	72.2	67.1	72.2	213.4	0.0	0.410	6,821	1133.68
33.00	6.73		5.000	74.500	5.31	N/A	72.2	67.1	72.2	180.6	0.0	0.347	7,535	1252.35
38.50	6.73		5.000	74.500	5.31	N/A	72.2	67.1	72.2	148.3	0.0	0.285	8,063	1340.11
44.00	6.73		5.000	74.500	5.31	N/A	72.2	67.1	72.2	116.7	0.0	0.224	8,381	1392.96
49.50	6.73		5.000	74.500	5.31	N/A	72.2	67.1	72.2	85.7	0.0	0.165	8,494	1411.75
54.50	6.73		5.000	74.500	5.31	N/A	72.2	67.1	72.2	118.4	0.0	0.228	8,456	1405.43
55.00	6.73		5.000	74.500	5.31	N/A	72.2	67.1	72.2	121.3	0.0	0.233	8,440	1402.77
60.50	6.73		5.000	74.500	5.31	N/A	72.2	67.1	72.2	153.5	0.0	0.295	8,163	1356.73
66.00	6.73		5.000	74.500	5.31	N/A	72.2	67.1	72.2	185.7	0.0	0.357	7,690	1278.12
71.50	6.73		5.000	74.500	5.31	N/A	72.2	67.1	72.2	217.9	0.0	0.419	7,027	1167.92
77.00	6.73		5.000	74.500	5.31	N/A	72.2	67.1	72.2	250.0	0.0	0.481	6,180	1027.15
82.50	6.73		5.000	74.500	5.31	N/A	72.2	67.1	72.2	282.0	0.0	0.542	5,158	857.29
88.00	5.81		5.158	74.342	4.60	N/A	72.4	66.9	72.4	313.8	0.0	0.602	3,966	657.35
93.50	5.81	15.31	3.980	75.520	8.58	N/A	72.3	68.0	72.3	345.4	0.0	0.663	-393	-65.23
99.00	4.90	15.31	3.980	75.520	8.58	N/A	72.3	68.0	72.3	376.8	0.0	0.724	-1,535	-254.77
102.50	4.90	15.31	3.980	75.520	8.58	N/A	72.3	68.0	72.3	396.6	0.0	0.762	-2,489	-413.11

Table 5.7-1 Shear Analysis at Different Sections (cont'd)

Dist. <sup>(1)</sup>	A <sub>psfpo</sub> (2,13)	ε <sub>s</sub> <sup>(14)</sup>	θ <sup>(15)</sup>	β <sup>(15)</sup>	V <sub>c</sub>	S <sub>Req'd</sub> <sup>(17)</sup>	S <sub>Prov'd</sub> <sup>(17)</sup>	V <sub>s</sub> (comp.)	φV <sub>n</sub>	φV <sub>n</sub> /V <sub>u</sub>	T <sup>(16)</sup>	Nom. Flex. Res.	Nom. Flex.> T?
(ft.)	(kips)	(strain)	(deg)		(kips)	(in.)	(in.)	(kips)	(kips)		(kips)	(kips)	
7.00	926.1	0.00000	29.00	4.80	215.41	19.3	6.0	196.2	370.5	1.09	N/A	1147.1	N/A
11.00	994.1	0.00000	29.00	4.80	215.34	23.3	16.0	195.9	370.0	1.17	1017.3	1357.3	OK
16.50	1,098.1	0.00000	29.00	4.80	215.16	32.4	18.0	174.2	350.4	1.25	1193.7	1484.7	OK
22.00	1,098.1	0.00076	31.67	3.05	136.91	20.5	20.0	140.9	250.1	1.01	1307.7	1532.3	OK
27.50	1,272.0	0.00039	30.36	3.72	166.31	41.8	20.0	147.8	282.4	1.32	1412.2	1718.0	OK
33.00	1,272.0	0.00084	31.92	2.95	132.01	40.5	24.0	115.8	222.8	1.23	1481.4	1764.1	OK
38.50	1,272.0	0.00112	32.94	2.60	116.49	55.4	24.0	111.4	204.9	1.38	1508.4	1781.3	OK
44.00	1,272.0	0.00123	33.32	2.49	111.47	145.0	24.0	109.8	198.9	1.70	1506.7	1781.3	OK
49.50	1,272.0	0.00117	33.10	2.56	114.31	139.3	24.0	110.7	202.3	2.36	1472.9	1781.3	OK
54.50	1,272.0	0.00131	33.58	2.42	108.36	112.6	24.0	108.7	195.1	1.65	1521.7	1781.3	OK
55.00	1,272.0	0.00131	33.59	2.42	108.31	98.6	24.0	108.7	195.1	1.61	1523.8	1781.3	OK
60.50	1,272.0	0.00124	33.33	2.49	111.34	44.5	24.0	109.7	198.7	1.29	1532.5	1781.3	OK
66.00	1,272.0	0.00100	32.49	2.75	122.90	32.6	24.0	113.3	212.4	1.14	1513.0	1781.3	OK
71.50	1,272.0	0.00059	31.07	3.33	148.86	30.9	24.0	119.8	241.5	1.11	1470.3	1781.3	OK
77.00	1,272.0	0.00002	29.08	4.72	210.95	46.6	24.0	129.7	306.1	1.22	1409.7	1755.7	OK
82.50	1,272.0	0.00000	29.00	4.80	214.73	31.7	24.0	130.3	310.4	1.10	1305.1	1709.7	OK
88.00	1,098.1	0.00000	29.00	4.80	215.16	23.5	11.0	285.0	450.2	1.43	1029.3	1523.6	OK
93.50	1,098.1 <sup>(2)</sup>	0.00092	32.24	2.83	126.89	10.7	11.0 <sup>(18)</sup>	250.1	339.3	0.98 <sup>(18)</sup>	N/A	1476.0	N/A
99.00	926.1 <sup>(2)</sup>	0.00142	33.98	2.32	103.98	8.2	7.0	367.8	424.6	1.13	N/A	1295.5	N/A
102.50	926.1 <sup>(2)</sup>	0.00182	35.38	2.03	90.76	7.0	6.0	407.2	448.2	1.13	N/A	1106.1	N/A

Notes:

- (1) Distance measured from the centerline of the end support. Calculations for Span 1 are shown. From symmetry, Span 2 is a mirror image of Span 1.
- (2) Prestressing steel is on the compression side of the section in the negative moment region of the girder (intermediate pier region). This prestressing steel is ignored where the area of steel in an equation is defined as the area of steel on the tension side of the section.
- (3) Area of continuity reinforcement, i.e., the longitudinal reinforcement of the deck slab within the effective flange width of the girder in the girder negative moment region.
- (4) Distance from the centroid of the tension steel reinforcement to the extreme tension fiber of the section. In the positive moment region, this is the distance from the centroid of prestressing strands to the bottom of the prestressed beam. In the negative moment region, this is the distance from the centroid of the longitudinal deck slab reinforcement to the top of the structural deck slab (ignore the thickness of the integral wearing surface).
- (5) Effective depth of the section equals the distance from the centroid of the tension steel reinforcement to the extreme compression fiber of the section. In the positive moment region, this is the distance from the centroid of the prestressing strands to the top of the structural deck slab (ignore the thickness of the integral wearing surface). In the negative moment region, this is the distance from the centroid of the longitudinal deck slab reinforcement the bottom of the prestressed beam. The effective depth is calculated as the total depth of the section (which equals the depth of precast section, 72 in. + structural deck thickness, 7.5 in. = 79.5 in.) minus the quantity defined in note (4) above.
- (6) Distance from the extreme compression fiber to the neutral axis calculated assuming rectangular behavior using Eq. S5.7.3.1.1-4. Prestressing steel, effective width of slab and slab compressive strength are considered in the positive moment region. The slab longitudinal reinforcement, width of the girder bottom (compression) flange and girder concrete strength are considered in the negative moment region.
- (7) Distance from the extreme compression fiber to the neutral axis calculated assuming T-section behavior using Eq. S5.7.3.1.1-3. Only applicable if the rectangular section behavior proves untrue.



- (8) Effective depth for shear calculated using S5.8.2.9.
- (9) Maximum applied factored load effects obtained from the beam load analysis.
- (10) Vertical component of prestressing which is 0.0 for straight strands.
- (11) The applied shear stress,  $v_u$ , calculated as the applied factored shear force divided by the product of the web width,  $b$ , and effective shear depth,  $d_v$ .
- (12) Only the controlling case (positive moment or negative moment) is shown.
- (13) In the positive moment region, the parameter  $f_{po}$  is taken equal to  $0.7f_{pu}$  of the prestressing steel as allowed by S5.8.3.4.2. This value is reduced within the transfer length of the strands to account for the lack of full development.
- (14) Value of the parameter  $\epsilon_s$  calculated using Eq. S5.8.3.4.2-4. The quantity  $\epsilon_s$  is always taken as positive. Where  $\epsilon_s$  is negative, it shall be taken as 0.
- (15) The values of  $\beta$  and  $\theta$  as calculated according to the general procedure in S5.8.3.4.2.
- (16) Force in longitudinal reinforcement including the effect of the applied shear (S5.8.3.5)
- (17)  $S_{req'd}$  is the theoretical required spacing of the shear stirrup reinforcement at each analysis location, in inches.  $S_{prov'd}$  is the chosen stirrup spacing at each analysis location, in inches. In cases where the theoretical spacing exceeds 24", the maximum spacing allowed of 24" controls.
- (18) Since a crack must cross through the height of the girder, it must cross multiple stirrups in the process. At this analysis point the chosen stirrup spacing changes from 7 in to 11 in. If an average spacing of 9 in is used to investigate shear strength at this location, the capacity is computed as 389 kips, resulting in an adequate design.

### Design Step 5.7.1 Critical section for shear near the end support

*According to S5.8.3.2, where the reaction force in the direction of the applied shear introduces compression into the end region of a member, the location of the critical section for shear shall be taken as  $d_v$  from the internal face of the support. Otherwise, the design section shall be taken at the internal face of the support. Where the beam-type element extends on both sides of the reaction area, the design section on each side of the reaction shall be determined separately based upon the loads on each side of the reaction and whether their respective contribution to the total reaction introduces tension or compression into the end region.*

Therefore, the critical section near the end support is taken as  $d_v$ .

### Design Step 5.7.2 Shear analysis for a section in the positive moment region. Sample Calculations: Section $d_v$ from the centerline of the end bearing

#### Design Step 5.7.2.1

Determine the effective depth for shear,  $d_v$

- $d_v$  = effective shear depth taken as the distance, measured perpendicular to the neutral axis, between the resultants of the tensile and compressive forces due to flexure; it need not be taken to be less than the greater of  $0.9d_e$  or  $0.72h$  (S5.8.2.9)
- $h$  = total depth of beam (in.)  
= depth of the precast beam + structural slab thickness

$$= 72 + 7.5 = 79.5 \text{ in. (notice that the depth of the haunch was ignored in this calculation)}$$

$$\begin{aligned} d_e &= \text{distance from the extreme compression fiber to the center of gravity of the} \\ &\text{prestressing steel at the section (in.). From Figure 2-6,} \\ &= 79.5 - 5.375 = 74.125 \text{ in.} \end{aligned}$$

Assuming rectangular section behavior with no compression steel or mild tension reinforcement, the distance from the extreme compression fiber to the neutral axis,  $c$ , may be calculated as:

$$c = A_{ps}f_{pu}/[\alpha_1 f'_c\beta_1b + kA_{ps}(f_{pu}/d_p)] \quad (S5.7.3.1)$$

$$\alpha_1 = 0.85 \text{ for 4 ksi slab concrete (not exceeding 10 ksi) (S5.7.2.2)}$$

$$\beta_1 = 0.85 \text{ for 4 ksi slab concrete (not exceeding 4 ksi) (S5.7.2.2)}$$

$$\begin{aligned} b &= \text{effective flange width} \\ &= 116 \text{ in. (calculated in Section 2.2)} \end{aligned}$$

Area of prestressing steel that is bonded at the section under consideration (see Figure 2-5) =  $32(0.153) = 4.896 \text{ in}^2$

$$c = 4.896(270)/[0.85(4)(0.85)(116) + 0.28(4.896)(270/74.125)]$$

$$= 3.89 \text{ in.} < \text{structural slab thickness} = 7.5 \text{ in.}$$

The assumption of the section behaving as a rectangular section is correct.

Depth of compression block,  $a = \beta_1c = 0.85(3.89) = 3.30 \text{ in.}$

$$\begin{aligned} \text{Distance between the resultants of the tensile and compressive forces due to flexure:} &= \\ d_e - a/2 &= 74.125 - 3.30/2 \\ &= 72.5 \text{ in. (1)} \end{aligned}$$

$$\begin{aligned} 0.9d_e &= 0.9(74.125) \\ &= 66.71 \text{ in. (2)} \end{aligned}$$

$$\begin{aligned} 0.72h &= 0.72(79.5) \\ &= 57.24 \text{ in. (3)} \end{aligned}$$

$$d_v = \text{largest of (1), (2) and (3)} = 72.5 \text{ in.}$$

Notice that 0.72h is always less than the other two values for all sections of this beam. This value is not shown in Table 5.7-1 for clarity.

**Design Step 5.7.2.2**Minimum required transverse reinforcement

*Limits on maximum factored shear stresses for sections without transverse reinforcement are presented in S5.8.2.4. Traditionally, transverse reinforcement satisfying the minimum transverse reinforcement requirements of S5.8.2.5 is provided along the full length of the beam.*

Minimum transverse reinforcement,  $A_v$ :

$$A_v \geq 0.0316\sqrt{f'_c} \frac{b_v s}{f_y} \quad (\text{S5.8.2.5-1})$$

$f'_c$  = compressive strength of the web concrete = 6.0 ksi

$f_y$  = yield strength of the transverse reinforcement = 60 ksi

Assume that #4 bars are used for the stirrups.  $A_v$  = area of 2 legs of a #4 bar = 0.4 in<sup>2</sup>  
Substitute 0.4 in<sup>2</sup> to determine “s”, the maximum allowable spacing of #4 bars (2-leg stirrups).

$$0.4 \geq 0.0316(2.449)(8/60)s$$

$$s \leq 38.77 \text{ in.}$$

**Design Step 5.7.2.3**Maximum spacing for transverse reinforcement

The maximum spacing of transverse reinforcement is determined in accordance with S5.8.2.7. Depending on the level of applied factored shear stress,  $v_u$ , the maximum permitted spacing,  $s_{\max}$ , is determined as:

- If  $v_u < 0.125f'_c$ , then:

$$s_{\max} = 0.8d_v < 24.0 \text{ in.} \quad (\text{S5.8.2.7-1})$$

- If  $v_u \geq 0.125f'_c$ , then:

$$s_{\max} = 0.4d_v < 12.0 \text{ in.} \quad (\text{S5.8.2.7-2})$$

For the section under consideration,  $v_u$  is determined using Equation S5.8.2.9-1.  $v_u = 0.653 \text{ ksi} = 0.109f'_c$ . Therefore, the maximum permitted spacing,

$$\begin{aligned}
 s_{\max} &= 0.8d_v \\
 &= 0.8(72.5) \\
 &= 58.0 \text{ in.} > 24.0 \text{ in. Use maximum permitted stirrup spacing of 24 in.}
 \end{aligned}$$

#### Design Step 5.7.2.4

##### Shear strength

The shear strength provided by the concrete,  $V_c$ , is calculated using the following equation:

$$V_c = 0.0316\beta\sqrt{f'_c} b_v d_v \quad (\text{S5.8.3.3-3})$$

The values of  $\beta$  and the shear cracking inclination angle,  $\theta$ , are determined using the procedure outlined in S5.8.3.4.2.

The parameter  $\varepsilon_s$  is the net longitudinal tensile strain in the section at the centroid of the tension reinforcement. For sections containing at least the minimum transverse reinforcement calculated above,  $\varepsilon_s$  may be calculated using the following equations:

$$\varepsilon_s = \frac{\left( \frac{|M_u|}{d_v} + 0.5N_u + |V_u - V_p| - A_{ps}f_{po} \right)}{E_s A_s + E_p A_{ps}} \quad (\text{S5.8.3.4.2-4})$$

If the value of  $\varepsilon_s$  from Eq. S5.8.3.4.2-4 is negative, the strain may be recomputed or conservatively taken as 0. For this example, the simplification is used and negative strains will be taken as 0.

For this example, the value of both the applied factored axial load,  $N_u$ , and the vertical component of prestressing,  $V_p$ , are taken equal to 0.

For the section under consideration:

$$V_u = \text{maximum applied factored shear} = 340.4 \text{ kips}$$

$$M_u = \text{maximum factored moment at the section} = 2,241 \text{ k-ft, which must be } \geq V_u d_v = (340.4)(72.5)/12 = 2,057 \text{ k-ft. Use } M_u = 2,241 \text{ k-ft}$$

Notice that the maximum live load moment and the maximum live load shear at any section are likely to result from two different locations of the load along the length of the bridge. Conducting the shear analysis using the maximum factored shear and the concurrent factored moment is permitted. However, most computer programs list the maximum values of the moment and the maximum value of the shear without listing the concurrent forces. Therefore, hand calculations and most design computer programs typically conduct shear analysis using the maximum moment value instead of the moment concurrent with the maximum shear. This results in a conservative answer.

According to S5.8.3.4.2,  $f_{po}$  is defined as follows:

$f_{po}$  = a parameter taken as the modulus of elasticity of the prestressing tendons multiplied by the locked-in difference in strain between the prestressing tendons and the surrounding concrete (ksi). For the usual levels of prestressing, a value of  $0.7f_{pu}$  will be appropriate for both pretensioned and posttensioned members.

For pretensioned members, multiplying the modulus of elasticity of the prestressing tendons by the locked in difference in strain between the prestressing tendons and the surrounding concrete yields the stress in the strands when the concrete is poured around them, i.e., the stress in the strands immediately prior to transfer. For pretensioned members, SC5.8.3.4.2 allows  $f_{po}$  to be taken equal to the jacking stress. This value is typically larger than  $0.7f_{pu}$ . Therefore, using  $0.7f_{pu}$  is more conservative since it results in a larger value of  $\epsilon_s$ .

For this example,  $f_{po}$  is taken as  $0.7f_{pu}$ .

$$f_{po} = 0.7(270) = 189 \text{ ksi} \quad (\text{SC5.8.3.4.2})$$

$$N_u = 0.0 \text{ k}$$

Notice that, as required by Article S5.8.3.4.2, within the transfer length,  $f_{po}$  shall be increased linearly from zero at the location where the bond between the strands and concrete commences to its full value at the end of the transfer length.

$A_s$ ,  $E_s$ ,  $A_{ps}$  and  $E_{ps}$  are the area of mild tension reinforcement (0.0), modulus of elasticity of mild reinforcement (29,000 ksi), area of prestressing steel (4.896 in<sup>2</sup>) and modulus of elasticity of the prestressing strands (28,500 ksi), respectively.

$$\epsilon_s = \frac{\left( \frac{2241}{6.04} + 0.5(0.0) + 340.4 - 4.896(189) \right)}{28500(4.896)} = -0.001533 < 0$$

Therefore, use  $\epsilon_s = 0$ .

The value of  $\theta$  is found using S5.8.3.4.2-3:

$$\theta = 29 + 3500\epsilon_s = 29 + 0 = 29^\circ$$

$$\begin{aligned} A_{ps} &= \text{area of prestressed steel at the section on the flexural side of the member} \\ &= 32(0.153) \\ &= 4.896 \text{ in}^2 \end{aligned}$$

$$d_v = 72.5 \text{ in. (6.04 ft.)}$$

The value of  $\beta$  is found using S5.8.3.4.2-1:

$$\beta = \frac{4.8}{1 + 750\varepsilon_s} = 4.8 \quad (\text{S5.8.3.4.2-1})$$

Calculate the shear resistance provided by the concrete,  $V_c$ .

$$V_c = 0.0316\beta\sqrt{f'_c}b_vd_v \quad (\text{S5.8.3.3-3})$$

$$V_c = 0.0316(4.8)(2.449)(8)(72.5) = 215.5 \text{ k}$$

Calculate the shear resistance provided by the transverse reinforcement (stirrups),  $V_s$ .

$$V_s = [A_vf_yd_v(\cot \theta + \cot \alpha)\sin \alpha]/s \quad (\text{S5.8.3.3-4})$$

Assuming the stirrups are placed perpendicular to the beam longitudinal axis at 16 in. spacing and are comprised of #4 bars, each having two legs:

$$\begin{aligned} A_v &= \text{area of shear reinforcement within a distance "s" (in}^2\text{)} \\ &= 2(\text{area of \#4 bar}) \\ &= 2(0.2) \\ &= 0.4 \text{ in}^2 \end{aligned}$$

$$s = 16 \text{ in.}$$

$$\begin{aligned} \alpha &= \text{angle between the stirrups and the longitudinal axis of the beam} \\ &= 90 \text{ degrees} \end{aligned}$$

$$V_s = [0.4(60)(72.5)(\cot 29.0)]/16 = 196.2 \text{ k}$$

The nominal shear resistance,  $V_n$ , is determined as the lesser of:

$$V_n = V_c + V_s + V_p \quad (\text{S5.8.3.3-1})$$

$$V_n = 0.25f'_cb_vd_v + V_p \quad (\text{S5.8.3.3-2})$$

Notice that the purpose of the limit imposed by Eq. S5.8.3.3-2 is to ensure that the concrete in the web will not crush prior to yield of the transverse reinforcement.

$$V_p = 0.0 \text{ for straight strands}$$

$$V_n = \text{lesser of:}$$

$$\begin{aligned} V_c + V_s + V_p &= 215.5 + 196.2 + 0.0 = 411.7 \text{ k} \\ \text{and} \end{aligned}$$

$$0.25f'_c b_v d_v + V_p = 0.25(6)(8)(72.5) + 0.0 = 870.0 \text{ k}$$

Therefore,  $V_n = 411.7 \text{ k}$

The resistance factor,  $\phi$ , for shear in normal weight concrete is 0.9. (S5.5.4.2.1)

Shear factored resistance,  $V_r$ :

$$\begin{aligned} V_r &= \phi V_n && \text{(S5.8.2.1-2)} \\ &= 0.9(411.7) \\ &= 370.5 \text{ k} > \text{maximum applied factored shear, } V_u = 340.4 \text{ k} \quad \mathbf{OK} \end{aligned}$$

Check if transverse reinforcement is required (S5.8.2.4):

Transverse reinforcement shall be provided in prestressed concrete beams where:

$$V_u > 0.5\phi(V_c + V_p)$$

All of the terms have been defined above, therefore:

$$0.5(0.9)(215.5 + 0) = 97.0 \text{ k} < 340.4 \text{ k, transverse reinforcement is required}$$

### Design Step 5.7.3 Shear analysis for sections in the negative moment region

The critical section for shear near the intermediate pier may be determined using the same procedure as shown in Design Steps 5.7.1 and 5.7.2 for a section near the end support. Calculations for a section in the negative moment region are illustrated below for the section at 99 ft. from the centerline of the end bearing. This section is not the critical section for shear and is used only for illustrating the design process.

#### Sample Calculations: Section 99 ft. from the centerline of end bearings

##### Design Step 5.7.3.1

##### Difference in shear analysis in the positive and negative moment regions

1) *For the pier (negative moment) regions of precast simple span beams made continuous for live load, the prestressing steel near the piers is often in the compression side of the beam. The term  $A_{ps}$  in the equations for  $\epsilon_s$  is defined as the area of prestressing steel on the tension side of the member. Since the prestressing steel is on the compression side of the member, this steel is ignored in the analysis. This results in an increase in  $\epsilon_x$  and, therefore, a decrease in the shear resistance of the section. This approach gives conservative results and is appropriate for hand calculations.*

*A less conservative approach is to calculate  $\varepsilon_s$  as the average longitudinal strain in the web. This requires the calculation of the strain at the top and bottom of the member at the section under consideration at the strength limit state. This approach is more appropriate for computer programs.*

*The difference between the two approaches is insignificant in terms of the cost of the beam. The first approach requires more shear reinforcement near the ends of the beam. The spacing of the stirrups in the middle portion of the beam is often controlled by the maximum spacing requirements and, hence, the same stirrup spacing is often required by both approaches.*

*It is beneficial to use the second approach in the following situations:*

- *Heavily loaded girders where the first approach results in congested shear reinforcement*
- *Analysis of existing structures where the first approach indicates a deficiency in shear resistance.*

2) *In calculating the distance from the neutral axis to the extreme compression fiber “c”, the following factors need to be considered:*

- *The compression side is at the bottom of the beam. The concrete strength used to determine “c” is that of the precast girder*
- *The width of the bottom flange of the beam is substituted for “b”, the width of the member*
- *The area of the slab longitudinal reinforcement over the intermediate pier represents the reinforcement on the tension side of the member. The area and yield strength of this reinforcement should be determined in advance.*

The first approach is used in this example.

### Design Step 5.7.3.2

Determine the effective depth for shear,  $d_v$

$$h = 72 + 7.5 = 79.5 \text{ in.} \quad (\text{notice that the depth of the haunch was ignored in this calculation})$$

The center of gravity of the deck slab longitudinal reinforcement from the top of the structural thickness of the deck = 3.98 in. (see Design Step 5.6.5.1)

$$d_e = 79.5 - 3.98 = 75.52 \text{ in.}$$



The area of longitudinal slab reinforcement within the effective flange width of the beam is 15.31 in<sup>2</sup> (see Design Step 5.6.5.1)

Yield strength of the slab reinforcement = 60 ksi

Assuming rectangular section behavior with no compression or prestressing steel, the distance from the extreme compression fiber to the neutral axis,  $c$ , may be calculated as:

$$c = A_s f_y / (\alpha_1 f'_c \beta_1 b) \quad (\text{S5.7.3.1.1-4})$$

where:

$$\alpha_1 = 0.85 \text{ for 6 ksi beam concrete (S5.7.2.2)}$$

$$\beta_1 = 0.75 \text{ for 6 ksi beam concrete (S5.7.2.2)}$$

$$b = \text{precast beam bottom (compression) flange width (in.)} \\ = 28 \text{ in.}$$

$$f'_c = 6 \text{ ksi}$$

$$c = 15.31(60) / [0.85(6)(0.75)(28)] \\ = 8.58 \text{ in. } \approx \text{thickness of the beam bottom flange (8 in.)}$$

Therefore, the assumption of the section behaving as a rectangular section is considered correct.

*Notice that if the value of “c” is significantly larger than the beam bottom flange thickness, a rectangular behavior may be used after adjusting the beam bottom flange width to account for the actual beam area in compression. However, if “c” is not significantly larger than the beam bottom flange thickness, the effect on the results will be minor and the analysis may be continued without adjusting the beam bottom flange width. This reasoning is used in this example.*

Depth of compression block,  $a = \beta_1 c = 0.75(8.58) = 6.44 \text{ in.}$

$$\text{Distance between the resultants of the tensile and compressive forces due to flexure:} = d_e - a/2 \\ = 75.52 - 6.44/2 \\ = 72.30 \text{ in.} \quad (1)$$

$$0.9d_e = 0.9(75.52) \\ = 67.97 \text{ in.} \quad (2)$$

$$0.72h = 0.72(79.5) \\ = 57.24 \text{ in.} \quad (3)$$

$$d_v = \text{largest of (1), (2) and (3)} = 72.30 \text{ in.}$$

Notice that 0.72h is always less than the other two values for all sections of this beam. This value is not shown in Table 5.7-1 for clarity.

**Design Step 5.7.3.3**Shear stress on concrete

From Table 5.3-4, the factored shear stress at this section,  $V_u = 376.8$  kips

$$\phi = 0.9 \text{ (shear)} \quad (\text{S5.5.4.2.1})$$

$$b_v = \text{width of web} = 8 \text{ in.}$$

From Article S5.8.2.9, the shear stress on the concrete is:

$$v_u = (V_u - \phi V_p) / (\phi b_v d_v)$$

$$v_u = (376.8 - 0) / [0.9(8)(72.30)] = 0.724 \text{ ksi}$$

$$v_u / f'_c = 0.724 / 6.0 = 0.1207$$

**Design Step 5.7.3.4**Minimum required transverse reinforcement

Maximum allowable spacing for #4 stirrups with two legs per stirrup was calculated in Design Step 5.7.2.2.

$$s \leq 38.77 \text{ in.}$$

**Design Step 5.7.3.5**Maximum spacing for transverse reinforcement

The maximum spacing of transverse reinforcement is determined in accordance with S5.8.2.7. Depending on the level of applied factored shear stress,  $v_u$ , the maximum permitted spacing,  $s_{max}$ , is determined as:

- If  $v_u < 0.125f'_c$ , then:

$$s_{max} = 0.8d_v < 24.0 \text{ in.} \quad (\text{S5.8.2.7-1})$$

- If  $v_u \geq 0.125f'_c$ , then:

$$s_{max} = 0.4d_v < 12.0 \text{ in.} \quad (\text{S5.8.2.7-2})$$

For the section under consideration,  $v_u = 0.1207f'_c$ .

Therefore, the maximum permitted spacing,

$$\begin{aligned} s_{\max} &= 0.8d_v \\ &= 0.8(72.30) \\ &= 57.84 \text{ in.} > 24.0 \text{ in.} \quad \mathbf{NG} \end{aligned}$$

Assume maximum permitted stirrup spacing = 24 in.

### Design Step 5.7.3.6

#### Shear resistance

For sections in the negative moment region of the beam, calculate  $\epsilon_s$ , using Eq. S5.8.3.4.2-4 and assume there is no prestressing steel.

$$\epsilon_s = \frac{\left( \frac{|M_u|}{d_v} + 0.5N_u + |V_u - V_p| - A_{ps}f_{po} \right)}{(E_s A_s + E_p A_{ps})} \quad (\text{S5.8.3.4.2-4})$$

For this example, the value of both the applied factored axial load,  $N_u$ , and the vertical component of prestressing,  $V_p$ , are taken equal to 0.

$$\begin{aligned} V_u &= \text{maximum applied factored shear from Table 5.3-4} \\ &= 376.8 \text{ kips} \end{aligned}$$

$$\begin{aligned} M_u &= \text{maximum applied factored moment from Table 5.3-2} \\ &= -1,535 \text{ k-ft} \end{aligned}$$

Notice that the term  $M_u/d_v$  represents the force in the tension reinforcement due to the applied factored moment. Therefore,  $M_u/d_v$  is taken as a positive value regardless of the sign of the moment.

$$f_{po} = 0.0 \text{ ksi at this location (prestressing force ignored)}$$

$$\begin{aligned} A_s &= \text{area of longitudinal reinforcement in the deck at this section} \\ &= 15.31 \text{ in}^2 \end{aligned}$$

Notice that the area of deck longitudinal reinforcement used in this calculation is the area of the bars that extend at least one development length beyond the section under consideration. If the section lies within the development length of some bars, these bars may be conservatively ignored or the force in these bars be prorated based on the ratio between the full and available development length. Consideration should also be given to adjusting the location of the center of

*gravity of the reinforcement to account for the smaller force in the bars that are not fully developed.*

$$d_v = 72.30 \text{ in. (6.03 ft.)}$$

$$E_s = 29,000 \text{ ksi}$$

$$E_{ps} = 28,500 \text{ ksi}$$

$$A_{ps} = \text{area of prestressing steel on the } \underline{\text{tension side}} \text{ of the member} \\ = 0.0 \text{ in}^2$$

Substitute these variables in Eq. S5.8.3.4.2-4 to determine  $\epsilon_s$ :

$$\epsilon_s = [1,535(12)/72.30 + (376.8 - 0) - 0]/[(29,000)(15.31) + 0] \\ = 0.001422$$

Determine the values of  $\theta$  and  $\beta$  using Equations S5.8.3.4.2-1 and S5.8.3.4.2-3.

$$\beta = 4.8/[1+750(0.001422)] = 2.32 \quad (\text{S5.8.3.4.2-1})$$

$$\theta = 29 + 3500(0.001422) = 33.98^\circ \quad (\text{S5.8.3.4.2-3})$$

Calculate the shear resistance provided by concrete,  $V_c$ :

$$V_c = 0.0316\beta\sqrt{f'_c}b_v d_v \quad (\text{S5.8.3.3-3})$$

$$V_c = 0.0316(2.32)(2.449)(8)(72.30) = 103.9 \text{ k}$$

Calculate the shear resistance provided by the transverse reinforcement (stirrups),  $V_s$ :

$$V_s = [A_v f_y d_v (\cot \theta + \cot \alpha) \sin \alpha]/s \quad (\text{S5.8.3.3-4})$$

Assuming the stirrups are placed perpendicular to the beam longitudinal axis at 7 in. spacing and are comprised of #4 bars, each having two legs:

$$A_v = 2(\text{area of \#4 bar}) \\ = 2(0.2) \\ = 0.4 \text{ in}^2$$

$$s = 7 \text{ in.}$$

$$\alpha = 90 \text{ degrees}$$

$$V_s = 0.4(60)(72.30)(\cot 33.98)/7 = 367.8 \text{ k}$$

The nominal shear resistance,  $V_n$ , is determined as the lesser of:

$$V_n = V_c + V_s + V_p \quad (\text{S5.8.3.3-1})$$

$$V_n = 0.25f'_c b_v d_v + V_p \quad (\text{S5.8.3.3-2})$$

Notice that the purpose of the limit imposed by Eq. S5.8.3.3-2 is intended to eliminate excessive shear cracking.

$$V_p = 0.0 \text{ for straight strands}$$

$V_n$  is taken as the lesser of:

$$V_c + V_s + V_p = 103.9 + 367.8 + 0.0 = 471.7 \text{ k}$$

and

$$0.25f'_c b_v d_v + V_p = 0.25(6)(8)(72.30) + 0.0 = 867.6 \text{ k}$$

Therefore,  $V_n = 471.7 \text{ k}$

The resistance factor,  $\phi$ , for shear in normal weight concrete = 0.90 (S5.5.4.2.1)

Factored shear resistance:

$$\begin{aligned} V_r &= \phi V_n \\ &= 0.9(471.7) \\ &= 424.6 \text{ k} > \text{max. applied factored shear, } V_u = 376.8 \text{ k} \quad \mathbf{OK} \end{aligned}$$

#### Design Step 5.7.4 Factored splitting resistance (S5.10.10.1)

*The bursting resistance of the pretensioned anchorage zones is calculated according to S5.10.10.1 at the service limit state.*

$$P_r = f_s A_s \quad (\text{S5.10.10.1-1})$$

where:

$$f_s = \text{stress in the steel not exceeding 20 ksi}$$

$$A_s = \text{total area of vertical reinforcement located within the distance } h/4 \text{ from the end of the beam (in}^2\text{)}$$

$$H = \text{overall depth of the precast member (in.)}$$

*The resistance shall not be less than 4% of the prestressing force at transfer.*

From Design Step 5.4.4:

$$\begin{aligned} \text{Prestressing force at transfer at end of beam} &= 32(0.153)(188.8) \\ &= 924.4 \text{ kips} \end{aligned}$$

Determine the required area of steel to meet the minimum resistance using  $f_s = 20$  ksi (max).

Therefore,

$$\begin{aligned} 0.04(924.4) &= 20(A_s) \\ A_s &= 1.85 \text{ in}^2 \end{aligned}$$

Since one stirrup is  $0.4 \text{ in}^2$  (includes 2 legs), determine the number of stirrups required.

$$1.85/0.4 = 4.63 \quad \text{Say 5 stirrups required}$$

These stirrups must fit within  $h/4$  distance from the end of the beam.

$$\begin{aligned} h/4 &= 72/4 \\ &= 18 \text{ in.} \end{aligned}$$

Use 5 stirrups at 3 in. spacing as shown in Figure 5.7-1.

### Design Step 5.7.5 Confinement reinforcement (S5.10.10.2)

*For the distance of  $1.5d$  [ $1.5(72/12) = 9 \text{ ft.}$ ] from the end of the beam, reinforcement shall be placed to confine the prestressing steel in the bottom flange. The reinforcement is required to be not less than No. 3 deformed bars, with spacing not exceeding 6.0 in. and shaped to enclose the strands. The stirrups required to resist the applied shear and to satisfy the maximum stirrup requirements are listed in Table 5.7-1 for different sections. The maximum required spacings shown in Table 5.7-1 in the end zones of the beam are greater than 6 in. For a beam where all strands are located in the bottom flange, two different approaches may be utilized to provide the required confinement reinforcement:*

- 1) *Reduce the stirrup spacing in the end zone ( $1.5d$ ) to not greater than 6 in.*
- 2) *Place the main vertical bars of the stirrups at the spacing required by vertical shear analysis. Detail the vertical bars in the bottom of the beam to enclose the prestressing and place these bars at a spacing not greater than 6 in. within the end zones. The stirrups and the confinement bars in this approach will not be at the same spacing and pouring of the concrete may be difficult.*

*For a beam where some strands are located in the web approach (1) should be used.*

For this example, approach (1) was used. This is the basis for the stirrup distribution shown in Figure 5.7-1.

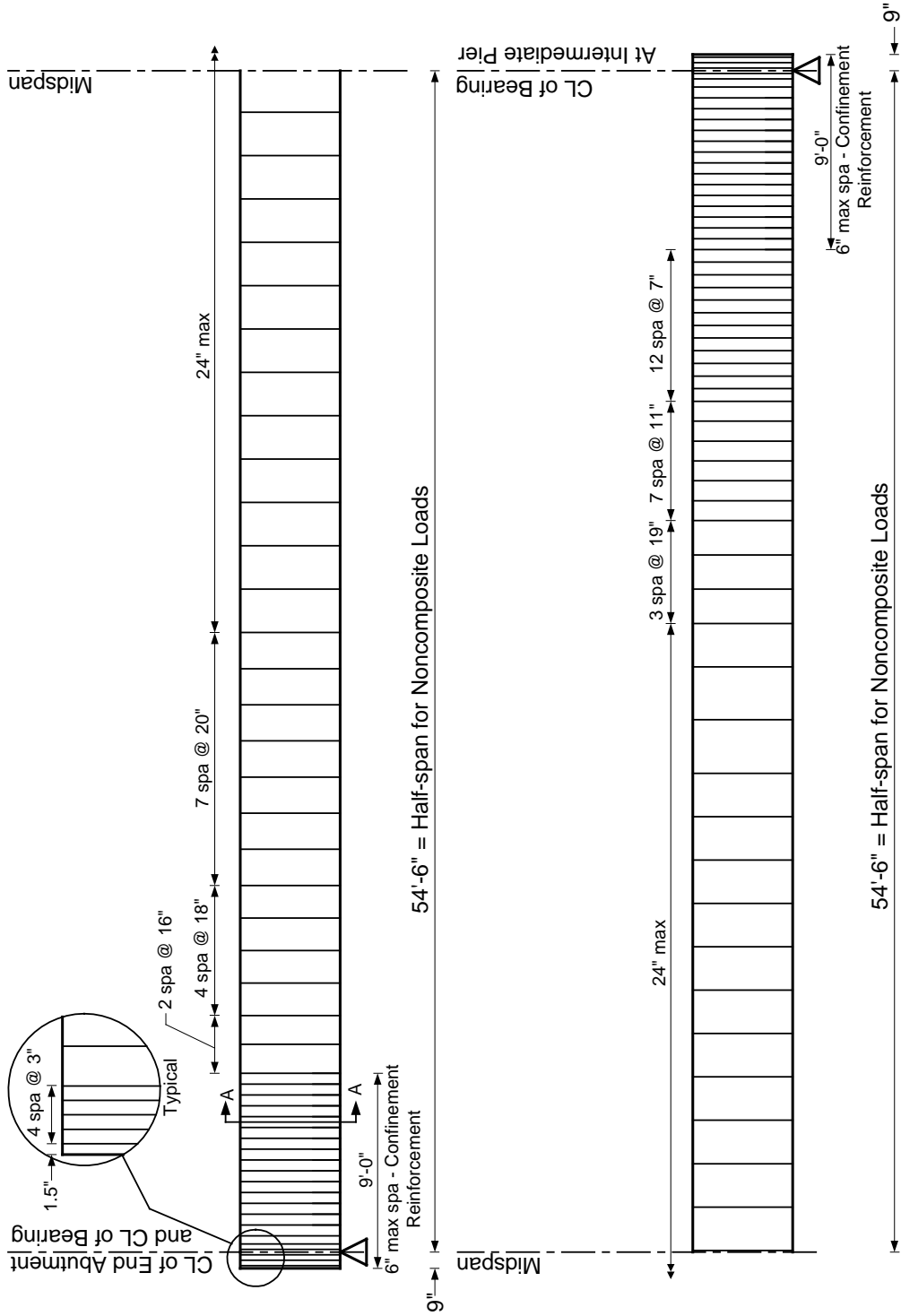


Figure 5.7-1 – Beam Transverse Reinforcement

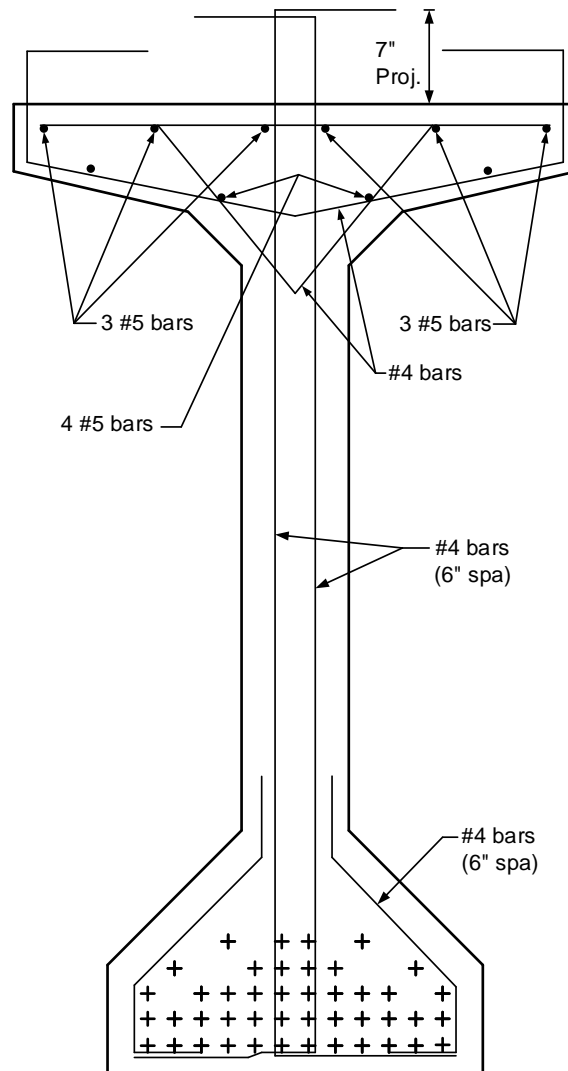


Figure 5.7-2 – Section A-A from Figure 5.7-1, Beam Cross Section Near the Girder Ends

**Design Step 5.7.6 Force in the longitudinal reinforcement including the effect of the applied shear (S5.8.3.5)**

*In addition to the applied moment,  $M_u$ , the following force effects contribute to the force in the longitudinal reinforcement:*

- *Applied shear forces,  $V_u$*
- *Vertical component of the prestressing force*
- *Applied axial force,  $N_u$*
- *The shear force resisted by the transverse reinforcement,  $V_s$*



To account for the effect of these force effects on the force in the longitudinal reinforcement, S5.8.3.5 requires that the longitudinal reinforcement be proportioned so that at each section, the tensile capacity of the reinforcement on the flexural tension side of the member, taking into account any lack of full development of that reinforcement, is greater than or equal to the force ( $A_{ps}f_{ps} + A_s f_y$ ) calculated as:

$$A_{ps}f_{ps} + A_s f_y = \frac{|M_u|}{d_v \phi_v} + 0.5 \frac{N_u}{\phi_c} + \left( \left| \frac{V_u}{\phi_v} - V_p \right| - 0.5V_s \right) \cot \theta \quad (S5.8.3.5-1)$$

where:

$V_s$  = shear resistance provided by the transverse reinforcement at the section under investigation as given by Eq. S5.8.3.3-4, except  $V_s$  shall not be taken as greater than  $V_u/\phi$  (kips)

$\theta$  = angle of inclination of diagonal compressive stresses used in determining the nominal shear resistance of the section under investigation as determined by S5.8.3.4 (degrees)

$\phi$  = resistance factors taken from S5.5.4.2 as appropriate for moment, shear, and axial resistance

This check is required for sections located no less than a distance equal to  $d_v \cot \theta$  from the support. The values for the critical section for shear near the end support are substituted for  $d_v$  and  $\theta$ .

$$0.5(72.5) \cot 29.00 = 65.40 \text{ in.} \approx 5.45 \text{ ft.}$$

The check for tension in the longitudinal reinforcement must be performed for all sections no closer than 5.45 ft. from the support.

Sample calculation: Section at 11.0 ft. from the centerline of bearing at the end support

Using information from Table 5.7-1:

Force in the longitudinal reinforcement at nominal flexural resistance, ( $A_{ps}f_{ps} + A_s f_y$ )

$$\begin{aligned} (A_{ps}f_{ps} + A_s f_y) &= \frac{3,393(12)}{[72.40(1.0)]} + 0 + \left[ \left| \frac{315.1}{0.9} - 0 \right| - 0.5(195.9) \right] \cot 29.00 \\ &= 1017.3 \text{ kips} \end{aligned}$$

From Table 5.5-1, the maximum strand resistance at this section at the nominal moment resistance is 1,295.5 kips > ( $A_{ps}f_{ps} + A_s f_y$ ) = 1017.3 kips **OK**

## Design Step 5.7.7 Horizontal shear between the beam and slab

Table 5.7-2 - Interface Shear Calculations

Dist.	$d_v$	$V_1 = V_u$	Max. Stirrup Spacing	Interface Reinf., $A_{vf}$	Horiz. Shear, $V_h$	Nominal Resistanc e	Factored Resistanc e	Resist./ Applied Load
(ft.)	(in.)	(kips)	(in.)	(in <sup>2</sup> /in.)	(k/in.)	(k/in.)	(k/in.)	> 1.0 OK
7.00	70.4	340.4	16.0	0.050	4.84	14.8	13.3	OK
11.00	70.5	315.1	18.0	0.044	4.47	14.4	13.0	OK
16.50	70.6	280.7	21.0	0.038	3.98	14.0	12.6	OK
22.00	70.6	246.7	20.0	0.040	3.49	14.2	12.7	OK
27.50	70.8	213.4	24.0	0.033	3.02	13.7	12.4	OK
33.00	70.8	180.6	24.0	0.033	2.55	13.7	12.4	OK
38.50	70.8	148.3	24.0	0.033	2.10	13.7	12.4	OK
44.00	70.8	116.7	24.0	0.033	1.65	13.7	12.4	OK
49.50	70.8	85.7	24.0	0.033	1.21	13.7	12.4	OK
54.50	70.8	118.4	24.0	0.033	1.67	13.7	12.4	OK
55.00	70.8	121.3	24.0	0.033	1.71	13.7	12.4	OK
60.50	70.8	153.5	24.0	0.033	2.17	13.7	12.4	OK
66.00	70.8	185.7	24.0	0.033	2.62	13.7	12.4	OK
71.50	70.8	217.9	24.0	0.033	3.08	13.7	12.4	OK
77.00	70.8	250.0	24.0	0.033	3.53	13.7	12.4	OK
82.50	70.8	282.0	21.0	0.033	3.99	13.7	12.4	OK
88.00	70.6	313.8	19.0	0.042	4.45	14.3	12.9	OK
93.50	75.5	345.4	11.0	0.073	4.57	16.1	14.5	OK
99.00	75.5	376.8	8.0	0.100	4.99	17.8	16.0	OK
102.50	75.5	396.6	7.0	0.114	5.25	18.6	16.7	OK

Sample calculations at 11 ft. from the centerline of bearing on the abutment (11 ft. – 9 in. from girder end)

Horizontal shear forces develop along the interface between the concrete girders and the deck. As an alternative to the classical elastic strength of materials approach, the value of these forces per unit length of the girders at the strength limit state can be taken as:

$$V_{hi} = V_1/d_v \quad (\text{SC5.8.4.2-7})$$

where:

$V_{hi}$  = factored horizontal shear per unit length of the girder (kips/length)

$V_1$  = the factored vertical shear (kips)  
= 315.1 k (From Table 5.7-2)

$$\begin{aligned}
 d_v &= \text{distance between the centroid of the tension steel and the mid-thickness of the} \\
 &\quad \text{slab to compute a factored interface shear (in.)} \\
 &= 72 + 7.5/2 - 5.279 = 70.47 \text{ in. (see Table 5.7-2)}
 \end{aligned}$$

$$\begin{aligned}
 V_{hi} &= 315.1/70.47 \\
 &= 4.47 \text{ k/in.}
 \end{aligned}$$

Stirrup spacing at this location = 18 in.

Assume that the stirrups extend into the deck. In addition, assume that there is another #4 bar with two legs extending into the deck as shown in Figure 5.7-2.

Area of reinforcement passing through the interface between the deck and the girder,  $A_{vf}$

$$\begin{aligned}
 A_{vf} &= 4 \text{ #4 bars} \\
 &= 4(0.2) \\
 &= 0.8 \text{ in}^2
 \end{aligned}$$

$A_{vf}$  per unit length of beam =  $0.8/18 = 0.044 \text{ in}^2/\text{in.}$  of beam length.

#### Check if the minimum interface shear reinforcement may be waived (S5.8.4.4)

For cast-in-place concrete decks on clean concrete girder surfaces free of laitance, the minimum interface shear reinforcement requirements may be waived provided that the beam top flange surface is roughened to an amplitude of 0.25 inches where the interface stress  $v_{ui}$  (Eq. S5.8.4.2-1) is less than 0.210 ksi and all transverse shear reinforcement required by S5.8.2.5 is extended across the beam/slab interface and adequately anchored in the slab.

$$\begin{aligned}
 \text{Shear stress on the interface} &= V_{hi}/\text{Area of interface per unit length} \\
 &= 4.47/42 \\
 &= 0.106 \text{ ksi} < 0.210 \text{ ksi}
 \end{aligned}$$

Therefore, the minimum reinforcement requirement may be waived.

*Note that the interface shear stress is only approximately half of the limiting value of 0.210 ksi. In typical design, the minimum shear interface reinforcement requirement would be waived. For this example, in order to provide a complete reference, the minimum reinforcement requirement will not be waived, and calculation will continue.*

#### Check the minimum interface shear reinforcement

$$\begin{aligned}
 A_{vf} &\geq 0.05A_{cv}/f_y = 0.05b_v/f_y && \text{(S5.8.4.4-1)} \\
 &= 0.05(42)/60
 \end{aligned}$$

$$= 0.035 \text{ in}^2/\text{in. of beam length} < A_s \text{ provided } \mathbf{OK}$$

Note that the minimum interface shear reinforcement,  $A_{vf}$ , also need not exceed the lesser of the amount determined using Eq. S5.8.4.4-1 ( $0.035 \text{ in}^2/\text{in}$ ) and the amount needed to resist  $1.33V_{ui}/\phi$  as determined using Eq. S5.8.4.1-3.

#### Factored interface shear resistance (S5.8.4.1)

The interface shear resistance of the interface has two components. The first component is due to the adhesion between the two surfaces. The second component is due to the friction. In calculating friction, the force acting on the interface is taken equal to the compression force on the interface plus the yield strength of the reinforcement passing through the interface. The nominal shear resistance of the interface plane,  $V_{ni}$ , is calculated using S5.8.4.1.

$$V_{ni} = cA_{cv} + \mu(A_{vf}f_y + P_c) \quad (\text{S5.8.4.1-1})$$

where:

$V_{ni}$  = nominal shear friction resistance (kips)

$A_{cv}$  = area of concrete engaged in shear transfer ( $\text{in}^2$ )

$A_{vf}$  = area of shear reinforcement crossing the shear plane ( $\text{in}^2$ )

$f_y$  = yield strength of reinforcement (ksi)  
(The strength of reinforcing steel crossing the interface shall not be taken as greater than 60.0 ksi)

$c$  = cohesion factor specified in S5.8.4.3 (ksi)

$\mu$  = friction factor specified in S5.8.4.3

$P_c$  = permanent net compressive force normal to the shear plane (kips)  
(If force is tensile,  $P_c = 0.0 \text{ k}$ )

Calculate the nominal shear resistance per unit length of beam.

For cast-in-place concrete slab on clean concrete girder surfaces, free of laitance with surface roughened to an amplitude of 0.25 in.:

$$c = 0.28 \text{ ksi}, \mu = 1.0, K_1 = 0.3, K_2 = 1.8 \text{ ksi} \quad (\text{S5.8.4.3}) \quad (\text{S5.8.4.3})$$

Ignore compression on the interface from loads on the deck:  $P_c = 0.0$

$$A_{cv} = 42 \text{ in}^2/\text{in. of beam length}$$

$$A_{vf} = 0.044 \text{ in}^2/\text{in. of beam length}$$

$$f_y = 60 \text{ ksi}$$

Therefore,

$$\begin{aligned} V_{ni} &= 0.28(42) + 1.0[0.044(60) + 0.0] \\ &= 14.4 \text{ k/in. of beam length} \end{aligned}$$

According to S5.8.4.1, the nominal shear resistance,  $V_{ni}$ , used in the design must also satisfy:

$$V_{ni} \leq K_1 f'_c A_{cv} \quad (\text{S5.8.4.1-4})$$

OR

$$V_{ni} \leq K_2 A_{cv} \quad (\text{S5.8.4.1-5})$$

where:

$$\begin{aligned} f'_c &= \text{the specified 28-day compressive strength of the weaker concrete on} \\ &\quad \text{either side of the interface (ksi)} \\ &= 4.0 \text{ ksi for slab concrete} \end{aligned}$$

$$V_{ni} \leq 0.3 f'_c A_{cv} = 0.3(4.0)(42) = 50.4 \text{ k/in. of beam length}$$

OR

$$V_{ni} \leq 1.8 A_{cv} = 1.8(42) = 75.6 \text{ k/in. of beam length}$$

Therefore,  $V_{ni}$  used for design = 14.8 k/in. of beam length.

$$\begin{aligned} V_{ri} &= \phi V_{ni} \\ &= 0.9(14.4) \\ &= 13.0 \text{ k/in. of beam length} > \text{applied force, } V_h = 4.47 \text{ k/in. } \quad \mathbf{OK} \end{aligned}$$

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## 6.0 DESIGN OF REINFORCED ELASTOMERIC BEARINGS

Design Step 6 - steel-reinforced elastomeric bearing design (S14)

### Design requirements (S14.5.3)

#### Movements during construction

*Where practicable, construction staging should be used to delay construction of abutments and piers located in or adjacent to embankments until the embankments have been placed and consolidated. Otherwise, deck joints should be sized to accommodate the probable abutment and pier movements resulting from embankment consolidation after their construction.*

*Closure pours may be used to minimize the effect of prestress-induced shortening on the width of seals and the size of bearings.*

### Characteristics (S14.6.2)

*The bearing chosen for a particular application has to have appropriate load and movement capabilities. Table S14.6.2-1 may be used as a guide when comparing different bearing systems.*

### Force effects resulting from restraint of movement at the bearing (S14.6.3)

*Restraint forces occur when any part of a movement is prevented. Forces due to direct loads include dead load of the bridge and loads due to traffic, earthquakes, water and wind. The applicable limit states must be considered.*

*Bearings are typically located in an area which collects large amounts of dirt and moisture and promotes problems of corrosion and deterioration. As a result, bearings should be designed and installed to have the maximum possible protection against the environment and to allow easy access for inspection.*

### Elastomeric bearing overview

*Shore A Durometer hardnesses of  $60 \pm 5$  are common, and they lead to shear modulus values in the range of 0.130 to 0.200 ksi. The shear stiffness of the bearing is its most important property since it affects the forces transmitted between the superstructure and substructure. Some states use a slightly different common range than stated above. See S14.7.5.2 and S14.7.6.2 for material requirements of neoprene bearing pads.*

*Elastomer may be used as a plain pad (PEP) or may be reinforced with steel. Steel reinforced elastomeric bearings are composed of layers of elastomer and steel plates bonded together with adhesive.*

*Elastomers are flexible under shear and uniaxial deformation, but they are very stiff against volume changes. This feature makes the design of a bearing that is stiff in compression but flexible in shear possible. Under uniaxial compression, the flexible elastomer would shorten significantly*

*and, to maintain constant volume, sustain large increases in its plan dimension, but the stiff steel layers of the steel reinforced elastomeric bearings restrain the lateral expansion.*

*Elastomers stiffen at low temperatures. The low temperature stiffening effect is very sensitive to the elastomer compound, and the increase in shear resistance can be controlled by selection of an elastomer compound which is appropriate for the climatic conditions.*

*The design of a steel reinforced elastomeric bearing requires an appropriate balance of compressive, shear and rotational stiffnesses. The shape factor, taken as the plan area divided by the area of the perimeter free to bulge, affects the compressive and rotational stiffnesses, but it has no impact on the translational stiffness or deformation capacity.*

*The bearing must be designed to control the stress in the steel reinforcement and the strain in the elastomer. This is done by controlling the elastomer layer thickness and the shape factor of the bearing. Fatigue, stability, delamination, yield and rupture of the steel reinforcement, stiffness of the elastomer, and geometric constraints must all be satisfied.*

### **Design methods**

*Two design methods are allowed by the AASHTO-LRFD Specifications. Method A, specified in S14.7.6, is applicable to plain, steel reinforced and fiber glass reinforced elastomeric pads as well as cotton duck pads. Method B, specified in S14.7.5, is applicable to steel reinforced elastomeric bearings. The following sections and the design example below are based on Method B. Flowcharts for the bearing design using both Method A and Method B are included in Section 3.*

### **General elastomer material properties and selection criteria (S14.7.5.2)**

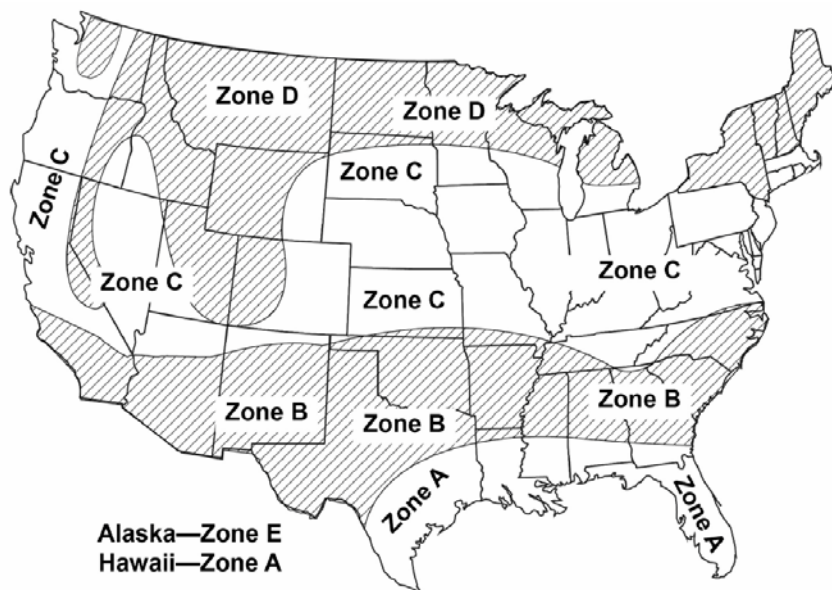
*Commonly used elastomers have a shear modulus between 0.080 and 0.175 ksi and a nominal hardness between 50 and 60 on the Shore A scale. The shear modulus of the elastomer at 73 °F is used as the basis for design. The elastomer may be specified by its shear modulus or hardness. If the elastomer is specified explicitly by its shear modulus, that value is used in design, and other properties are obtained from Table S14.7.6.2-1. If the material is specified by its hardness, the shear modulus is taken as the least favorable value from the range for that hardness given in Table S14.7.6.2-1. Intermediate values may be obtained by interpolation.*

*Elastomer grade is selected based on the temperature zone of the bridge location and by Table S14.7.5.2-1. The temperature zones are shown in Figure 6-1.*



**Table S14.7.5.2-1 – Low-Temperature Zones and Minimum Grades of Elastomer**

Low-Temperature Zone	A	B	C	D	E
50-year low temperature (°F)	0	-20	-30	-45	< -45
Maximum number of consecutive days when the temperature does not rise above 32°F	3	7	14	NA	NA
Minimum low-temperature elastomer grade	0	2	3	4	5
Minimum low-temperature elastomer grade when special force provisions are incorporated	0	0	2	3	5



**Figure 6-1 – Temperature Zones**

According to S14.7.5.2, any of the three design options listed below may be used to specify the elastomer:

- 1) Specify the elastomer with the minimum low-temperature grade indicated in Table S14.7.5.2-1 and determine the shear force transmitted by the bearing as specified in S14.6.3.1;

- 2) *Specify the elastomer with the minimum low-temperature grade for use when special force provisions are incorporated in the design but do not provide a low friction sliding surface, in which case the bridge shall be designed to withstand twice the design shear force specified in S14.6.3.1; or*
- 3) *Specify the elastomer with the minimum low-temperature grade for use when special force provisions are incorporated in the design but do not provide a low friction sliding surface, in which case the components of the bridge shall be designed to resist four times the design shear force as specified in S14.6.3.1.*

**Design Step 6.1 Design a steel reinforced elastomeric bearing for the interior girders at the intermediate pier**

*A typical elastomer with hardness 60 Shore A Durometer and a shear modulus of 0.150 ksi is assumed. The bearing reaction at the Service I limit state is equal to the shear at the end of Span 1 as shown in Tables 5.3-3 and -4. These values are shown in Table 6-1 below.*

**Table 6-1 – Design Forces on Bearings of Interior Girders at the Intermediate Pier**

	Max. factored reaction (k)	Max. reaction due to LL (k)	Reaction due to DL (k)
Service I	290.5	129.9	160.6

*Notice that:*

*The live load reaction per bearing is taken equal to the maximum girder live load end shear. Recognizing that the girder, which is continuous for live load, has two bearings on the intermediate pier, another acceptable procedure is to divide the maximum live load reaction on the pier equally between the two bearings. This will result in lower bearing loads compared to using the girder end shear to design the bearings. This approach was not taken in this example, rather, the girder end shear was applied to the bearing.*

**Design Step 6.1.1 Determine the minimum bearing area**

The bearing at the intermediate pier is fixed and is not subject to shear deformation due to the lack of movements.

The corners of the bottom flanges of the girder are usually chamfered. The bearing should be slightly narrower than the flat part of the flange unless a stiff sole plate is used to insure uniform distribution of the compressive stress and strain over the bearing area. The bearing should be as short along the length of the girder as practical to permit rotation about the transverse axis. This requires the bearing to be as wide as possible which is desirable when stabilizing the girder during erection. For a first estimate, choose a 24 in. width [28 in. wide girder bottom flange – 2(1 in. chamfer + 1 in. edge clearance)] and a 7.5 in. longitudinal dimension. The longitudinal translation is 0 in. for a fixed bearing. Notice that for a bearing subject to translation, i.e., movable bearing, the shear strains due to translation must be less than 0.5 in./in. to prevent rollover and excess fatigue damage. This means that the total elastomer thickness,  $h_{ri}$ , must be greater than two times the design translation,  $\Delta_s$ , where applicable. A preliminary shape factor should be calculated according to S14.7.5.1.

For this example, assume two interior elastomer layers, each with a thickness of 0.5 in. and a two cover elastomer layers, each with a thickness of 0.25 in.

**Design Step 6.1.2 Steel-reinforced elastomeric bearings – Method B (S14.7.5)**

For bridges at locations where the roadway has positive or negative grade, the thickness of the bearing may need to be varied along the length of the girder. This is typically accomplished through the use of a tapered steel top plate. In this example, the bridge is assumed to be at zero grade and, therefore, each elastomer and reinforcement layer has a constant thickness. All internal layers of elastomer shall be of the same thickness. For bearings with more than two elastomer layers, the top and bottom cover layers should be no thicker than 70 percent of the internal layers.

The shape factor of a layer of an elastomeric bearing,  $S_i$ , is taken as the plan area of the layer divided by the area of perimeter free to bulge. For rectangular bearings without holes, the shape factor of the layer may be taken as:

$$S_i = \frac{LW}{2h_{ri}(L+W)} \quad (\text{S14.7.5.1-1})$$

where:

- L = length of a rectangular elastomeric bearing (parallel to the longitudinal bridge axis) (in.)
- W = width of the bearing in the transverse direction (in.)
- $h_{ri}$  = thickness of  $i^{\text{th}}$  elastomeric layer in elastomeric bearing (in.)

The shape factor for the  $i$ th interior layer is:

$$S_i = 7.5(24) / [ 2(0.5)(7.5 + 24) ] \\ = 5.71$$

**Design Step 6.1.2.1**

Design Requirements (S14.7.5.3)

Compressive stress:

Design Method B no longer has a separate compressive stress check. It is now included in a combined compression, rotation and shear check described in S14.7.5.3.3 and calculated in Design Step 6.1.2.4.

**Design Step 6.1.2.2 Compressive deflection (S14.7.5.3.6)**

*This provision need only be checked if deck joints are present on the bridge. Since this design example is a jointless bridge, commentary for this provision is provided below, but no design is investigated.*

*Deflections of elastomeric bearings due to live load and long-term dead load will be considered separately.*

*Instantaneous live load deflection is be taken as:*

$$\delta_L = \sum \varepsilon_{Li} h_{ri} \tag{S14.7.5.3.6-1}$$

where:

$\varepsilon_{Li}$  = instantaneous live load compressive strain in  $i^{th}$  elastomer layer of a laminated bearing

$h_{ri}$  = thickness of  $i^{th}$  elastomeric layer in a laminated bearing (in.)

*Initial dead load deflection is taken as:*

$$\delta_d = \sum \varepsilon_{di} h_{ri} \tag{S14.7.5.3.6-2}$$

where:

$\varepsilon_{di}$  = initial dead load compressive strain in  $i^{th}$  elastomer layer of a laminated bearing

$h_{ri}$  = thickness of  $i^{th}$  elastomeric layer in a laminated bearing (in.)

*Values for  $\varepsilon_i$  are determined from test results or by analysis when considering long-term deflections. The effects of creep of the elastomer are added to the initial deflection:*

$$\delta_{it} = \delta_d + a_{cr}\delta_d \quad (S14.7.5.3.6-3)$$

where:

$a_{cr}$  = creep deflection divided by initial dead load deflection

Creep effects should be determined from information relevant to the elastomeric compound used. In the absence of material-specific data, the values given in STable 14.7.6.2-1 may be used.

### Design Step 6.1.2.3 Shear deformation (S14.7.5.3.2)

This provision need only be checked if the bearing is a movable bearing. Since the bearing under consideration is a fixed bearing, this provision does not apply. Commentary on this provision is provided below, but no design checks are performed.

The maximum horizontal movement of the bridge superstructure,  $\Delta_o$ , is taken as the extreme displacement caused by creep, shrinkage, and posttensioning combined with 65 percent of the design thermal movement range.

The maximum shear deformation of the bearing at the service limit state,  $\Delta_s$ , is taken as  $\Delta_o$ , modified to account for the substructure stiffness and construction procedures. If a low friction sliding surface is installed,  $\Delta_s$  need not be taken to be larger than the deformation corresponding to first slip.

The bearing is required to satisfy:

$$h_{rt} \geq 2\Delta_s \quad (S14.7.5.3.2-1)$$

where:

$h_{rt}$  = total elastomer thickness (sum of the thicknesses of all elastomer layers)  
(in.)

$\Delta_s$  = maximum shear deformation of the elastomer from applicable service load combinations (in.)

This limit on  $h_{rt}$  ensures that rollover at the edges and delamination due to fatigue will not take place. See SC14.7.5.3.2 for more stringent requirements when shear deformations are due to high cycle loading such as braking forces and vibrations.

**Design Step 6.1.2.4 Combined compression, rotation, and shear (S14.7.5.3.3)**

*Combinations of axial load, rotation, and shear at the service limit state shall satisfy:*

$$(\gamma_{a,st} + \gamma_{r,st} + \gamma_{s,st}) + 1.75(\gamma_{a,cy} + \gamma_{r,cy} + \gamma_{s,cy}) \leq 5.0 \quad (\text{S14.7.5.3.3-1})$$

*The static component of  $\gamma_a$  shall also satisfy:*

$$\gamma_{a,st} \leq 3.0 \quad (\text{S14.7.5.3.3-2})$$

*where:*

$\gamma_a$	= shear strain caused by axial load
$\gamma_r$	= shear strain caused by rotation
$\gamma_s$	= shear strain caused by shear displacement

*Subscripts "st" and "cy" indicate static and cyclic loading, respectively. Cyclic loads shall consist of loads induced by traffic. All other loads may be considered static.*

The shear strain due to axial load may be taken as:

$$\gamma_a = D_a \frac{\sigma_s}{GS_i} \quad (\text{S14.7.5.3.3-3})$$

where:

$D_a$	= dimensionless coefficient used to determine shear strain due to axial load, taken as 1.4 for rectangular bearings
$G$	= shear modulus of the elastomer (ksi)
$S_i$	= shape factor of the $i^{\text{th}}$ internal layer of an elastomeric bearing
$\sigma_s$	= average compressive stress due to total static or cyclic load from applicable service load combinations (ksi)

Calculate compressive stresses in bearing pad:

$$\sigma_s = \frac{P}{A}$$

$$\begin{aligned} \sigma_{s,st} &= 160.6 / (24(7.5)) \\ &= 0.892 \text{ ksi} \end{aligned}$$

$$\begin{aligned} \sigma_{s,cy} &= 129.9 / (24(7.5)) \\ &= 0.722 \text{ ksi} \end{aligned}$$

and the shear strain:

$$\begin{aligned}\gamma_{a,st} &= 1.4 ( 0.892 / ( 0.150 ( 5.71 ) ) ) \\ &= 1.45\end{aligned}$$

$$\begin{aligned}\gamma_{a,cy} &= 1.4 ( 0.722 / ( 0.150 ( 5.71 ) ) ) \\ &= 1.18\end{aligned}$$

Next, calculate the shear strain due to rotation:

$$\gamma_r = D_r \left( \frac{L}{h_{ri}} \right)^2 \frac{\theta_s}{n} \quad (\text{S14.7.5.3.3-6})$$

where:

- $D_r$  = dimensionless coefficient used to determine shear strain due to rotation, taken as 0.5 for a rectangular bearing
- $L$  = plan dimension of the bearing perpendicular to the axis of rotation under consideration (generally parallel to the global longitudinal bridge axis) (in.)
- $h_{ri}$  = thickness of the  $i^{\text{th}}$  internal elastomeric layer (in.)
- $\theta_s$  = maximum static or cyclic service rotation (rad.)

For this example,  $\theta_s$  will include the rotations due to live load and construction load (assume 0.005 rad.) only. As a result of camber under the prestressing force and permanent dead loads, prestressed beams typically have end rotation under permanent dead loads in the opposite direction than that of the live load end rotations. Conservatively assume the end rotations to be zero under the effect of the prestressing and permanent loads.

$$\theta_{s,st} = 0.005 \text{ rad. (assumed)}$$

$$\theta_{s,cy} = 0.000944 \text{ rad. (from a live load analysis program)}$$

$n$  = number of interior layers of elastomer, where interior layers are defined as those layers which are bonded on each face. Exterior layers are defined as those layers which are bonded only on one face. When the thickness of the exterior layer of elastomer is equal to or greater than one-half the thickness of an interior layer, the parameter,  $n$ , may be increased by one-half for each such exterior layer.

$$\begin{aligned}\gamma_{r,st} &= 0.5 ( 7.5 / 0.5 )^2 ( 0.005 / 2 ) \\ &= 0.281\end{aligned}$$

$$\begin{aligned}\gamma_{r,cy} &= 0.5 ( 7.5 / 0.5 )^2 ( 0.000944 / 2 ) \\ &= 0.0531\end{aligned}$$

Finally, calculate the shear strain due to shear deformation:

$$\gamma_s = \frac{\Delta_s}{h_{rt}} \quad (\text{S14.7.5.3.3-10})$$

where:

- $\Delta_s$  = maximum total static or cyclic shear deformation of the elastomer from applicable service load combinations (in.)
- $h_{rt}$  = total elastomer thickness (in.)

Since the bearing under consideration is a fixed bearing, there is no shear deformation or shear strain due to shear deformation.

Check total strains:

$$(\gamma_{a,st} + \gamma_{r,st} + \gamma_{s,st}) + 1.75(\gamma_{a,cy} + \gamma_{r,cy} + \gamma_{s,cy}) \leq 5.0 \quad (\text{S14.7.5.3.3-1})$$

$$(1.45 + 0.281 + 0.00) + 1.75(1.18 + 0.0531 + 0.00) = 3.89 < 5.0 \quad \text{OK}$$

also check static component of  $\gamma_a$ :

$$\gamma_{a,st} \leq 3.0 \quad (\text{S14.7.5.3.3-2})$$

$$1.45 < 3.0 \quad \text{OK}$$

Use 2 interior layers 0.5 in. thick each. Use exterior layers 0.25 in. thick each (< 70% of the thickness of the interior layer).

**Design Step 6.1.2.5 Stability of elastomeric bearings (S14.7.5.3.4)**

Bearings are investigated for instability at the service limit state load combinations specified in Table S3.4.1-1.

Bearings satisfying Eq. S14.7.5.3.4-1 are considered stable, and no further investigation of stability is required.

$$2A \leq B \quad (\text{S14.7.5.3.4-1})$$

for which:

$$A = \frac{1.92 \frac{h_{rt}}{L}}{\sqrt{1 + \frac{2.0L}{W}}} \quad (\text{S14.7.5.3.4-2})$$

$$B = \frac{2.67}{(S_i + 2.0) \left(1 + \frac{L}{4.0W}\right)} \quad (\text{S14.7.5.3.4-3})$$



where:

$$L = 7.5 \text{ in.}$$

$$W = 24 \text{ in.}$$

$$\begin{aligned} h_{rt} &= \text{total thickness of the elastomer in the bearing (in.)} \\ &= 2(0.25) + 2(0.5) \\ &= 1.5 \text{ in.} \end{aligned}$$

For a rectangular bearing where L is greater than W, stability will be investigated by interchanging L and W in Eqs. S14.7.5.3.4-2 and -3.

$$\begin{aligned} A &= \frac{1.92 \left( \frac{1.5}{7.5} \right)}{\sqrt{1 + \frac{2.0(7.5)}{24}}} \\ &= 0.301 \end{aligned}$$

$$\begin{aligned} B &= \frac{2.67}{(5.71 + 2.0) \left( 1 + \frac{7.5}{4.0(24)} \right)} \\ &= 0.321 \end{aligned}$$

Check  $2A \leq B$

$$2(0.301) = 0.602 > 0.321, \text{ therefore, the bearing is not stable and Eqs. S14.7.5.3.4-4 and -5 need to be checked.}$$

For bridge decks fixed against translation, the following equation needs to be satisfied to ensure stability.

$$\sigma_s \leq GS_i / (A - B) \quad (\text{S14.7.5.3.4-5})$$

However, if  $A - B \leq 0$ , then the bearing is considered stable (CS14.7.5.3.4).

$$\begin{aligned} A - B &= 0.301 - 0.321 \\ &= -0.02 \end{aligned}$$

Therefore, the bearing is stable.

**Design Step 6.1.2.6 Reinforcement (S14.7.5.3.5)**

The reinforcement should sustain the tensile stresses induced by compression on the bearing. With the present load limitations, the minimum steel plate thickness practical for fabrication will usually provide adequate strength. The minimum thickness of steel reinforcement,  $h_s$ , shall be 0.0625 in, as specified in Article 4.5 of AASHTO M 251.

At the service limit state:

$$h_s \geq 3h_{ri}\sigma_s/F_y \quad (\text{S14.7.5.3.5-1})$$

where:

$$h_{ri} = \text{thickness of } i^{\text{th}} \text{ internal elastomeric layer (in.)} \\ = 0.5 \text{ in.}$$

$$\sigma_s = 0.892 + 0.722 = 1.614 \text{ ksi}$$

$$F_y = \text{yield strength of steel reinforcement (ksi)} \\ = 36 \text{ ksi}$$

$$h_{s(\text{serv})} \geq 3(0.5)(1.614)/36 \\ \geq 0.067 \text{ in.}$$

At the fatigue limit state:

$$h_s \geq 2.0h_{ri}\sigma_L/\Delta F_{TH} \quad (\text{S14.7.5.3.5-2})$$

where:

$$h_{ri} = 0.5 \text{ in.}$$

$$\sigma_L = 0.722 \text{ ksi}$$

$$\Delta F_{TH} = \text{constant amplitude fatigue threshold for Category A} \\ \text{as specified in Table S6.6.1.2.5-3 (ksi)} \\ = 24 \text{ ksi}$$

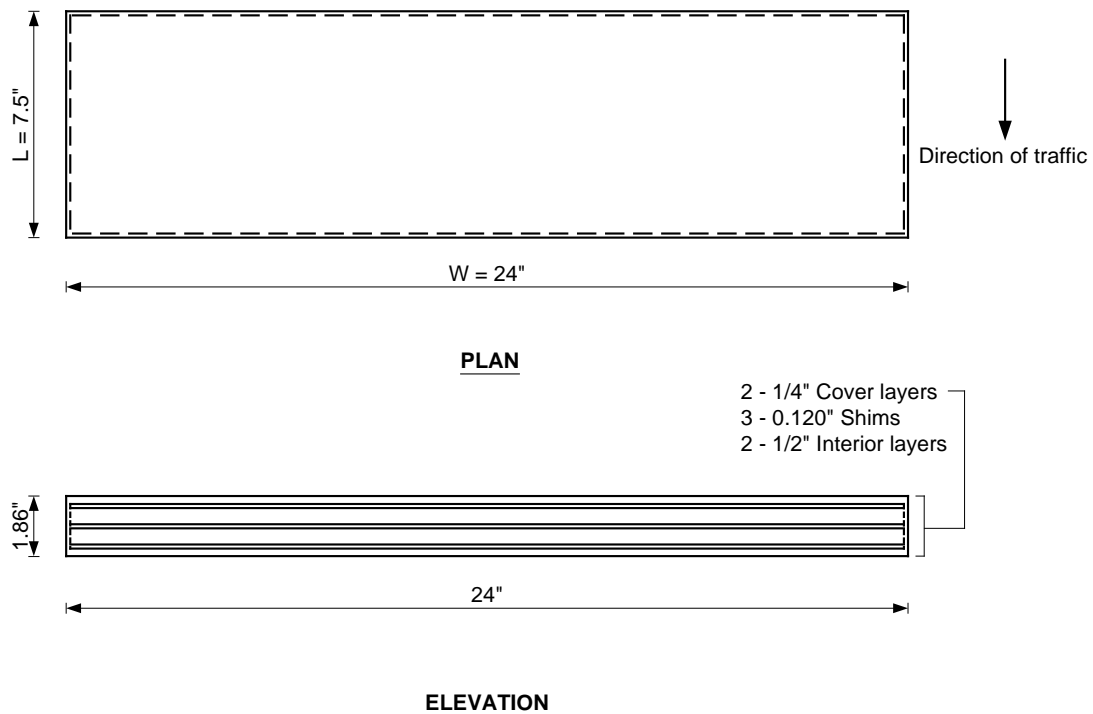
$$h_{s(\text{fatg})} \geq 2(0.5)(0.722)/24 \\ \geq 0.030 \text{ in.}$$

Use  $h_s = 0.120$  in. thick steel reinforcement plates; this is an 11 gage shim.

*If holes exist in the reinforcement, the minimum thickness is increased by a factor equal to twice the gross width divided by the net width. Holes in the reinforcement cause stress concentrations. Their use should be discouraged. The required increase in steel thickness accounts for both the material removed and the stress concentrations around the hole.*

The total height of the bearing,  $h_{rt}$ :

$$\begin{aligned}
 h_{rt} &= \text{cover layers} + \text{elastomer layers} + \text{shim thicknesses} \\
 &= 2(0.25) + 2(0.5) + 3(0.120) \\
 &= 1.86 \text{ in.}
 \end{aligned}$$



**Figure 6-2 – Dimensions of Elastomeric Bearing**

Notes:

1. 11 gage steel shim thickness is held constant for all bearings
2. All cover layers and edge covers are to be 1/4-inch thick.
3. Total bearing thickness will include the summation of a masonry plate, a sole plate, and the laminated elastomeric pad thickness.
4. Elastomer in all bearings shall have grade 60 Shore A Durometer hardness.
5. Pad shall be vulcanized to masonry plate and sole plate in the shop
6. Pad thickness shown is uncompressed.

A shear key between the bent cap and the concrete diaphragm will provide the movement restraint in the longitudinal direction. See Figure 6-3.

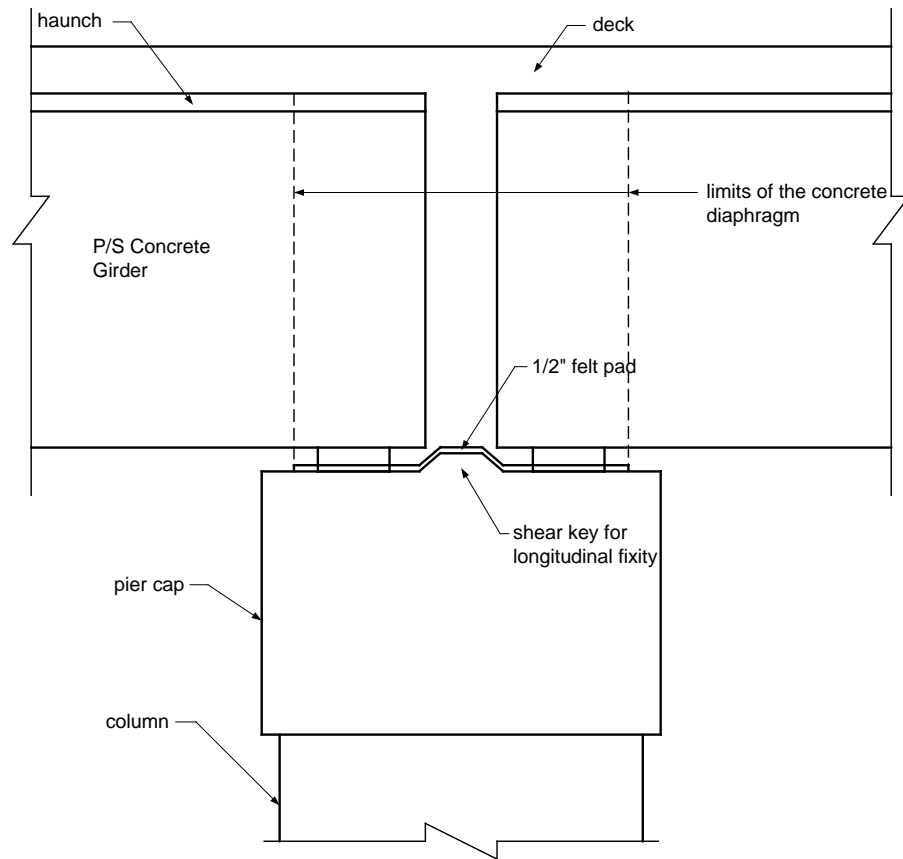


Figure 6-3 – Longitudinal Fixity at Intermediate Bent

**Design Step 7.1 INTEGRAL ABUTMENT DESIGN (This section has not been updated in 2015.)****General considerations and common practices**

*Integral abutments are used to eliminate expansion joints at the end of a bridge. They often result in “Jointless Bridges” and serve to accomplish the following desirable objectives:*

- *Long-term serviceability of the structure*
- *Minimal maintenance requirements*
- *Economical construction*
- *Improved aesthetics and safety considerations*

*A jointless bridge concept is defined as any design procedure that attempts to achieve the goals listed above by eliminating as many expansion joints as possible. The ideal jointless bridge, for example, contains no expansion joints in the superstructure, substructure or deck.*

*Integral abutments are generally founded on one row of piles made of steel or concrete. The use of one row of piles reduces the stiffness of the abutment and allows the abutment to translate parallel to the longitudinal axis of the bridge. This permits the elimination of expansion joints and movable bearings. Because the earth pressure on the two end abutments is resisted by compression in the superstructure, the piles supporting the integral abutments, unlike the piles supporting conventional abutments, do not need to be designed to resist the earth loads on the abutments.*

*When expansion joints are completely eliminated from a bridge, thermal stresses must be relieved or accounted for in some manner. The integral abutment bridge concept is based on the assumption that due to the flexibility of the piles, thermal stresses are transferred to the substructure by way of a rigid connection, i.e. the uniform temperature change causes the abutment to translate without rotation. The concrete abutment contains sufficient bulk to be considered as a rigid mass. A positive connection to the girders is generally provided by encasing girder ends in the reinforced concrete backwall. This provides for full transfer of forces due to thermal movements and live load rotational displacement experienced by the abutment piles.*

**Design criteria**

*Neither the AASHTO-LRFD Specifications nor the AASHTO-Standard Specifications contain detailed design criteria for integral abutments. In the absence of universally-accepted design criteria, many states have developed their own design guidelines. These guidelines have evolved over time and rely heavily on past experience with integral abutments at a specific area. There are currently two distinctive approaches used to design integral abutments:*

- *One group of states design the piles of an integral abutment to resist only gravity loads applied to the abutment. No consideration is given to the effect of the horizontal displacement of the abutment on the pile loads and/or pile resistance. This approach is*

*simple and has been used successfully. When the bridge is outside a certain range set by the state, e.g. long bridges, other considerations are taken into account in the design.*

- *The second approach accounts for effects of different loads, in addition to gravity loads, when calculating pile loads. It also takes into account the effect of the horizontal movements on the pile load resistance. One state that has detailed design procedures following this approach is Pennsylvania.*

*The following discussion does not follow the practices of a specific state; it provides a general overview of the current state-of-practice.*

### **Bridge length limits**

*Most states set a limit on the bridge length of jointless bridges beyond which the bridge is not considered a “typical bridge” and more detailed analysis is taken into account. Typically, the bridge length is based on assuming that the total increase of the bridge length under uniform temperature change from the extreme low to the extreme high temperature is 4 inches. This means that the movement at the top of the pile at each end is 2 inches or, when the bridge is constructed at the median temperature, a 1 inch displacement in either direction. This results in a maximum bridge length of 600 ft. for concrete bridges and 400 ft. for steel bridges at locations where the climate is defined as “Moderate” in accordance to S3.12.2.1. The maximum length is shorter for regions defined as having a “cold” climate.*

### **Soil conditions**

*The above length limits assume that the soil conditions at the bridge location and behind the abutment are such that the abutment may translate with relatively low soil resistance. Therefore, most jurisdictions specify select granular fill for use behind integral abutments. In addition, the fill within a few feet behind the integral abutment is typically lightly compacted using a vibratory plate compactor (jumping jack). When bedrock, stiff soil and/or boulders exist in the top layer of the soil (approximately the top 12 to 15 ft.), it is typically required that oversized holes be drilled to a depth of approximately 15 ft.; the piles are then installed in the oversized holes. Subsequently, the holes are filled with sand. This procedure is intended to allow the piles to translate with minimal resistance.*

### **Skew angle**

*Earth pressure acts in a direction perpendicular to the abutments. For skewed bridges, the earth pressure forces on the two abutments produce a torque that causes the bridge to twist in plan. Limiting the skew angle reduces this effect. For skewed, continuous bridges, the twisting torque also results in additional forces acting on intermediate bents.*

*In addition, sharp skews are suspected to have caused cracking in some abutment backwalls due to rotation and thermal movements. This cracking may be reduced or eliminated by limiting the skew. Limiting the skew will also reduce or eliminate design uncertainties, backfill compaction*

*difficulty and the additional design and details that would need to be worked out for the abutment U-wingwalls and approach slab.*

*Currently, there are no universally accepted limits on the degree of skew for integral abutment bridges.*

### **Horizontal alignment and bridge plan geometry**

*With relatively few exceptions, integral abutments are typically used for straight bridges. For curved superstructures, the effect of the compression force resulting from the earth pressure on the abutment is a cause for concern. For bridges with variable width, the difference in the length of the abutments results in unbalanced earth pressure forces if the two abutments are to move the same distance. To maintain force equilibrium, it is expected that the shorter abutment will deflect more than the longer one. This difference should be considered when determining the actual expected movement of the two abutments as well as in the design of the piles and the expansion joints at the end of the approach slabs (if used).*

### **Grade**

*Some jurisdictions impose a limit on the maximum vertical grade between abutments. These limits are intended to reduce the effect of the abutment earth pressure forces on the abutment vertical reactions.*

### **Girder types, maximum depth and placement**

*Integral abutments have been used for bridges with steel I-beams, concrete I-beams, concrete bulb tees and concrete spread box beams.*

*Deeper abutments are subjected to larger earth pressure forces and, therefore, less flexible. Girder depth limits have been imposed by some jurisdictions based on past successful practices and are meant to ensure a reasonable level of abutment flexibility. Soil conditions and the length of the bridge should be considered when determining maximum depth limits. A maximum girder depth of 6 ft. has been used in the past. Deeper girders may be allowed when the soil conditions are favorable and the total length of the bridge is relatively short.*

### **Type and orientation of piles**

*Integral abutments have been constructed using steel H-piles, concrete-filled steel pipe piles and reinforced and prestressed concrete piles. For H-piles, there is no commonly used orientation of the piles. In the past, H-piles have been placed both with their strong axis parallel to the girder's longitudinal axis and in the perpendicular direction. Both orientations provide satisfactory results.*

### Consideration of dynamic load allowance in pile design

*Traditionally, dynamic load allowance is not considered in foundation design. However, for integral abutment piles, it may be argued that the dynamic load allowance should be considered in the design of the top portion of the pile. The rationale for this requirement is that the piles are almost attached to the superstructure, therefore, the top portions of the piles do no benefit from the damping effect of the soil.*

### Construction sequence

*Typically, the connection between the girders and the integral abutment is made after the deck is poured. The end portion of the deck and the backwall of the abutment are usually poured at the same time. This sequence is intended to allow the dead load rotation of the girder ends to take place without transferring these rotations to the piles.*

*Two integral abutment construction sequences have been used in the past:*

- *One-stage construction:*

*In this construction sequence, two piles are placed adjacent to each girder, one pile on each side of the girder. A steel angle is connected to the two piles and the girder is seated on the steel angle. The abutment pier cap (the portion below the bottom of the beam) and the end diaphragm or backwall (the portion encasing the ends of the beams) are poured at the same time. The abutment is typically poured at the time the deck in the end span is poured.*

- *Two-stage construction:*

#### *Stage 1:*

*A pile cap supported on one row of vertical piles is constructed. The piles do not have to line up with the girders. The top of the pile cap reaches the bottom of the bearing pads under the girders. The top of the pile cap is required to be smooth in the area directly under the girders and a strip approximately 4 in. wide around this area. Other areas are typically roughened (i.e. rake finished).*

#### *Stage 2:*

*After pouring the entire deck slab, except for the portions of the deck immediately adjacent to the integral abutment (approximately the end 4 ft. of the deck from the front face of the abutment) the end diaphragm (backwall) encasing the ends of the bridge girders is poured. The end portion of the deck is poured simultaneously with the end diaphragm.*

### Negative moment connection between the integral abutment and the superstructure

*The rigid connection between the superstructure and the integral abutment results in the development of negative moments at this location. Some early integral abutments showed signs of deck cracking parallel to the integral abutments in the end section of the deck due to the lack of*



*proper reinforcement to resist this moment. This cracking was prevented by specifying additional reinforcement connecting the deck to the back (fill) face of the abutment. This reinforcement may be designed to resist the maximum moment that may be transferred from the integral abutment to the superstructure. This moment is taken equal to the sum of the plastic moments of the integral abutment piles. The section depth used to design these bars may be taken equal to the girder depth plus the deck thickness. The length of the bars extending into the deck is typically specified by the bridge owner. This length is based on the length required for the superstructure dead load positive moment to overcome the connection negative moment.*

### **Wingwalls**

*Typically, U-wingwalls (wingwalls parallel to the longitudinal axis of the bridge) are used in conjunction with integral abutments. A chamfer (typically 1 ft.) is used between the abutment and the wingwalls to minimize concrete shrinkage cracking caused by the abrupt change in thickness at the connection.*

### **Approach slab**

*Bridges with integral abutments were constructed in the past with and without approach slabs. Typically, bridges without approach slabs are located on secondary roads that have asphalt pavements. Traffic and seasonal movements of the integral abutments cause the fill behind the abutment to shift and to self compact. This often caused settlement of the pavement directly adjacent to the abutment.*

*Providing a reinforced concrete approach slab tied to the bridge deck moves the expansion joint away from the end of the bridge. In addition, the approach slab bridges cover the area where the fill behind the abutment settles due to traffic compaction and movements of the abutment. It also prevents undermining of the abutments due to drainage at the bridge ends. Typically, approach slabs are cast on polyethylene sheets to minimize the friction under the approach slab when the abutment moves.*

*The approach slab typically rests on the abutment at one end and on a sleeper slab at the other. The approach slab differs from typical roadway pavement since the soil under the approach slab is more likely to settle unevenly resulting in the approach slab bridging a longer length than expected for roadway pavement. Typically, the soil support under the approach slab is ignored in the design and the approach slab is designed as a one-way slab bridging the length between the integral abutment and the sleeper slab. The required length of the approach slab depends on the total depth of the integral abutment. The sleeper slab should be placed outside the area where the soil is expected to be affected by the movement of the integral abutment. This distance is a function of the type of fill and the degree of compaction.*

*Due to the difference in stiffness between the superstructure and the approach slab, the interface between the integral abutment and the approach slab should preferably allow the approach slab to rotate freely at the end connected to the abutment. The reinforcement bars connecting the abutment to the approach slab should be placed such that the rotational restraint provided by these bars is minimized.*

*A contraction joint is placed at the interface between the approach slab and the integral abutment. The contraction joint at this location provides a controlled crack location rather than allowing a random crack pattern to develop.*

### **Expansion joints**

*Typically, no expansion joints are provided at the interface between the approach slab and the roadway pavement when the bridge total length is relatively small and the roadway uses flexible pavement. For other cases, an expansion joint is typically used.*

### **Bearing pads**

*Plain elastomeric bearing pads are placed under all girders when the integral abutment is constructed using the two-stage sequence described above. The bearing pads are intended to act as leveling pads and typically vary from 1/2 to 3/4 in. thick. The pad length parallel to the girder's longitudinal axis varies depending on the bridge owner's specifications and the pad length in the perpendicular direction varies depending on the width of the girder bottom flange and the owner's specifications. It is recommended to block the area under the girders that is not in contact with the bearing pads using backer rods. Blocking this area is intended to prevent honeycombing of the surrounding concrete. Honeycombing will take place when the cement paste enters the gap between the bottom of girder and the top of the pile cap in the area under the girders not in contact with the bearing pads.*

## **Design Step 7.1.1 Gravity loads**

### Interior girder: unfactored loads

(See Table 5.3-3 for girder end shears)

#### Noncomposite:

Girder	= 61.6 k
Slab and haunch	= 62.2 k
<u>Exterior diaphragm</u>	<u>= 2.5 k</u>
Total NC	= 126.4 k

#### Composite:

Parapets	= 8.9 k
Future wearing surface	= 12.0 k

#### Live load:

Maximum truck per lane (without impact or distribution factors)	= 64.42 k
Minimum truck per lane (without impact or distribution factors)	= -6.68 k
Maximum lane per lane	= 30.81 k
Minimum lane per lane	= -4.39 k

Exterior girder: unfactored loads

(See Table 5.3-7 for girder end shears)

## Noncomposite:

Girder	= 61.6 k
Slab and haunch	= 55.1 k
<u>Exterior diaphragm</u>	<u>= 1.3 k</u>
Total NC	= 117.9 k

## Composite:

Parapets	= 8.9 k
Future wearing surface	= 8.1 k

## Live load:

Maximum truck per lane (without impact or distribution factors)	= 64.42 k
Minimum truck per lane (without impact or distribution factors)	= -6.68 k
Maximum lane per lane	= 30.81 k
Minimum lane per lane	= -4.39 k

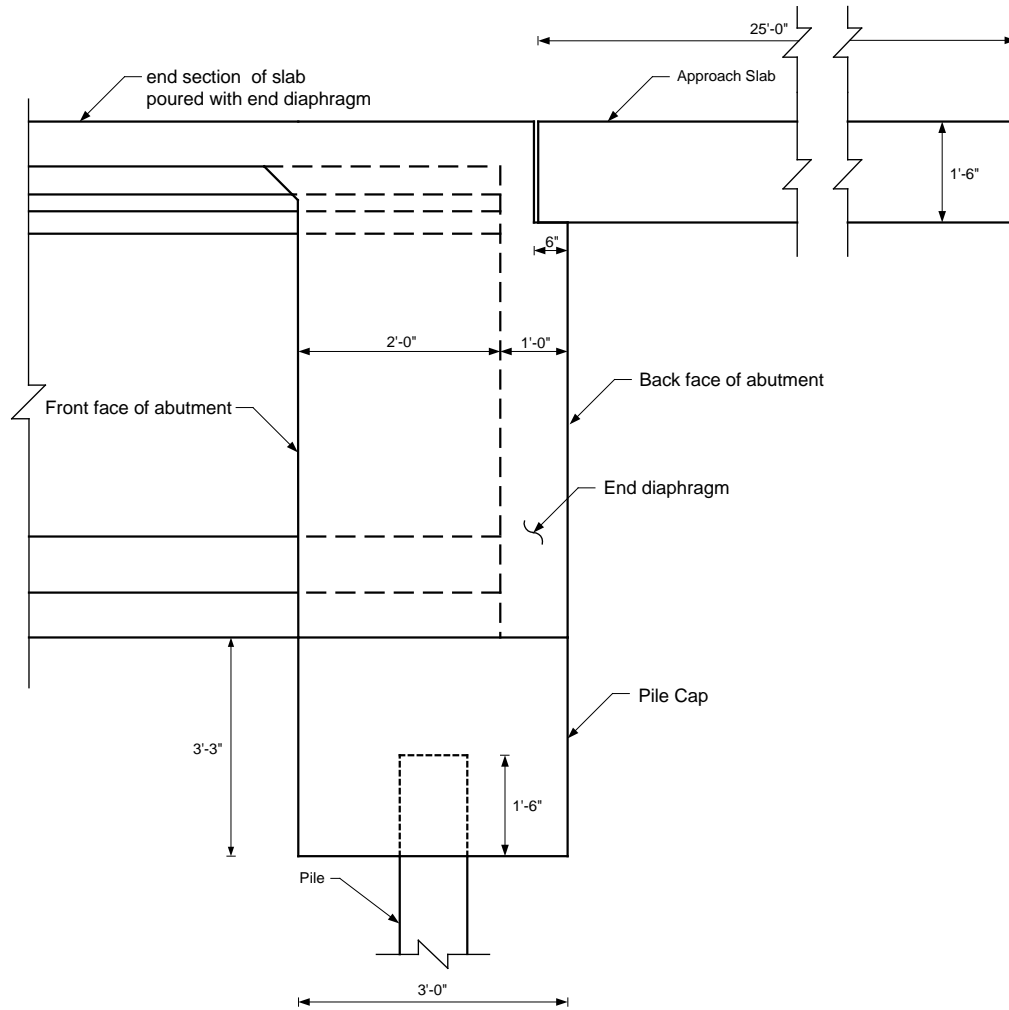


Figure 7.1-1 – General View of an Integral Abutment Showing Dimensions Used for the Example

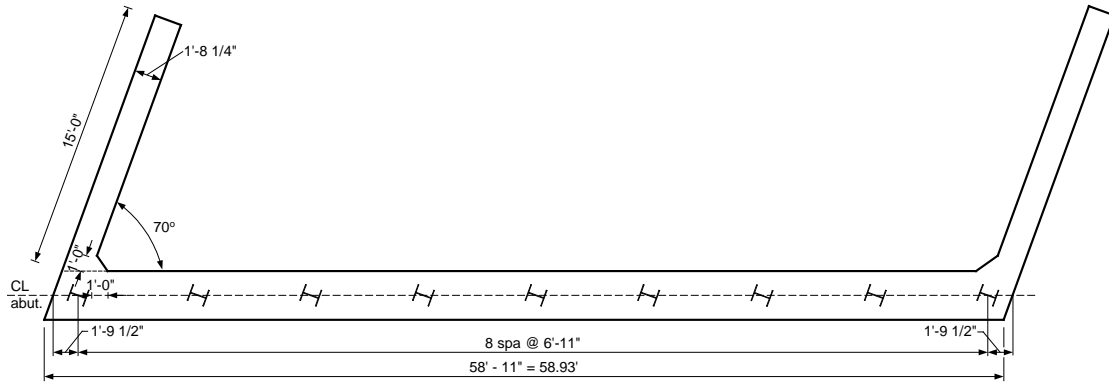


Figure 7.1-2 – Plan View of the Integral Abutment

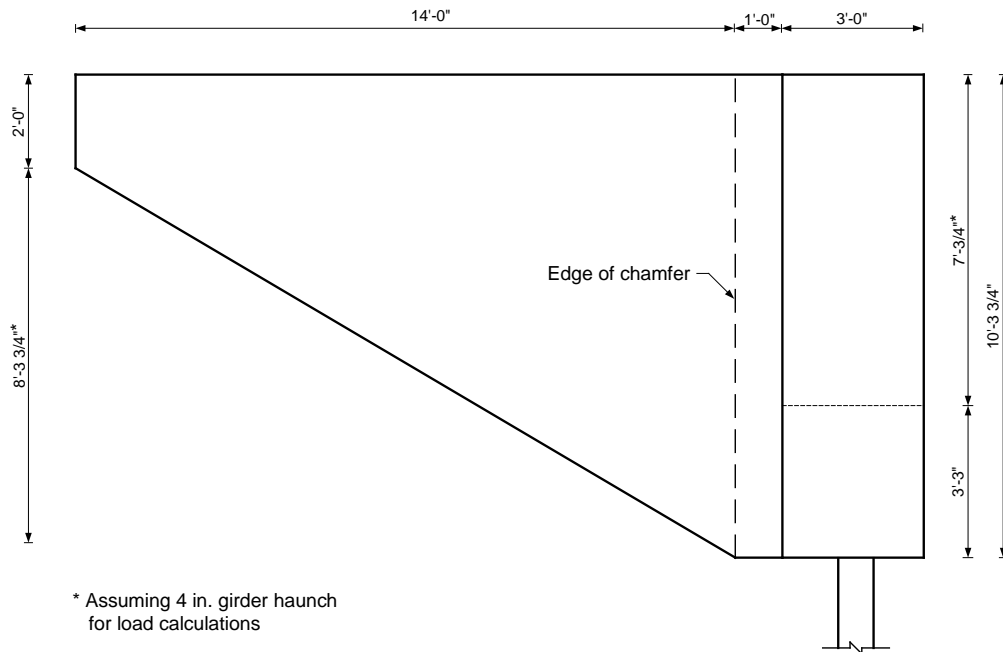


Figure 7.1-3 – Elevation View of Integral Abutment and Tapered Wingwall

In the next section, “w” and “P” denote the load per unit length and the total load, respectively. The subscripts denote the substructure component. Dimensions for each component are given in Figures 7.1-1 through 7.1-3.

Pile cap: unfactored loading

$$\begin{aligned} \text{Pile cap length along the skew} &= 55.354/\cos 20 \\ &= 58.93 \text{ ft.} \end{aligned}$$

$$\begin{aligned} w_{\text{cap}} &= 3.25(3)(0.150) \\ &= 1.46 \text{ k/ft} \end{aligned}$$

OR

$$\begin{aligned} P_{\text{cap}} &= 1.46(58.93) \\ &= 86.0 \text{ k} \end{aligned}$$

Concrete weight from the end diaphragm (approximate, girder volume not removed): unfactored loading

Assuming bearing pad thickness of  $\frac{3}{4}$  in., girder height of 72 in., haunch thickness of 4 in., and deck thickness of 8 in.:

$$\begin{aligned} w_{\text{end dia}} &= 3[(0.75 + 72 + 4 + 8)/12](0.150) \\ &= 3.18 \text{ k/ft} \end{aligned}$$

OR

$$\begin{aligned} P_{\text{end dia}} &= 3.18(58.93) \\ &= 187.4 \text{ k} \end{aligned}$$

Wingwall: unfactored load

$$\begin{aligned} A_{\text{wing}} &= (123.75/12)(15) - \frac{1}{2} (14)(99.75/12) \\ &= 96.5 \text{ ft}^2 \end{aligned}$$

$$\begin{aligned} \text{Wingwall thickness} &= \text{parapet thickness at the base} \\ &= 20.25 \text{ in. (given in Section 4)} \end{aligned}$$

$$\begin{aligned} \text{Wingwall weight} &= 96.5(20.25/12)(0.150) \\ &= 24.43 \text{ k} \end{aligned}$$

$$\begin{aligned} \text{Chamfer weight} &= (123.75/12)(1.0)(1.0)(0.150)/2 \\ &= 0.77 \text{ k} \end{aligned}$$

Notice that the chamfer weight is insignificant and is not equal for the two sides of the bridge due to the skew. For simplicity, it was calculated based on a right angle triangle and the same weight is used for both sides.

$$\begin{aligned} \text{Weight of two wingwalls plus chamfer} &= 2(24.43 + 0.77) \\ &= 50.4 \text{ k} \end{aligned}$$

$$\text{Parapet weight} = 0.65 \text{ k/ft (given in Section 5.2)}$$

$$\begin{aligned}\text{Parapet length on wingwall and abutment} &= 15 + 3/\sin 70 \\ &= 18.19 \text{ ft.}\end{aligned}$$

$$\begin{aligned}P_{\text{parapet}} &= 2(0.650)(18.19) \\ &= 23.65 \text{ k total weight}\end{aligned}$$

Approach slab load acting on the integral abutment: unfactored loading

Approach slab length = 25 ft.

$$\begin{aligned}\text{Approach slab width between parapets} &= 58.93 - 2[(20.25/12)/\sin 70] \\ &= 55.34 \text{ ft.}\end{aligned}$$

Self weight of the approach slab:

$$\begin{aligned}W_{\text{approach slab}} &= \frac{1}{2} (25)(1.5)(0.150) \\ &= 2.81 \text{ k/ft}\end{aligned}$$

OR

$$\begin{aligned}P_{\text{approach slab}} &= 2.81(55.34) \\ &= 155.5 \text{ k}\end{aligned}$$

Future wearing surface acting on the approach slab (assuming 25 psf):

$$\begin{aligned}w_{\text{FWS}} &= \frac{1}{2} (0.025)(25) \\ &= 0.31 \text{ k/ft}\end{aligned}$$

OR

$$\begin{aligned}P_{\text{FWS}} &= 0.31(55.34) \\ &= 17.2 \text{ k}\end{aligned}$$

Live load on the approach slab, reaction on integral abutment:

$$\begin{aligned}P_{\text{lane load}} &= \frac{1}{2} (0.64)(25) && (\text{S3.6.1.2.4}) \\ &= 8.0 \text{ k (one lane)}\end{aligned}$$

Notice that one truck is allowed in each traffic lane and that the truck load is included in the girder reactions. Therefore, no trucks were assumed to exist on the approach slab and only the uniform load was considered.

### Design Step 7.1.2 Pile cap design

*The girder reactions, interior and exterior, are required for the design of the abutment pile cap. Notice that neither the piles nor the abutment beam are infinitely rigid. Therefore, loads on the piles due to live loads are affected by the location of the live load across the width of the integral abutment. Moving the live load reaction across the integral abutment and trying to maximize the load on a specific pile by changing the number of loaded traffic lanes is not typically done when*

*designing integral abutments. As a simplification, the live load is assumed to exist on all traffic lanes and is distributed equally to all girders in the bridge cross section. The sum of all dead and live loads on the abutment is then distributed equally to all piles supporting the abutment.*

The maximum number of traffic lanes allowed on the bridge based on the available width (52 ft. between gutter lines) is:

$$N_{\text{lanes}} = 52 \text{ ft.}/12 \text{ ft. per lane} = 4.33 \text{ say } 4 \text{ lanes}$$

Factored dead load plus live load reactions for one interior girder, Strength I limit state controls (assume the abutment is poured in two stages as discussed earlier):

Maximum reaction Stage I:

$$\begin{aligned} P_{\text{SI(I)}} &= 1.25(\text{girder} + \text{slab} + \text{haunch}) \\ &= 1.25(126.4) \\ &= 158 \text{ k} \end{aligned}$$

Notice that construction loads should be added to the above reaction if construction equipment is allowed on the bridge before pouring the backwall (Stage II).

Maximum reaction for Final Stage:

Including the dynamic load allowance (for design of the pile cap top portion of the piles):

$$\begin{aligned} P_{\text{FNL(I)}} &= 1.25(\text{DC}) + 1.50(\text{DW}) + 1.75(\text{LL} + \text{IM})(N_{\text{lanes}})/N_{\text{girders}} \\ &= 1.25(126.4 + 8.9) + 1.5(12.0) + 1.75[1.33(64.42) + 30.81](4)/6 \\ &= 323 \text{ k} \end{aligned}$$

Without the dynamic load allowance (for design of the lower portion of the piles):

$$P_{\text{FNL(I)}} = 298.3 \text{ k}$$

Factored dead load plus live load reactions for one exterior girder, Strength I limit state controls:

Maximum reaction Stage I:

$$\begin{aligned} P_{\text{SI(E)}} &= 1.25(117.9) \\ &= 147.4 \text{ k} \end{aligned}$$

Notice that construction loads should be added to the above reaction.

Maximum reaction for Final Stage:



Including the dynamic load allowance:

$$\begin{aligned} P_{\text{FNL(E)}} &= 1.25(\text{DC}) + 1.50(\text{DW}) + 1.75(\text{LL} + \text{IM})(N_{\text{lanes}})/N_{\text{girders}} \\ &= 1.25(117.9 + 8.9) + 1.5(8.1) + 1.75[1.33(64.42) + 30.81](4)/6 \\ &= 306.6 \text{ k} \end{aligned}$$

Without the dynamic load allowance:

$$P_{\text{FNL(E)}} = 281.8 \text{ k}$$

### Design Step 7.1.3 Piles

*Typically, integral abutments may be supported on end bearing piles or friction piles. Reinforced and prestressed concrete piles, concrete-filled steel pipe piles or steel H-piles may be used. Steel H-piles will be used in this example.*

*Typically, the minimum distance between the piles and the end of the abutment, measured along the skew, is taken as 1'-6" and the maximum distance is usually 2'-6". These distances may vary from one jurisdiction to another. The piles are assumed to be embedded 1'-6" into the abutment. Maximum pile spacing is assumed to be 10 ft. The minimum pile spacing requirements of S10.7.1.5 shall apply.*

- *From S10.7.1.5, the center-to-center pile spacing shall not be less than the greater of 30.0 in. or 2.5 pile diameters (or widths). The edge distance from the side of any pile to the nearest edge of the footing shall be greater than 9.0 in.*
- *According to S10.7.1.5, where a reinforced concrete beam is cast-in-place and used as a bent cap supported by piles, the concrete cover at the sides of the piles shall be greater than 6.0 in., plus an allowance for permissible pile misalignment, and the piles shall project at least 6.0 in. into the cap. This provision is specifically for bent caps, therefore, keep 1'-6" pile projection for integral abutment to allow the development of moments in the piles due to movements of the abutment without distressing the surrounding concrete.*

*From Figure 7.1-2, steel H-piles are shown to be driven with their weak axis perpendicular to the centerline of the beams. As discussed earlier, piles were also successfully driven with their strong axis perpendicular to the centerline of the beams in the past.*

*According to S10.7.4.1, the structural design of driven concrete, steel, and timber piles must be in accordance with the provisions of Sections S5, S6, and S8 respectively. Articles S5.7.4, S5.13.4, S6.15, S8.4.13, and S8.5.2.2 contain specific provisions for concrete, steel, and wood piles. Design of piles supporting axial load only requires an allowance for unintended eccentricity. For the steel H-piles used in this example, this has been accounted for by the resistance factors in S6.5.4.2 for steel piles.*

General pile design

As indicated earlier, piles in this example are designed for gravity loads only.

*Generally, the design of the piles is controlled by the minimum capacity as determined for the following cases:*

- *Case A - Capacity of the pile as a structural member according to the procedures outlined in S6.15. The design for combined moment and axial force will be based on an analysis that takes the effect of the soil into account.*
- *Case B - Capacity of the pile to transfer load to the ground.*
- *Case C - Capacity of the ground to support the load.*

*For piles on competent rock, only Case A needs to be investigated.*

**Design Step 7.1.3.1**Pile compressive resistance (S6.15 and S6.9.2)

The factored resistance of components in compression,  $P_r$ , is taken as:

$$P_r = \phi P_n \quad (\text{S6.9.2.1-1})$$

where:

$P_n$  = nominal compressive resistance specified in S6.9.4 and S6.9.5 (kip)

$\phi_c$  = resistance factor for axial compression, steel only as specified in S6.5.4.2  
= 0.5 for H-piles assuming severe driving conditions

Check the width/thickness requirements per S6.9.4.2. Assume HP12x53 piles.

Slenderness of plates must satisfy:

$$\frac{b}{t} \leq k \sqrt{\frac{E}{F_y}} \quad (\text{S6.9.4.2-1})$$

where:

$k$  = plate buckling coefficient as specified in Table S6.9.4.2-1  
= 0.56 for flanges and projecting legs or plates

$b$  = width of plate equals one-half of flange width as specified in Table S6.9.4.2-1 (in.)

$$= 12.045/2$$

$$= 6.02 \text{ in.}$$

$$t = \text{flange thickness (in.)}$$

$$= 0.435 \text{ in.}$$

$$\frac{b}{t} = 6.02/0.435$$

$$= 13.8$$

$$k \sqrt{\frac{E}{F_y}} = 0.56 \sqrt{\frac{29,000}{36}}$$

$$= 15.9 > 13.8$$

Therefore, use S6.9.4.1 to calculate the compressive resistance.

*(Notice that the b/t ratio for the webs of HP sections is always within the limits of Table S6.9.4.2-1 for webs and, therefore, need not be checked.)*

For piles fully embedded in soil, the section is considered continuously braced and Eq. S6.9.4.1-1 is reduced to  $P_n = F_y A_s$ .

$$P_n = 36(15.5)$$

$$= 558 \text{ k}$$

Therefore, the factored resistance of components in compression,  $P_r$ , is taken as:

$$P_r = \phi P_n$$

$$= 0.5(558)$$

$$= 279 \text{ k}$$

*The above capacity applies to the pile at its lower end where damage from driving may have taken place. At the top of the pile, higher resistance factors that do not account for damage may be used. For piles designed for gravity loads only, as in this example, the resistance at the lower end will always control due to the lower resistance factor regardless if the dynamic load allowance is considered in determining the load at the top of the pile or not (notice that the dynamic load allowance is not considered in determining the load at the bottom of the pile).*

## Design Step 7.1.3.2

Determine the number of piles required

Maximum total girder reactions for Stage I (detailed calculations of girder reactions shown earlier):

$$\begin{aligned} P_{SI(Total)} &= 2(147.4) + 4(158) \\ &= 926.8 \text{ k} \end{aligned}$$

Maximum total girder reaction for final stage not including the dynamic load allowance (detailed calculations of girder reactions shown earlier):

$$\begin{aligned} P_{FNL(Total)} &= 2(281.8) + 4(298.3) \\ &= 1,756.8 \text{ k} \end{aligned}$$

Maximum factored DL + LL on the abutment, Strength I limit state controls:

$$\begin{aligned} P_{Str. I} &= P_{FNL(Total)} + 1.25(DC) + 1.50(DW) + 1.75(LL_{max})(N_{lanes}) \\ &= 1,756.8 + 1.25(86.0 + 187.4 + 50.4 + 23.65 + 155.5) + 1.5(17.2) \\ &\quad + 1.75(8.0)(4) \\ &= 1,756.8 + 710.5 \\ &= 2,467 \text{ k} \end{aligned}$$

where:

“ $P_{FNL(Total)}$ ” is the total factored DL + LL reaction of the bridge girders on the abutment.

“DC” includes the weight of the pile cap, diaphragm, wingwalls, approach slab and parapet on the wingwalls.

“DW” includes the weight of the future wearing surface on the approach slab.

“ $LL_{max}$ ” is the live load reaction from the approach slab transferred to the abutment (per lane)

“ $N_{lanes}$ ” is the maximum number of traffic lanes that fit on the approach slab, 4 lanes.

Therefore, the number of piles required to resist the applied dead and live loads is:

$$\begin{aligned} N_{piles} &= P_{Str. I} / P_r \\ &= 2,467 / 279 \\ &= 8.84 \text{ piles, say 9 piles} \end{aligned}$$

**Design Step 7.1.3.3**Pile spacing

Total length of the pile cap = 58.93 ft.

Assume pile spacing is 6'-11" (6.917 ft.) which provides more than the recommended edge distance of 1'-6" for the piles.

$$\begin{aligned}\text{Pile end distance} &= [58.93 - 8(6.917)]/2 \\ &= 1.80 \text{ ft. (1'-9 1/2" )}\end{aligned}$$

**Design Step 7.1.4 Backwall design**

The thickness of the abutment backwall is taken to be 3 ft.

Design of the pier cap for gravity loads

*For an integral abutment constructed in two stages, the abutment is designed to resist gravity loads as follows:*

- *Case A - The first stage of the abutment, i.e., the part of the abutment below the bearing pads, is designed to resist the self weight of the abutment, including the diaphragm, plus the reaction of the girders due to the self weight of the girder plus the deck slab and haunch.*
- *Case B - The entire abutment beam, including the diaphragm, is designed under the effect of the full loads on the abutment.*

Instead of analyzing the abutment beam as a continuous beam supported on rigid supports at pile locations, the following simplification is common in conducting these calculations and is used in this example:

- *Calculate moments assuming the abutment beam acting as a simple span between piles and then taking 80% of the simple span moment to account for the continuity. The location of the girder reaction is often assumed at the midspan for moment calculations and near the end for shear calculations. This assumed position of the girders is meant to produce maximum possible load effects. Due to the relatively large dimensions of the pile cap, the required reinforcement is typically light even with this conservative simplification.*

Required information:

Concrete compressive strength,  $f'_c = 3$  ksi

Reinforcing steel yield strength,  $F_y = 60$  ksi

Pile spacing = 6.917 ft.

### CASE A

The maximum factored load due to the girders and slab (from the interior girder):

$$\begin{aligned} P_u &= 1.5(126.4) \\ &= 189.6 \text{ k} \end{aligned}$$

Factored load due to the self weight of the pile cap and diaphragm:

$$\begin{aligned} w_u &= 1.5(1.46 + 3.18) \\ &= 6.96 \text{ k/ft} \end{aligned}$$

*Notice that only dead loads exist at this stage. The 1.5 load factor in the above equations is for Strength III limit state, which does not include live loads.*

### Flexural design for Case A

The maximum positive moment,  $M_u$ , assuming a simple span girder, is at midspan between piles. The simple span moments are reduced by 20% to account for continuity:

$$\begin{aligned} M_u &= P_u \ell / 4 + w_u \ell^2 / 8 \\ &= 0.8[189.6(6.917)/4 + 6.96(6.917)^2/8] \\ &= 295.6 \text{ k-ft} \end{aligned}$$

Determine the required reinforcing at the bottom of the pile cap.

$$M_r = \phi M_n \quad (\text{S5.7.3.2.1-1})$$

The nominal flexural resistance,  $M_n$ , is calculated using Eq. (S5.7.3.2.2-1).

$$M_n = A_s f_y (d_s - a/2) \quad (\text{S5.7.3.2.2-1})$$

where:

$$\begin{aligned} A_s &= \text{area of nonprestressed tension reinforcement (in}^2\text{), notice that available room will} \\ &\quad \text{only allow four bars, two on either side of the piles. Use 4 \#8 bars.} \\ &= 4(0.79) \\ &= 3.16 \text{ in}^2 \end{aligned}$$

$$\begin{aligned} f_y &= \text{specified yield strength of reinforcing bars (ksi)} \\ &= 60 \text{ ksi} \end{aligned}$$

$$\begin{aligned} d_s &= \text{distance from the extreme compression fiber to the centroid of the nonprestressed} \\ &\quad \text{tensile reinforcement (in.)} \\ &= \text{depth of pile cap} - \text{bottom cover} - \frac{1}{2} \text{ diameter bar} \end{aligned}$$

$$\begin{aligned}
 &= 3.25(12) - 3 - \frac{1}{2}(1.0) \\
 &= 35.5 \text{ in.}
 \end{aligned}$$

$$\begin{aligned}
 A &= c\beta_1, \text{ depth of the equivalent stress block (in.)} \\
 &= A_s f_y / 0.85 f'_c b \quad (\text{S5.7.3.1.1-4}) \\
 &= 3.16(60) / [0.85(3)(3.0)(12)] \\
 &= 2.07 \text{ in.}
 \end{aligned}$$

$$\begin{aligned}
 M_n &= 3.16(60)(35.5 - 2.07/2) / 12 \\
 &= 544.5 \text{ k-ft}
 \end{aligned}$$

Therefore,

$$\begin{aligned}
 M_r &= 0.9(544.5) \\
 &= 490 \text{ k-ft} > M_u = 295.6 \text{ k-ft} \quad \mathbf{OK}
 \end{aligned}$$

Negative moment over the piles is taken equal to the positive moment. Use the same reinforcement at the top of the pile cap as determined for the bottom (4 #8 bars).

By inspection:

- $M_r > 4/3(M_u)$ . This means the minimum reinforcement requirements of S5.7.3.3.2 are satisfied.
- The depth of the compression block is small relative to the section effective depth. This means that the maximum reinforcement requirements of S5.7.3.3.1 are satisfied.

#### Shear design for Case A

The maximum factored shear due to the construction loads assuming the simple span condition and girder reaction at the end of the span:

$$\begin{aligned}
 V_u &= P_u + w_u \ell / 2 \\
 &= 189.6 + 6.96(6.917) / 2 \\
 &= 213.7 \text{ k}
 \end{aligned}$$

The factored shear resistance,  $V_r$ , is calculated as:

$$V_r = \phi V_n \quad (\text{S5.8.2.1-2})$$

The nominal shear resistance,  $V_n$ , is calculated according to S5.8.3.3 and is the lesser of:

$$V_n = V_c + V_s \quad (\text{S5.8.3.3-1})$$

OR

$$V_n = 0.25f'_c b_v d_v \quad (\text{S5.8.3.3-2})$$

where:

$$V_c = 0.0316\beta\sqrt{f'_c}b_v d_v \quad (\text{S5.8.3.3-3})$$

$\beta$  = factor indicating ability of diagonally cracked concrete to transmit tension as specified in S5.8.3.4  
= 2.0

$f'_c$  = specified compressive strength of the concrete (ksi)  
= 3.0 ksi

$b_v$  = effective shear width taken as the minimum web width within the depth  $d_v$  as determined in S5.8.2.9 (in.)  
= 36 in.

$d_v$  = effective shear depth as determined in S5.8.2.9 (in.)

S5.8.2.9 states that  $d_v$  is not to be taken less than the greater of  $0.9d_e$  or  $0.72h$

$$\begin{aligned} d_v &= d_e - a/2 \\ &= 35.5 - (2.07/2) \\ &= 34.47 \text{ in.} \end{aligned}$$

$$\begin{aligned} 0.9d_e &= 0.9(35.5) \\ &= 31.95 \text{ in.} \end{aligned}$$

$$\begin{aligned} 0.72h &= 0.72[3.25(12)] \\ &= 28.08 \text{ in.} \end{aligned}$$

Therefore,  $d_v$  should be taken as 34.47 in.

$$\begin{aligned} V_c &= 0.0316(2.0)\sqrt{3}(36)(34.47) \\ &= 135.8 \text{ k} \end{aligned}$$

Assuming shear reinforcement is #5 @ 10 in. spacing perpendicular to the pier cap longitudinal axis.

$$V_s = A_v f_y d_v / s \quad (\text{S5.8.3.3-4})$$

where:

$$\begin{aligned} A_v &= \text{area of shear reinforcement within a distance "s" (in}^2\text{)} \\ &= 2 \text{ legs}(0.31) \\ &= 0.62 \text{ in}^2 \end{aligned}$$



$$s = \text{spacing of stirrups (in.)} \\ = 10 \text{ in.}$$

$$V_s = 0.62(60)(34.47)/10 \\ = 128.2 \text{ k}$$

The nominal shear resistance,  $V_n$ , is taken as the smaller of:

$$V_n = 135.8 + 128.2 \\ = 264 \text{ k}$$

OR

$$V_n = 0.25(3)(36)(34.47) \\ = 930.7 \text{ k}$$

Therefore, use the shear resistance due to the concrete and transverse steel reinforcement.

$$V_r = \phi V_n \\ = 0.9(264) \\ = 237.6 \text{ k} > V_u = 213.7 \text{ k} \quad \mathbf{OK}$$

### CASE B

The maximum factored load due to all applied dead and live loads which include the approach slab, live load on approach slab, etc. The load due to the wingwalls is not included since its load minimally affects the responses at the locations where girder reactions are applied.

Point load:

$$P_{\text{Str-I}} = \text{maximum factored girder reaction calculated earlier} \\ = 323 \text{ k}$$

Notice that the 323 k assumes that the live load is distributed equally to all girders. This approximation is acceptable since this load is assumed to be applied at the critical location for moment and shear. Alternately, the maximum reaction from the tables in Section 5.3 may be used.

Distributed load:

$$w_{\text{Str-I}} = 1.25(\text{cap self wt.} + \text{end diaph.} + \text{approach slab}) + 1.5(\text{approach FWS}) + \\ 1.75(\text{approach slab lane load})(N_{\text{lanes}})/L_{\text{abutment}} \\ = 1.25(1.46 + 3.18 + 2.81) + 1.5(0.31) + 1.75(8.0)(4)/58.93 \\ = 10.73 \text{ k/ft}$$

Flexural design for Case B

The maximum positive moment is calculated assuming the girder reaction is applied at the midspan between piles and taking 80% of the simple span moment.

$$\begin{aligned} M_u &= 0.8[323(6.917)/4 + 10.73(6.917)^2/8] \\ &= 498.2 \text{ k-ft} \end{aligned}$$

Determine the required reinforcing at the bottom of the pile cap.

$$M_r = \phi M_n \quad (\text{S5.7.3.2.1-1})$$

and

$$M_n = A_s f_y (d_s - a/2) \quad (\text{S5.7.3.2.2-1})$$

where:

$$\begin{aligned} A_s &= \text{use 4 \#8 bars} \\ &= 4(0.79) \\ &= 3.16 \text{ in}^2 \end{aligned}$$

$$f_y = 60 \text{ ksi}$$

$$\begin{aligned} d_s &= \text{total depth of int. abut. (no haunch) – bottom cover – } \frac{1}{2} \text{ bar diameter} \\ &= 119.75 - 3 - \frac{1}{2}(1.0) \\ &= 116.25 \text{ in.} \end{aligned}$$

$$\begin{aligned} a &= A_s f_y / 0.85 f'_c b \quad (\text{S5.7.3.1.1-4}) \\ &= 3.16(60) / [0.85(3)(3.0)(12)] \\ &= 2.07 \text{ in.} \end{aligned}$$

$$\begin{aligned} M_n &= 3.16(60)(116.25 - 2.07/2)/12 \\ &= 1,820 \text{ k-ft} \end{aligned}$$

Therefore,

$$\begin{aligned} M_r &= 0.9(1,820) \\ &= 1,638 \text{ k-ft} > M_u = 498.2 \text{ k-ft} \quad \mathbf{OK} \end{aligned}$$

Negative moment over the piles is taken equal to the positive moment. Use the same reinforcement at the top of the abutment beam as determined for the bottom (4 #8 bars).

By inspection:

- $M_r > 4/3(M_u)$ .
- The depth of the compression block is small relative to the section effective depth.

Shear design for Case B

Assume the girder reaction is adjacent to the pile.

The maximum factored shear due to all applied loading:

$$\begin{aligned} V_u &= P_u + w_u \ell / 2 \\ &= 323 + 10.73(6.917) / 2 \\ &= 360.1 \text{ k} \end{aligned}$$

The factored shear resistance,  $V_r$ , is calculated as:

$$V_r = \phi V_n \quad (\text{S5.8.2.1-2})$$

The nominal shear resistance,  $V_n$ , is calculated according to S5.8.3.3 and is the lesser of:

$$V_n = V_c + V_s \quad (\text{S5.8.3.3-1})$$

OR

$$V_n = 0.25 f'_c b_v d_v \quad (\text{S5.8.3.3-2})$$

where:

$$V_c = 0.0316 \beta \sqrt{f'_c} b_v d_v \quad (\text{S5.8.3.3-3})$$

$$\beta = 2.0$$

$$f'_c = 3.0 \text{ ksi}$$

$$b_v = 36 \text{ in.}$$

$$d_v = d_e - a/2$$

$$d_e = 116.25 \text{ in. (calculated earlier)}$$

$$d_v = 116.25 - (2.07/2)$$

$$= 115.2 \text{ in.}$$

$$0.9d_e = 0.9(116.25)$$

$$= 104.6 \text{ in.}$$

$$0.72h = 0.72(119.75)$$

$$= 86.22 \text{ in.}$$

Therefore,  $d_v$  should be taken as 115.2 in.

The nominal shear resistance,  $V_n$ , is taken as the lesser of:

$$\begin{aligned}V_c &= 0.0316(2.0)\sqrt{3}(36)(115.2) \\ &= 454.0 \text{ k}\end{aligned}$$

Notice that  $V_c$  is large enough, relative to the applied load, that the contribution of the transverse shear reinforcement,  $V_s$ , is not needed.

OR

$$\begin{aligned}V_n &= 0.25(3)(36)(115.2) \\ &= 3,110.4 \text{ k}\end{aligned}$$

Therefore, use the shear resistance due to the concrete,  $V_c$

$$\begin{aligned}V_r &= \phi V_n \\ &= 0.9(454.0) \\ &= 408.6 \text{ k} > V_u = 360.1 \text{ k} \quad \mathbf{OK}\end{aligned}$$

Typical reinforcement details of the abutment beam are shown in Figures 7.1-4 through 7.1-7. Notice that bar shapes vary depending on the presence of girders and/or piles at the section.

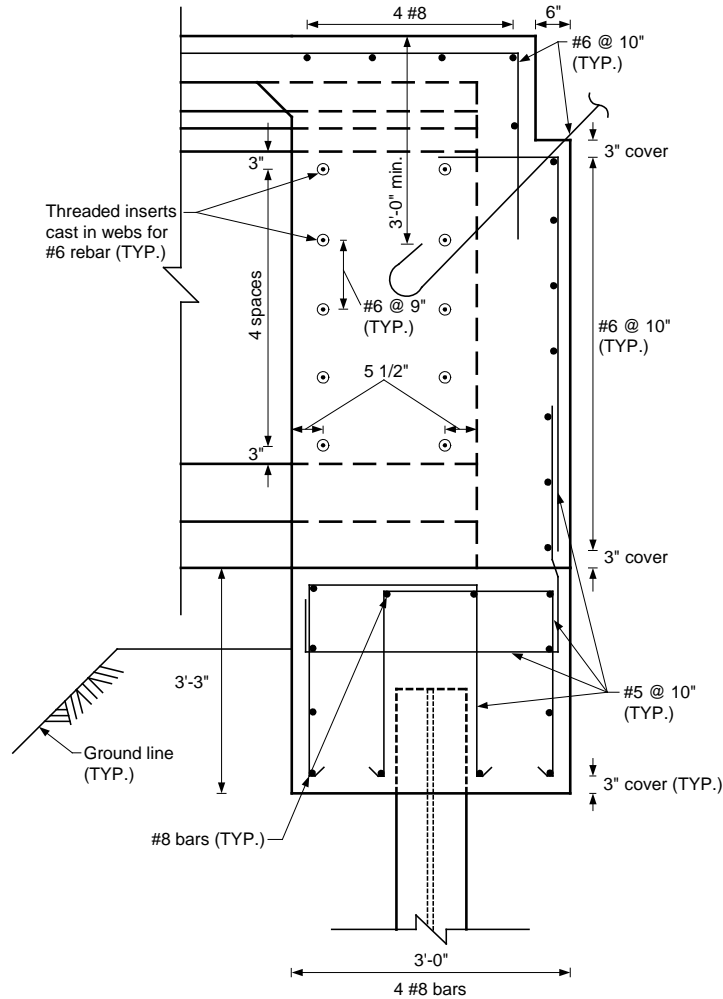


Figure 7.1-4 – Integral Abutment Reinforcement, Girder and Pile Exist at the Same Section

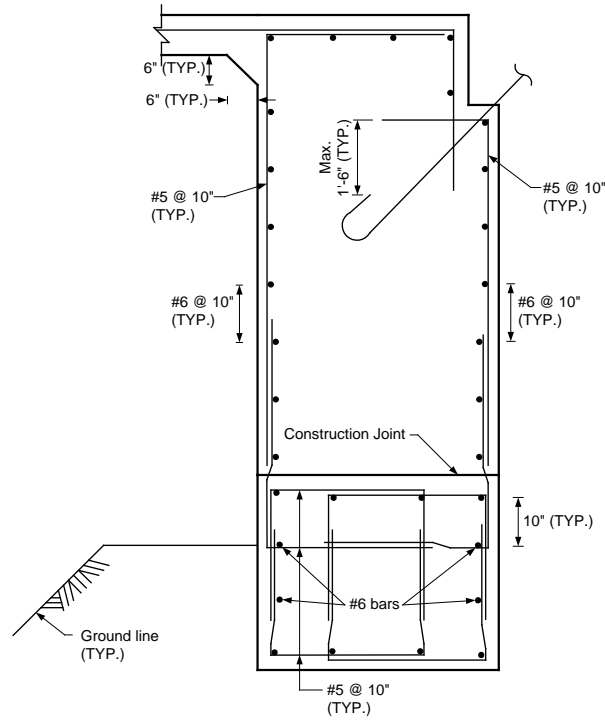


Figure 7.1-5 – Integral Abutment Reinforcement, No Girder and No Pile at the Section

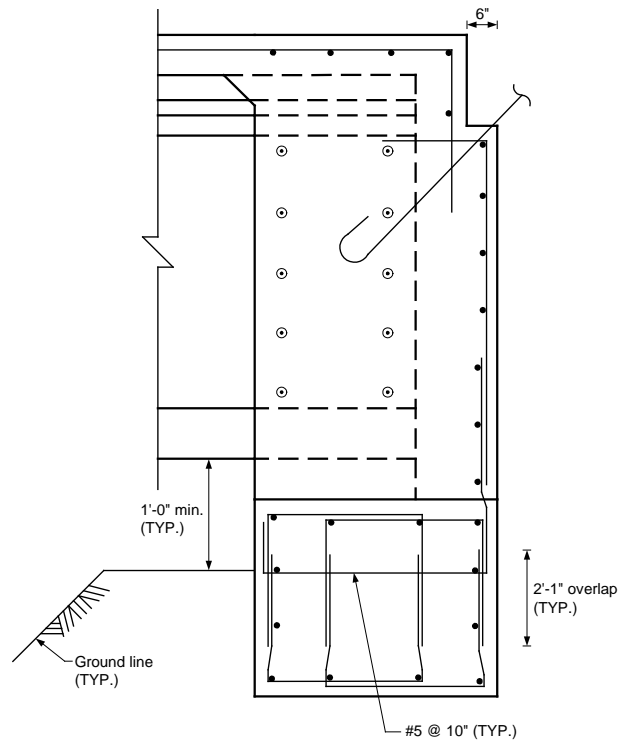


Figure 7.1-6 – Integral Abutment Reinforcement, Girder, No Pile at the Section

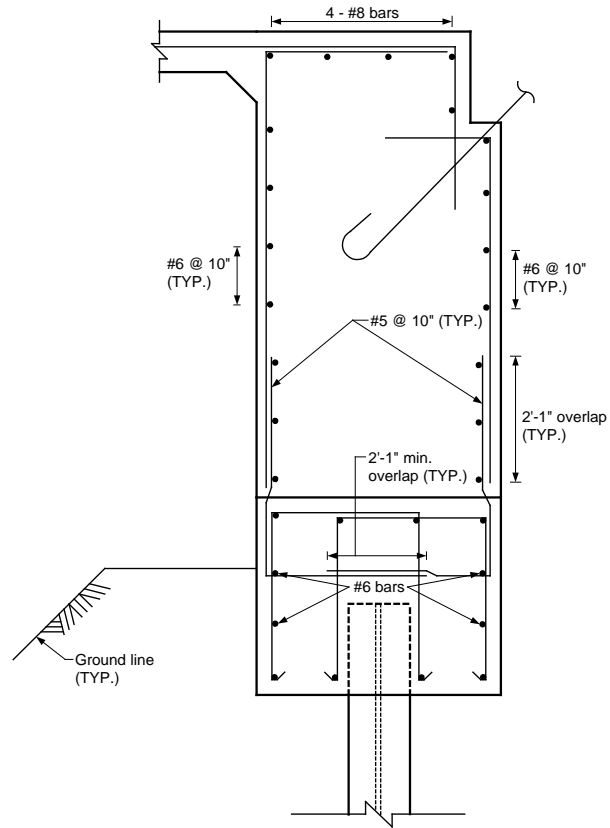
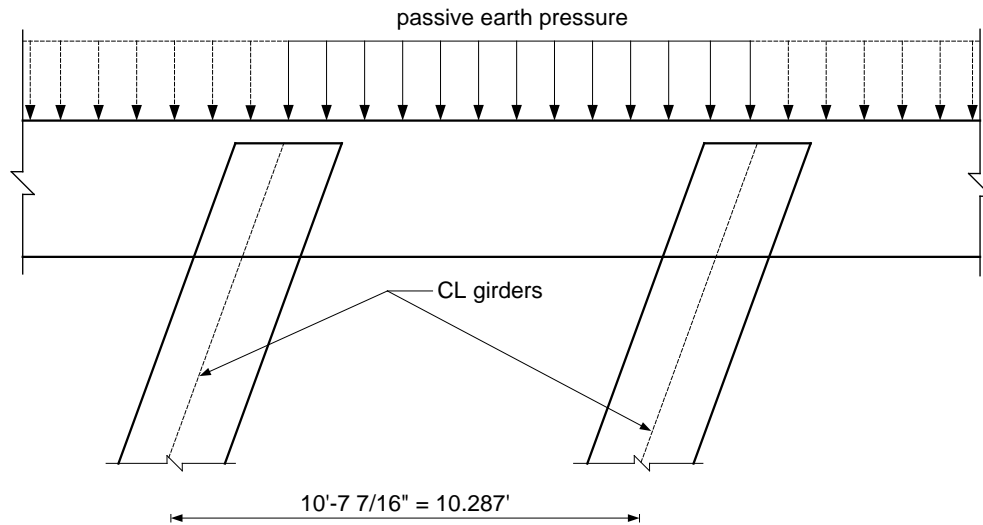


Figure 7.1-7 – Integral Abutment Reinforcement, Pile Without Girder

Design Step 7.1.4.1

Design the backwall as a horizontal beam resisting passive earth pressure



**Figure 7.1-8 – Passive Earth Pressure Applied to Backwall**

Calculate the adequacy of the backwall to resist passive pressure due to the abutment backfill material.

Passive earth pressure coefficient,  $k_p = (1 + \sin \phi)/(1 - \sin \phi)$   
 (Notice that  $k_p$  may also be obtained from Figure S3.11.5.4-1)

$$w_p = \frac{1}{2} \gamma z^2 k_p \quad (S3.11.5.1-1)$$

where:

$w_p$  = passive earth pressure per unit length of backwall (k/ft)

$\gamma$  = unit weight of soil bearing on the backwall (kcf)  
 = 0.130 kcf

$Z$  = height of the backwall from the bottom of the approach slab to the bottom of the pile cap (ft.)  
 = slab + haunch + girder depth + bearing pad thickness + pile cap depth – approach slab thickness  
 = (8/12) + (4/12) + 6 + (0.75/12) + 3.25 – 1.5  
 = 8.81 ft.



$\phi$  = internal friction of backfill soil assumed to be 30°

$$\begin{aligned} w_p &= \frac{1}{2} (0.130)(8.81)^2 [(1 + \sin 30)/(1 - \sin 30)] \\ &= 15.1 \text{ k/ft of wall} \end{aligned}$$

*Notice that developing full passive earth pressure requires relatively large displacement of the structure (0.01 to 0.04 of the height of the structure for cohesionless fill). The expected displacement of the abutment is typically less than that required to develop full passive pressure. However, these calculations are typically not critical since using full passive pressure is not expected to place high demand on the structure or cause congestion of reinforcement.*

No load factor for passive earth pressure is specified in the LRFD specifications. Assume the load factor is equal to that of the active earth pressure ( $\phi = 1.5$ ).

$$\begin{aligned} w_u &= \phi_{EH} w_p \\ &= 1.5(15.1) \\ &= 22.65 \text{ k/ft of wall} \end{aligned}$$

The backwall acts as a continuous horizontal beam supported on the girders, i.e., with spans equal to the girder spacing along the skew.

$$\begin{aligned} M_u &\cong w_u \ell^2 / 8 \\ &= 22.65(9.667/\cos 20)^2 / 8 \\ &= 300 \text{ k-ft/ft} \end{aligned}$$

Calculate the nominal flexural resistance,  $M_r$ , of the backwall.

$$M_r = \phi M_n \quad (\text{S5.7.3.2.1-1})$$

and

$$M_n = A_s f_y (d_s - a/2) \quad (\text{S5.7.3.2.2-1})$$

where:

$$\begin{aligned} A_s &= \text{area of the longitudinal reinforcement bars at front face (tension side) of} \\ &\quad \text{the abutment (9 \#6 bars)} \\ &= 9(0.44) \\ &= 3.96 \text{ in}^2 \end{aligned}$$

$$f_y = 60 \text{ ksi}$$

$$\begin{aligned} d_s &= \text{width of backwall} - \text{concrete cover} - \text{vertical bar dia.} - \frac{1}{2} \text{ bar dia.} \\ &= 3.0(12) - 3 - 0.625 - \frac{1}{2}(0.75) \\ &= 32.0 \text{ in.} \end{aligned}$$

$$A = A_s f_y / 0.85 f'_c b \quad (\text{S5.7.3.1.1-4})$$

where "b" is the height of the component

$$= 3.96(60)/[0.85(3)(119.75)]$$

$$= 0.78 \text{ in.}$$

$$M_n = 3.96(60)(32.0 - 0.78/2)/12$$

$$= 626 \text{ k-ft/ft}$$

Therefore, the factored flexural resistance, where  $\phi = 0.9$  for flexure (S5.5.4.2.1), is taken as:

$$M_r = 0.9(626)$$

$$= 563 \text{ k-ft/ft} > M_u = 300 \text{ k-ft/ft} \quad \mathbf{OK}$$

By inspection:

- $M_r > 4/3(M_u)$ .
- The depth of the compression block is small relative to the depth.

Check shear for the section of backwall between girders:

$$V_u = P_u \ell / 2$$

$$= 22.65(9.667 / \sin 20) / 2$$

$$= 116.5 \text{ k/ft}$$

The factored shear resistance,  $V_r$ , is calculated as:

$$V_r = \phi V_n \quad (\text{S5.8.2.1-2})$$

The nominal shear resistance,  $V_n$ , is calculated according to S5.8.3.3 and is the lesser of:

$$V_n = V_c + V_s \quad (\text{S5.8.3.3-1})$$

OR

$$V_n = 0.25f'_c b_v d_v \quad (\text{S5.8.3.3-2})$$

where:

$$V_c = 0.0316\beta\sqrt{f'_c}b_v d_v \quad (\text{S5.8.3.3-3})$$

$$\beta = 2.0$$

$$f'_c = 3.0 \text{ ksi}$$

$$b_v = \text{effective horizontal beam width taken as the abutment depth (in.)}$$

$$= 119.75 \text{ in.}$$

$$d_v = d_e - a/2$$

$$= 32.0 - (0.78/2)$$

$$= 31.61 \text{ in.}$$

$$\begin{aligned} 0.9d_e &= 0.9(32.0) \\ &= 28.8 \text{ in.} \end{aligned}$$

$$\begin{aligned} 0.72h &= 0.72(36) \\ &= 25.92 \text{ in.} \end{aligned}$$

Therefore,  $d_v$  should be taken as 31.61 in.

Ignore the contribution of the transverse reinforcement to the shear resistance (i.e.,  $V_s = 0$ ),  $V_n$  is taken as the smaller of:

$$\begin{aligned} V_c &= 0.0316(2.0)\sqrt{3}(119.75)(31.61) \\ &= 414.4 \text{ k/ft} \end{aligned}$$

OR

$$\begin{aligned} V_n &= 0.25(3)(119.75)(31.61) \\ &= 2,839 \text{ k/ft} \end{aligned}$$

Therefore, use the shear resistance due to the concrete,  $V_c$

$$\begin{aligned} V_r &= \phi V_n \\ &= 0.9(414.4) \\ &= 373.0 \text{ k/ft} > V_u = 116.5 \text{ k/ft} \quad \mathbf{OK} \end{aligned}$$

### Design Step 7.1.5 Wingwall design

*There is no widely accepted method of determining design loads for the wingwalls of integral abutments. The following design procedure will result in a conservative design as it takes into account maximum possible loads.*

*Two load cases are considered:*

*Load Case 1:*

*The wingwall is subjected to passive earth pressure. This case accounts for the possibility of the bridge moving laterally and pushing the wingwall against the fill. It is not likely that the displacement will be sufficient to develop full passive pressure. However, there is no available method to determine the expected pressure with certainty. This load case is considered under strength limit state.*

*Load Case 2:*

*The wingwall is subjected to active pressure and collision load on the parapet. Active pressure was considered instead of passive to account for the low probability that a collision load and passive pressure will exist simultaneously. This load case is considered at the extreme event limit state, i.e.  $\phi = 1.0$  (Table S3.4.1-1)*

Required information:

Angle of internal friction of fill, $\phi$	= 30 degrees
Coefficient of active earth pressure, $k_a$	= $(1 - \sin \phi)/(1 + \sin \phi)$ = 0.333
Coefficient of passive earth pressure, $k_p$	= $(1 + \sin \phi)/(1 - \sin \phi)$ = 3
$k_a/k_p$	= $0.333/3$ = 0.111

Load Case 1

From Figure 7.1-9 and utilizing properties of a right angle pyramid [volume =  $1/3(\text{base area})(\text{height})$  and the center of gravity (applied at a distance measured from the vertical leg of the right angle pyramid) =  $1/4$  base length].

Moment at the critical section for moment under passive pressure:

$$M_p = 0.2(14)(0.5)(14/2) + 0.2[14(8.31/2)](14/3) + (1/3)[3.24(8.31)(14/2)](14/4) = 284 \text{ k-ft}$$

Minimum required factored flexural resistance,  $M_r = 284 \text{ k-ft}$ .

$$M_r = \phi M_n \quad (\text{S5.7.3.2.1-1})$$

where:

$$M_n = \text{nominal resistance (k-ft)} \\ = M_p$$

$$\phi = 0.9 \text{ for flexure at the strength limit state (S5.5.4.2)}$$

$$\text{Min. required } M_n = 284/0.9 \\ = 316 \text{ k-ft}$$

Load Case 2

Moment on the critical section for moment under active pressure:

$$M_a = 0.111(284) \\ = 31.5 \text{ k-ft}$$

Moment from collision load on the parapet:

From SA13.2 for Test Level 5, the crash load on the parapet is equal to 124 kips and is applied over a length of 8 ft.

Maximum collision moment on the critical section:

$$\begin{aligned} M &= 124(14 - 8/2) \\ &= 1,240 \text{ k-ft} \end{aligned}$$

$$\begin{aligned} \text{Total moment for Load Case 2, } M_{\text{total}} &= 1,240 + 31.5 \\ &= 1,271.5 \text{ k-ft} \end{aligned}$$

The minimum required factored flexural resistance,  $M_r = 1,271.5 \text{ k-ft}$

$$M_r = \phi M_n \quad (\text{S5.7.3.2.1-1})$$

where:

$$\phi = 1.0 \text{ for flexure at the extreme event limit state}$$

$$\begin{aligned} \text{Min. required } M_n &= 1,271.5/1.0 \\ &= 1,271.5 \text{ k-ft} \end{aligned}$$

From the two cases of loading:

$$M_n \text{ required} = 1,271.5 \text{ k-ft}$$

Develop a section that provides the minimum nominal flexural resistance

Required information:

Assuming reinforcement of #8 @ 6 in.

Number of bars within the 10.3125 ft. height of the wing wall = 22 bars

Section thickness = parapet thickness at base  
= 20.25 in.

Concrete cover = 3 in.

The nominal flexural resistance,  $M_n$ , is taken as:

$$M_n = A_s f_y (d_s - a/2) \quad (\text{S5.7.3.2.2-1})$$

where:

$$d_s = \text{section thickness} - \text{cover} - \frac{1}{2} \text{ bar diameter}$$

$$= 20.25 - 3 - \frac{1}{2}(1.0)$$

$$= 16.75 \text{ in.}$$

$$A_s = 22(0.79)$$

$$= 17.38 \text{ in}^2$$

$$a = A_s f_y / 0.85 f'_c b \quad (\text{S5.7.3.1.1-4})$$

$$= 17.38(60) / [0.85(3)(123.75)]$$

$$= 3.30 \text{ in.}$$

$$M_n = A_s f_y (d_s - a/2)$$

$$= 17.38(60)(16.75 - 3.30/2) / 12$$

$$= 1,312 \text{ k-ft} > 1,271.5 \text{ k-ft required OK}$$

Secondary reinforcement of the wingwall is not by design, it is only meant for shrinkage. Use #6 @ 12 in. spacing as shown in Figure 7.1-10.

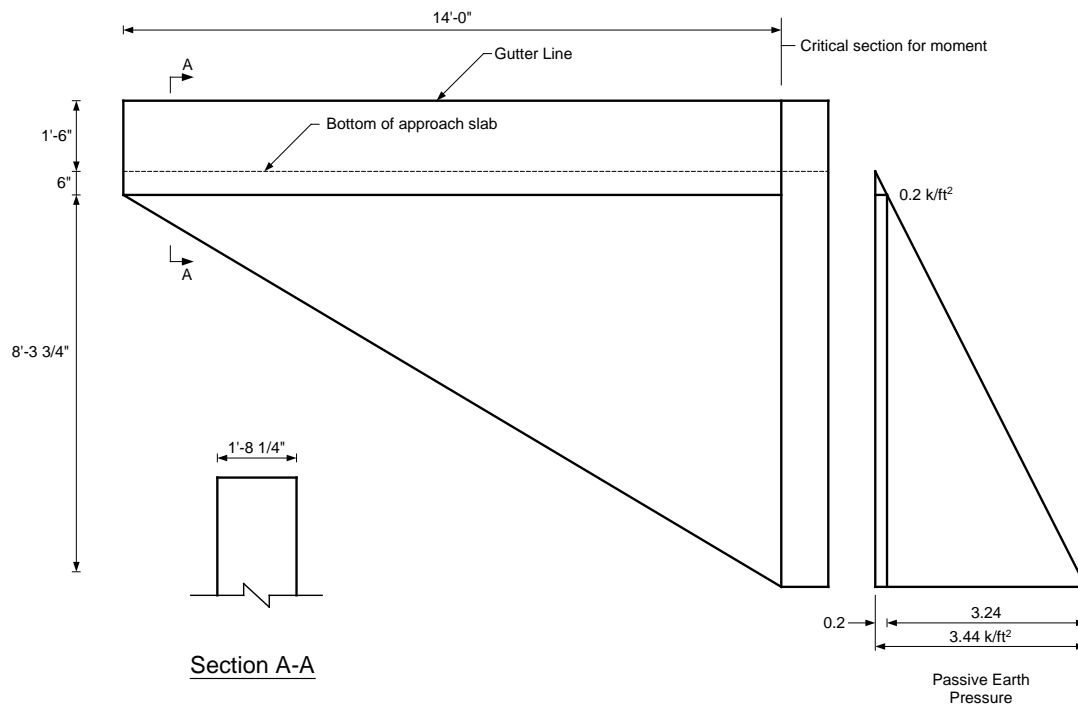


Figure 7.1-9 – Wingwall Dimensions

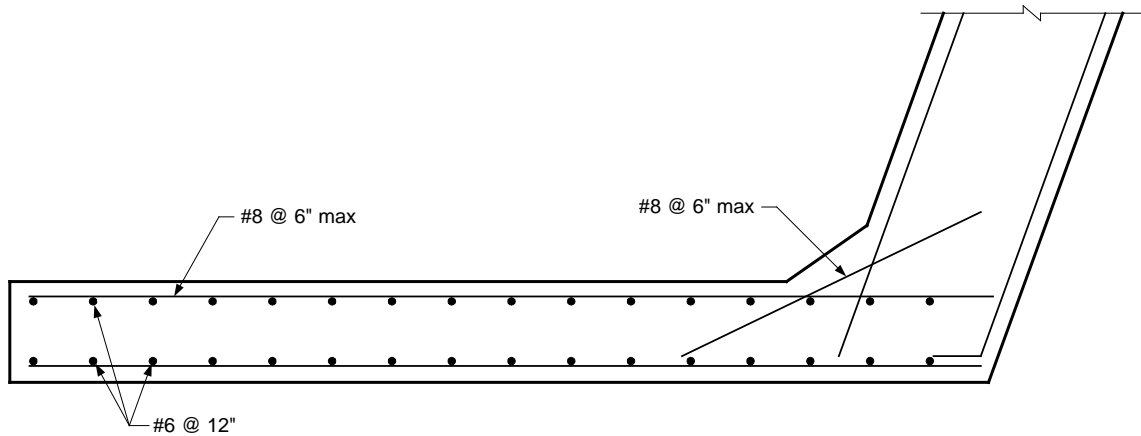


Figure 7.1-10 – Wingwall Reinforcement

**Design Step 7.1.6 Design of approach slab**

Approach slab loading for a 1 ft. wide strip:

$$\begin{aligned}
 W_{self} &= 0.15(1.5) \\
 &= 0.225 \text{ k/ft}
 \end{aligned}$$

$$W_{FWS} = 0.025 \text{ k/ft}$$

Factored distributed dead loading:

$$\begin{aligned}
 W_{Str I} &= 1.25(0.225) + 1.50(0.025) \\
 &= 0.32 \text{ k/ft}
 \end{aligned}$$

Live load distribution width (S4.6.2.3)

The equivalent strip width of longitudinal strips per lane for both shear and moment is calculated according to the provisions of S4.6.2.3.

- For single lane loaded

$$E = 10 + 5\sqrt{L_1 W_1} \quad (S4.6.2.3-1)$$

- For multiple lanes loaded

$$E = 84.0 + 1.44\sqrt{L_1 W_2} \leq \frac{12.0W}{N_L} \quad (\text{S4.6.2.3-2})$$

where:

$E$  = equivalent width (in.)

$L_1$  = modified span length taken equal to the lesser of the actual span or 60.0 ft. (ft.)

$W_1$  = modified edge-to-edge width of bridge taken to be equal to the lesser of the actual width or 60.0 ft. for multilane lading, or 30.0 ft. for single-lane loading (ft.)

$W$  = physical edge-to-edge width of bridge (ft.)

$N_L$  = number of design lanes as specified in S3.6.1.1.1

$$\begin{aligned} E_{\text{single}} &= 10 + 5\sqrt{25(30)} \\ &= 146.9 \text{ in.} \end{aligned}$$

$$\begin{aligned} E_{\text{mult.}} &= 84.0 + 1.44\sqrt{25(55.34)} \\ &= 137.6 \text{ in.} \end{aligned}$$

$$\frac{12.0(55.34)}{4} = 166.02 \text{ in.}$$

Therefore, the equivalent strip width is:

$$E = 137.6 \text{ in.}$$

Live load maximum moment:

$$\begin{aligned} \text{Lane load: max moment} &= 0.64(25)^2/8 \\ &= 50 \text{ k-ft} \end{aligned}$$

Truck load: max moment = 207.4 k-ft (from live load analysis output for a 25 ft. simple span)

$$\begin{aligned} \text{Total LL + IM} &= 50 + 1.33(207.4) \\ &= 325.8 \text{ k-ft} \end{aligned}$$

$$\begin{aligned} \text{Total LL + IM moment per unit width of slab} &= 325.8/(137.6/12) \\ &= 28.4 \text{ k-ft/ft} \end{aligned}$$

Maximum factored positive moment per unit width of slab due to dead load plus live load:



$$\begin{aligned}
 M_u &= w\ell^2/8 + 1.75(\text{LL} + \text{IM moment}) \\
 &= 0.32(25)^2/8 + 1.75(28.4) \\
 &= 74.7 \text{ k-ft}
 \end{aligned}$$

The factored flexural resistance,  $M_r$ , is taken as:

$$M_r = \phi M_n \quad (\text{S5.7.3.2.1-1})$$

and

$$M_n = A_s f_y (d - a/2) \quad (\text{S5.7.3.2.2-1})$$

where:

$$\begin{aligned}
 A_s &= \text{use \#9 bars at 9 in. spacing} \\
 &= 1.0(12/9) \\
 &= 1.33 \text{ in}^2 \text{ per one foot of slab}
 \end{aligned}$$

$$f_y = 60 \text{ ksi}$$

$$\begin{aligned}
 d &= \text{slab depth} - \text{cover (cast against soil)} - \frac{1}{2} \text{ bar diameter} \\
 &= 1.5(12) - 3 - \frac{1}{2}(1.128) \\
 &= 14.4 \text{ in.}
 \end{aligned}$$

$$\begin{aligned}
 a &= A_s f_y / 0.85 f'_c b \quad (\text{S5.7.3.1.1-4}) \\
 &= 1.33(60) / [0.85(3)(12)] \\
 &= 2.61 \text{ in.}
 \end{aligned}$$

$$\begin{aligned}
 M_n &= 1.33(60)(14.4 - 2.61/2)/12 \\
 &= 87.1 \text{ k-ft}
 \end{aligned}$$

Therefore,

$$\begin{aligned}
 M_r &= 0.9(87.1) \\
 &= 78.4 \text{ k-ft} > M_u = 74.7 \text{ k-ft} \quad \mathbf{OK}
 \end{aligned}$$

### Design Step 7.1.6.1

#### Bottom distribution reinforcement (S9.7.3.2)

For main reinforcement parallel to traffic, the minimum distribution reinforcement is taken as a percentage of the main reinforcement:

$$100/\sqrt{S} \leq 50\%$$

where:

$$S = \text{the effective span length taken as equal to the effective length specified in S9.7.2.3 (ft.)}$$

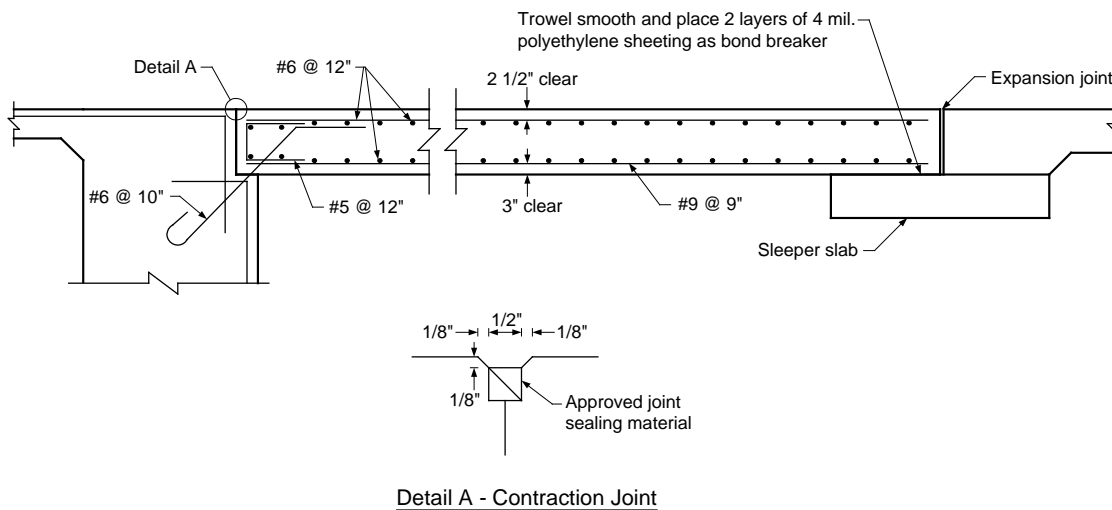
Assuming “S” is equal to the approach slab length,

$$100/\sqrt{25} = 20\%$$

Main reinforcement: #9 @ 9 in. = 1.0(12/9)  
 = 1.33 in<sup>2</sup>/ft

Required distribution reinforcement = 0.2(1.33)  
 = 0.27 in<sup>2</sup>/ft

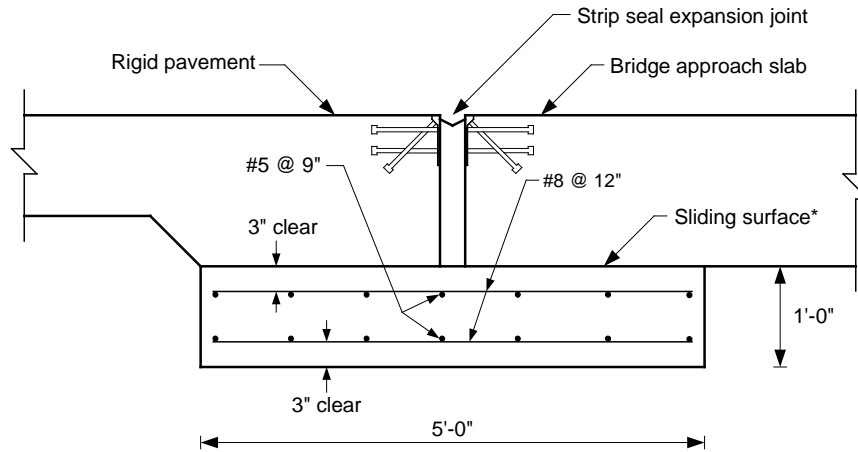
Use #6 @ 12 in. = 0.44 in<sup>2</sup>/ft > required reinforcement **OK**



**Figure 7.1-11 – Typical Approach Slab Reinforcement Details**

**Design Step 7.1.7 Sleeper slab**

*No design provisions are available for sleeper slabs. The reinforcement is typically shown as a standard detail. If desired, moment in the sleeper slab may be determined assuming the wheel load is applied at the midpoint of a length assumed to bridge over settled fill, say a 5 ft. span length.*



\* Trowel smooth and place 2 layers of 4 mil. polyethylene sheeting as bond breaker

**Figure 7.1-12 – Sleeper Slab Details Used by the Pennsylvania Department of Transportation**

**Design Step 7.2 INTERMEDIATE PIER DESIGN (This section has not been updated in 2015.)****Design Step 7.2.1 Substructure loads and load application**

*In the following sections, the word “pier” is used to refer to the intermediate pier or intermediate bent.*

Dead load

*Notice that the LRFD specifications include a maximum and minimum load factor for dead load. The intent is to apply the maximum or the minimum load factors to all dead loads on the structure. It is not required to apply maximum load factors to some dead loads and minimum load factors simultaneously to other dead loads to obtain the absolute maximum load effects.*

Live load transmitted from the superstructure to the substructure

*Accurately determining live load effects on intermediate piers always represented an interesting problem. The live load case of loading producing the maximum girder reactions on the substructure varies from one girder to another and, therefore, the case of loading that maximizes live load effects at any section of the substructure also varies from one section to another. The equations used to determine the girder live load distribution produce the maximum possible live load distributed to a girder without consideration to the live load distributed concurrently to the surrounding girders. This is adequate for girder design but is not sufficient for substructure design. Determining the concurrent girder reactions requires a three-dimensional modeling of the structure. For typical structures, this will be cumbersome and the return, in terms of more accurate results, is not justifiable. In the past, different jurisdictions opted to incorporate some simplifications in the application of live loads to the substructure and these procedures, which are independent of the design specifications, are still applicable under the AASHTO-LRFD design specifications. The goal of these simplifications is to allow the substructure to be analyzed as a two-dimensional frame. One common procedure is as follows:*

- *Live load reaction on the intermediate pier from one traffic lane is determined. This reaction from the live load uniform load is distributed over a 10 ft. width and the reaction from the truck is applied as two concentrated loads 6 ft. apart. This means that the live load reaction at the pier location from each traffic lane is a line load 10 ft. wide and two concentrated loads 6 ft. apart. The loads are assumed to fit within a 12 ft. wide traffic lane. The reactions from the uniform load and the truck may be moved within the width of the traffic lane, however, neither of the two truck axle loads may be placed closer than 2 ft. from the edge of the traffic lane.*
- *The live load reaction is applied to the deck at the pier location. The load is distributed to the girders assuming the deck acts as a series of simple spans supported on the girders. The girder reactions are then applied to the pier. In all cases, the appropriate multiple presence factor is applied.*

- *First, one lane is loaded. The reaction from that lane is moved across the width of the bridge. To maximize the loads, the location of the 12 ft. wide traffic lane is assumed to move across the full width of the bridge between gutter lines. Moving the traffic lane location in this manner provides for the possibility of widening the bridge in the future and/or eliminating or narrowing the shoulders to add additional traffic lanes. For each load location, the girder reactions transmitted to the pier are calculated and the pier itself is analyzed.*
- *Second, two traffic lanes are loaded. Each of the two lanes is moved across the width of the bridge to maximize the load effects on the pier. All possible combinations of the traffic lane locations should be included.*
- *The calculations are repeated for three lanes loaded, four lanes loaded and so forth depending on the width of the bridge.*
- *The maximum and minimum load effects, i.e. moment, shear, torsion and axial force, at each section from all load cases are determined as well as the other concurrent load effects, e.g. maximum moment and concurrent shear and axial loads. When a design provision involves the combined effect of more than one load effect, e.g. moment and axial load, the maximum and minimum values of each load effect and the concurrent values of the other load effects are considered as separate load cases. This results in a large number of load cases to be checked. Alternatively, a more conservative procedure that results in a smaller number of load cases may be used. In this procedure, the envelopes of the load effects are determined. For all members except for the columns and footings, the maximum values of all load effects are applied simultaneously. For columns and footings, two cases are checked, the case of maximum axial load and minimum moment and the case of maximum moment and minimum axial load.*

*This procedure is best suited for computer programs. For hand calculations, this procedure would be cumbersome. In lieu of this lengthy process, a simplified procedure used satisfactorily in the past may be utilized.*

#### Load combinations

*The live load effects are combined with other loads to determine the maximum factored loads for all applicable limit states. For loads other than live, when maximum and minimum load factors are specified, each of these two factored loads should be considered as separate cases of loading. Each section is subsequently designed for the controlling limit state.*

#### Temperature and shrinkage forces

*The effects of the change in superstructure length due to temperature changes and, in some cases, due to concrete shrinkage, are typically considered in the design of the substructure.*

*In addition to the change in superstructure length, the substructure member lengths also change due to temperature change and concrete shrinkage. The policy of including the effects of the substructure length change on the substructure forces varies from one jurisdiction to another. These effects on the pier cap are typically small and may be ignored without measurable effect on the design of the cap. However, the effect of the change in the pier cap length may produce a significant force in the columns of multiple column bents. This force is dependant on:*

- *The length and stiffness of the columns: higher forces are developed in short, stiff columns*
- *The distance from the column to the point of equilibrium of the pier (the point that does not move laterally when the pier is subjected to a uniform temperature change): Higher column forces develop as the point of interest moves farther away from the point of equilibrium. The point of equilibrium for a particular pier varies depending on the relative stiffness of the columns. For a symmetric pier, the point of equilibrium lies on the axis of symmetry. The column forces due to the pier cap length changes are higher for the outer columns of multi-column bents. These forces increase with the increase in the width of the bridge.*

### Torsion

*Another force effect that some computer design programs use in pier design is the torsion in the pier cap. This torsion is applied to the pier cap as a concentrated torque at the girder locations. The magnitude of the torque at each girder location is calculated differently depending on the source of the torque.*

- *Torque due to horizontal loads acting on the superstructure parallel to the bridge longitudinal axis: The magnitude is often taken equal to the horizontal load on the bearing under the limit state being considered multiplied by the distance from the point of load application to mid-height of the pier cap, e.g. braking forces are assumed to be applied 6 ft. above the deck surface.*
- *Torque due to noncomposite dead load on simple spans made continuous for live load: Torque at each girder location is taken equal to the difference between the product of the noncomposite dead load reaction and the distance to the mid-width of the cap for the two bearings under the girder line being considered.*

*According to SC5.8.2.1, if the factored torsional moment is less than one-quarter of the factored pure torsional cracking moment, it will cause only a very small reduction in shear capacity or flexural capacity and, hence, can be neglected. For pier caps, the magnitude of the torsional moments is typically small relative to the torsional cracking moments and, therefore, is typically ignored in hand calculations.*

*For the purpose of this example, a computer program that calculates the maximum and minimum of each load effect and the other concurrent load effects was used. Load effects due to substructure temperature expansion/contraction and concrete shrinkage were not included in the*

design. The results are listed in Appendix C. Selected values representing the controlling case of loading are used in the sample calculations.

**Superstructure dead load**

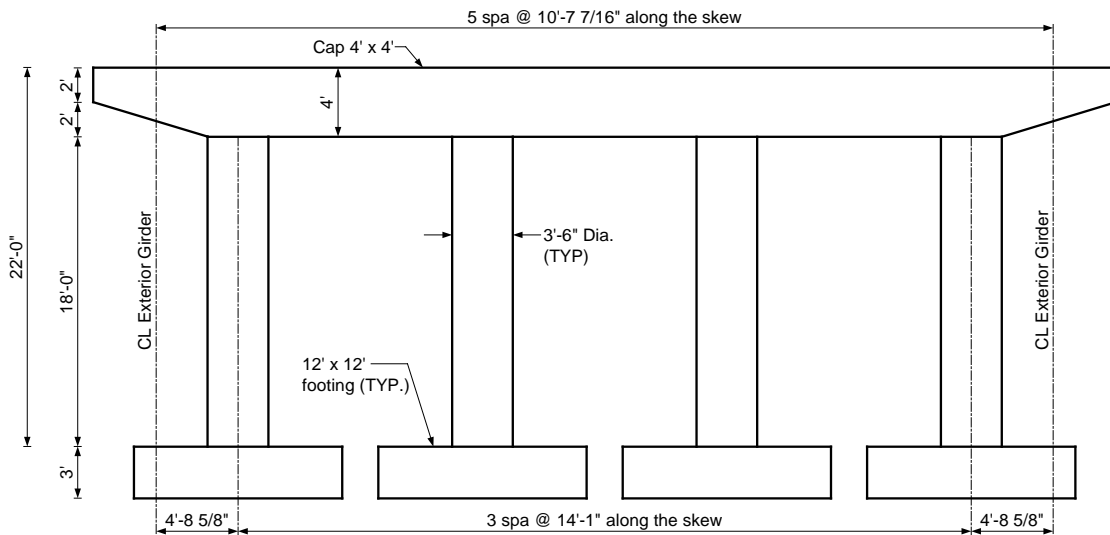
These loads can be obtained from Section 5.2 of the superstructure portion of this design example.

Summary of the unfactored loading applied vertically at each bearing (12 bearings total, 2 per girder line):

Girders (E/I)	= 61.6 k
Deck slab and haunch (E)	= 55.1 k
Deck slab and haunch (I)	= 62.2 k
Intermediate diaphragm (E)	= 1.3 k
Intermediate diaphragm (I)	= 2.5 k
Parapets (E/I)	= 14.8 k
Future wearing surface (E)	= 13.4 k
Future wearing surface (I)	= 19.9 k

- (E) – exterior girder
- (I) – interior girder

**Substructure dead load**



**Figure 7.2-1 – General Pier Dimensions**

Pier cap unfactored dead load

$$w_{\text{cap}} = (\text{cap cross-sectional area})(\text{unit weight of concrete})$$

Varying cross-section at the pier cap ends:

$$w_{\text{cap1}} = \text{varies linearly from } 2(2)(0.150) = 0.6 \text{ k/ft}$$

$$\text{to } 4(4)(0.150) = 2.4 \text{ k/ft}$$

Constant cross-section:

$$w_{\text{cap2}} = 4(4)(0.150)$$

$$= 2.4 \text{ k/ft}$$

OR

$$P_{\text{cap}} = 2.4(45.75) + [(2 + 4)/2](0.150)(13.167)$$

$$= 115.7 \text{ k}$$

Single column unfactored dead load

$$w_{\text{column}} = (\text{column cross sectional area})(\text{unit weight of concrete})$$

$$= \pi(1.75)^2(0.150)$$

$$= 1.44 \text{ k/ft}$$

OR

$$P_{\text{column}} = 1.44(18)$$

$$= 25.9 \text{ k}$$

Single footing unfactored dead load

$$w_{\text{footing}} = (\text{footing cross sectional area})(\text{unit weight of concrete})$$

$$= 12(12)(0.150)$$

$$= 21.6 \text{ k/ft}$$

OR

$$P_{\text{footing}} = 21.6(3)$$

$$= 64.8 \text{ k}$$

**Live load from the superstructure**

*Use the output from the girder live load analysis to obtain the maximum unfactored live load reactions for the interior and exterior girder lines.*

Summary of HL-93 live load reactions, without distribution factors or impact, applied vertically to each bearing (truck pair + lane load case governs for the reaction at the pier, therefore, the 90% reduction factor from S3.6.1.3.1 is applied):

$$\text{Maximum truck} = 59.5 \text{ k}$$



Minimum truck	= 0.0 k
Maximum lane	= 43.98 k
Minimum lane	= 0.0 k

**Braking force (BR) (S3.6.4)**

According to the specifications, the braking force shall be taken as the greater of:

*25 percent of the axle weight of the design truck or design tandem*

OR

*5 percent of the design truck plus lane load or 5 percent of the design tandem plus lane load*

*The braking force is placed in all design lanes which are considered to be loaded in accordance with S3.6.1.1.1 and which are carrying traffic headed in the same direction. These forces are assumed to act horizontally at a distance of 6 ft. above the roadway surface in either longitudinal direction to cause extreme force effects. Assume the example bridge can be a one-way traffic bridge in the future. The multiple presence factors in S3.6.1.1.2 apply.*

$$\begin{aligned} BR_1 &= 0.25(32 + 32 + 8)(4 \text{ lanes})(0.65)/1 \text{ fixed support} \\ &= 46.8 \text{ k} \end{aligned}$$

OR

$$\begin{aligned} BR_{2A} &= 0.05[72 + (110 + 110)(0.64)] \\ &= 10.6 \text{ k} \end{aligned}$$

$$\begin{aligned} BR_{2B} &= 0.05[(25 + 25) + 220(0.64)] \\ &= 9.54 \text{ k} \end{aligned}$$

where the subscripts are defined as:

- 1 – use the design truck to maximize the braking force
- 2A – check the design truck + lane
- 2B – check the design tandem + lane

Therefore, the braking force will be taken as 46.8 k (3.9 k per bearing or 7.8 k per girder) applied 6 ft. above the top of the roadway surface.

$$\begin{aligned} \text{Moment arm} &= 6 \text{ ft.} + \text{deck thickness} + \text{haunch} + \text{girder depth} \\ &= 6 + 0.667 + 0.333 + 6 \\ &= 13.0 \text{ ft. above the top of the bent cap} \end{aligned}$$

Apply the moment  $2(3.9)(13.0) = 101.4$  k-ft at each girder location.

**Wind load on superstructure (S3.8.1.2)**

*The pressures specified in the specifications are assumed to be caused by a base wind velocity,  $V_B$ , of 100 mph.*

*Wind load is assumed to be uniformly distributed on the area exposed to the wind. The exposed area is the sum of all component surface areas, as seen in elevation, taken perpendicular to the assumed wind direction. This direction is varied to determine the extreme force effects in the structure or in its components. Areas that do not contribute to the extreme force effect under consideration may be neglected in the analysis.*

*Base design wind velocity varies significantly due to local conditions. For small or low structures, such as this example, wind usually does not govern.*

*Pressures on windward and leeward sides are to be taken simultaneously in the assumed direction of wind.*

The direction of the wind is assumed to be horizontal, unless otherwise specified in S3.8.3. The design wind pressure, in KSF, may be determined as:

$$\begin{aligned} P_D &= P_B(V_{DZ}/V_B)^2 && \text{(S3.8.1.2.1-1)} \\ &= P_B(V_{DZ}^2/10,000) \end{aligned}$$

where:

$P_B$  = base wind pressure specified in Table S3.8.1.2.1-1 (ksf)

Since the bridge component heights are less than 30 ft. above the ground line,  $V_B$  is taken to be 100 mph.

#### Wind load transverse to the superstructure

$$F_{T \text{ Super}} = p_{wT}(H_{\text{wind}})[(L_{\text{back}} + L_{\text{ahead}})/2]$$

where:

$H_{\text{wind}}$  = the exposed superstructure height (ft.)  
 = girder + haunch + deck + parapet  
 = 6 + 0.333 + 0.667 + 3.5  
 = 10.5 ft.

$p_{wT}$  = transverse wind pressure values (ksf)  
 =  $P_B$  (use Table S3.8.1.2.2-1)

$L_{\text{back}}$  = span length to the deck joint, or end of bridge, back station from pier (ft.)  
 = 110 ft.

$L_{\text{ahead}}$  = span length to the deck joint, or end of bridge, ahead station from pier (ft.)  
 = 110 ft.

$$F_{T \text{ Super}} = 0.05(10.5)[(110 + 110)/2] = 57.8 \text{ k} \quad (0 \text{ degrees})$$

= 0.044(1,155)	= 50.8 k	(15 degrees)
= 0.041(1,155)	= 47.4 k	(30 degrees)
= 0.033(1,155)	= 38.1 k	(45 degrees)
= 0.017(1,155)	= 19.6 k	(60 degrees)

Wind load along axes of superstructure (longitudinal direction)

The longitudinal wind pressure loading induces forces acting parallel to the longitudinal axis of the bridge.

$$F_{L \text{ Super}} = p_{wL}(H_{\text{wind}})(L_{\text{back}} + L_{\text{ahead}})/n_{\text{fixed piers}}$$

where:

$$H_{\text{wind}} = 10.5 \text{ ft.}$$

$$p_{wL} = \text{Longitudinal wind pressure values (ksf)} \\ = P_B \text{ (use Table S3.8.1.2.2-1)}$$

$$L_{\text{back}} = 110 \text{ ft.}$$

$$L_{\text{ahead}} = 110 \text{ ft.}$$

$F_{L \text{ Super}} = 0.0(10.5)[(110 + 110)]/1$	= 0 k	(0 degrees)
= 0.006(2,310)/1	= 13.9 k	(15 degrees)
= 0.012(2,310)/1	= 27.7 k	(30 degrees)
= 0.016(2,310)/1	= 37.0 k	(45 degrees)
= 0.019(2,310)/1	= 43.9 k	(60 degrees)

The transverse and longitudinal pressures should be applied simultaneously.

Resultant wind load along axes of pier

*The transverse and longitudinal superstructure wind forces, which are aligned relative to the superstructure axis, are resolved into components that are aligned relative to the pier axes.*

Load perpendicular to the plane of the pier:

$$F_{L \text{ Pier}} = F_{L \text{ Super}} \cos(\theta_{\text{skew}}) + F_{T \text{ Super}} \sin(\theta_{\text{skew}})$$

At 0 degrees:

$$F_{L \text{ Pier}} = 0 \cos 20 + 57.8 \sin 20 \\ = 19.8 \text{ k}$$

At 60 degrees:

$$F_{L \text{ Pier}} = 43.9 \cos 20 + 19.6 \sin 20 \\ = 48.0 \text{ k}$$

Load in the plane of the pier (parallel to the line connecting the columns):

$$F_{T \text{ Pier}} = F_{L \text{ Super}} \sin(\theta_{\text{skew}}) + F_{T \text{ Super}} \cos(\theta_{\text{skew}})$$

At 0 degrees:

$$\begin{aligned} F_{T \text{ Pier}} &= 0 \sin 20 + 57.8 \cos 20 \\ &= 54.3 \text{ k} \end{aligned}$$

At 60 degrees:

$$\begin{aligned} F_{T \text{ Pier}} &= 43.9 \sin 20 + 19.6 \cos 20 \\ &= 33.4 \text{ k} \end{aligned}$$

The superstructure wind load acts at  $10.5/2 = 5.25$  ft. from the top of the pier cap.

*The longitudinal and transverse forces applied to each bearing are found by dividing the forces above by the number of girders. If the support bearing line has expansion bearings, the  $F_{L \text{ Super}}$  component in the above equations is zero.*

### Wind load on substructure (S3.8.1.2.3)

*The transverse and longitudinal forces to be applied directly to the substructure are calculated from an assumed base wind pressure of 0.040 ksf (S3.8.1.2.3). For wind directions taken skewed to the substructure, this force is resolved into components perpendicular to the end and front elevations of the substructures. The component perpendicular to the end elevation acts on the exposed substructure area as seen in end elevation, and the component perpendicular to the front elevation acts on the exposed areas and is applied simultaneously with the wind loads from the superstructure.*

$$W_{\text{wind on sub}} = W_{\text{cap}} + W_{\text{column}}$$

Transverse wind on the pier cap (wind applied perpendicular to the longitudinal axis of the superstructure):

$$\begin{aligned} W_{\text{cap}} &= 0.04(\text{cap width}) \\ &= 0.04(4) \\ &= 0.16 \text{ k/ft of cap height} \end{aligned}$$

Longitudinal wind on the pier cap (wind applied parallel to the longitudinal axis of the superstructure):

$$\begin{aligned} W_{\text{cap}} &= 0.04(\text{cap length along the skew}) \\ &= 0.04(58.93) \\ &= 2.36 \text{ k/ft of cap height} \end{aligned}$$

Transverse wind on the end column, this force is resisted equally by all columns:

$$\begin{aligned} W_{T, \text{column}} &= 0.04(\text{column diameter})/n_{\text{columns}} \\ &= 0.04(3.5)/4 \\ &= 0.035 \text{ k/ft of column height above ground} \end{aligned}$$

Longitudinal wind on the columns, this force is resisted by each of the columns individually:

$$\begin{aligned} W_{L, \text{column}} &= 0.04(\text{column diameter}) \\ &= 0.04(3.5) \\ &= 0.14 \text{ k/ft of column height above ground} \end{aligned}$$

There is no wind on the footings since they are assumed to be below ground level.

Total wind load on substructure:

$$W_{T \text{ wind on sub}} = 0.16 + 0.035 = 0.20 \text{ k/ft}$$

$$W_{L \text{ wind on sub}} = 2.36 + 0.14 = 2.50 \text{ k/ft}$$

### Wind on live load (S3.8.1.3)

*When vehicles are present, the design wind pressure is applied to both the structure and vehicles. Wind pressure on vehicles is represented by an interruptible, moving force of 0.10 klf acting normal to, and 6.0 ft. above, the roadway and is transmitted to the structure.*

*When wind on vehicles is not taken as normal to the structure, the components of normal and parallel force applied to the live load may be taken as follows with the skew angle taken as referenced normal to the surface.*

Use Table S3.8.1.3-1 to obtain  $F_w$  values,

$$F_{T \text{ Super}} = F_{WT}(L_{\text{back}} + L_{\text{ahead}})/2$$

$$\begin{aligned} F_{T \text{ Super}} &= 0.100(110 + 110)/2 &&= 11 \text{ k} && (0 \text{ degrees}) \\ &= 0.088(110) &&= 9.68 \text{ k} && (15 \text{ degrees}) \\ &= 0.082(110) &&= 9.02 \text{ k} && (30 \text{ degrees}) \\ &= 0.066(110) &&= 7.26 \text{ k} && (45 \text{ degrees}) \\ &= 0.034(110) &&= 3.74 \text{ k} && (60 \text{ degrees}) \end{aligned}$$

$$F_{L \text{ Super}} = F_{WL}(L_{\text{back}} + L_{\text{ahead}})/n_{\text{fixed piers}}$$

$$\begin{aligned} F_{L \text{ Super}} &= 0(110 + 110)/1 &&= 0 \text{ k} && (0 \text{ degrees}) \\ &= 0.012(220) &&= 2.64 \text{ k} && (15 \text{ degrees}) \\ &= 0.024(220) &&= 5.28 \text{ k} && (30 \text{ degrees}) \end{aligned}$$

$$\begin{aligned} &= 0.032(220) &= 7.04 \text{ k} & (45 \text{ degrees}) \\ &= 0.038(220) &= 8.36 \text{ k} & (60 \text{ degrees}) \end{aligned}$$

$$F_{WLL} = 11 \text{ k} \quad (\text{transverse direction, i.e., perpendicular to longitudinal axis of the superstructure})$$

**Temperature force (S3.12.2)**

*Due to the symmetry of the bridge superstructure, no force is developed at the intermediate bent due to temperature expansion/shrinkage of the superstructure.*

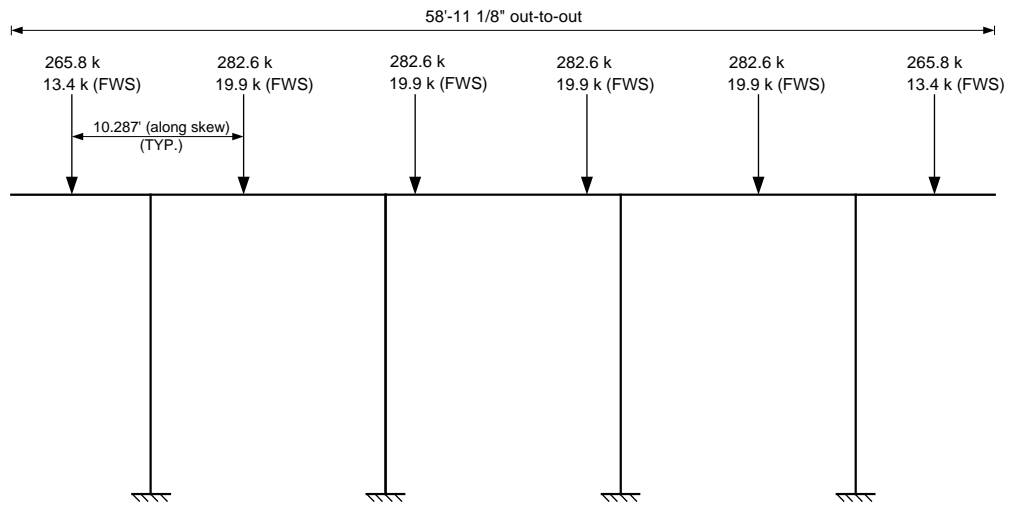
**Shrinkage (S3.12.4)**

*Due to the symmetry of the bridge superstructure, no force is developed at the intermediate bent due to shrinkage of the superstructure.*

**Load combinations**

Figures 7.2-2 and 7.2-3 show the unfactored loads applied to the bent from the superstructure and wind.

Superstructure Dead Loads



Substructure Dead Loads

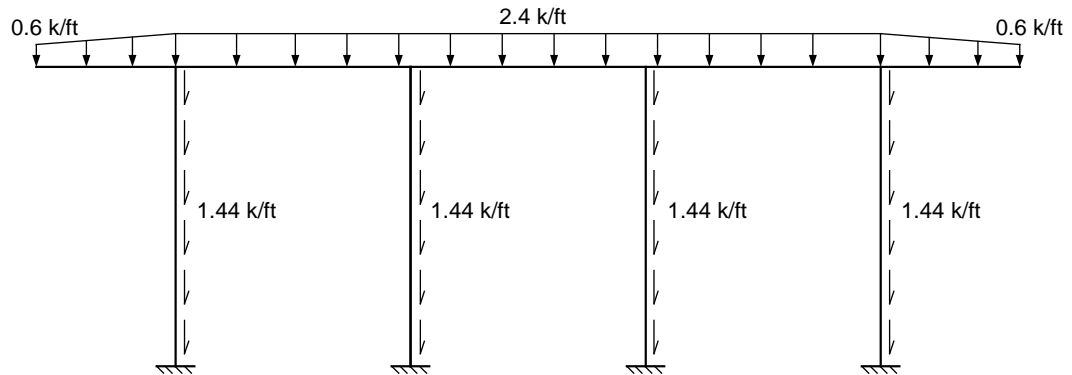
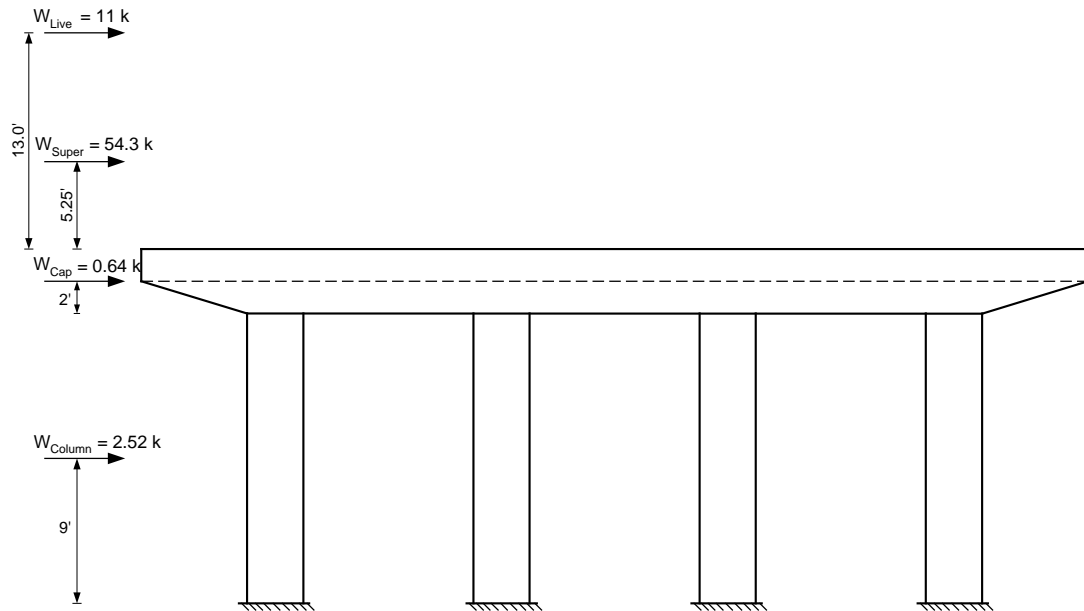


Figure 7.2-2 – Super- and Substructure Applied Dead Loads

Transverse Wind on Structure



Longitudinal Wind on Structure

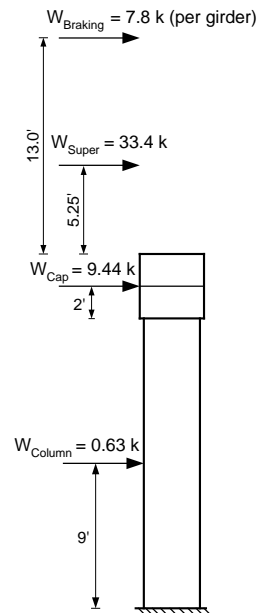


Figure 7.2-3 – Wind and Braking Loads on Super- and Substructure



## Design Step 7.2.2 Pier cap design

Required information:

General (these values are valid for the entire pier cap):

$$f'_c = 3.0 \text{ ksi}$$

$$\beta_1 = 0.85$$

$$f_y = 60 \text{ ksi}$$

Cap width = 4 ft.

Cap depth = 4 ft. (varies at ends)

No. stirrup legs = 6

Stirrup diameter = 0.625 in. (#5 bars)

Stirrup area = 0.31 in<sup>2</sup> (per leg)

Stirrup spacing = varies along cap length

Side cover = 2 in. (Table S5.12.3-1)

Cap bottom flexural bars:

No. bars in bottom row, positive region = 9 (#8 bars)

Positive region bar diameter = 1.0 in.

Positive region bar area,  $A_s = 0.79 \text{ in}^2$

Bottom cover = 2 in. (Table S5.12.3-1)

Cap top flexural bars:

No. bars in top row, negative region = 14 (7 sets of 2 #9 bars bundled horizontally)

Negative region bar diameter = 1.128 in.

Negative region bar area,  $A_s = 1.0 \text{ in}^2$

Top cover = 2 in. (Table S5.12.3-1)

From the analysis of the different applicable limit states, the maximum load effects on the cap were obtained. These load effects are listed in Table 7.2-1. The maximum factored positive moment occurs at 44.65 ft. from the cap end under Strength I limit state.

**Table 7.2-1 – Strength I Limit State for Critical Locations in the Pier Cap (Maximum Positive Moment, Negative Moment and Shear)**

	Location*	Unfactored Responses				Str-I
		DC	DW	LL + IM	BR	
Max Pos M (k-ft)	44.65 ft.	147.5	37.1	437.9	5.2	1,015.5
Max Neg M (k-ft)	6.79 ft.	-878.5	-84.9	-589.0	-1.9	-2,259.4
Max Shear (k)	34.96 ft.	292.9	39.5	210.4	2.8	798.3

\*measured from the end of the cap

Notes:

- DC: superstructure dead load (girders, slab and haunch, diaphragms, and parapets) plus the substructure dead load (all components)
- DW: dead load due to the future wearing surface
- LL + IM: live load + impact transferred from the superstructure
- BR: braking load transferred from the superstructure
- Str-I: load responses factored using Strength I limit state load factors

**Design Step 7.2.2.1**Pier cap flexural resistance (S5.7.3.2)

The factored flexural resistance,  $M_r$ , is taken as:

$$M_r = \phi M_n \quad (\text{S5.7.3.2.1-1})$$

where:

$$\begin{aligned} \phi &= \text{flexural resistance factor as specified in S5.5.4.2} \\ &= 0.9 \end{aligned}$$

$$M_n = \text{nominal resistance (k-in)}$$

*For calculation of  $M_n$ , use the provisions of S5.7.3.2.3 which state, for rectangular sections subjected to flexure about one axis, where approximate stress distribution specified in S5.7.2.2 is used and where the compression flange depth is **not** less than “c” as determined in accordance with Eq. S5.7.3.1.1-3, the flexural resistance  $M_n$  may be determined by using Eq. S5.7.3.1.1-1 through S5.7.3.2.2-1, in which case “ $b_w$ ” is taken as “b”.*

*Rectangular section behavior is used to design the pier cap. The compression reinforcement is neglected in the calculation of the flexural resistance.*

## Design Step 7.2.2.2

Maximum positive moment

Applied Strength I moment,  $M_u = 1,015.5$  k-ft

Applied Service I moment,  $M_s = 653.3$  k-ft (from computer software)

Axial load on the pier cap is small, therefore, the effects of axial load is neglected in this example.

Check positive moment resistance (bottom steel)

Calculate the nominal flexural resistance according to S5.7.3.2.3.

$$M_n = A_s f_y (d_s - a/2) \quad (\text{S5.7.3.2.2-1})$$

Determine  $d_s$ , the corresponding effective depth from the extreme fiber to the centroid of the tensile force in the tensile reinforcement.

$$d_s = \text{cap depth} - \text{CSG}_b$$

where:

$$\begin{aligned} \text{CSG}_b &= \text{distance from the centroid of the bottom bars to the bottom of the cap (in.)} \\ &= \text{cover} + \text{stirrup diameter} + \frac{1}{2} \text{ bar diameter} \\ &= 2 + 0.625 + \frac{1}{2} (1.0) \\ &= 3.125 \text{ in.} \end{aligned}$$

$$\begin{aligned} d_s &= 4(12) - 3.125 \\ &= 44.875 \text{ in.} \end{aligned}$$

$$\begin{aligned} A_s &= (\text{nbars Tension})(A_s \text{ bar}) \\ &= 9(0.79) \\ &= 7.1 \text{ in}^2 \end{aligned}$$

Determine “a” using Eq. S5.7.3.1.1-4

$$\begin{aligned} a &= A_s f_y / 0.85 f'_c b \quad (\text{S5.7.3.1.1-4}) \\ &= 7.1(60) / [0.85(3)(48)] \\ &= 3.48 \text{ in.} \end{aligned}$$

Calculate the nominal flexural resistance,  $M_n$

$$\begin{aligned} M_n &= A_s f_y (d_s - a/2) \quad (\text{S5.7.3.2.2-1}) \\ &= 7.1(60)[44.875 - (3.48/2)]/12 \\ &= 1,531 \text{ k-ft} \end{aligned}$$

Therefore, the factored flexural resistance,  $M_r$ , can be calculated as follows:

$$\begin{aligned} M_r &= 0.9(1,531) \\ &= 1,378 \text{ k-ft} > M_u = 1015.5 \text{ k-ft} \quad \mathbf{OK} \end{aligned}$$

Limits for reinforcement (S5.7.3.3)

Check if the section is over-reinforced.

The maximum amount of nonprestressed reinforcement shall be such that:

$$c/d_e \leq 0.42 \quad (\text{S5.7.3.3.1-1})$$

where:

$$\begin{aligned} c &= a/\beta_1 \\ &= 3.48/0.85 \\ &= 4.1 \text{ in.} \end{aligned}$$

$$\begin{aligned} d_e &= d_s \\ &= 44.875 \text{ in.} \end{aligned}$$

$$\begin{aligned} c/d_e &= 4.1/44.875 \\ &= 0.091 < 0.42 \quad \mathbf{OK} \end{aligned}$$

Check the minimum reinforcement requirements (S5.7.3.3.2)

Unless otherwise specified, at any section of a flexural component, the amount of nonprestressed tensile reinforcement must be adequate to develop a factored flexural resistance,  $M_r$ , at least equal to the lesser of:

$$1.2M_{cr} = 1.2f_r S$$

where:

$$\begin{aligned} f_r &= 0.24\sqrt{f'_c} && (\text{S5.4.2.6}) \\ &= 0.24\sqrt{3} \\ &= 0.42 \text{ ksi} \end{aligned}$$

$$\begin{aligned} S &= bh^2/6 \\ &= 4(12)[4(12)]^2/6 \\ &= 18,432 \text{ in}^3 \end{aligned}$$

$$1.2M_{cr} = 1.2(0.42)(18,432)/12 \\ = 774.1 \text{ k-ft}$$

OR

$$1.33M_u = 1.33(1,015.5) \\ = 1,351 \text{ k-ft}$$

Minimum required section resistance = 774.1 k-ft

Provided section resistance = 1,378 k-ft > 774.1 k-ft **OK**Check the flexural reinforcement distribution (S5.7.3.4)Check allowable stress,  $f_s$ 

$$f_{s, \text{allow}} = Z/[(d_c A)^{1/3}] \leq 0.6f_y \quad (\text{S5.7.3.4-1})$$

where:

$$Z = \text{crack width parameter (k/in)} \\ = 170 \text{ k/in (moderate exposure conditions are assumed)}$$

$$d_c = \text{distance from the extreme tension fiber to the center of the closest bar (in.)} \\ = \text{clear cover} + \text{stirrup diameter} + \frac{1}{2} \text{ bar diameter}$$

The cover on the bar under investigation cannot exceed 2.0 in., therefore, the stirrup diameter is not taken into account for  $d_c$  is:

$$= 2 + \frac{1}{2}(1.0) \\ = 2.5 \text{ in.}$$

$A$  = area having the same centroid as the principal tensile reinforcement and bounded by the surfaces of the cross-section and a straight line parallel to the neutral axis, divided by the number of bars ( $\text{in}^2$ )

$$= 2d_c(\text{cap width})/n_{\text{bars}} \\ = 2(2.5)(48)/9 \\ = 26.7 \text{ in}^2$$

$$f_{s, \text{allow}} = Z/[(d_c A)^{1/3}] \\ = 170/[(2.5)(26.7)]^{1/3} \\ = 41.9 \text{ ksi} > 0.6(60) = 36 \text{ ksi} \text{ therefore, } f_{s, \text{allow}} = 36 \text{ ksi}$$

Check service load applied steel stress,  $f_{s, \text{actual}}$ For 3.0 ksi concrete, the modular ratio,  $n = 9$  (see S6.10.3.1.1b or calculate by dividing the steel modulus of elasticity by the concrete and rounding up as required by S5.7.1)

Assume the stresses and strains vary linearly.

From the load analysis of the bent:

Dead load + live load positive service load moment = 653.3 k-ft

The transformed moment of inertia is calculated assuming elastic behavior, i.e., linear stress and strain distribution. In this case, the first moment of area of the transformed steel on the tension side about the neutral axis is assumed equal to that of the concrete in compression.

Assume the neutral axis at a distance “y” from the compression face of the section.

The section width equals 48 in.

Transformed steel area = (total steel bar area)(modular ratio) = 7.1(9) = 63.9 in<sup>2</sup>

By equating the first moment of area of the transformed steel about that of the concrete, both about the neutral axis:

$$63.9(44.875 - y) = 48y(y/2)$$

Solving the equation results in  $y = 9.68$  in.

$$\begin{aligned} I_{\text{transformed}} &= A_{\text{ts}}(d_s - y)^2 + by^3/3 \\ &= 63.9(44.875 - 9.68)^2 + 48(9.68)^3/3 \\ &= 93,665 \text{ in}^4 \end{aligned}$$

Stress in the steel,  $f_{s, \text{actual}} = (M_s c/I)n$ , where M is the moment action on the section.

$$\begin{aligned} f_{s, \text{actual}} &= [653.3(12)(35.195)/93,665]9 \\ &= 26.5 \text{ ksi} < f_{s, \text{allow}} = 36 \text{ ksi} \quad \mathbf{OK} \end{aligned}$$

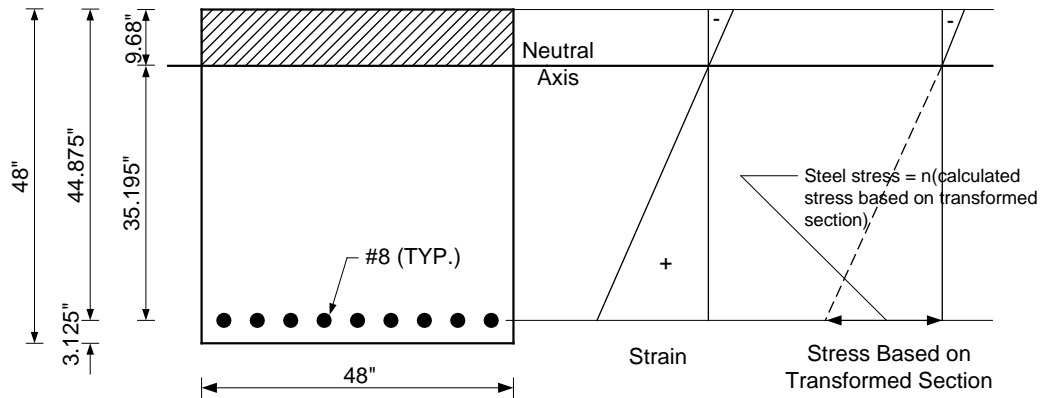


Figure 7.2-4 – Crack Control for Positive Reinforcement Under Service Load

Design Step 7.2.2.3

Maximum negative moment

From the bent analysis, the maximum factored negative moment occurs at 6.79 ft. from the cap edge under Strength I limit state:

Applied Strength I moment,  $M_u = -2,259.4$  k-ft

Applied Service I moment,  $M_s = -1,572.4$  k-ft (from computer analysis)

Check negative moment resistance (top steel)

Calculate  $M_n$  using Eq. S5.7.3.2.2-1.

Determine  $d_s$ , the corresponding effective depth from the extreme fiber to the centroid of the tensile force in the tensile reinforcement. The compressive reinforcement is neglected in the calculation of the nominal flexural resistance.

$$d_s = \text{cap depth} - CGS_t$$

where:

$$\begin{aligned} CGS_t &= \text{distance from the centroid of the top bars to the top of the cap (in.)} \\ &= \text{cover} + \text{stirrup diameter} + \frac{1}{2} \text{ bar diameter} \\ &= 2 + 0.625 + \frac{1}{2} (1.128) \\ &= 3.189 \text{ in.} \end{aligned}$$

$$\begin{aligned}d_s &= 4(12) - 3.189 \\ &= 44.81 \text{ in.}\end{aligned}$$

$$\begin{aligned}A_s &= (n_{\text{bars Tension}})(A_{s \text{ bar}}) \\ &= 14(1.0) \\ &= 14.0 \text{ in}^2\end{aligned}$$

Determine “a” using Eq. S5.7.3.1.1-4

$$\begin{aligned}a &= A_s f_y / 0.85 f'_c b && \text{(S5.7.3.1.1-4)} \\ &= 14.0(60) / [(0.85)(3)(4)(12)] \\ &= 6.86 \text{ in.}\end{aligned}$$

Calculate the nominal flexural resistance,  $M_n$

$$\begin{aligned}M_n &= 14.0(60)[44.81 - (6.86/2)]/12 \\ &= 2,897 \text{ k-ft}\end{aligned}$$

Therefore, the factored flexural resistance,  $M_r$ :

$$\begin{aligned}M_r &= 0.9(2,897) \\ &= 2,607 \text{ k-ft} > M_u = |-2,259.4| \text{ k-ft} \quad \mathbf{OK}\end{aligned}$$

#### Limits for reinforcement (S5.7.3.3)

Check if the section is over-reinforced.

The maximum amount of nonprestressed reinforcement shall be such that:

$$c/d_e \leq 0.42 \quad \text{(S5.7.3.3.1-1)}$$

where:

$$\begin{aligned}c &= a/\beta_1 \\ &= 6.86/0.85 \\ &= 8.07 \text{ in.}\end{aligned}$$

$$\begin{aligned}d_e &= d_s \\ &= 44.81 \text{ in.}\end{aligned}$$

$$\begin{aligned}c/d_e &= 8.07/44.81 \\ &= 0.18 < 0.42 \quad \mathbf{OK}\end{aligned}$$



Check minimum reinforcement (S5.7.3.3.2)

Unless otherwise specified, at any section of a flexural component, the amount of nonprestressed tensile reinforcement shall be adequate to develop a factored flexural resistance,  $M_r$ , at least equal to the lesser of:

$$1.2M_{cr} = 1.2f_r S$$

where:

$$\begin{aligned} f_r &= 0.24\sqrt{f'_c} && \text{(S5.4.2.6)} \\ &= 0.24\sqrt{3} \\ &= 0.42 \text{ ksi} \end{aligned}$$

$$\begin{aligned} S &= bh^2/6 \\ &= 4(12)[4(12)]^2/6 \\ &= 18,432 \text{ in}^3 \end{aligned}$$

$$\begin{aligned} 1.2M_{cr} &= 1.2(0.42)(18,432)/12 \\ &= 774.1 \text{ k-ft} \end{aligned}$$

OR

$$\begin{aligned} 1.33M_u &= 1.33(-2,259.4) \\ &= |-3,005| \text{ k-ft} \end{aligned}$$

Minimum required section resistance = 774.1 k-ft

Provided section resistance = 2,607 k-ft > 774.1 k-ft **OK**

Check the flexural reinforcement distribution (S5.7.3.4)

Check the allowable stress,  $f_s$

$$f_{s, \text{allow}} = Z/[(d_c A)^{1/3}] \leq 0.6f_y \quad \text{(S5.7.3.4-1)}$$

where:

$$Z = 170 \text{ k/in. (moderate exposure conditions are assumed)}$$

$$\begin{aligned} d_c &= 2 + \frac{1}{2}(1.128) \\ &= 2.56 \text{ in.} \end{aligned}$$

A = area having the same centroid as the principal tensile reinforcement and bounded by the surfaces of the cross-section and a straight line parallel to the neutral axis, divided by the number of bars ( $\text{in}^2$ )

$$\begin{aligned} &= 2d_c(\text{cap width})/n_{\text{bars}} \\ &= 2(2.56)(48)/14 \\ &= 17.6 \text{ in}^2 \end{aligned}$$

$$\begin{aligned}
 f_{s, \text{allow}} &= Z/[(d_c A)^{1/3}] \\
 &= 170/[2.56(17.6)]^{1/3} \\
 &= 47.8 \text{ ksi} > 0.6(60) = 36 \text{ ksi} \quad \mathbf{OK}, \text{ therefore, use } f_{s, \text{allow}} = 36 \text{ ksi}
 \end{aligned}$$

Check the service load applied steel stress,  $f_{s, \text{actual}}$

For 3.0 ksi concrete, the modular ratio,  $n = 9$

Assume the stresses and strains vary linearly.

From the load analysis of the bent:

Dead load + live load negative service load moment = -1,572.4 k-ft

The transformed moment of inertia is calculated assuming elastic behavior, i.e., linear stress and strain distribution. In this case, the first moment of area of the transformed steel on the tension side about the neutral axis is assumed equal to that of the concrete in compression.

Assume the neutral axis at a distance “y” from the compression face of the section.

Section width = 48 in.

Transformed steel area = (total steel bar area)(modular ratio) = 14.0(9) = 126 in<sup>2</sup>

By equating the first moment of area of the transformed steel about that of the concrete, both about the neutral axis:

$$126(44.81 - y) = 48y(y/2)$$

Solving the equation results in  $y = 12.9$  in.

$$\begin{aligned}
 I_{\text{transformed}} &= A_{ts}(d_s - y)^2 + by^3/3 \\
 &= 126(44.81 - 12.9)^2 + 48(12.9)^3/3 \\
 &= 162,646 \text{ in}^4
 \end{aligned}$$

Stress in the steel,  $f_{s, \text{actual}} = (M_s c/I)n$ , where M is the moment action on the section.

$$\begin{aligned}
 f_{s, \text{actual}} &= [|-1,572.4|(12)(31.91)/162,646]9 \\
 &= 33.3 \text{ ksi} < f_{s, \text{allow}} = 36 \text{ ksi} \quad \mathbf{OK}
 \end{aligned}$$

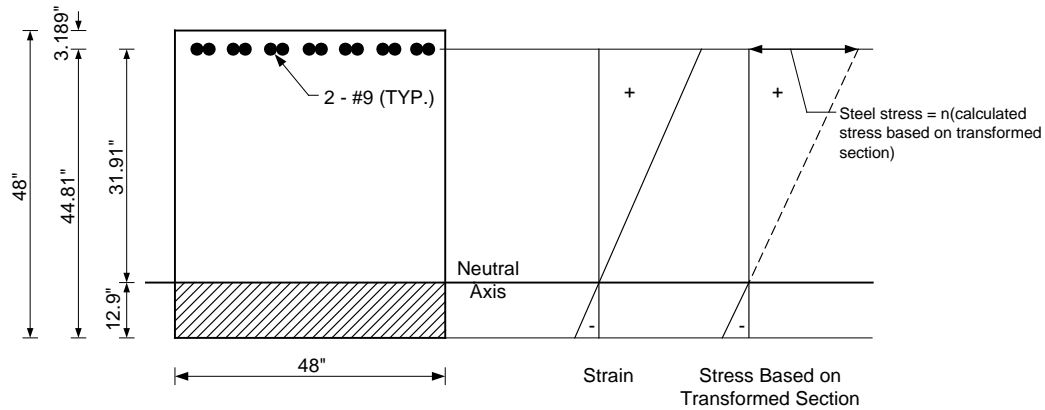


Figure 7.2-5 – Crack Control for Negative Reinforcement Under Service Load

Design Step 7.2.2.4

Check minimum temperature and shrinkage steel (S5.10.8)

Reinforcement for shrinkage and temperature stresses is provided near the surfaces of the concrete exposed to daily temperature changes and in structural mass concrete. Temperature and shrinkage reinforcement is added to ensure that the total reinforcement on exposed surfaces is not less than that specified below.

Using the provisions of S5.10.8.2,

$$A_{s, \min 1} = 0.11A_g/f_y \quad (S5.10.8.2-1)$$

where:

$$\begin{aligned} A_g &= \text{gross area of section (in}^2\text{)} \\ &= [4(12)]^2 \\ &= 2,304 \text{ in}^2 \end{aligned}$$

$$\begin{aligned} A_{s, \min 1} &= 0.11(2,304)/60 \\ &= 4.2 \text{ in}^2 \end{aligned}$$

This area is to be divided between the two faces, i.e., 2.1 in<sup>2</sup> per face. Shrinkage and temperature reinforcement must not be spaced farther apart than 3.0 times the component thickness or 18.0 in.

Use 4 #7 bars per face.

$$\begin{aligned} A_{s \text{ provided}} &= 4(0.6) \\ &= 2.4 \text{ in}^2 > 2.1 \text{ in}^2 \text{ OK} \end{aligned}$$

Design Step 7.2.2.5

Skin reinforcement (S5.7.3.4)

If the effective depth,  $d_e$ , of the reinforced concrete member exceeds 3 ft., longitudinal skin reinforcement is uniformly distributed along both side faces of the component for a distance of  $d/2$  nearest the flexural tension reinforcement. The area of skin reinforcement ( $\text{in}^2/\text{ft}$  of height) on each side of the face is required to satisfy:

$$A_{sk} \geq 0.012(d_e - 30) \leq (A_s + A_{ps})/4 \quad (\text{S5.7.3.4-4})$$

where:

$A_{ps}$  = area of prestressing ( $\text{in}^2$ )

$d_e$  = flexural depth taken as the distance from the compression face of the centroid of the steel, positive moment region (in.)

$$\begin{aligned} A_{sk} &= 0.012(44.875 - 30) \\ &= 0.179 \text{ in}^2/\text{ft} \leq 14.0/4 = 3.5 \text{ in}^2/\text{ft} \end{aligned}$$

Required  $A_{sk}$  per face =  $0.179(4) = 0.72 \text{ in}^2 < 2.4 \text{ in}^2$  provided **OK**

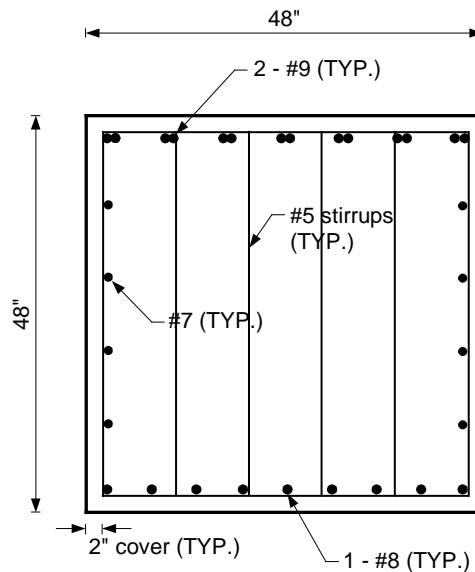


Figure 7.2-6 - Cap Cross-Section

## Design Step 7.2.2.6

Maximum shear

From analysis of the bent, the maximum factored shear occurs at 34.96 ft. from the cap end under Strength I limit state:

Shear,  $V_u = 798.3$  k

Calculate the nominal shear resistance using S5.8.3.3.

The factored shear resistance,  $V_r$

$$V_r = \phi V_n \quad (\text{S5.8.2.1-2})$$

where:

$\phi = 0.9$ , shear resistance factor as specified in S5.5.4.2

$V_n =$  nominal shear resistance (k)

The nominal shear resistance,  $V_n$ , shall be determined as the lesser of:

$$V_n = V_c + V_s + V_p \quad (\text{S5.8.3.3-1})$$

OR

$$V_n = 0.25f'_c b_v d_v + V_p \quad (\text{S5.8.3.3-2})$$

where:

$$\begin{aligned} V_c &= \text{shear resistance due to concrete (k)} \\ &= 0.0316\beta\sqrt{f'_c} b_v d_v \quad (\text{S5.8.3.3-3}) \end{aligned}$$

where:

$b_v =$  effective web width taken as the minimum web width within the depth  $d_v$  as determined in S5.8.2.9 (in.)  
= 48 in.

$d_v =$  effective shear depth as determined in S5.8.2.9 (in.). It is the distance, measured perpendicular to the neutral axis between the resultants of the tensile and compressive force due to flexure. It need not be taken less than the greater of  $0.9d_e$  or  $0.72h$ .  
=  $d_e - a/2$   
=  $44.81 - (6.86/2)$   
= 41.4 in.

$$0.9d_e = 0.9(44.81)$$

$$= 40.3 \text{ in.}$$

$$0.72h = 0.72(48)$$

$$= 34.56 \text{ in.}$$

Therefore, use  $d_v = 41.4 \text{ in.}$  for  $V_c$  calculation.

$\beta$  = factor indicating ability of diagonally cracked concrete to transmit tension as specified in S5.8.3.4  
= for nonprestressed sections,  $\beta$  may be taken as 2.0

$$V_c = 0.0316(2.0)\sqrt{3}(48)(41.4)$$

$$= 217.5 \text{ k}$$

$$V_s = \text{shear resistance due to steel (k)}$$

$$= [A_v f_y d_v (\cot \theta + \cot \alpha) \sin \alpha] / s \quad (\text{S5.8.3.3-4})$$

where:

$s$  = spacing of stirrups (in.)  
= assume 7 in.

$\theta$  = angle of inclination of diagonal compressive stresses as determined in S5.8.3.4 (deg)  
= 45 deg for nonprestressed members

$\alpha$  = angle of inclination of transverse reinforcement to longitudinal axis (deg)  
= 90 deg for vertical stirrups

$$A_v = (6 \text{ legs of \#5 bars})(0.31)$$

$$= 1.86 \text{ in}^2$$

$$V_s = [1.86(60)(41.4)(1/\tan 45)]/7$$

$$= 660.0 \text{ k}$$

$V_p$  = component in the direction of the applied shear of the effective prestressing force; positive if resisting the applied shear (k), not applicable in the pier cap  
= 0.0 for nonprestressed members

Therefore,  $V_n$  is the lesser of:

$$V_n = 217.5 + 660.0 + 0$$

$$= 877.5 \text{ k}$$

OR

$$V_n = 0.25(3)(48)(41.4) + 0$$

$$= 1,490.4 \text{ k}$$

Use  $V_n = 877.5$  k

Therefore,

$$\begin{aligned} V_r &= \phi V_n \\ &= 0.9(877.5) \\ &= 789.8 \text{ k} > V_u = 798.3 \text{ k} \quad \mathbf{OK} \end{aligned}$$

Check the minimum transverse reinforcement (S5.8.2.5)

*A minimum amount of transverse reinforcement is required to restrain the growth of diagonal cracking and to increase the ductility of the section. A larger amount of transverse reinforcement is required to control cracking as the concrete strength is increased.*

Where transverse reinforcement is required, as specified in S5.8.2.4, the area of steel must satisfy:

$$A_v = 0.0316\sqrt{f'_c}b_v s/f_y \quad (\text{S5.8.2.5-1})$$

where:

$b_v$  = width of web adjusted for the presence of ducts as specified in S5.8.2.9 (in.)

$$\begin{aligned} A_v &= 0.0316\sqrt{3}(48)(7)/60 \\ &= 0.307 \text{ in}^2 < 1.86 \text{ in}^2 \text{ provided } \mathbf{OK} \end{aligned}$$

Check the maximum spacing of the transverse reinforcement (S5.8.2.7)

The spacing of the transverse reinforcement must not exceed the maximum permitted spacing,  $s_{\max}$ , determined as:

$$\begin{aligned} \text{If } v_u < 0.125f'_c, \text{ then} \\ s_{\max} &= 0.8d_v \leq 24.0 \text{ in.} \quad (\text{S5.8.2.7-1}) \end{aligned}$$

$$\begin{aligned} \text{If } v_u \geq 0.125f'_c, \text{ then:} \\ s_{\max} &= 0.4d_v \leq 12.0 \text{ in.} \quad (\text{S5.8.2.7-2}) \end{aligned}$$

The shear stress on the concrete,  $v_u$ , is taken to be:

$$\begin{aligned} v_u &= V_u/(\phi b_v d_v) \quad (\text{S5.8.2.9-1}) \\ &= 798.3/[0.9(48)(41.4)] \\ &= 0.446 \text{ ksi} > 0.125(3) = 0.375 \text{ ksi} \end{aligned}$$

Therefore, use Eq. S5.8.2.7-2

$$s_{\max} = 0.4(41.4) = 16.6 \text{ in. } s_{\max} \text{ cannot exceed 12 in., therefore, use 12 in. as maximum}$$

$$s_{\text{actual}} = 7 \text{ in.} < 12 \text{ in. } \mathbf{OK}$$

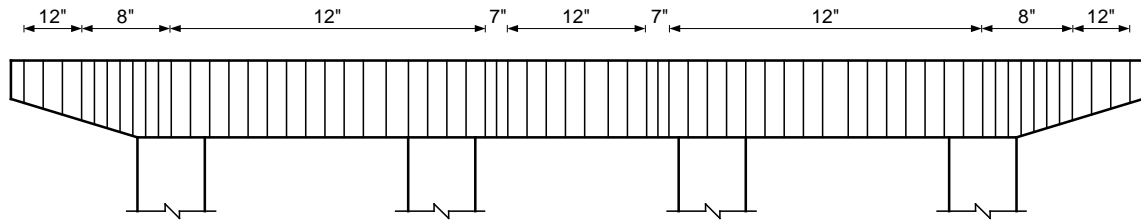


Figure 7.2-7 – Stirrup Distribution in the Bent Cap

### Design Step 7.2.3 Column design

#### Required information:

General:

$$\begin{aligned} f'_c &= 3.0 \text{ ksi} \\ E_c &= 3,321 \text{ ksi (S5.4.2.4)} \\ n &= 9 \\ f_y &= 60 \text{ ksi} \end{aligned}$$

Circular Columns:

$$\begin{aligned} \text{Column diameter} &= 3.5 \text{ ft.} \\ \text{Column area, } A_g &= 9.62 \text{ ft}^2 \\ \text{Side cover} &= 2 \text{ in. (Table S5.12.3-1)} \\ \text{Vertical reinforcing bar diameter (\#8)} &= 1.0 \text{ in.} \\ \text{Steel area} &= 0.79 \text{ in}^2 \\ \text{Number of bars} &= 16 \\ \text{Total area of longitudinal reinforcement} &= 12.64 \text{ in}^2 \\ \text{Type of transverse reinforcement} &= \text{ties} \\ \text{Tie spacing} &= 12 \text{ in.} \\ \text{Transverse reinforcement bar diameter (\#3)} &= 0.375 \text{ in. (S5.10.6.3)} \\ \text{Transverse reinforcement area} &= 0.11 \text{ in}^2/\text{bar} \end{aligned}$$



The example bridge is in Seismic Zone 1, therefore, a seismic investigation is not necessary for the column design. Article S5.10.11 provides provisions for seismic design where applicable.

Applied moments and shears

The maximum biaxial responses occur on column 1 at 0.0 ft. from the bottom (top face of footing).

From the load analysis of the bent, the maximum load effects at the critical location were obtained and are listed in Table 7.2-2.

**Table 7.2-2 – Maximum Factored Load Effects and the Concurrent Load Effects for Strength Limit States**

Load effect maximized	Limit State	$M_t$ (k-ft)	$M_l$ (k-ft)	$P_u$ (k)	$M_u$ (k-ft)
Positive $M_t$	Strength V	342	352	1,062	491
Negative $M_t$	Strength V	-129	-216	682	252
Positive $M_l$	Strength V	174	822	1,070	840
Negative $M_l$	Strength V	116	-824	1,076	832
Axial Load P	Strength I	90	-316	1,293	329

where:

$M_t$ : Factored moment about the transverse axis

$M_l$ : Factored moment about the longitudinal axis

$P_u$ : Factored axial load

Sample hand calculations are presented for the case of maximum positive  $M_l$  from Table 7.2-2.

Maximum shear occurs on column 1 at 0.0 ft. from the bottom (top face of footing)

Factored shears – strength limit state:

$$V_t = 44.8 \text{ k (Str-V)}$$

$$V_l = 26.0 \text{ k (Str-V)}$$

Check limits for reinforcement in compression members (S5.7.4.2)

The maximum area of nonprestressed longitudinal reinforcement for non-composite compression components shall be such that:

$$A_s/A_g \leq 0.08 \quad (\text{S5.7.4.2-1})$$

where:

$A_s$  = area of nonprestressed tension steel (in<sup>2</sup>)

$A_g$  = gross area of section (in<sup>2</sup>)

$$12.64/[9.62(144)] = 0.009 < 0.08 \quad \mathbf{OK}$$

The minimum area of nonprestressed longitudinal reinforcement for noncomposite compression components shall be such that:

$$\begin{aligned} A_s f_y / A_g f'_c &\geq 0.135 && (\text{S5.7.4.2-3}) \\ &= 12.64(60)/[9.62(144)(3)] \\ &= 0.182 > 0.135 \quad \mathbf{OK} \end{aligned}$$

Therefore, the column satisfies the minimum steel area criteria, do not use a reduced effective section. For oversized columns, the required minimum longitudinal reinforcement may be reduced by assuming the column area is in accordance with S5.7.4.2.

Strength reduction factor,  $\phi$ , to be applied to the nominal axial resistance (S5.5.4.2)

For compression members with flexure, the value of  $\phi$  may be increased linearly from axial (0.75) to the value for flexure (0.9) as the factored axial load resistance,  $\phi P_n$ , decreases from  $0.10f'_c A_g$  to zero. The resistance factor is incorporated in the interaction diagram of the column shown graphically in Figure 7.2-8 and in tabulated form in Table 7.2-3.

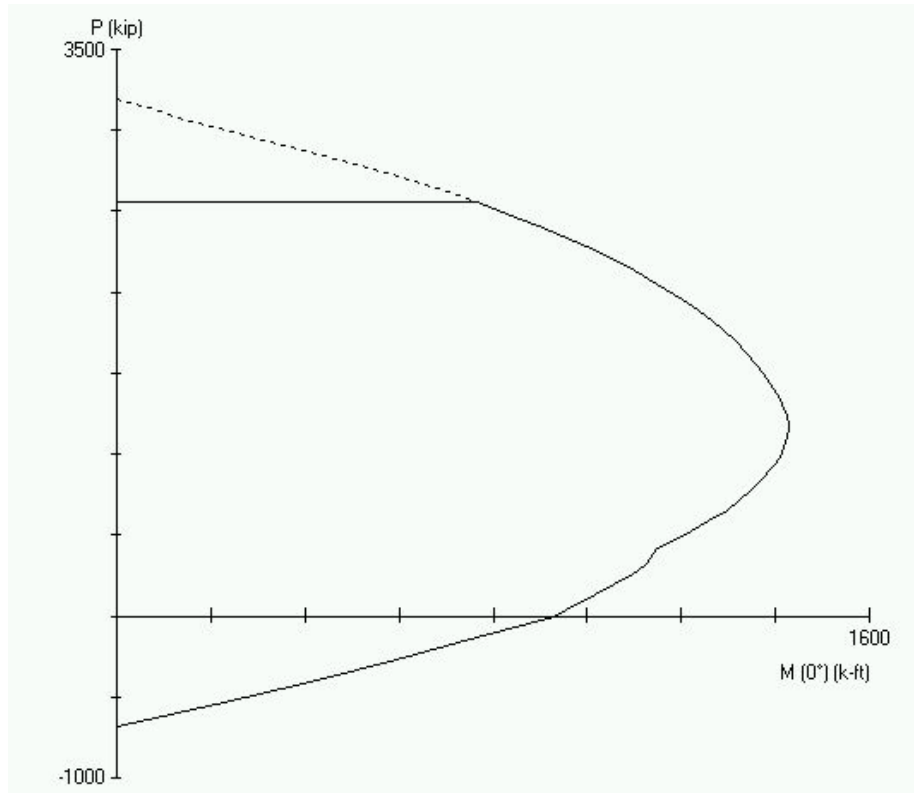


Figure 7.2-8 – Column Interaction Diagram

Table 7.2-3 – Column Interaction Diagram in Tabulated Form

P (k)	M (k-ft)	P (k) (cont.)	M (k-ft) (cont.)
$P_{max} = 2,555$	764	799	1,354
2,396	907	639	1,289
2,236	1,031	479	1,192
2,076	1,135	319	1,124
1,917	1,222	160	1,037
1,757	1,291	0	928
1,597	1,348	-137	766
1,437	1,389	-273	594
1,278	1,419	-410	410
1,118	1,424	-546	212
958	1,404	-683	0

## Design Step 7.2.3.1

Slenderness effects

*The effective length factor,  $K$ , is taken from S4.6.2.5. The slenderness moment magnification factors are typically determined in accordance with S4.5.3.2.2. Provisions specific to the slenderness of concrete columns are listed in S5.7.4.3.*

*Typically, the columns are assumed unbraced in the plane of the bent with the effective length factor,  $K$ , taken as 1.2 to account for the high rigidity of the footing and the pier cap. In the direction perpendicular to the bent  $K$  may be determined as follows:*

- *If the movement of the cap is not restrained in the direction perpendicular to the bent, the column is considered not braced and the column is assumed to behave as a free cantilever.  $K$  is taken equal to 2.1 (see Table SC4.6.2.5-1)*
- *If the movement of the cap is restrained in the direction perpendicular to the bent, the column is considered braced in this direction and  $K$  is taken equal to 0.8 (see Table SC4.6.2.5-1)*

For the example, the integral abutments provide restraint to the movements of the bent in the longitudinal direction of the bridge (approximately perpendicular to the bent). However, this restraint is usually ignored and the columns are considered unbraced in this direction, i.e.  $K = 2.1$ .

The slenderness ratio is calculated as  $K\ell_u/r$

where:

$K$  = effective length factor taken as 1.2 in the plane of the bent and 2.1 in the direction perpendicular to the bent

$\ell_u$  = unbraced length calculated in accordance with S5.7.4.3 (ft.)  
 = distance from the top of the footing to the bottom of the cap  
 = 18 ft.

$r$  = radius of gyration (ft.)  
 =  $\frac{1}{4}$  the diameter of circular columns  
 = 0.875 ft.

*For a column to be considered slender,  $K\ell_u/r$  should exceed 22 for unbraced columns and, for braced columns, should exceed  $34-12(M_1/M_2)$  where  $M_1$  and  $M_2$  are the smaller and larger end moments, respectively. The term  $(M_1/M_2)$  is positive for single curvature flexure (S5.7.4.3)*

Slenderness ratio in the plane of the bent

$$K\ell_u/r = 1.2(18)/(0.875)$$

= 24.7 > 22 therefore, the column is slightly slender

Slenderness ratio out of the plane of the bent

$$\begin{aligned} K\ell_u/r &= 2.1(18)/(0.875) \\ &= 43.2 > 22 \text{ therefore, the column is slender} \end{aligned}$$

With the column slender in both directions, effect of slenderness needs to be considered.

Moment magnification in the bent

Longitudinal direction:

$$M_{cl} = \delta_b M_{2b} + \delta_s M_{2s} \quad (\text{S4.5.3.2.2b-1})$$

where:

$$\delta_b = C_m/[1 - (P_u/\phi P_e)] \geq 1.0 \quad (\text{S4.5.3.2.2b-3})$$

$$\delta_s = 1/[1 - \Sigma P_u/\phi \Sigma P_e] \quad (\text{S4.5.3.2.2b-4})$$

where:

$C_m$  = parameter of the effect of moment-curvature  
= 1.0 for members not braced for sidesway (S4.5.3.2.2b)

$P_u$  = factored axial load for critical case, see Table 7.2-2 (k)  
= 1,070 k

$P_e$  = Euler buckling load (k)

$\phi$  = 0.75, resistance factor for axial compression (S5.5.4.2)

$M_{2b}$  = moment on compression member due to factored gravity loads that result in no appreciable sidesway calculated by conventional first-order elastic frame analysis, always positive (k-ft)

$M_{2s}$  = moment on compression member due to factored lateral or gravity loads that result in sidesway,  $\Delta$ , greater than  $\ell_u/1500$ , calculated by conventional first-order elastic frame analysis, always positive (k-ft)

Calculate  $P_e$ ,

$$P_e = \pi^2 EI / (K\ell_u)^2 \quad (\text{S4.5.3.2.2b-5})$$

where:

$EI$  = column flexural stiffness calculated using the provisions of S5.7.4.3 and is taken as the greater of:

$$EI = [E_c I_g / 5 + E_s I_s] / (1 + \beta_d) \quad (S5.7.4.3-1)$$

AND

$$EI = [E_c I_g / 2.5] / (1 + \beta_d) \quad (S5.7.4.3-2)$$

where:

$$\begin{aligned} E_c &= \text{modulus of elasticity of concrete per S5.4.2.4 (ksi)} \\ &= 33,000 w_c^{1.5} \sqrt{f'_c} = 33,000 (0.150)^{1.5} \sqrt{3} \\ &= 3,321 \text{ ksi} \end{aligned}$$

$$\begin{aligned} I_g &= \text{moment of inertia of gross concrete section about the} \\ &\quad \text{centroidal axis (in}^4\text{)} \\ &= \pi r^4 / 4 = \pi [1.75(12)]^4 / 4 \\ &= 152,745 \text{ in}^4 \end{aligned}$$

$$\begin{aligned} \beta_d &= \text{ratio of the maximum factored permanent load moment to the maximum} \\ &\quad \text{factored total load moment, always positive. This can be determined} \\ &\quad \text{for each separate load case, or for simplicity as shown here, it can be} \\ &\quad \text{taken as the ratio of the maximum factored permanent load from all} \\ &\quad \text{cases to the maximum factored total load moment from all cases at the} \\ &\quad \text{point of interest.} \\ &= M_{I \text{ permanent}} / M_{I \text{ total}} \\ &= 118.3 / 822 \\ &= 0.144 \end{aligned}$$

As a simplification, steel reinforcement in the column is ignored in calculating  $EI$ , therefore, neglect Eq. S5.7.4.3-1.

$$\begin{aligned} EI &= [3,321(152,745) / 2.5] / (1 + 0.144) \\ &= 1.77 \times 10^8 \text{ k-in}^2 \end{aligned}$$

$$\begin{aligned} K &= \text{effective length factor per Table SC4.6.2.5-1} \\ &= 2.1 \end{aligned}$$

$$\begin{aligned} \ell_u &= \text{unsupported length of the compression member (in.)} \\ &= 18(12) \\ &= 216 \text{ in.} \end{aligned}$$

$$\begin{aligned} P_e &= \pi^2 (1.77 \times 10^8) / [2.1(216)]^2 \\ &= 8,490 \text{ k} \end{aligned}$$

Therefore, the moment magnification factors  $\delta_b$  and  $\delta_s$  can be calculated.

$$\begin{aligned}\delta_b &= 1.0/[1 - (1,070/[0.75(8,490)])] \\ &= 1.20\end{aligned}$$

$$\delta_s = 1/[1 - \Sigma P_u / \phi \Sigma P_e]$$

$\Sigma P_u$  and  $\Sigma P_e$  are the sum of the applied factored loads and the sum of the buckling loads of all columns in the bent, respectively. For hand calculations, it is not feasible to do calculations involving several columns simultaneously. Therefore, in this example,  $P_u$  and  $P_e$  of the column being designed are used instead of  $\Sigma P_u$  and  $\Sigma P_e$ .

$$\delta_s = 1.20$$

Therefore, the magnified moment in the longitudinal direction is taken as:

$$\begin{aligned}M_{cl} &= \delta_b M_{2b} + \delta_s M_{2s} && (S4.5.3.2.2b-1) \\ &= 1.20(M_{2b} + M_{2s}) \\ &= 1.20(\text{total factored moment, } M_l) \\ &= 1.20(822) \\ &= 986.4 \text{ k-ft}\end{aligned}$$

Transverse direction:

$$M_{ct} = \delta_b M_{2b} + \delta_s M_{2s} \quad (S4.5.3.2.2b-1)$$

Calculate  $P_e$ ,

$$P_e = \pi^2 EI / (K\ell_u)^2 \quad (S4.5.3.2.2b-5)$$

where:

$EI$  = column flexural stiffness calculated using the provisions of S5.7.4.3 and is taken as the greater of:

$$EI = [E_c I_g / 5 + E_s I_s] / (1 + \beta_d) \quad (S5.7.4.3-1)$$

AND

$$EI = [E_c I_g / 2.5] / (1 + \beta_d) \quad (S5.7.4.3-2)$$

where:

$$E_c = 3,321 \text{ ksi}$$

$$I_g = 152,745 \text{ in}^4$$

$$\begin{aligned}\beta_d &= M_{t \text{ permanent}} / M_{t \text{ total}} \\ &= 101.7 / 342 \\ &= 0.30\end{aligned}$$

For simplification, steel reinforcement in the column is ignored in calculating EI, therefore, neglect Eq. S5.7.4.3-1.

$$\begin{aligned} EI &= [3,321(152,745)/2.5]/(1 + 0.30) \\ &= 1.56 \times 10^8 \text{ k-in}^2 \end{aligned}$$

$$K = 1.2$$

$$\ell_u = 216 \text{ in.}$$

$$\begin{aligned} P_e &= \pi^2(1.56 \times 10^8)/[1.2(216)]^2 \\ &= 22,917 \text{ k} \end{aligned}$$

Therefore, the moment magnification factors  $\delta_b$  and  $\delta_s$  can be calculated.

$$\begin{aligned} \delta_b &= 1.0/[1 - (1,070/[0.75(23,064)])] \\ &= 1.07 \end{aligned}$$

$$\delta_s = 1/[1 - \Sigma P_u/\phi \Sigma P_e]$$

$$\begin{aligned} &\text{Similar to longitudinal, use } P_u \text{ and } P_e \text{ instead of } \Sigma P_u \text{ and } \Sigma P_e. \\ &= 1.07 \end{aligned}$$

Therefore, the magnified moment in the transverse direction is taken as:

$$\begin{aligned} M_{ct} &= 1.07(\text{total factored moment, } M_t) \\ &= 1.07(174) \\ &= 186 \text{ k-ft} \end{aligned}$$

The combined moment  $M_u$  is taken as:

$$\begin{aligned} M_u &= \sqrt{M_{cl} + M_{ct}} \\ &= \sqrt{986.4^2 + 186^2} \\ &= 1004 \text{ k-ft} \end{aligned}$$

Factored axial load on the column for the load case being checked = 1,070 k

By inspection, from the column interaction diagram Figure 7.2-8 or Table 7.2-3, the applied factored loads ( $M = 1,004 \text{ k-ft}$  and  $P = 1,070 \text{ k}$ ) are within the column resistance.



## Design Step 7.2.3.2

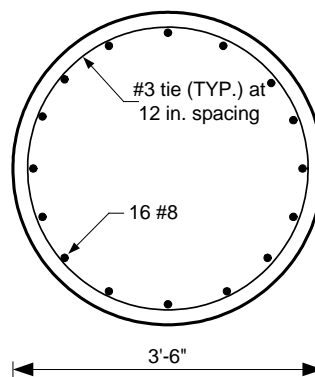
Transverse reinforcement for compression members (S5.10.6)

*Transverse reinforcement for compression members may consist of either spirals or ties. Ties are used in this example. In tied compression members, all longitudinal bars are enclosed by lateral ties. Since the longitudinal bars are #8, use #3 bars for the ties (S5.10.6.3).*

*The spacing of ties is limited to the least dimension of the compression member or 12.0 in., therefore, the ties are spaced at 12.0 in. center-to-center.*

*Ties are located vertically no more than half a tie spacing above the footing and not more than half a tie spacing below the lowest horizontal reinforcement in the cap.*

Figure 7.2-9 shows the column cross-section.



**Figure 7.2-9 – Column Cross-Section**

## Design Step 7.2.4 Footing design

Based on the intermediate bent load analysis, the critical footing is Footing 1 supporting Column 1

Required information:

General:

$$f'_c = 3.0 \text{ ksi}$$

$$f_y = 60 \text{ ksi}$$

Side concrete cover = 3 in. (Table S5.12.3-1)

Top concrete cover = 3 in.

Bottom concrete cover = 3 in.

Top bars (T)ransverse or (L)ongitudinal in bottom mat = L

Direction of bottom bars in bottom mat = T

A preliminary analysis of the footing yielded the following information:

Footing depth = 3.0 ft.

Footing width,  $W = 12.0$  ft.

Footing length,  $L = 12.0$  ft.

Top mat reinforcing bar diameter, #5 bars = 0.625 in.

Top mat reinforcing bar area, #5 bars =  $0.31 \text{ in}^2$

Bottom mat reinforcing bar diameter, #9 bars = 1.128 in.

Bottom mat reinforcing bar area, #9 bars =  $1.0 \text{ in}^2$

Number of bars = 13 bars in each direction in both the top and bottom mats

#### Location of critical sections

*According to S5.13.3.6.1, the critical section for one-way shear is at a distance  $d_v$ , the shear depth calculated in accordance with S5.8.2.9, from the face of the column. For two-way shear, the critical section is at a distance of  $d_v/2$  from the face of the column.*

*For moment, the critical section is taken at the face of the column in accordance with S5.13.3.4.*

For the circular column in this example, the face of the column is assumed to be located at the face of an equivalent square area concentric with the circular column in accordance with S5.13.3.4.

#### Determine the critical faces along the y-axis for moment

Since the column has a circular cross-section, the column may be transformed into an effective square cross-section for the footing analysis.

Critical face in y-direction =  $\frac{1}{2}$  footing width,  $W - \frac{1}{2}$  equivalent column width

Equivalent column width =  $\sqrt{\text{shaft area}}$   
 =  $\sqrt{9.62}$   
 = 3.10 ft.

Critical face in y-direction =  $\frac{1}{2}$  footing width,  $W - \frac{1}{2}$  equivalent column width  
 =  $\frac{1}{2}(12) - \frac{1}{2}(3.10)$   
 = 4.45 ft.

Critical faces in the y-direction = 4.45 ft. and 7.55 ft.

#### Determine the critical faces along the x-axis for moment

For a square footing with an equivalent square column:

Critical face in the x-direction = Critical face in the y-direction  
 = 4.45 ft.

Critical faces in the x-direction = 4.45 ft. and 7.55 ft.

See Figure 7.2-10 for a schematic showing the critical sections for moments.

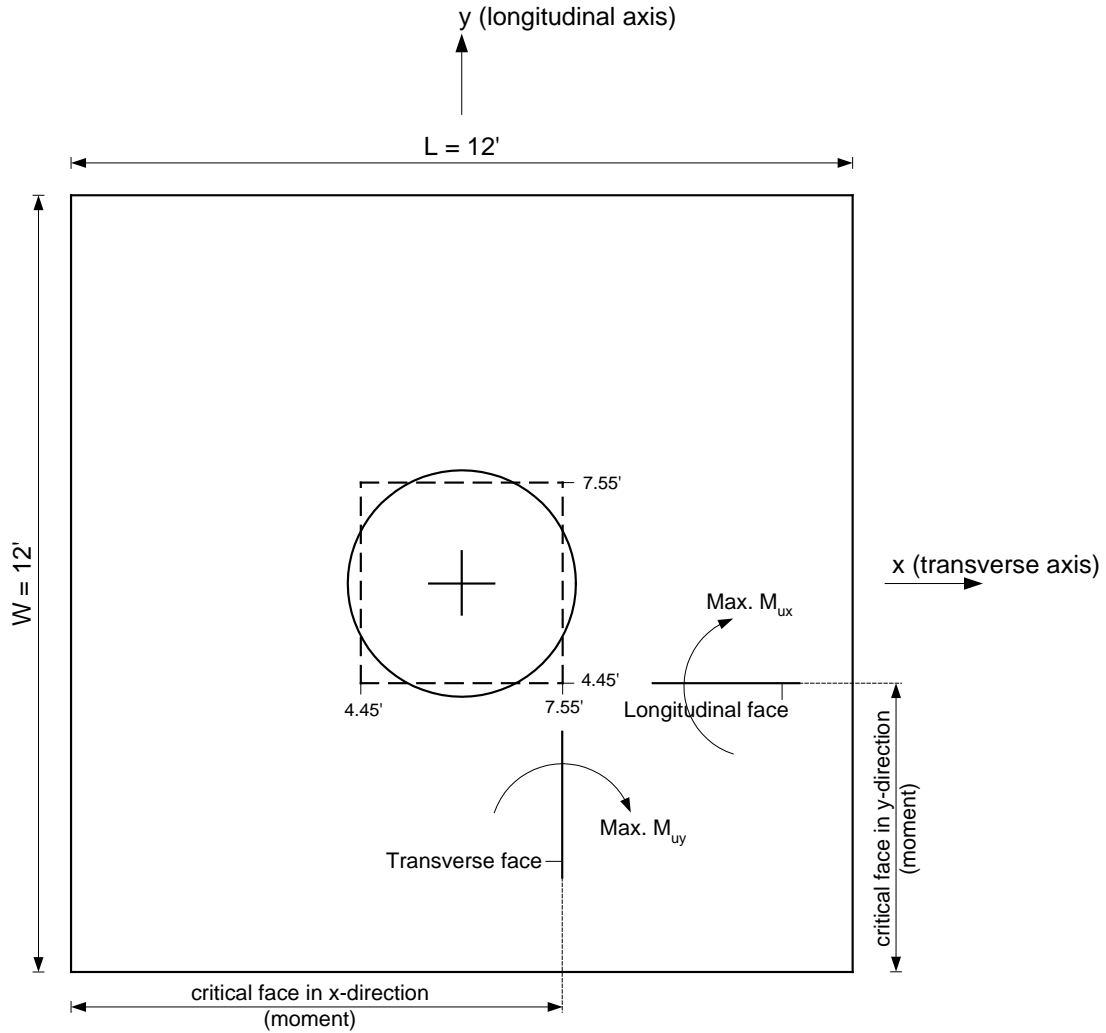


Figure 7.2-10 – Critical Sections for Moment

Design factored loads at the critical section

From the analysis of the intermediate bent computer program, the cases of loading that produced maximum load effects and the other concurrent load effects on the footing are shown in Table 7.2-4.

Table 7.2-4 – Loads on Critical Footing (Footing Supporting Column 1)

Load effect maximized	Limit State	$M_t$ (k-ft)	$M_l$ (k-ft)	$P_u$ (k)
Positive $M_t$	Strength V	423	377	1,143
Negative $M_t$	Strength III	-154	-197	628
Positive $M_l$	Strength V	232	895	1,151
Negative $M_l$	Strength V	158	-897	1,157
Axial Load P	Strength I	121	-363	1,374

Each row in Table 7.2-4 represents the maximum value of one load effect (max.  $+M_t, -M_l$ , etc.). The corresponding concurrent load effects are also given. Many engineers design the footing for the above listed cases. However, computer design programs are able to check many more cases of loading to determine the most critical case. For example, a load case that does not produce maximum axial load or maximum moment may still produce the maximum combined effects on the footing. From the output of a footing design program, the critical case for the footing design was found to produce the following factored footing loads under Strength I limit state:

$$\begin{aligned} P_u &= 1,374 \text{ k} \\ M_t &= -121 \text{ k-ft} \\ M_l &= 626 \text{ k-ft} \end{aligned}$$

The critical Service I loads:

$$\begin{aligned} P_u &= 891 \text{ k} \\ M_{t,s} &= 176 \text{ k-ft} \\ M_{l,s} &= 620 \text{ k-ft} \end{aligned}$$

For the sample calculations below, the factored loads listed above for the critical case of loading were used.

Sample calculations for the critical footing under the critical case of loading

If  $M/P < L/6$  then the soil under the entire area of the footing is completely in compression and the soil stress may be determined using the conventional stress formula (i.e.  $\sigma = P/A \pm Mc/I$ ).

$$\begin{aligned} M_t/P_u &= 121/1,374 \\ &= 0.088 < 12/6 = 2 \quad \mathbf{OK} \end{aligned}$$

$$\begin{aligned} M_1/P_u &= 626/1,374 \\ &= 0.456 < 2 \quad \mathbf{OK} \end{aligned}$$

Therefore, the soil area under the footing is under compression.

### Moment

For  $M_{ux}$  (k-ft/ft), where  $M_{ux}$  is the maximum factored moment per unit width of the footing due to the combined forces at a longitudinal face, see Figure 7.2-10:

$$\sigma_1, \sigma_2 = P/LW \pm M_1(L/2)/(L^3W/12)$$

where:

$\sigma_1$  = stress at beginning of footing in direction considered (see Figure 7.2-10) (ksf)

$\sigma_2$  = stress at end of footing in direction considered (ksf)

$P$  = axial load from above (k)

$M_1$  = moment on longitudinal face from above (k-ft)

$L$  = total length of footing (ft.)

$W$  = total width of footing (ft.)

$$\begin{aligned} \sigma_1 &= 1,374/[12(12)] + 626(12/2)/[12^3(12)/12] \\ &= 9.54 + 2.17 \\ &= 11.71 \text{ ksf} \end{aligned}$$

$$\begin{aligned} \sigma_2 &= 9.54 - 2.17 \\ &= 7.37 \text{ ksf} \end{aligned}$$

Interpolate to calculate  $\sigma_3$ , the stress at critical location for moment (at face of column, 4.45 ft. from the end of the footing along the width).

$$\sigma_3 = 10.10 \text{ ksf}$$

Therefore,

$$M_{ux} = \sigma_3 L_1(L_1/2) + 0.5(\sigma_1 - \sigma_3)(L_1)(2L_1/3)$$

where:

$L_1$  = distance from the edge of footing to the critical location (ft.)

$$\begin{aligned} M_{ux} &= 10.10(4.45)(4.45/2) + 0.5(11.71 - 10.10)(4.45)[2(4.45)/3] \\ &= 100.0 + 10.63 \\ &= 110.6 \text{ k-ft/ft} \end{aligned}$$

For  $M_{uy}$  (k-ft/ft), where  $M_{uy}$  is the maximum factored moment per unit length from the combined forces at a transverse face acting at 4.45 ft. from the face of the column (see Figure 7.2-10):

$$\sigma_5, \sigma_6 = P/LW \pm M_t(W/2)/(W^3L/12)$$

where:

$M_t$  = moment on transverse face from above (k-ft)

$$\begin{aligned}\sigma_5 &= 1,374/[12(12)] - (-121)(12/2)/[12^3(12)/12] \\ &= 9.54 - (-0.420) \\ &= 9.96 \text{ ksf}\end{aligned}$$

$$\begin{aligned}\sigma_6 &= 9.54 + (-0.420) \\ &= 9.12 \text{ ksf}\end{aligned}$$

Interpolate to calculate  $\sigma_7$ , the stress at critical location for moment (at face of column, 4.45 ft. from the end of the footing along the length).

$$\sigma_7 = 9.65 \text{ ksf}$$

Therefore,

$$\begin{aligned}M_{uy} &= \sigma_7 L_3(L_3/2) + 0.5(\sigma_5 - \sigma_7)(L_3)(2L_3/3) \\ &= 9.65(4.45)(4.45/2) + 0.5(9.96 - 9.65)(4.45)[2(4.45)/3] \\ &= 95.54 + 2.05 \\ &= 97.6 \text{ k-ft/ft}\end{aligned}$$

Factored applied design moment, Service I limit state, calculated using the same method as above:

$$\begin{aligned}M_{ux,s} &= 75.9 \text{ k-ft/ft} \\ M_{uy,s} &= 72.0 \text{ k-ft/ft}\end{aligned}$$

Where  $M_{ux,s}$  is the maximum service moment from combined forces at a longitudinal face at 4.45 ft. along the width and  $M_{uy,s}$  is the maximum service moment from combined forces at a transverse face at 7.55 ft. along the length.

### Shear

Factored applied design shear.

For  $V_{ux}$  (k/ft), where  $V_{ux}$  is the shear per unit length at a longitudinal face:

$$V_{ux} = \sigma_4 L_2 + 0.5(\sigma_1 - \sigma_4) L_2$$

where:

$L_2$  = distance from the edge of footing to a distance  $d_v$  from the effective column (ft.)

Based on the preliminary analysis of the footing,  $d_v$  is estimated as 30.3 in. Generally, for load calculations,  $d_v$  may be assumed equal to the effective depth of the reinforcement minus 1 inch. Small differences between  $d_v$  assumed here for load calculations and the final  $d_v$  will not result in significant difference in the final results.

The critical face along the y-axis  $= 4.45 - 30.3/12$   
 $= 1.925$  ft. from the edge of the footing

By interpolation between  $\sigma_1$  and  $\sigma_2$ ,  $\sigma_4 = 11.01$  ksf

$$\begin{aligned} V_{ux} &= 11.01(1.925) + 0.5(11.71 - 11.01)(1.925) \\ &= 21.19 + 0.67 \\ &= 21.9 \text{ k/ft} \end{aligned}$$

For  $V_{uy}$  (k/ft), where  $V_{uy}$  is the shear per unit length at a transverse face:

$$V_{ux} = \sigma_8 L_4 + 0.5(\sigma_5 - \sigma_8) L_4$$

where:

$d_v = 31.4$  in. for this direction (from preliminary design). Alternatively, for load calculations,  $d_v$  may be assumed equal to the effective depth of the reinforcement minus 1 inch).

The critical face along the x-axis  $= 4.45 - 31.4/12$   
 $= 1.833$  ft. from the edge of the footing

By interpolation between  $\sigma_5$  and  $\sigma_6$ ,  $\sigma_8 = 9.83$  ksf

$$\begin{aligned} V_{ux} &= 9.83(1.83) + 0.5(9.96 - 9.83)(1.83) \\ &= 17.99 + 0.12 \\ &= 18.1 \text{ k/ft} \end{aligned}$$

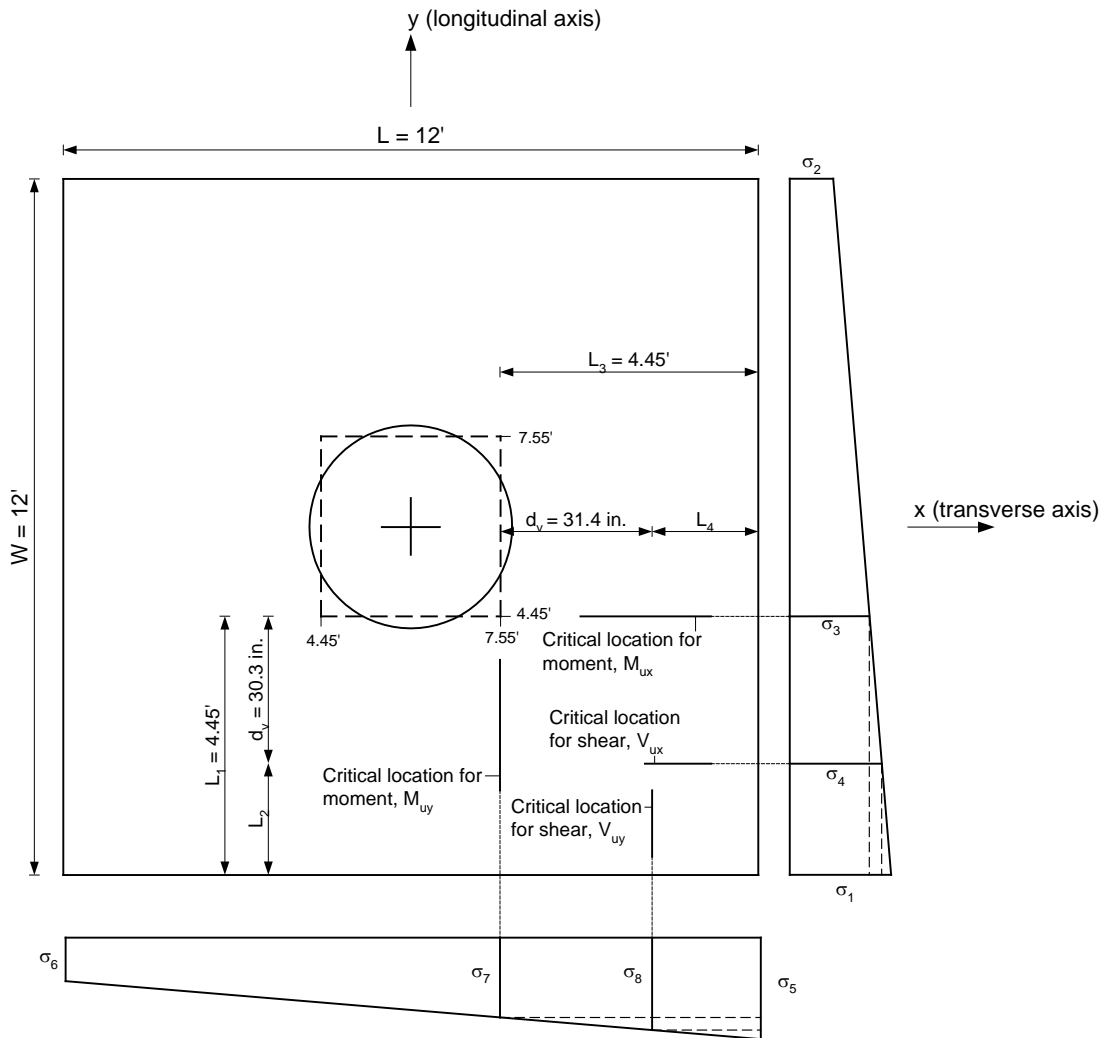


Figure 7.2-11 – Stress at Critical Locations for Moment and Shear

**Design Step 7.2.4.1 Flexural resistance (S5.7.3.2)**

Check the design moment strength (S5.7.3.2)

Article S5.13.3.5 allows the reinforcement in square footings to be uniformly distributed across the entire width of the footing.

Check the moment resistance for moment at the critical longitudinal face (S5.13.3.4)

The critical section is at the face of the effective square column (4.45 ft. from the edge of the footing along the width). In the case of columns that are not rectangular, the critical section is taken at the side of the concentric rectangle of equivalent area as in this example.



$$M_{rx} = \phi M_{nx} \quad (S5.7.3.2.1-1)$$

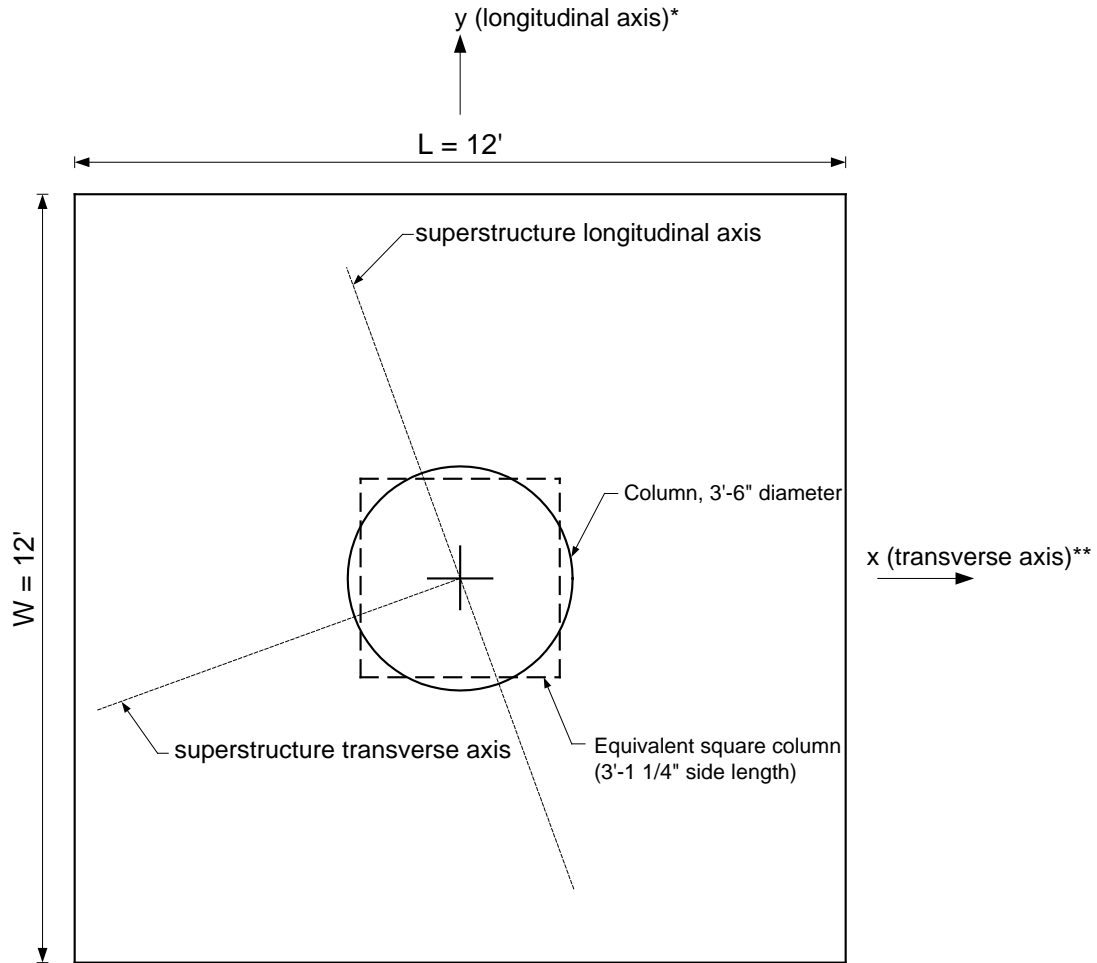
where:

$$\phi = 0.9 \quad (S5.5.4.2.1)$$

$$M_{nx} = A_s f_y (d_{sx} - a/2) \quad (S5.7.3.2.2-1)$$

Determine  $d_{sx}$ , the distance from the top bars of the bottom reinforcing mat to the compression surface.

$$\begin{aligned} d_{sx} &= \text{footing depth} - \text{bottom cvr} - \text{bottom bar dia.} - \frac{1}{2} \text{ top bar dia. in bottom mat} \\ &= 3(12) - 3 - 1.128 - \frac{1}{2} (1.128) \\ &= 31.3 \text{ in.} \end{aligned}$$



\* perpendicular to the bent  
 \*\* in the plane of the bent

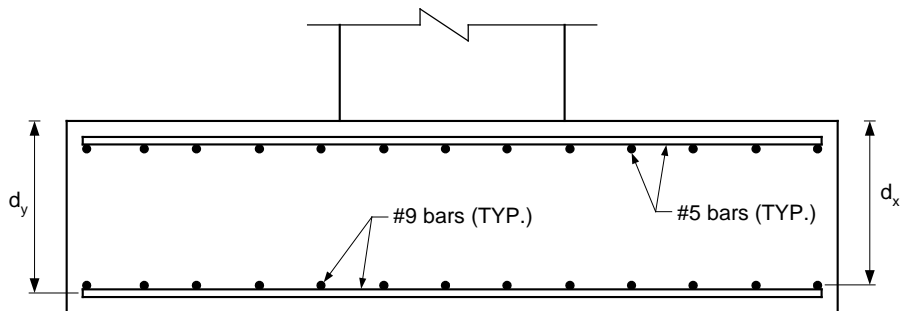


Figure 7.2-12 – Footing Reinforcement Locations

Determine  $A_s$  per foot of length. The maximum bar spacing across the width of the footing is assumed to be 12.0 in. in each direction on all faces (S5.10.8.2). Use 13 #9 bars and determine the actual spacing.

$$\begin{aligned}\text{Actual bar spacing} &= [L - 2(\text{side cover}) - \text{bar diameter}] / (n_{\text{bars}} - 1) \\ &= [12 - 2(3)/12 - 1.128/12] / (13 - 1) \\ &= 11.41 \text{ in.}\end{aligned}$$

$$\begin{aligned}A_s &= 1.0(12/11.41) \\ &= 1.05 \text{ in}^2\end{aligned}$$

Determine “a”, the depth of the equivalent stress block.

$$a = A_s f_y / 0.85 f'_c b \quad (\text{S5.7.3.1.1-4})$$

for a strip 12 in. wide,  $b = 12$  in. and  $A_s = 1.05 \text{ in}^2$

$$\begin{aligned}a &= 1.05(60) / [0.85(3)(12)] \\ &= 2.06 \text{ in.}\end{aligned}$$

Calculate  $\phi M_{nx}$ , the factored flexural resistance.

$$\begin{aligned}M_{rx} &= \phi M_{nx} \\ &= 0.9[1.05(60)(31.3 - 2.06/2)] / 12 \quad (\text{S5.7.3.2.2-1}) \\ &= 143.0 \text{ k-ft/ft} > \text{applied factored moment, } M_{ux} = 110.6 \text{ k-ft/ft} \quad \mathbf{OK}\end{aligned}$$

#### Check minimum temperature and shrinkage steel (S5.10.8)

*According to S5.10.8.1, reinforcement for shrinkage and temperature stresses shall be provided near surfaces of concrete exposed to daily temperature changes and in structural mass concrete. Footings are not exposed to daily temperature changes and, therefore, are not checked for temperature and shrinkage reinforcement. Nominal reinforcement is provided at the top of the footing to arrest possible cracking during the concrete early age before the footing is covered with fill.*

#### **Design Step 7.2.4.2 Limits for reinforcement (S5.7.3.3)**

##### Check maximum reinforcement (S5.7.3.3.1)

$$c/d_e \leq 0.42 \quad (\text{S5.7.3.3.1-1})$$

where:

$$\begin{aligned}c &= a/\beta_1 \\ &= 2.06/0.85 \\ &= 2.42 \text{ in/ft}\end{aligned}$$

$$\begin{aligned}c/d_e &= 2.42/31.3 \\ &= 0.077 < 0.42 \quad \mathbf{OK}\end{aligned}$$

Minimum reinforcement check (S5.7.3.3.2)

Unless otherwise specified, at any section of a flexural component, the amount of nonprestressed tensile reinforcement shall be adequate to develop a factored flexural resistance,  $M_r$ , at least equal to the lesser of:

$$1.2M_{cr} = 1.2f_r S$$

where:

$$\begin{aligned} f_r &= 0.24\sqrt{f'_c} && \text{(S5.4.2.6)} \\ &= 0.24\sqrt{3} \\ &= 0.42 \text{ ksi} \end{aligned}$$

For a 1 ft. wide strip, 3 ft. thick,

$$\begin{aligned} S &= bh^2/6 \\ &= [1(12)][3(12)]^2/6 \\ &= 2,592 \text{ in}^3/\text{ft} \end{aligned}$$

$$\begin{aligned} 1.2M_{cr} &= 1.2(0.42)(2,592)/12 \\ &= 108.9 \text{ k-ft/ft} \end{aligned}$$

OR

$$\begin{aligned} 1.33M_{ux} &= 1.33(110.6) \\ &= 147.1 \text{ k-ft/ft} \end{aligned}$$

Therefore, the minimum required section moment resistance = 108.9 k-ft/ft  
 Provided moment resistance = 143.0 k-ft/ft > 108.9 k-ft/ft **OK**

Check the moment resistance for moment at the critical transverse face

The critical face is at the equivalent length of the shaft (7.55 ft. from the edge of the footing along the length). In the case of columns that are not rectangular, the critical section is taken at the side of the concentric rectangle of equivalent area.

$$\begin{aligned} M_{ry} &= \phi M_{ny} \\ &= \phi [A_s f_y (d_{sy} - a/2)] && \text{(S5.7.3.2.2-1)} \end{aligned}$$

Determine  $d_{sy}$ , the distance from the bottom bars of the bottom reinforcing mat to the compression surface.

$$\begin{aligned} d_{sy} &= \text{footing depth} - \text{cover} - \frac{1}{2} (\text{bottom bar diameter}) \\ &= 3(12) - 3 - \frac{1}{2} (1.128) \\ &= 32.4 \text{ in.} \end{aligned}$$

Determine  $A_s$  per foot of length

$$\begin{aligned}\text{Actual bar spacing} &= [W - 2(\text{side cover}) - \text{bar diameter}]/(n_{\text{bars}} - 1) \\ &= [12 - 2(3)/12 - 1.128/12]/(13 - 1) \\ &= 11.41 \text{ in.}\end{aligned}$$

$$\begin{aligned}A_s &= 1.0(12/11.41) \\ &= 1.05 \text{ in}^2\end{aligned}$$

Determine “a”, depth of the equivalent stress block.

$$a = A_s f_y / (0.85 f'_c b)$$

For a strip 12 in. wide,  $b = 12$  in. and  $A_s = 1.05 \text{ in}^2$

$$\begin{aligned}a &= 1.05(60)/[0.85(3)(12)] \\ &= 2.06 \text{ in.}\end{aligned}$$

Calculate  $\phi M_{ny}$ , the factored flexural resistance

$$\begin{aligned}M_{ry} &= \phi M_{ny} \\ &= 0.9[1.05(60)(32.4 - 2.06/2)]/12 \\ &= 148.2 \text{ k-ft/ft} > M_{uy} = 97.6 \text{ k-ft/ft} \quad \mathbf{OK}\end{aligned}$$

#### Design Step 7.2.4.3 Control of cracking by distribution of reinforcement (S5.7.3.4)

Check distribution about footing length, L

$$f_{s, \text{allow}} = Z/(d_c A)^{1/3} \leq 0.60 f_y \quad (\text{S5.7.3.4-1})$$

where:

$Z = 170$  k/in. (moderate exposure conditions assumed, no dry/wet cycles and no harmful chemicals in the soil)

Notice that the value of the of the crack control factor,  $Z$ , used by different jurisdictions varies based on local conditions and past experience.

$$\begin{aligned}d_c &= \text{bottom cover} + \frac{1}{2} \text{ bar diameter} \\ &= 2 + \frac{1}{2}(1.128) \\ &= 2.56 \text{ in.}\end{aligned}$$

$$\begin{aligned}A &= 2d_c(\text{bar spacing}) \\ &= 2(2.56)(11.41) \\ &= 58.4 \text{ in}^2\end{aligned}$$

$$\begin{aligned}f_{s, \text{allow}} &= Z/[(d_c A)^{1/3}] \\ &= 170/[2.56(58.4)]^{1/3}\end{aligned}$$

$$= 32.0 \text{ ksi} < 0.6(60) = 36 \text{ ksi} \text{ therefore, use } f_{s, \text{allow}} = 32.0 \text{ ksi}$$

Check actual steel stress,  $f_{s, \text{actual}}$

For 3.0 ksi concrete, the modular ratio,  $n = 9$

Maximum service load moment as shown earlier = 77.3 k-ft

The transformed moment of inertia is calculated assuming elastic behavior, i.e., linear stress and strain distribution. In this case, the first moment of area of the transformed steel on the tension side about the neutral axis is assumed equal to that of the concrete in compression.

Assume the neutral axis at a distance “y” from the compression face of the section.

Section width = bar spacing = 11.41 in.

Transformed steel area = (bar area)(modular ratio) = 1.0(9) = 9.0 in<sup>2</sup>

By equating the first moment of area of the transformed steel about that of the concrete, both about the neutral axis:

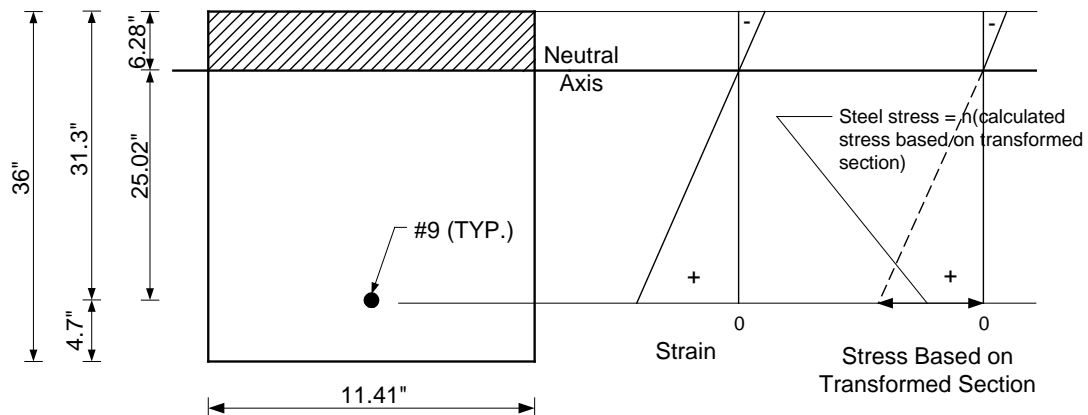
$$9.0(31.3 - y) = 11.41y(y/2)$$

Solving the equation results in  $y = 6.28$  in.

$$\begin{aligned} I_{\text{transformed}} &= A_{ts}(d_{sx} - y)^2 + by^3/3 \\ &= 9.0(31.3 - 6.28)^2 + 11.41(6.28)^3/3 \\ &= 6,576 \text{ in}^4 \end{aligned}$$

Stress in the steel,  $f_{s, \text{actual}} = (M_s c/I)n$ , where  $M_s$  is the moment acting on the 11.41 in. wide section.

$$\begin{aligned} f_{s, \text{actual}} &= [77.3(11.41)(31.3 - 6.28)/6,576]9 \\ &= 30.2 \text{ ksi} < f_{s, \text{allow}} = 32.0 \text{ ksi} \quad \mathbf{OK} \end{aligned}$$



**Figure 7.2-13 – Crack Control for Top Bar Reinforcement Under Service Load**

Check distribution about footing width, W

This check is conducted similarly to the check shown above for the distribution about the footing length and the reinforcement is found to be adequate.

**Design Step 7.2.4.4 Shear analysis**

Check design shear strength (S5.8.3.3)

According to S5.13.3.6.1, the most critical of the following conditions shall govern the design for shear:

- *One-way action, with a critical section extending in a plane across the entire width and located at a distance taken as specified in S5.8.3.2.*
- *Two-way action, with a critical section perpendicular to the plane of the slab and located so that its perimeter,  $b_o$ , is a minimum but not closer than  $0.5d_v$  to the perimeter of the concentrated load or reaction area.*

The subscripts “x” and “y” in the next section refer to the shear at a longitudinal face and shear at a transverse face, respectively.

Determine the location of the critical face along the y-axis

Since the column has a circular cross-section, the column may be transformed into an effective square cross-section for the footing analysis.

As stated previously, the critical section for one-way shear is at a distance  $d_v$ , the shear depth calculated in accordance with S5.8.2.9, from the face of the column and for two-way shear at a distance of  $d_v/2$  from the face of the column.

Determine the effective shear depth,  $d_{vx}$ , for a longitudinal face.

$$\begin{aligned} d_{vx} &= \text{effective shear depth for a longitudinal face per S5.8.2.9 (in.)} \\ &= d_{sx} - a/2 && \text{(S5.8.2.9)} \\ &= 31.3 - 2.06/2 \\ &= 30.3 \text{ in.} \end{aligned}$$

but not less than:

$$\begin{aligned} 0.9d_{sx} &= 0.9(31.3) \\ &= 28.2 \text{ in.} \\ 0.72h &= 0.72(36) \\ &= 25.9 \text{ in.} \end{aligned}$$

Therefore, use  $d_{vx} = 30.3$  in.

$$\begin{aligned} \text{The critical face along the y-axis} &= 4.45 - 30.3/12 \\ &= 1.925 \text{ ft. from the edge of the footing} \end{aligned}$$

#### Determine the location of the critical face along the x-axis

Determine the effective shear depth,  $d_{vy}$ , for a transverse face.

$$\begin{aligned} d_{vy} &= \text{effective shear depth for a transverse face per S5.8.2.9 (in.)} \\ &= d_{sy} - a/2 && \text{(S5.8.2.9)} \\ &= 32.4 - 2.06/2 \\ &= 31.4 \text{ in.} \end{aligned}$$

but not less than:

$$\begin{aligned} 0.9d_{sy} &= 0.9(32.4) \\ &= 29.2 \text{ in.} \\ 0.72h &= 0.72(36) \\ &= 25.9 \text{ in.} \end{aligned}$$

Therefore, use  $d_{vy} = 31.4$  in.

$$\begin{aligned} \text{The critical face along the x-axis} &= 4.45 - 31.4/12 \\ &= 1.833 \text{ ft. from the edge of the footing} \end{aligned}$$

See Figure 7.2-14 for locations of the critical sections.



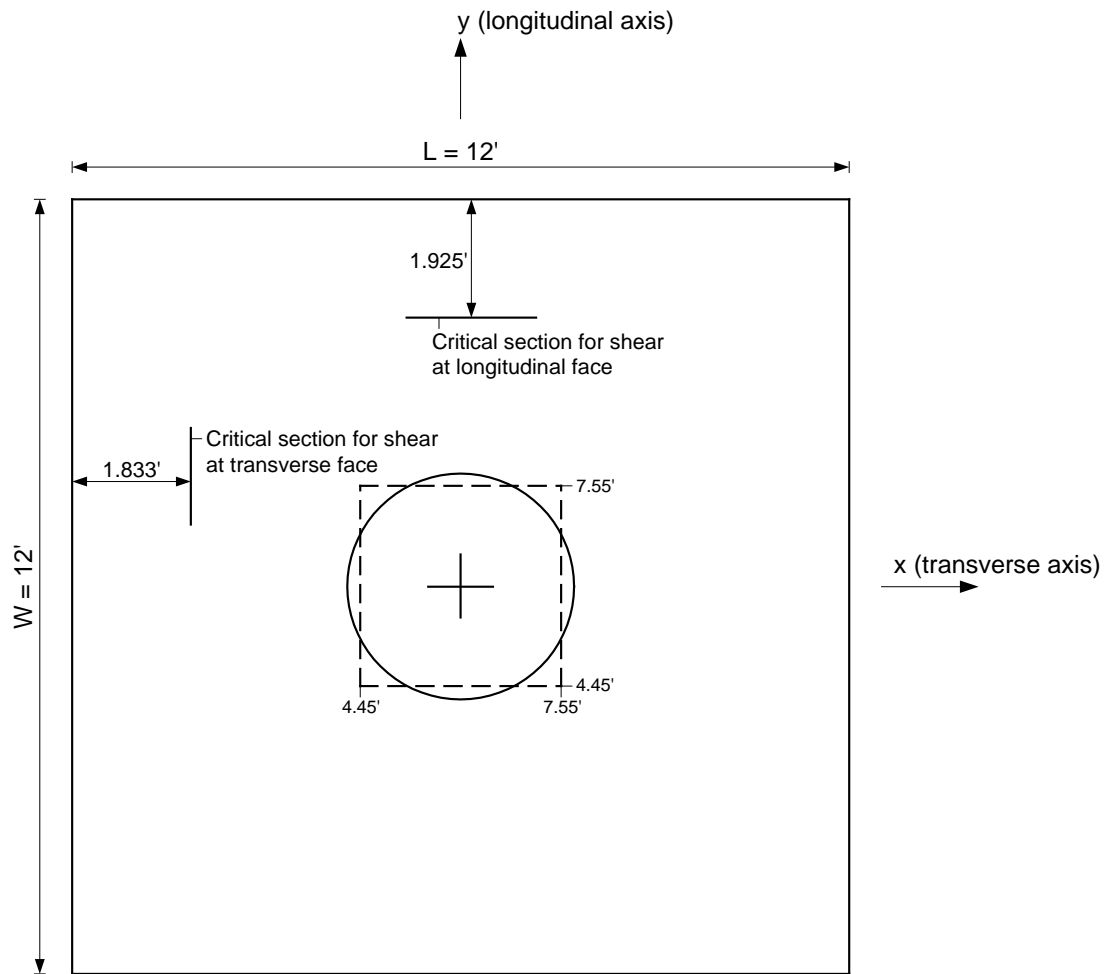


Figure 7.2-14 – Critical Sections for Shear

Determine one-way shear capacity for longitudinal face (S5.8.3.3)

For one-way action, the shear resistance of the footing of slab will satisfy the requirements specified in S5.8.3.

$$V_{rx} = \phi V_{nx} \quad (S5.8.2.1-2)$$

The nominal shear resistance,  $V_{nx}$ , is taken as the lesser of:

$$V_{nx} = V_c + V_s + V_p \quad (S5.8.3.3-1)$$

OR

$$V_{nx} = 0.25f'_c b_v d_{vx} + V_p \quad (S5.8.3.3-2)$$

$$V_c = 0.0316\beta\sqrt{f'_c}b_v d_{vx} \quad (\text{S5.8.3.3-3})$$

where:

$$\beta = 2.0$$

$$b_v = 12 \text{ in. (to obtain shear per foot of footing)}$$

$$d_{vx} = \text{effective shear depth for a longitudinal face per S5.8.2.9 (in.)} \\ = 30.3 \text{ in. from above}$$

$$V_p = 0.0 \text{ k}$$

The nominal shear resistance is then taken as the lesser of:

$$V_{nx} = 0.0316(2.0)\sqrt{3}(12)(30.3) \\ = 39.8 \text{ k/ft}$$

AND

$$V_{nx} = 0.25f'_c b_v d_v \\ = 0.25(3)(12)(30.3) \\ = 272.7 \text{ k/ft}$$

Therefore, use  $V_{nx} = 39.8 \text{ k/ft}$

$$V_{rx} = \phi V_{nx} \\ = 0.9(39.8) \\ = 35.8 \text{ k/ft} > \text{applied shear, } V_{ux} = 21.9 \text{ k/ft (calculated earlier) OK}$$

Determine one-way shear capacity for transverse face

$$V_{ry} = \phi V_{ny} \quad (\text{S5.8.2.1-2})$$

The nominal shear resistance,  $V_{nx}$ , is taken as the lesser of:

$$V_{ny} = V_c + V_s + V_p \quad (\text{S5.8.3.3-1})$$

OR

$$V_{ny} = 0.25f'_c b_v d_{vy} + V_p \quad (\text{S5.8.3.3-2})$$

$$V_c = 0.0316\beta\sqrt{f'_c}b_v d_{vy} \quad (\text{S5.8.3.3-3})$$

where:

$$\beta = 2.0$$

$$\begin{aligned} b_v &= 12 \text{ in. (to obtain shear per foot of footing)} \\ d_{vy} &= \text{effective shear depth for a transverse face per S5.8.2.9 (in.)} \\ &= 31.4 \text{ in. from above} \end{aligned}$$

$$V_p = 0.0 \text{ k}$$

The nominal shear resistance is then taken as the lesser of:

$$\begin{aligned} V_{cy} &= 0.0316(2.0)\sqrt{3}(12)(31.4) \\ &= 41.2 \text{ k/ft} \end{aligned}$$

AND

$$\begin{aligned} V_{ny} &= 0.25f'_c b_v d_v \\ &= 0.25(3)(12)(31.4) \\ &= 282.6 \text{ k/ft} \end{aligned}$$

Therefore, use  $V_{ny} = 41.2 \text{ k/ft}$

$$\begin{aligned} V_{ry} &= \phi V_{ny} \\ &= 0.9(41.2) \\ &= 37.1 \text{ k/ft} > \text{applied shear, } V_{uy} = 18.1 \text{ k/ft (calculated earlier) OK} \end{aligned}$$

Determine two-way (punching) shear capacity at the column (S5.13.3.6.3)

For two-way action for sections without transverse reinforcement, the nominal shear resistance,  $V_n$  in kips, of the concrete shall be taken as:

$$V_n = (0.063 + 0.126/\beta_c)\sqrt{f'_c}b_o d_v \leq 0.126\sqrt{f'_c}b_o d_v \quad (\text{S5.13.3.6.3-1})$$

where:

$$\begin{aligned} \beta_c &= \text{ratio of long side to short side of the rectangular through which the} \\ &\quad \text{concentrated load or reaction force is transmitted} \\ &= (\text{column equivalent length}) / (\text{column equivalent width}) \\ &= 3.10/3.10 \\ &= 1.0 \text{ (notice, for circular columns this ratio is always 1.0)} \end{aligned}$$

$$\begin{aligned} d_v &= \text{average effective shear depth (in.)} \\ &= (d_{vx} + d_{vy})/2 \\ &= (30.3 + 31.4)/2 \\ &= 30.9 \text{ in.} \end{aligned}$$

$$\begin{aligned} b_o &= \text{perimeter of the critical section (in.), the critical section is } 0.5d_v \text{ from the} \\ &\quad \text{reaction area (S5.13.3.6.1). Use the circular column cross-section and} \\ &\quad \text{cylindrical surface for punching shear.} \\ &= 2\pi(42/2 + 30.9/2) \\ &= 229 \text{ in.} \end{aligned}$$

$$\begin{aligned} V_n &= (0.063 + 0.126/1.0)\sqrt{3}(229)(30.9) \\ &= 2,316 \text{ k} \end{aligned}$$

The nominal shear resistance,  $V_n$ , cannot exceed  $0.126\sqrt{f'_c}b_o d_v$

$$\begin{aligned} V_n &= 0.126\sqrt{3}(229)(30.9) \\ &= 1,544 \text{ k} \end{aligned}$$

Therefore,

$$\begin{aligned} V_r &= 0.9(1,544) \\ &= 1,390 \text{ k} \end{aligned}$$

The maximum factored vertical force for punching shear calculations equals the maximum factored axial load on the footing minus the factored weight of the footing.

$$\begin{aligned} P_{2 \text{ way}} &= 1,374 - 1.25[12(12)(3)](0.150) \\ &= 1,293 \text{ k} \end{aligned}$$

The maximum shear force for punching shear calculations for a footing with the entire footing area under compression and the column at the center of the footing:

$$\begin{aligned} V_{2 \text{ way}} &= P_{2 \text{ way}}(1 - \text{area within punching shear perimeter/footing area}) \\ &= 1,293[1 - \pi((42/2 + 30.9/2)/12)^2 / 12(12)] \\ &= 1,293(1 - 0.201) \\ &= 1,033 \text{ k} < V_r = 1,390 \text{ k} \quad \mathbf{OK} \end{aligned}$$

For footings with eccentric columns or with tension under some of the footing area, the design force for punching shear is calculated as the applied load,  $P_{2 \text{ way}}$ , minus the soil load in the area within the perimeter of the punching shear failure.

### Design Step 7.2.4.5 Foundation soil bearing resistance at the Strength Limit State (S10.6.3)

#### Foundation assumptions:

- Footings rest on dry cohesionless soil
- Angle of internal friction of the soil ( $\phi_f$ ) = 32 degrees
- Depth of the bottom of the footing from the ground surface = 6 ft.
- Soil density = 120 lb/ft<sup>3</sup>
- Footing plan dimensions are 12 ft. by 12 ft.

Footing effective dimensions

According to S10.6.3.1.1, where loads are eccentric, the effective footing dimensions  $L'$  and  $B'$ , as specified in S10.6.3.1.5, shall be used instead of the overall dimensions  $L$  and  $B$  in all equations, tables, and figures pertaining to bearing capacity.

Therefore, for each load case shown in Table 7.2-4, a unique combination of the footing effective dimensions is used. In the following section, the case of maximum axial load on the footing will be used to illustrate the bearing capacity calculations.

The footing effective dimensions are calculated using S10.6.3.1.5 and Figure SC10.6.3.1.5-1 (shown below).

$$B' = B - 2e_B \quad (\text{S10.6.3.1.5-1})$$

where:

$e_B$  = eccentricity parallel to dimension  $B$  (ft.)

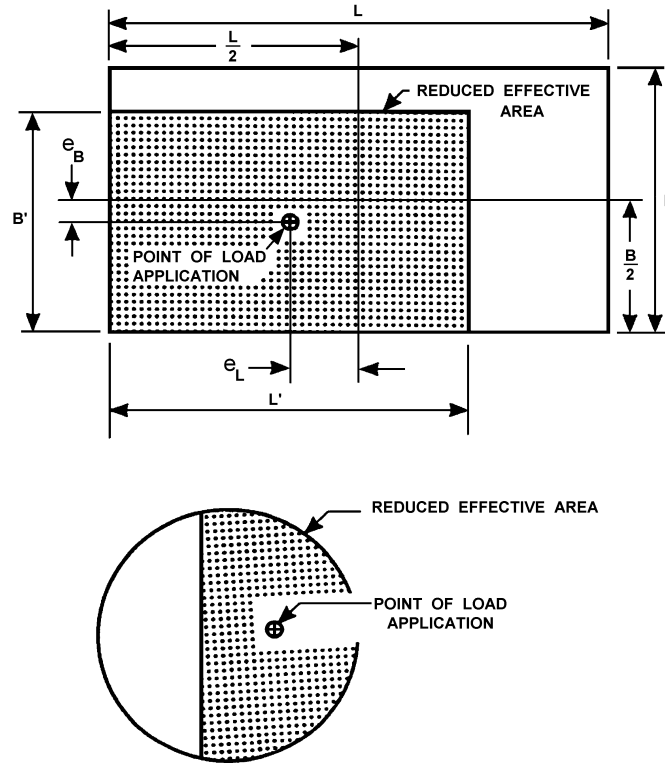
$$\begin{aligned} B' &= 12 - 2(121/1,374) \\ &= 11.82 \text{ ft.} \end{aligned}$$

$$L' = L - 2e_L \quad (\text{S10.6.3.1.5-2})$$

where:

$e_L$  = eccentricity parallel to dimension  $L$  (ft.)

$$\begin{aligned} L' &= 12 - 2(626/1,374) \\ &= 11.09 \text{ ft.} \end{aligned}$$



**Figure SC10.6.3.1.5-1 – Reduced Footing Dimensions (Reproduced from the Specifications)**

According to S10.6.3.1.2c, for cohesionless soil, the nominal bearing resistance of a layer of the soil in TSF may be determined as:

$$q_{ult} = 0.5\gamma BC_{w1}N_{\gamma m} + \gamma C_{w2}D_f N_{qm} \quad (S10.6.3.1.2c-1)$$

where:

$D_f$  = depth of footing from ground level (ft.)  
 = 6 ft.

$\gamma$  = total, i.e., moist density of sand or gravel (TCF)  
 = 120/2,000  
 = 0.06 TCF

$B$  = footing width (ft.)  
 = smaller of 11.82 and 11.09 ft.  
 = 11.09 ft.

$C_{w1}, C_{w2}$  = coefficients as specified in Table S10.6.3.1.2c-1 as a function of  $D_w$  (dimensionless)  
 = for dry soil with a large depth,  $C_{w1} = C_{w2} = 1.0$

$D_w$  = depth to water surface taken from the ground surface (ft.)  
 = assume a large distance relative to the footing dimensions

$N_{\gamma m}, N_{qm}$  = modified bearing capacity factor (dimensionless)

Substituting in Eq. S10.6.3.1.2c-1:

$$\begin{aligned} q_{ult} &= 0.5(0.06)(11.09)(1.0)N_{\gamma m} + 0.06(1.0)(6)N_{qm} \\ &= 0.334N_{\gamma m} + 0.36N_{qm} \end{aligned}$$

From Eqs. S10.6.3.1.2c-2 and -3

$$N_{\gamma m} = N_{\gamma} S_{\gamma} c_{\gamma} i_{\gamma} \quad (\text{S10.6.3.1.2c-2})$$

$$N_{qm} = N_q S_q c_q i_q d_q \quad (\text{S10.6.3.1.2c-3})$$

where:

$N_{\gamma}$  = bearing capacity factor as specified in Table S10.6.3.1.2c-2 for footings on relatively level ground

$N_q$  = bearing capacity factor as specified in Table S10.6.3.1.2c-2 for relatively level ground

$S_q, S_{\gamma}$  = shape factors specified in Tables S10.6.3.1.2c-3 and -4, respectively

$c_q, c_{\gamma}$  = soil compressibility factors specified in Tables S10.6.3.1.2c-5

$i_q, i_{\gamma}$  = load inclination factors specified in Tables S10.6.3.1.2c-7 and -8

$d_q$  = depth factor specified in Table S10.6.3.1.2c-9

From Table S10.6.3.1.2c-2:  $N_{\gamma} = 30$  for  $\phi_f = 32$  degrees

From Table S10.6.3.1.2c-2:  $N_q = 23$  for  $\phi_f = 32$  degrees

$$\begin{aligned} L'/B' &= 11.82/11.09 \\ &= 1.07 \end{aligned}$$

Interpolate between  $L'/B' = 1$  and 2. However, using values corresponding to  $L'/B' = 1.0$  will not lead to significant change because  $L'/B' \approx 1.0$ .

From Table S10.6.3.1.2c-3:  $S_q = 1.62$  for  $L'/B' = 1.0$  and  $\phi_f = 30$  degrees

From Table S10.6.3.1.2c-4:  $S_\gamma = 0.6$  for  $L'/B' = 1.0$  and  $\phi_f = 30$  degrees

Soil stress at the footing depth before excavation,  $q = 0.06(6) = 0.36$  TSF

For Tables S10.6.3.1.2c-5 and -6, either interpolate between  $q = 0.25$  and  $q = 0.5$  or, as a conservative approach, use the value corresponding to  $q = 0.5$ . For this example, the value corresponding to  $q = 0.5$  TSF is used.

From Table S10.6.3.1.2c-5:  $c_q, c_\gamma = 1.0$  for  $q = 0.5$  and  $\phi_f = 32$  degrees

The maximum factored horizontal load on the bottom of the column from the bent analysis equals 46.0 and 26.0 kips in the transverse and longitudinal directions, respectively. In Table S10.6.3.1.2c-7, it is intended to use the unfactored horizontal and vertical loads. However, due to the small ratio of horizontal to vertical loads, using the factored loads does not affect the results.

Horizontal-to-vertical load ratio:

$$\begin{aligned} H/V &= 44.8/1,374 \\ &= 0.033 \text{ in the transverse direction} \end{aligned}$$

$$\begin{aligned} H/V &= 26.0/1,374 \\ &= 0.019 \text{ in the longitudinal direction} \end{aligned}$$

Table S10.6.3.1.2-7 lists values for  $i_q, i_\gamma$  that correspond to horizontal-to-vertical load ratios of 0.0 and 0.1. Interpolation between the two values is acceptable. A more conservative approach is to use the value corresponding to  $H/V = 0.1$ .

From Table S10.6.3.1.2c-7:  $i_q = 0.85$  for square footing with  $H/V = 0.1$

From Table S10.6.3.1.2c-7:  $i_\gamma = 0.77$  for square footing with  $H/V = 0.1$

Table S10.6.3.1.2c-9 lists values for  $d_q$  that correspond to a friction angle,  $\phi_f \geq 32$  degrees and for  $D_f/B \geq 1.0$ . For this example,  $\phi_f = 30$  degrees and  $D_f/B = 6/11.13 = 0.54$

By extrapolation from Table S10.6.3.1.2c-9, use  $d_q = 1.05$

Substituting in Eqs. S10.6.3.1.2c-2 and -3:

$$\begin{aligned} N_{(m)} &= N_\gamma S_\gamma c_\gamma i_\gamma && \text{(S10.6.3.1.2c-2)} \\ &= 30(0.6)(1.0)(0.77) \\ &= 13.86 \end{aligned}$$

$$\begin{aligned} N_{qm} &= N_q S_q c_q i_q d_q && \text{(S10.6.3.1.2c-3)} \\ &= 23(1.62)(1.0)(0.85)(1.05) \end{aligned}$$



$$= 33.3$$

Therefore,

$$\begin{aligned} q_{ult} &= 0.333N_{\gamma m} + 0.36N_{qm} \\ &= 0.333(13.86) + 0.36(33.3) \\ &= 16.6 \text{ TSF} \end{aligned}$$

### Resistance factor

From Table S10.5.5-1, several resistance factors are listed for cohesionless soil (sand). The selection of a particular resistance factor depends on the method of soil exploration used to determine the soil properties. Assuming that  $\phi$  was estimated from SPT data, the resistance factor = 0.35

According to S10.6.3.1.1,

$$\begin{aligned} q_R &= \phi q_n = \phi q_{ult} \\ &= 0.35(16.6) \\ &= 5.81 \text{ TSF} \end{aligned}$$

$$\begin{aligned} \text{Footing load resistance} &= (q_R)(\text{footing effective area}) \\ &= 5.81(11.82)(11.09) \\ &= 762.0 \text{ Tons} \\ &= 1,524 \text{ k} > 1,374 \text{ k applied } \mathbf{OK} \end{aligned}$$

The soil load resistance check may be repeated using the same procedures for other load cases.

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**APPENDIX A – This appendix is obsolete**

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## APPENDIX B – General guidelines for refined analysis of deck slabs

### B.1 General guidelines

Traditionally, deck slabs have been analyzed using approximate methods. The approximate methods are based on calculating moments per unit width of the deck and design the reinforcement to resist these moments. This approach has been used successfully for many decades. However, the approximate methods were generally based on laboratory testing and/or refined analysis of typical decks supported on parallel girders and no skew. In case of deck slabs with unusual geometry, such as sharply skewed decks, the results of the approximate methods may not be accurate. For example, negative moments may develop at the acute corner of a sharply skewed deck. These moments are not accounted for in the approximate methods as they rely on assuming that the deck is behaving as a continuous beam.

In cases of unusual deck geometry, bridge designers may find it beneficial to employ refined methods of analysis. Typically the use of the refined methods of analysis is meant for the design of both of the girders and the deck slab. The design method of analysis most used is the finite element analysis. However, for deck slabs, other methods such as the yield line method and the finite differences method may be used. Following is a general description of the use of the finite elements in analyzing deck slabs.

### B.2 Finite element modeling of decks

#### B.2.1 Type of elements

The finite element method is based on dividing a component into a group of small components or “finite elements”. Depending on the type of the element, the number of displacements (translations and rotations) varies at each end or corner of the element varies. The displacements are typically referred to as ‘degrees of freedom’. The basic output of the analysis is the displacements at each node. These displacements are then converted into forces at the nodes. The force output corresponding to a rotational degrees of freedom is in the form of a moment while forces correspond to translational degrees of freedom. Following are the types of elements typically used to model a plate structure and the advantage and disadvantages of each type.

#### Plate elements

Plate elements are developed assuming that the thickness of the plate component is small relative to the other two dimensions. The plate is modeled by its middle surface. Each element typically has four corners or nodes. Most computer programs have the ability of handling three-node or triangular plate elements, which are typically treated as a special case of the four-node basic element. Following the general plate theory, plate elements are assumed have three allowed displacements at each node; translation perpendicular to the plate and rotations about two perpendicular axes in the plane of the plate. The typical output includes the moments (usually given as moment per unit width of the face of the elements) and the shear in the plate. This form of output is convenient because the moments may be directly used to design the deck.

The main disadvantage of plate elements is that they do not account for the forces in the plane of the plate. This results in ignoring the stiffness of the plate elements in this plane. This precludes them from being used as part of a three-dimensional model to analyze both the deck and the girders.

The deck supports are modeled as rigid supports along the lines of the supporting components, i.e. girders, diaphragms and/or floor beams. Where it is desirable to consider the effect of the flexibility of the supporting components on the deck moments, the model may include these components that are typically modeled as beams. As the plate elements, theoretically, have no in-plane stiffness, the effect of the composite action on the stiffness of the beams should be considered when determining the stiffness of the beam elements.

### Shell elements

Shell elements are also developed assuming that the thickness of the component is small relative to the other two dimensions and are also modeled by their middle surface. They differ from plate elements in that they are considered to have six degrees of freedom at each node, three translations and three rotations. Typically the rotation about the axis perpendicular to the surface at a node is eliminated leaving only five degrees of freedom per node. Shell elements may be used to model two dimensional (plate) components or three-dimensional (shell) components. Commercially available computer programs typically allow three-node and four-node elements. The typical output includes the moments (usually given as moment per unit width of the face of the elements) and the shear and axial loads in the element. This form of output is convenient because the moments may be directly used to design the deck.

Due to the inclusion of the translations in the plane of the elements, shell elements may be used as part of a three-dimensional model to analyze both the deck and the girders. When the supporting components are modeled using beam elements, only the stiffness of the noncomposite beams is introduced when defining the stiffness of the beams. The effect of the composite action between the deck and the supporting components is automatically included due to the presence of the in-plane stiffness of the shell elements representing the deck.

### Solid elements

Solid elements may be used to model both thin and thick components. The thickness of the component may be divided into several layers or, for thin components such as decks, may be modeled using one layer. The solid elements are developed assuming three translations at each node and the rotations are not considered in the development. The typical output includes the forces in the direction of the three degrees of freedom at the nodes. Most computer programs have the ability to determine the surface stresses of the solid elements. This form of output is not convenient because these forces or stresses need to be converted to moments that may be used to design the deck. Notice that, theoretically, there should be no force perpendicular to the free surface of an element. However, due to rounding off errors, a small force is typically calculated.

Similar to shell elements, due to the inclusion of all translations in the development of the elements, solid elements may be used as part of a three-dimensional model to analyze both the deck and the

girders. When the supporting components are modeled using beam elements, only the stiffness of the noncomposite beams is introduced when defining the stiffness of the beams.

### **B.2.2 Element size and aspect ratio**

The accuracy of the results of a finite element model increases as the element size decreases. The required size of elements is smaller at areas where high loads exist such as location of applied concentrated loads and reactions. For a deck slab, the dividing the width between the girders to five or more girders typically yields accurate results. The aspect ratio of the element (length-to-width ratio for plate and shell elements and longest-to-shortest side length ratio for solid elements) and the corner angles should be kept within the values recommended by the developer of the computer program. Typically an aspect ratio less than 3 and corner angles between 60 and 120 degrees are considered acceptable. In case the developer recommendations are not followed, the inaccurate results are usually limited to the nonconformant elements and the surrounding areas. When many of the elements do not conform to the developer recommendation, it is recommended that a finer model be developed and the results of the two models compared. If the difference is within the acceptable limits for design, the coarser model may be used. If the difference is not acceptable, a third, finer model should be developed and the results are then compared to the previous model. This process should be repeated until the difference between the results of the last two models is within the acceptable limits.

For deck slabs with constant thickness, the results are not very sensitive to element size and aspect ratio.

### **B.2.3 Load application**

Local stress concentrations take place at the locations of concentrated loads applied to a finite element model. For a bridge deck, wheel loads should preferably be applied as uniform load distributed over the tire contact area specified in Article S3.6.1.2.5. To simplify live load application to the deck model, the size of the elements should be selected to eliminate the partial loading of some finite elements, i.e. the tire contact area preferably match the area of one or a group of elements.

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## APPENDIX C Calculations of Creep and Shrinkage Effects

See Design Step 5.3 for the basic information about creep and shrinkage effects. Design Step 5.3 also contains the table of fixed end moments used in this appendix.

### Design Step C1.1 Analysis of creep effects on the example bridge

Calculations are shown for Span 1 for a deck slab cast 450 days after the beams are made. Span 2 calculations are similar. See the tables at the end of this appendix for the final results for a case of the slab and continuity connection cast 30 days after the beams are cast. All calculations are made following the procedures outlined in the publication entitled “Design of Continuous Highway Bridges with Precast, Prestressed Concrete Girders” published by the Portland Cement Association (PCA) in August 1969.

The distance from the composite neutral axis to the bottom of the beam is 51.96 in. from Section 2. Therefore, the prestressing force eccentricity at midspan is:

$$\begin{aligned} e_c &= NA_{\text{bottom}} - CGS \\ &= 51.96 - 5.0 \\ &= 46.96 \text{ in.} \end{aligned}$$

### Design Step C1.2

Calculate the creep coefficient,  $\psi_{(t, t_i)}$ , for the beam at infinite time according to S5.4.2.3.2.

Calculate the humidity factor,  $k_{hc}$

$$\begin{aligned} k_{hc} &= 1.56 - 0.008H && \text{(S5.4.2.3.2-3)} \\ k_{hc} &= 1.56 - 0.008(70) = 1.00 \end{aligned}$$

where,

$$H = \text{relative humidity} = 70\%$$

Calculate the concrete strength factor,  $k_f$

$$\begin{aligned} k_f &= \frac{5}{1 + f_{ci}} && \text{(S5.4.2.3.2-4)} \\ &= 5/[1 + 4.8] \\ &= 0.862 \end{aligned}$$

Calculate the time development factor,  $k_{td}$

$$k_{td} = \left( \frac{t}{12((100 - 4f'_{ci})/(f'_{ci} + 20)) + t} \right) \quad (S5.4.2.3.2-5)$$

For final age,  $t = t_f - t_i = t_i =$  age of concrete when load is initially applied  
 = 1 day

$t_f =$  infinite days

$$k_{td} = \frac{\infty}{12((100 - 4(4.8))/(4.8 + 20)) + \infty} = 1.0$$

Calculate the volume to surface area factor,  $k_c$

$$k_c = \left[ \frac{\left( \frac{t}{26e^{0.36(V/S)_b} + t} \right)}{\left( \frac{t}{45 + t} \right)} \right] \left[ \frac{1.80 + 1.77e^{-0.54(V/S)_b}}{2.587} \right] \quad (SC5.4.2.3.2-1)$$

where:

$t =$  maturity of concrete  
 = infinite days

$e =$  natural log base (approx. 2.71828)

$(V/S)_b =$  volume to surface ratio for the beam  
 = beam surface area is 2,955.38 in<sup>2</sup>/ft (see Figure 2-3 for beam dimensions) and the volume is 13,020 in<sup>3</sup>/ft  
 = (13,020/2,955.38)  
 = 4.406 in.

$$k_c = \left[ 1 \right] \left[ \frac{1.80 + 1.77e^{-0.54(4.406)}}{2.587} \right]$$

$$k_c = 0.759$$

The creep coefficient is the ratio between creep strain and the strain due to permanent stress (SC5.4.2.3.2)

Calculate creep coefficient according to Eq. S5.4.2.3.2-1.

$$\Psi(t, t_i) = 1.9k_c k_{hc} k_f k_{td} t_i^{-0.118} \quad (S5.4.2.3.2-1)$$

$$\Psi(\infty, 1) = 1.9k_c k_{hc} k_f k_{td} t_i^{-0.118}$$

where:

$$k_c = 0.759 \text{ (see above)}$$

$$k_{hc} = 1.000 \text{ (see above)}$$

$$k_f = 0.862 \text{ (see above)}$$

$$k_{td} = 1.000 \text{ (see above)}$$

$$\Psi_b(t_f, t_i) = 1.9(0.759)(1.00)(0.862)(1.0)(1)^{-0.118}$$

$$\Psi_{(\infty, 1)} = 1.243$$

### Design Step C1.3

Calculate the creep coefficient,  $\psi_{(t, t_i)}$ , in the beam at the time the slab is cast according to S5.4.2.3.2.

Calculate the time development factor,  $k_{td}$

$$k_{td} = \left( \frac{t}{12((100 - 4f'_{ci})/(f'_{ci} + 20)) + t} \right) \quad (\text{S5.4.2.3.2-5})$$

For final age,  $t = t_f - t_i = t_i =$  age of concrete when load is initially applied  
= 1 day

$$t_f = 450 \text{ days}$$

$$k_{td} = \frac{450}{12((100 - 4(4.8))/(4.8 + 20)) + 450} = 0.920$$

Calculate the volume to surface area factor,  $k_c$

$$k_c = \left[ \frac{\left( \frac{t}{26e^{0.36(V/S)_b} + t} \right)}{\left( \frac{t}{45 + t} \right)} \right] \left[ \frac{1.80 + 1.77e^{-0.54(V/S)_b}}{2.587} \right] \quad (\text{SC5.4.2.3.2-1})$$

where:

$$t = 450 \text{ days}$$

$$e = \text{natural log base (approx. 2.71828)}$$

$$(V/S)_b = 4.406 \text{ in.}$$

$$k_c = \left[ \frac{\left( \frac{450}{26e^{0.36(4.406)} + 450} \right)}{\left( \frac{450}{45 + 450} \right)} \right] \left[ \frac{1.80 + 1.77e^{-0.54(4.406)}}{2.587} \right]$$

$$k_c = 0.651$$

Calculate the creep coefficient,  $\Psi_{(t,t_i)}$ , according to Eq. S5.4.2.3.2-1.

$$\Psi(t, t_i) = 1.9k_c k_{hc} k_f k_{td} t_i^{-0.118} \quad (\text{S5.4.2.3.2-1})$$

$$\Psi_{(450,1)} = 1.9k_c k_{hc} k_f k_{td} t_i^{-0.118}$$

where:

$$k_c = 0.651 \text{ (see above)}$$

$$k_{hc} = 1.000 \text{ (see above)}$$

$$k_f = 0.862 \text{ (see above)}$$

$$k_{td} = 0.920 \text{ (see above)}$$

$$t_i = 1 \text{ day}$$

$$t = 450 \text{ days}$$

$$\Psi_b(t_f, t_i) = 1.9(0.651)(1.00)(0.862)(0.920)(1)^{-0.118}$$

$$\Psi_{(450,1)} = 0.981$$

Calculate the restrained creep coefficient in the beam,  $\phi$ , as the creep coefficient for creep that takes place after the continuity connection has been established.

$$\begin{aligned} \phi &= \Psi_\infty - \Psi_{450} \text{ (from PCA publication referenced in Step 5.3.2.2)} \\ &= 1.243 - 0.981 \\ &= 0.26 \end{aligned}$$

#### Design Step C1.4 Calculate the prestressed end slope, $\theta$ .

For straight strands (debonding neglected). Calculate the end slope,  $\theta$ , for a simple beam under constant moment.

$$\text{Moment} = P_e e_c$$

$$\theta = P_e e_c L_{\text{span}} / 2E_c I_c$$

where:

$$P_e = \text{effective prestressing force after all losses (kips)}$$

$$= 1,118 \text{ kips (see Design Step 5.4 for detailed calculations of the prestressing force)}$$

$$e_c = 46.96 \text{ in. (calculated above)}$$

$$L_{\text{span}} = 110.5 \text{ ft. (1,326 in.) (taken equal to the continuous beam span length)}$$

$$E_c = \text{the modulus of elasticity of the beam at final condition (ksi)} \\ = 4,696 \text{ ksi}$$

$$I_c = \text{moment of inertia of composite beam (in}^4\text{)} \\ = 1,402,024 \text{ in}^4$$

$$\theta = [1,118(46.96)(1,326)]/[2(4,696)(1,402,024)] \\ = 0.0053 \text{ rads}$$

### Design Step C1.5 Calculate the prestressed creep fixed end action for Span 1

The equation is taken from Table 5.3-9 of PCA Publication for prestressed creep FEA, left end span, right moment.

$$\text{FEM}_{\text{cr}} = 3E_c I_c \theta / L_{\text{span}} \\ = [3(4,696)(1,402,024)(0.0053)]/1,326 \\ = 78,947/12 \\ = 6,579 \text{ k-ft}$$

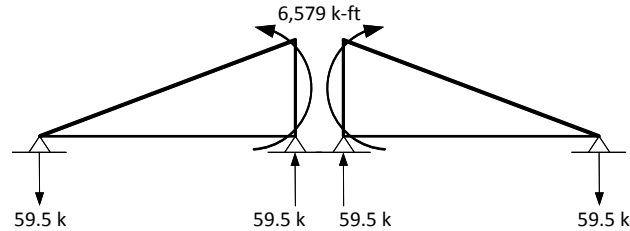
End forces due to prestress creep in Span 1:

$$\text{Left reaction} = R1_{\text{PScr}} \\ = -\text{FEM}_{\text{cr}}/L_{\text{span}} \\ = -(6,579)/110.5 \\ = -59.5 \text{ k}$$

$$\text{Right reaction} = R2_{\text{PScr}} \\ = -R1_{\text{PScr}} \\ = 59.5 \text{ k}$$

$$\text{Left moment} = M1_{\text{PScr}} \\ = 0.0 \text{ k-ft}$$

$$\text{Right moment} = M2_{\text{PScr}} \\ = \text{FEM}_{\text{cr}} \\ = 6,579 \text{ k-ft}$$



**Figure C1 – Prestress Creep Restraint Moment**

**Design Step C1.6 Calculate dead load creep fixed end actions**

Calculate the total dead load moment at the midspan

$$\begin{aligned} \text{Noncomposite DL moment} &= M_{DNC} \\ &= 42,144 \text{ k-in (3,512 k-ft)} \quad (\text{see Section 5.3}) \end{aligned}$$

$$\begin{aligned} \text{Composite DL moment} &= M_{DC} \\ &= 4,644 \text{ k-in (387 k-ft)} \quad (\text{see Section 5.3}) \end{aligned}$$

$$\begin{aligned} \text{Total DL moment} &= M_{DL} \\ &= M_{DNC} + M_{DC} \\ &= 42,144 + 4,644 \\ &= 46,788/12 \\ &= 3,899 \text{ k-ft} \end{aligned}$$

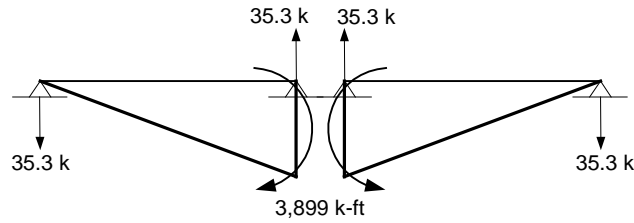
End forces due to dead load creep in Span 1:

$$\begin{aligned} \text{Left reaction} &= R1_{DLcr} \\ &= -M_{DL}/L_{span} \\ &= -3,899/110.5 \\ &= -35.3 \text{ k} \end{aligned}$$

$$\begin{aligned} \text{Right reaction} &= R2_{DLcr} \\ &= -R1_{DLcr} \\ &= 35.3 \text{ k} \end{aligned}$$

$$\begin{aligned} \text{Left moment} &= M1_{DLcr} \\ &= 0.0 \text{ k-ft} \end{aligned}$$

$$\begin{aligned} \text{Right moment} &= M2_{DLcr} \\ &= -M_{DL} \\ &= -3,899 \text{ k-ft} \end{aligned}$$



**Figure C2 – Dead Load Creep Restraint Moment**

Calculate the creep correction factor,  $C_{cr}$

$$C_{cr} = 1 - e^{-\phi} \text{ (from PCA publication referenced in Step 5.3.2.2)} = 1 - e^{-0.26} = 0.229$$

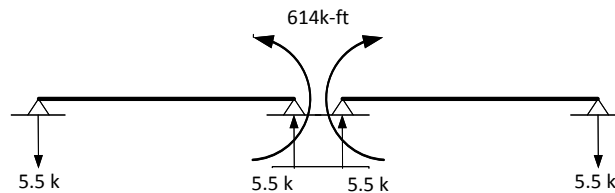
Calculate the total creep (prestress + dead load) fixed end actions for 450 days.

$$\begin{aligned} \text{Left reaction} &= R1_{cr} \\ &= C_{cr}(R1_{PS_{cr}} + R2_{DL_{cr}}) \\ &= 0.229(-59.5 + 35.3) \\ &= -5.54 \text{ k} \end{aligned}$$

$$\begin{aligned} \text{Right reaction} &= R2_{cr} \\ &= -R1_{cr} \\ &= 5.54 \text{ k} \end{aligned}$$

$$\begin{aligned} \text{Left moment} &= M1_{cr} \\ &= 0.0 \text{ k-ft} \end{aligned}$$

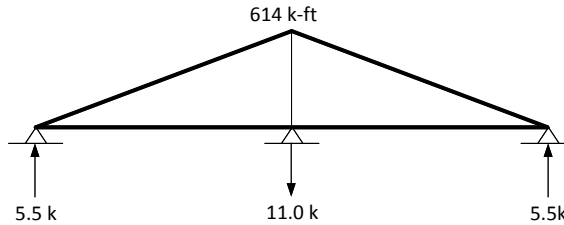
$$\begin{aligned} \text{Right moment} &= M2_{cr} \\ &= C_{cr}(M2_{PS_{cr}} + M2_{DL_{cr}}) \\ &= 0.229[6,579 + (-3,899)] \\ &= 614 \text{ k-ft} \end{aligned}$$



**Figure C3 – Total Creep Fixed End Actions**

**Design Step C1.7 Creep final effects**

The fixed end moments shown in Figure C3 are applied to the continuous beam. The beam is analyzed to determine the final creep effects. Due to the symmetry of the two spans of the bridge, the final moments at the middle support are the same as the applied fixed end moments. For a bridge with more than two spans or a bridge with two unequal spans, the magnitude of the final moments would be different from the fixed end moments.



**Figure C4 – Creep Final Effects for a Deck and Continuity Connection Cast 450 Days After the Beams were Cast**

**Design Step C2.1 Analysis of shrinkage effects on the example bridge**

Calculate shrinkage strain in beam at infinite time according to S5.4.2.3.3

Calculate the size factor,  $k_s$ .

$$k_s = \left[ \frac{\left( \frac{t}{26e^{0.36(V/S)_b} + t} \right)}{\left( \frac{t}{45 + t} \right)} \right] \left[ \frac{1,064 - 94(V/S)_b}{923} \right] \quad (SC5.4.2.3.2-2)$$

where:

$t$  = drying time  
= infinite days

$e$  = natural log base (approx. 2.71828)

$(V/S)_b = 4.406$  in.

$$k_s = [1] \left[ \frac{1,064 - 94(4.406)}{923} \right]$$



$$k_s = 0.704$$

Calculate the humidity factor,  $k_{hs}$

$$k_{hs} = 2.00 - 0.014H \quad (S5.4.2.3.3-2)$$

$$k_{hs} = 2.00 - 0.014(70) = 1.02$$

Assume the beam will be steam cured and devoid of shrinkage-prone aggregates, therefore, the shrinkage strain in the beam at infinite time is calculated as:

$$\epsilon_{sh} = k_s k_{hs} k_f k_{td} 0.48 \times 10^{-3} \quad (S5.4.2.3.3-1)$$

$$\epsilon_{sh,b,\infty} = k_s k_{hs} k_f k_{td} (0.48 \times 10^{-3})$$

where:

$$k_s = 0.704 \text{ (from above)}$$

$$k_{hs} = 1.020 \text{ for 70\% humidity}$$

$$t = \text{infinite days}$$

$$\epsilon_{sh,b,\infty} = (0.704)(1.020)(0.862)(1.000)(0.48 \times 10^{-3})$$

$$= 0.000297$$

### Design Step C2.2

Calculate shrinkage strain in the beam at the time the slab is cast (S5.4.2.3.3)

$$t = \text{time the slab is cast}$$

$$= 450 \text{ days (maximum value)}$$

Calculate the size factor,  $k_s$ .

$$k_s = \left[ \frac{\left( \frac{t}{26e^{0.36(V/S)_b} + t} \right)}{\left( \frac{t}{45 + t} \right)} \right] \left[ \frac{1,064 - 94(V/S)_b}{923} \right] \quad (SC5.4.2.3.2-2)$$

where:

$$t = 450 \text{ days}$$

$$e = \text{natural log base (approx. 2.71828)}$$

$$(V/S)_b = 4.406 \text{ in.}$$

$$k_s = \left[ \frac{\left( \frac{450}{26e^{0.36(4.406)} + 450} \right)}{\left( \frac{450}{45 + 450} \right)} \right] \left[ \frac{1,064 - 94(4.406)}{923} \right]$$

$$k_s = 0.604$$

Assume the beam will be steam cured and devoid of shrinkage-prone aggregates, therefore, the shrinkage strain in the beam at infinite time is calculated as:

$$\begin{aligned} \epsilon_{sh} &= k_s k_{hs} k_f k_{td} 0.48 \times 10^{-3} && \text{(S5.4.2.3.3-1)} \\ \epsilon_{sh,b,450} &= k_s k_{hs} k_f k_{td} (0.48 \times 10^{-3}) \end{aligned}$$

where:

- $k_s = 0.604$
- $k_h = 1.020$  for 70% humidity
- $t = 450$  days

$$\begin{aligned} \epsilon_{sh,b,450} &= (0.604)(1.020)(0.862)(0.920)(0.48 \times 10^{-3}) \\ &= 0.000235 \end{aligned}$$

**Design Step C2.3**

Calculate the shrinkage strain in the slab at infinite time (S5.4.2.3.3)

Calculate the size factor,  $k_s$

$$k_s = \left[ \frac{\left( \frac{t}{26e^{0.36(V/S)_s} + t} \right)}{\left( \frac{t}{45 + t} \right)} \right] \left[ \frac{1,064 - 94(V/S)_b}{923} \right] \quad \text{(SC5.4.2.3.2-2)}$$

where:

- $t =$  infinite days
- $e =$  natural log base (2.71828)

Compute the volume to surface area ratio for the slab.

$$(V/S)_s = (b_{slab})(t_{slab}) / (2b_{slab} - w_{tf})$$

where:

$b_{slab} =$  slab width taken equal to girder spacing (in.)

$t_{\text{slab}}$  = slab structural thickness (in.)

$w_{\text{tf}}$  = beam top flange width (in.)

$$\begin{aligned} (V/S)_s &= 116(7.5)/[2(116) - 42] \\ &= 4.58 \text{ in.} \end{aligned}$$

$$k_s = \left[ 1 + \frac{1,064 - 94(4.58)}{923} \right]$$

$$k_s = 0.686$$

The slab will *not* be steam cured, therefore, use

$$\varepsilon_{sh} = k_s k_{hs} k_f k_{td} 0.48 \times 10^{-3} \quad (\text{S5.4.2.3.3-1})$$

$$\varepsilon_{sh,s,\infty} = k_s k_{hs} k_f k_{td} (0.48 \times 10^{-3})$$

where:

Calculate the concrete strength factor,  $k_f$

$$\begin{aligned} k_f &= \frac{5}{1 + f'_{ci}} && (\text{S5.4.2.3.2-4}) \\ &= 5/[1 + 3.2] \\ &= 1.19 \end{aligned}$$

$$k_s = 0.686$$

$$k_h = 1.020 \text{ for 70\% humidity}$$

$$t = \text{infinite days}$$

$$\begin{aligned} \varepsilon_{sh,s,\infty} &= (0.686)(1.020)(1.19)(1.0)(0.48 \times 10^{-3}) \\ &= 0.000400 \end{aligned}$$

### Design Step C2.4

Calculate the differential shrinkage strain as the difference between the deck total shrinkage strain and the shrinkage strain of the beam due to shrinkage that takes place after the continuity connection is cast.

$$\begin{aligned} \Delta\varepsilon_{sh} &= \varepsilon_{sh,s,\infty} - (\varepsilon_{sh,b,\infty} - \varepsilon_{sh,b,450}) \\ &= 4.00 \times 10^{-4} - [2.97 \times 10^{-4} - 2.35 \times 10^{-4}] \\ &= 3.38 \times 10^{-4} \end{aligned}$$

Design Step C2.5

Calculate the shrinkage driving end moment,  $M_s$

$$M_s = \Delta\epsilon_{sh} E_{cs} A_{slab} e' \quad (\text{from PCA publication referenced in Design Step 5.3.2.2})$$

where:

- $\Delta\epsilon_{sh}$  = differential shrinkage strain
- $E_{cs}$  = elastic modulus for the deck slab concrete (ksi)
- $A_{slab}$  = cross-sectional area of the deck slab (in<sup>2</sup>)
- $e'$  = the distance from the centroid of the slab to the centroid of the composite section (in.)  
 $= d_{beam} + t_{slab}/2 - NA_{beam \text{ bottom}}$   
 $= 72 + 7.5/2 - 51.96$   
 $= 23.79 \text{ in.}$

$$\begin{aligned} M_s &= (3.38 \times 10^{-4})(3,834)(116)(7.5)(23.79) \\ &= 26,821/12 \\ &= 2,235 \text{ k-ft (see notation in Table 5.3-9 for sign convention)} \end{aligned}$$

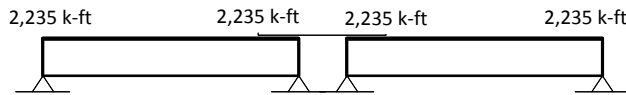


Figure C5 – Shrinkage Driving Moment

For beams under constant moment along their full length, the restraint moment may be calculated as shown above for the case of creep due to prestressing force or according to Table 5.3-9.

$$\begin{aligned} \text{Shrinkage fixed end actions} &= -1.5M_s = -1.5(2,235) \\ &= -3,353 \text{ k-ft} \end{aligned}$$

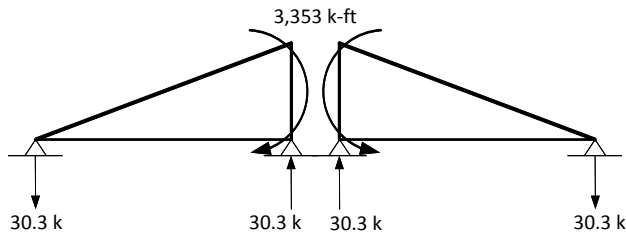
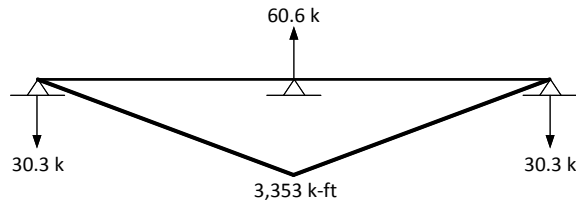


Figure C6 – Shrinkage Fixed End Actions

**Design Step C2.6**Analyze the beam for the fixed end actions

Due to symmetry of the spans, the moments under the fixed end moments shown in Figure C6 are the same as the final moments (shown in Fig. C7). For bridges with three or more spans and for bridges with two unequal spans, the continuity moments will be different from the fixed end moments.



**Figure C7 – Shrinkage Continuity Moments**

**Design Step C2.7**

Calculate the correction factor for shrinkage.

$$\begin{aligned} C_{sh} &= (1 - e^{-\phi})/\phi \text{ (from PCA publication referenced in Step 5.3.2.2)} \\ &= [1 - e^{-0.26}]/0.26 \\ &= 0.881 \end{aligned}$$

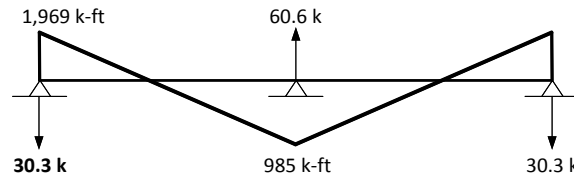
**Design Step C2.8**

Calculate the shrinkage final moments by applying the correction factor for shrinkage to the sum of the shrinkage driving moments (Figure C5) and the shrinkage continuity moment (Figure C7) fixed end actions.

End moments, Span 1:

$$\begin{aligned} \text{Left end moment} &= M_{1sh} \\ &= C_{sh}(M_{sh,dr} + \text{shrinkage continuity moment}) \\ &= 0.881(2,235 + 0) \\ &= 1,969 \text{ k-ft} \end{aligned}$$

$$\begin{aligned} \text{Right end moment} &= M_{2sh} \\ &= C_{sh}(M_{sh,dr} + \text{shrinkage continuity moment}) \\ &= 0.881(2,235 - 3,353) \\ &= -985 \text{ k-ft} \end{aligned}$$



**Figure C8 – Final Total Shrinkage Effect**

Tables C1 and C2 provide a summary of the final moments for the case of the deck poured 30 days after the beams were cast.

**Table C1 - 30 Day Creep Final Moments**

Span	M1 (k-ft)	M2 (k-ft)	R1 (k)	R2 (k)
1	0	1,684	-15.2	15.2
2	-1,684	0	15.2	-15.2

**Table C2 - 30 Day Shrinkage Final Moments**

Span	M1 (k-ft)	M2 (k-ft)	R1 (k)	R2 (k)
1	504	-252	-15.4	15.4
2	-252	504	15.4	-15.4

*When a limit state calls for inclusion of the creep and shrinkage effects and/or the design procedures approved by the bridge owner calls for their inclusion, the final creep and shrinkage effects should be added to other load effect at all sections. The positive moment connection at the bottom of the beams at the intermediate support is designed to account for the creep and shrinkage effects since these effects are the major source of these moments.*

*Notice that when combining creep and shrinkage effects, both effects have to be calculated using the same age of beam at the time the continuity connection is established.*